CHAPTER

5 Teaching mathematics in the light of STEM

The 'Marmite' subject

In England there is an advertisement for a savoury spread called Marmite in which consumers responses are depicted as either 'love it' or 'hate it'. School mathematics is sometimes known as the 'Marmite' subject; pupils either love it or hate it. For those who hate mathematics, it is almost incomprehensible to them that someone could like it so much that they give their career over to being a professional mathematician. The popular image of the mathematician is not dissimilar to the popular image of the scientist mentioned in Chapter 1: male, elderly, unfashionable, untidy, withdrawn into world that only he, and I stress he, is interested in or understands and which he can't explain to others in everyday language. Yet, mathematics is the product of the human mind. Unfortunately, the mathematics we learn at school tells us little if anything of the mathematicians who produced it. Vera John Steiner and Reuben Hersh write compellingly about the 'life mathematical' enjoyed by those who commit to mathematics. They acknowledge that it is certainly not an easy life. It is full of intellectual struggle accompanied by a roller coaster of emotional highs and lows as ideas which seem promising turn out to be false and must be discarded in an ever more ruthless pursuit of truth. Among mathematicians there is fierce rivalry as well as intense friendship and loyalty, played out within a domain that few others can appreciate. However, Steiner and Hersh do acknowledge that for many the 'life mathematical in school' is a very different affair.

This phobia of mathematics is compounded in many western societies with an almost perverse pride in not being able to 'do' mathematics. Whereas those who cannot read go to great lengths to hide this failing, a large number of the population are more than happy to admit that they weren't good at mathematics at school. And those that did enjoy mathematics at school are more than likely to keep this accomplishment hidden from friends and colleagues. It seems only English has the popular expression 'too clever by half'. How does this situation arise? Steiner and Hersh have no doubt as to the answer. They write, 'People aren't born disliking math. They learn to dislike it at school!' This is not to decry the efforts of teachers but to acknowledge that the content of current mathematics courses in conjunction with their significance in high-stakes testing and examination success needed to gain access to college or university courses puts a very heavy burden on students who find the subject bemusing. This is not a particularly new insight. In 1902, Betrand Russell wrote:

> In the beginning of algebra, even the most intelligent child finds, as a rule, very great difficulty. The use of letters is a mystery, which seems to have no purpose except mystification. It's almost impossible, at first, not to think that every letter stands for some particular number, if only the teacher would reveal what number it stands for. The fact is, that in algebra the mind is first taught to consider general truths, truths which are not asserted to hold only this or that particular thing but of any one of a whole group of things. […] Usually the method that has been adopted in arithmetic is continued: rules are set forth, with no adequate explanation of their grounds; the pupil learns to use the rules blindly, and presently, when he is able to obtain the answer that the teacher desires, he feels that he has mastered the difficulties of the subject. But of inner comprehension of the processes employed he has probably acquired almost nothing.
>
> (Russell, 1902, p. 60)

In fairness, we should acknowledge that Bertrand Russell's attempts at school teaching were not successful and the school he set up with Ludwig Wittgenstein was a complete failure. But we cannot deny that there is considerable concern over many young peoples' dislike of school mathematics and the resultant poor levels of attainment. We will now discuss this concern .

Causes for concern

Politicians need yardsticks with which to measure the educational achievements of their young citizens and compare them to that of young people in other countries. One such yardstick is the Programme for International Student Assessment (PISA). Since the year 2000, every three years, a randomly selected group of 15 year olds take tests in the key subjects: reading, mathematics and science, with focus given to one subject in each year of assessment. The students and their school principals also fill in questionnaires to provide information on the students' family backgrounds and the way their schools are run. Some countries and economies also choose to have parents fill in a questionnaire. PISA is unique because it develops tests which are not directly linked to school curricula and provides context through the background questionnaires which can help analysts interpret the results. The tests are designed to assess to what extent students near the end of compulsory education can apply their knowledge to real-life situations and be equipped for full participation in society. It is generally agreed that PISA data provides governments with a powerful tool to shape policy making.

The performance in mathematics in 2009 did not make encouraging reading for politicians in either the UK or the USA, with Shanghai-China, Singapore and Hong Kong-China in the top three positions. The United Kingdom came 28th with a score not statistically different from the OECD average and the USA came 31st with a score statistically significantly below the OECD average. President Obama was reported as being extremely disappointed by the results, vowing to improve US schools' performance. US Education Secretary Arne Duncan pulled no punches in his response, quoting President Obama, 'the nation that out-educates us today will out-compete us tomorrow' and acknowledging with brutal honesty that the PISA results 'show that a host of developed nations were out-educating the USA'. His view was that the big picture from PISA was one of educational stagnation at a time of fast-rising demand for highly-educated workers and that 'the mediocre performance of America's students is a problem we cannot afford to accept and cannot afford to ignore'. Michael Gove, the Minister for Education in England in 2011 was also a great admirer of the data generated by PISA, citing in a speech to the World Education Forum that the fall of student performance in mathematics from 8th to 28th in the past 10 years as a significant statistic that needed to be addressed by the coalition government. The theme is unrepentantly instrumental – education for economic success – but all the more difficult to achieve during a global recession. It is worth noting that the Trends in International Mathematics and Science Study (TIMSS) collects data concerned with the performance of students in primary (elementary) and secondary (junior high) schools. These tests are more closely aligned to the school curriculum and recently schools in England have shown improved performance. However, overall such tests give little comfort to politicians in England and the USA who are concerned at the disparity between the performance of young people in their countries compared to that of young people in other jurisdictions.

In England, the most recent Ofsted Report ('Mathematics Made to Measure', 2012) into the teaching of mathematics offers little comfort. As with all such reports the sample is limited. Inspectors visited 160 primary and 160 secondary schools and observed more than 470 primary and 1200 secondary mathematics lessons, but there is little reason to suspect that the findings are not typical of the wider picture. The report highlights the failure of much teaching to develop pupils' conceptual understanding alongside their fluent recall of knowledge, and a lack of confidence in problem solving indicating that too much teaching concentrated on the acquisition of disparate skills that might have enabled pupils to pass tests and examinations but did not equip them adequately for the next stage of education, work and life. The report emphasised the problems of a poor start which doubtless goes a long way to explain the 'hate it' as opposed to 'love it' response of many pupils:

> The 10% who do not reach the expected standard at age 7 doubles to 20% by age 11, and nearly doubles again by 16
>
> (Ofsted, 2012, p. 4)

The report noted that this is compounded by the lack of curricular guidance and professional development in enhancing subject knowledge and effective pedagogy. In his introduction to the report, the Chief Inspector of Schools Sir Michael Wilshaw returns to the instrumental theme:

> Our failure to stretch some of our most able pupils threatens the future supply of well-qualified mathematicians, scientists and engineers.
>
> (Ofsted, 2012, p. 4)

Another report, 'A world class mathematics education for all our young people', developed by a task force chaired by Carol Vorderman and lead author Roger Porkess, paints a bleak picture of the disenchantment felt by many with regard to their mathematics experience and lack of success at schools:

> If you imagine those 300,000+ young people who have failed holding hands in a line, then that line of students would stretch the entire length of the M1 from London to Leeds. These are just this year's GCSE students who, after 11 years of being taught mathematics, have learnt to fear and hate the subject. And next year there will be another such line of students, and the year after, and so on.
>
> (Porkess et al., 2011, p. 4)

The report makes a wide range of recommendations for improving the situation, many of which have been welcomed by the current political administration and to some extent implemented, particularly those concerned with making teaching more attractive to mathematics graduates and raising the level of mathematics qualifications required by all those entering teaching whatever subject they might teach. The report also cites a publication by the Nuffield Foundation, 'Is the UK an outlier? An international comparison of upper secondary mathematics education'. The findings of this report are stark. In England, Wales and Northern Ireland fewer than one in five students study any mathematics after the age of 16 (Scotland does slightly better). In 18 of the 24 countries studied (mainly from the OECD) more than half of students in the age group study mathematics; in 14 of these, the participation rate is over 80%; and in eight of these every student studies mathematics. When it comes to the mathematics education of its upper secondary students the UK is out on a limb. The Vorderman-Porkess report indicates strongly that this situation should be addressed. It is important to note that Vorderman and Porkess do not in any way place blame on students, teachers or schools. They see these actors as victims of a system over which they have no control. Rather, the report identifies a lack of innovation in syllabuses; qualifications and provision, it argues, have been stifled by prevailing regulation and accountability procedures and pose a significant problem. This, the report suggests, has prevented the necessary developments to improve the experience of school mathematics such that it is rewarding and challenging without being daunting and such that it could provide qualifications of value which could also identify the suitability of young people for particular employment and further or higher education. Such developments could reveal the significance and potential of teaching mathematics in the light of STEM. Given the extent of these concerns it is important to consider how they are being addressed.

Responding to concern

The mathematical education community is not complacent and has responded vigorously and positively to the problems identified in the previous section. The report by the Advisory Committee on Mathematics Education (ACME) on the mathematical needs of learners is particularly insightful (ACME, 2011). The report is critical of the assessment regime that fosters 'teaching to the test' at the expense of developing a genuine understanding based on conceptual development. A particularly useful feature of the report is the development of a model for mapping a curriculum for learners' mathematical needs. To some extent the approach parallels that of Wynne Harlen and colleagues in identifying key ideas in science that we saw in Chapter 3. The approach relies on identifying BIG mathematical ideas associated with a particular domain and making suggestions how such ideas might be used in a particular contexts and practices. The model of progression involves starting with a wide range of early ideas, their reduction to a few key ideas, the development of new meanings which lead to a wider and richer set of later ideas. This framework is shown in Figure 5.1a. As proof of concept, two examples were worked through (1) the development of multiplicative thinking and (2) understanding of measurement. These are shown in Figures 5.1b and 5.1c respectively. Inspection of the framework

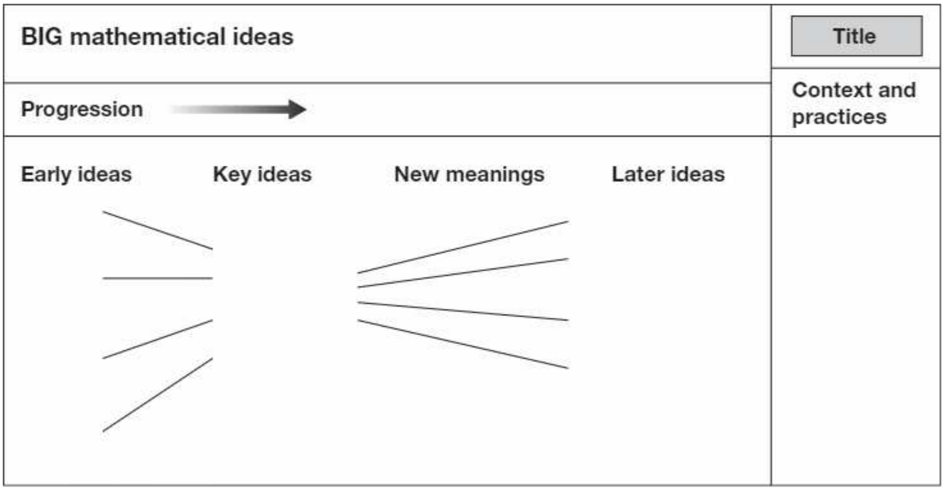

FIGURE 5.1a Framework for describing progression in the learning of BIG mathematical ideas.
Adapted from http://www.acme-uk.org/media/7627/acme_theme_b_final.pdf

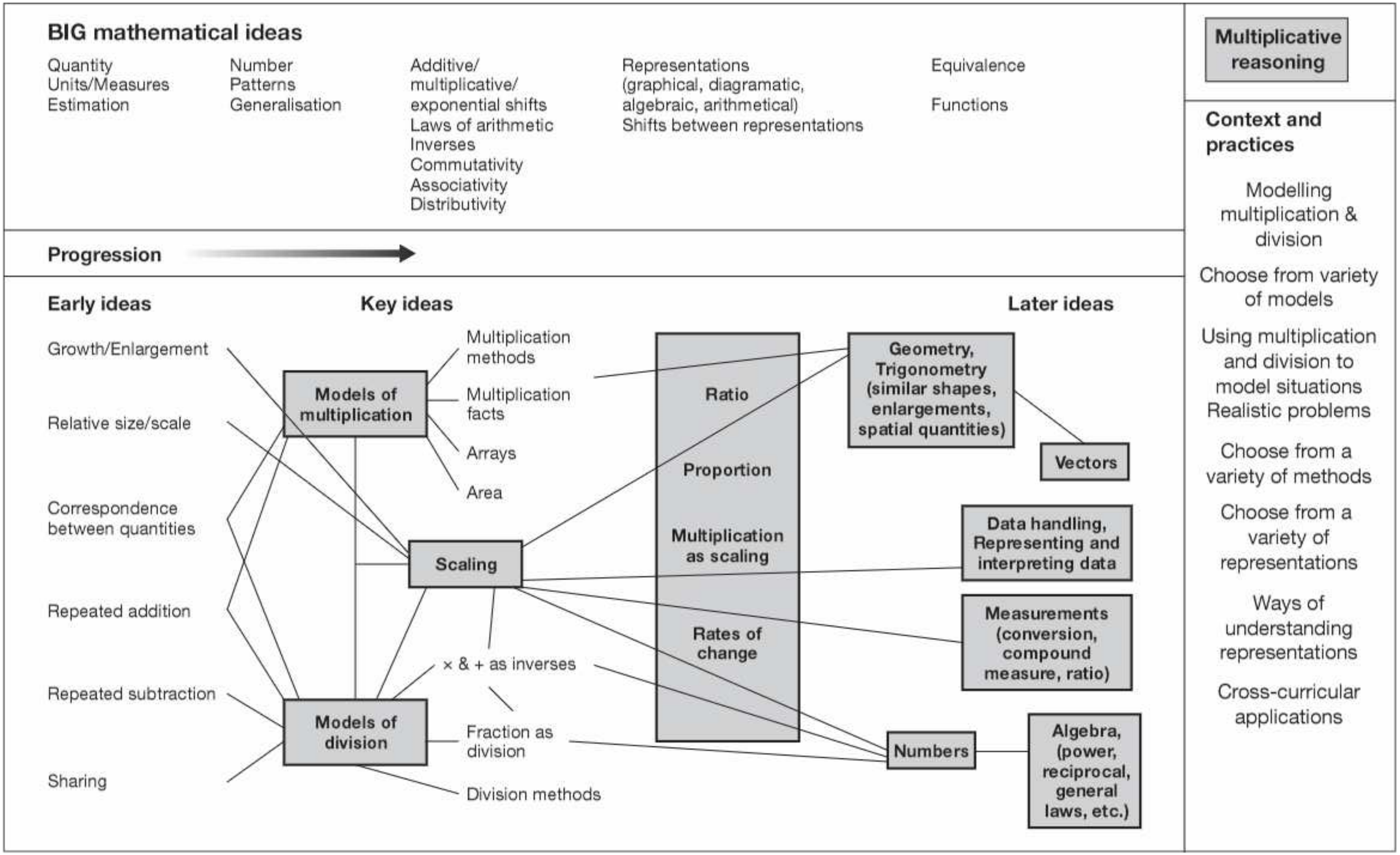

FIGURE 5.1b Description of progression in multiplicative reasoning

Adapted from http://www.acme-uk.org/media/7627/acme_theme_b_final.pdf

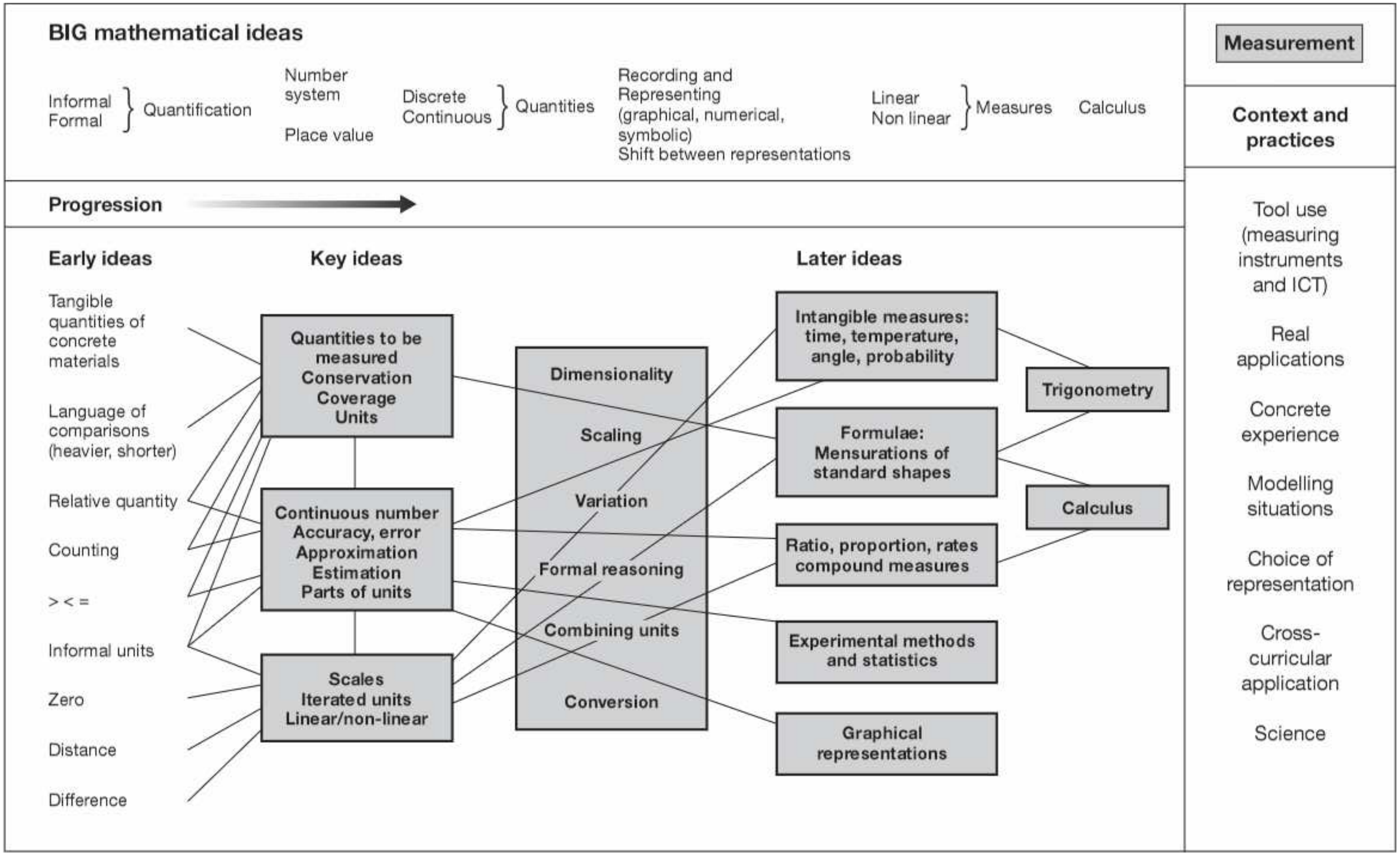

FIGURE 5.1c Description of progression in understanding measurement

Adapted from http://www.acme-uk.org/media/7627/acme_theme_b_final.pdf

and the examples indicate that they have the potential for providing particularly rich starting points for teaching mathematics in the light of STEM. Exploiting the dynamic relationship between the progressive development of ideas and their application as indicated in context and practices provides a flexible structure for ensuring that such cross-curricular endeavours will lead to the sort of in-depth conceptual understanding that is required to give both proficiency and intellectual satisfaction to students.

Building on good practice

Of course, there are schools in England in which mathematics is taught exceptionally well and an important strategy in improving the overall situation is to identify this practice and make it available to other schools. This was the rationale driving the case study approach of the National Centre for Excellence in the Teaching of Mathematics (NCETM). The report produced by the NCETM, 'Developing Mathematics in Secondary Schools' (2009) discussed five important issues: (1) what mathematics brings to the students and to the school; (2) recruitment; (3) retention; (4) continuing professional development; and (5) leadership and management. This chapter will concentrate on the first of these issues, what mathematics brings to students and schools. As far as the schools that taught mathematics well were concerned, the value of the subject lay well beyond public examination performance. They ensured that pupils appreciated the career value of mathematics. They used mathematics, particularly communicating mathematical ideas in class discussions to build students' self-confidence. They did not shy away from demanding a rigorous approach for all pupils whatever their level of achievement. They celebrated the enjoyment and love of mathematics as something for all. In addition, in some cases mathematics provided a language that pupils who were not adept at using spoken or written English could access and use fluently. And, particularly important from our STEM perspective, they valued the connection of mathematics with other subjects in the curriculum.

Linking research and teaching

The Targeted Initiative in Science and Mathematics Education (TISME) works with teachers to provide access to research-informed approaches to teaching. One particular project, Increasing Competence and Confidence in Algebra and Multiplicative Structures (ICCAMS), is of particular relevance to this chapter. Teachers who have taken part in this project have reported that the approach requires them to spend time enabling the pupils to have mathematical conversations and to 'listen in' on these conversations so that they are able to ask questions that provoke deeper thinking from the students such that they are more able to engage in conversations with peers. The word 'conversation' is significant here in that it implies both speaking *and* listening and also describes a learning environment in which talking and listening are important features. Given that taking tests in silence is one of the memories that haunt those who learned to dislike mathematics at school, the move to a 'conversational mathematics classroom' would seem a step in the right direction! The teachers confided that it was not easy to change practice from the traditional didactic model that had dominated their previous approaches but they were unanimous in their opinion that enabling conversations paid big dividends both in pupils' understanding of algebra and in their overall mathematical confidence. Pupils who are used to having mathematical conversations in mathematics might well be able to engage in such conversations in scientific and technological contexts. Hence such an approach might be conducive to teaching mathematics in the light of STEM – talking brings the learning of concepts and the exploration of new ideas into the open so that pupils can 'hear what they think'.

Developing new approaches to assessment

It is well known that employers and universities demand that school leavers are able to apply their mathematical knowledge to problem solving in varied and unfamiliar contexts. It is also acknowledged that most mathematics examinations have neglected this aspect and, in responding to the requirement for examination success, classroom teaching ignores this as well. Some attempts have been made to remedy the situation. In 2010, GCSE assessment for mathematics required the use of functional skills to respond to unstructured questions. There was an expectation that teachers might change their teaching to meet this requirement. More recently, Ian Jones of Loughborough University has been exploring the use of comparative judgement as a means of assessing pupils' mathematical problem-solving skills. His initial investigations reveal that comparative judgement has the potential to provide valid and reliable assessment of mathematical problem solving. In comparative judgement, the assessor compares two answers from different candidates and puts them in an order to show which candidate showed greater mathematical problem-solving ability. Given enough judgements across a large enough sample the results are valid and reliable and create an order of proficiency across the sample. Such an approach is a radical departure from traditional marking in which there is a marking scheme which gives particular marks for each aspect of each question on an examination paper. Where grade boundaries might be drawn is not decided by this method and the method does not give marks to each candidate – just their position in the sample. In assessing other mathematical qualities it is likely that more conventional approaches might be used. It is worth emphasising the concern over current assessment of mathematics. Roger Porkess (lead author of the aforementioned report 'A world class mathematics education for all our young people'), and colleagues have commented:

> Many really important aspects and ideas in mathematics cannot be assessed by exam only, for example, sampling and data collection in statistics, mathematical

> modelling, numerical analysis, extended problem solving and appropriate use of computer software. Although these are written into the learning outcomes of many qualifications…the present regulations ensure that they are assessed… superficially.
>
> (Porkess et al., 2011, p. 97)

Ian Jones and his colleagues envisage that a comparative judgement approach would enable the reliable assessment of an unstructured component that dealt with problem solving within a diverse range of assessment formats carefully chosen to appropriately assess the full range of valued competencies from technical fluency and conceptual understanding, through to problem-solving processes. Although there is still much to be done to establish the comparative judgement approach in mathematics assessment, its explicit use in assessing mathematical problem solving would encourage teachers to teach mathematical problem solving.

This aspect of mathematics would of course lend itself to teaching mathematics in the light of STEM as both science and design & technology are a rich source of problems for which mathematical problem solving provides useful insights.

The Khan Academy

Salman Khan has caused a considerable stir in the mathematics education community. A highly successful hedge fund manager, he has posted some 3,300 videos on YouTube many of which can be used to learn mathematics. The phenomenon is known as the Khan Academy. *Time* Magazine has reported the following statistics: 600 million exercises completed, some 2 million per day, 15,000 classrooms in which the Khan Academy is used in some form, 5 million unique users per month, 160 million videos watched since 2006 and 234 countries and territories where the Khan Academy is used. The Khan Academy has gained significant funding from the Gates Foundation, Google and Netflix. The Khan Academy is an example of the 'flipped classroom'. Pupils use online videos and similar resources at home after the school day – complete with assessment, feedback and guidance as to next steps. Pupils then use what they have learned at home to complete exercises at school the following day. This leaves the teacher free from whole-class instruction and able to deal with individuals with their particular learning problems according to what they had learned at home the previous evening. Pupils would be able to progress at their own pace through the mathematics curriculum and traditional whole-class, didactic teaching would become a thing of the past.

Some mathematics educators are unconvinced by the approach. Karim Kai Ani, a former middle school teacher and mathematics tutor, voices a typical criticism suggesting that there are basic errors in teaching simple concepts and these set the pupil on a path of misunderstanding which hampers their long term mathematical development, although it will give short-term success in high-stakes assessment. They do not decry Khan's achievement in creating such a vast and varied library. They acknowledge that he deserves to be recognised and to be praised. They see him as a good guy with a good mission – but a bad teacher whose teaching adopts a 'do this then do this approach' that presents mathematics as a meaningless series of steps.

Michael Pershan has contrasted the Khan Academy approach with that used to teach mathematics in Japan. A key point for him is the extent to which Japanese teachers allow their pupils to struggle with conceptual problems in contrast to the way American teachers spend little or almost no time on such activity. He argues for 'getting smarter by struggling' and notes that there is little if any struggle in the Khan Academy approach and that this is an inherent limitation in the approach.

If a school uses the Khan Academy approach to teaching mathematics where would that leave teaching mathematics in the light of STEM? One position might be that it forces mathematics into its own silo with a limited and perhaps even an inadequate pedagogy, which would make such teaching difficult if not impossible. The collective discussions so highly valued in the ICCAMS project could be difficult to facilitate with different pupils following their online, individual, isolated paths. The opposite position might be that with some orchestration pupils' conversations with the teacher in the flipped classroom could just as easily relate mathematical learning to science and design & technology.

Having considered both concerns with regard to mathematics education in schools and some of the ways in which the education community is responding to these concerns, we now consider how to address these through teaching mathematics in the light of pupil's learning in science and design & technology.

How might teaching mathematics in the light of STEM help?

In this section we will consider how pupils' learning in science and design & technology might be used to enhance their learning mathematics. Our approach will be to identify the mathematical requirements of some topics taught in science and design & technology. An obvious requirement for both subjects is the need to quantify observations for length, area, volume, mass, time and to develop compound measures such as speed, density and strength. In the case of science, it is important to be able to find patterns in data and use these to develop explanations so that these can be used in turn to inform design decisions in design & technology. Let us begin with chemistry and the move from a qualitative to a quantitative approach.

Deriving chemical formulae

A common experiment is to burn a weighed amount of magnesium ribbon in a lidded crucible and then weigh the amount of magnesium oxide formed. This is a tricky experiment to perform well. First, the crucible and lid should be weighed, then the magnesium ribbon added to the crucible in the form of an open coil so that it can burn and the whole assembly reweighed. Then, the crucible must be heated to cause the magnesium to ignite and the lid must occasionally be raised ever so slightly to allow more air into the crucible to ensure complete combustion. But it is important not to allow any of the oxide being formed to escape.

Once the reaction appears to be complete then the whole assembly must be left to cool before it is reweighed. The weighing data can be used to find the mass of oxygen that has combined with the starting mass of magnesium. The challenge now is to turn this data about combining masses into a formula for magnesium oxide. There are several simple possibilities. If one atom of magnesium combined with one atom of oxygen the formula would be MgO. If one atom of magnesium combined with two atoms of oxygen the formula would be MgO_2. If two atoms of magnesium combined with one atom of oxygen the formula would be Mg_2O. The calculation also has to take into account the differing atomic masses of magnesium and oxygen. Magnesium has an atomic mass of 24 whereas oxygen has an atomic mass of 16. Hence for a formula of MgO the ratio of combination would be 24:16 equivalent to 3:2, whereas for a formula of MgO_2 would require the ratio of combination to be 24:32 equivalent to 3:4. Given that pupils will be handing masses of magnesium between 0.5 and 1.5 g the arithmetic can become complicated. One way forward here is to plot the data obtained on a chart which shows line graphs for different possible formulae of magnesium oxide. Such a graph is shown on Figure 5.2. Pupils in the class can share their results and then plot them on the graph. The result will be a scatter of points and the position of this scatter should enable the class to decide which of the three possible formulae is correct according to their experiments. Of course, the chemistry teacher will want her pupils to know and remember the correct formula of magnesium oxide (MgO) but the point of this experiment is to reinforce the idea that formulae are derived from experimental data. Later in the chemistry course the pupils will learn about atomic structure and valency and be able to use these ideas to explain why the formula of magnesium oxide is MgO and not Mg_2O or MgO_2. Clearly, the pupils will be using significant mathematics in this experiment.

It is possible to view the data manipulation and presentation as a series of mathematical operations. Each operation is not complicated but the overall sequence of operations is demanding. This requires a clear understanding of the purpose of the endeavour, experimental skill in obtaining the required data, competence in presenting the data graphically, an understanding of ratios and the ability to interpret the data once presented graphically. It is a worthwhile opportunity for pupils to use their mathematics in a chemistry lesson. Conversations between the chemistry teacher and the mathematics teacher are necessary to ensure that pupils are encouraged to use mathematical thinking and justify their procedures as

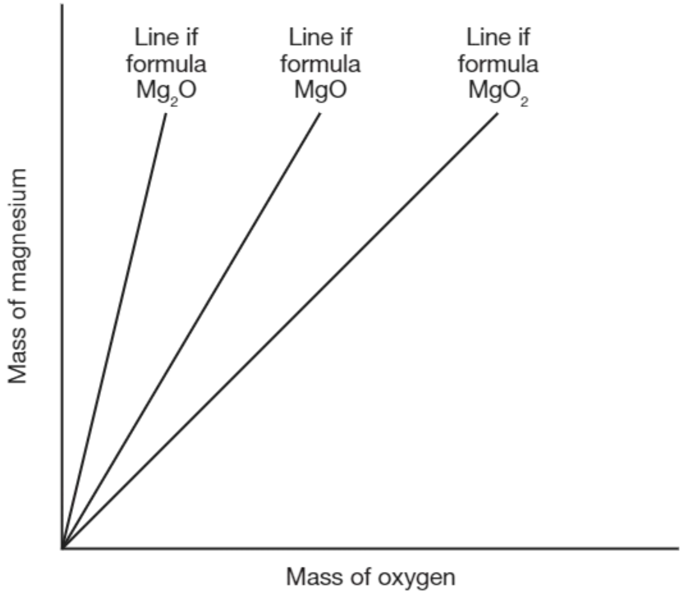

FIGURE 5.2 Graphical presentation of possible formulae for magnesium oxide.

opposed to simply following instructions. It might even be possible for the mathematics teacher to include the 'work up' of the results in a mathematics lessons. This would require a deliberate intervention to teach mathematics in the light of STEM. If the mathematics teacher felt that the pupils had sufficient mathematical knowledge and skill before such an intervention then the 'work up' lesson can be seen as an opportunity for revision and assessment of previous learning. If the pupils are going to be using unfamiliar mathematics then the work up session provides a novel way to introduce such topics.

Calculations from equations

In attempts to show the application of chemistry in everyday life, teachers often consider simple medicines such as indigestion remedies. The basis of such remedies is sometimes the neutralisation of excess stomach acid. Such products are often called 'antacid tablets' and the key ingredient will be a substance that reacts with the acid in the stomach. In some cases this is a carbonate. The reaction of the carbonate with the acid causes the production of carbon dioxide so that the reduction of acid content in the stomach is accompanied by the formation of a gas. In such cases those taking the tablets often 'burp'. In other cases the key ingredient is a hydroxide. Here the neutralisation of the acid is not accompanied by the production of carbon dioxide.

For those producing indigestion remedies it is important to know how much of the antacid ingredient to put in each tablet. Too little and the remedy will fail; too much and the reduction of acid in the stomach will be so great that food cannot be properly digested. The acid in the stomach is hydrochloric acid and the equation for the reaction of magnesium hydroxide with hydrochloric acid is as follows:

$$Mg(OH)_2(s) + 2HCl(aq) \rightarrow MgCl_2(aq) + 2H_2O(l)$$

The challenge is to use this equation to calculate how much magnesium hydroxide is needed to neutralise some of the acid in the stomach. Typically, an adult's stomach contains about 200ml of gastric juice with a concentration of 0.1 mol/l. To ensure that not too much of the acid is neutralized we can assume that only half the acid in the stomach should be neutralized. What are the steps in the calculation?

a) Decide on the amount of acid to be neutralised.
b) Use the equation to decide on the number of moles of magnesium hydroxide and hydrochloric acid that are reacting together.
c) Use the answer from a) to decide on the number of moles of hydrochloric acid that need to be neutralized.
d) Use the answers from b) and c) to decide on the number of moles of magnesium hydroxide needed to neutralize the hydrochloric acid.
e) Convert the answer to d) into grams of magnesium hydroxide needed for a single antacid tablet.

Underpinning this lengthy calculation are the important ideas of ratio and proportion. The question for the chemistry teacher is how the use of ratio and proportion in the calculation relates to the way this is taught in mathematics lessons. The question for the mathematics teacher is how might the way pupils are taught about ratio and proportion in mathematics support their thinking in such chemistry calculations. It is clear that a conversation is necessary to explore these questions and develop an approach in which the learning in both chemistry and mathematics is enhanced. Let us now move on to physics.

Understanding waves

Waves are a fundamental concept in physics and the behaviour of waves is used to explain a wide variety of phenomena. These include earthquakes, (seismic waves), sound waves, the electromagnetic spectrum and the properties of light (propagation, refraction and diffraction). Fundamental to understanding the behaviour of waves is the wave equation

Wave speed	=	Frequency	x	Wavelength
(metres per second, m/s)		(hertz, Hz)		(metres, m)

The unit hertz refers to cycles per second, a unit named in honour of the physicist Heinrich Hertz who discovered radio waves. This is sometimes abbreviated to

$$v = f \times \lambda$$

where

$$v = \text{wave speed}$$
$$f = \text{frequency}$$
$$\lambda = \text{wavelength}$$

This can be shown diagrammatically as a sine wave (see Figure 5.3). The amplitude of the wave can change without any change to the wavelength or frequency.

FIGURE 5.3 Sine wave.
Adapted from Millar et al. (2011)

It is possible to use this wave equation to calculate any one of the features in the equation if values are known for the other two features. The equation can be used to explain the colour of visible light. The speed of light can be treated as a constant, hence the frequency × wavelength is also constant. But if the frequency is decreased then the wavelength must increase. Similarly, if the frequency is increased then the wavelength must decrease. In the visible spectrum blue light has a higher frequency and lower wavelength than red light. The numbers get scary because of the units. Blue light has a frequency 606–668 THz and a wavelength 450–495 nm. Red light has a frequency 400–484 THz and a wavelength of 620–750 nm. T stands for tera which means x million million – 1 000 000 000 000 or 10^{12}, so the frequency of blue light is in the region of 460 million million cycles per second (or Hz); n stands for nano which means one thousand millionth – 0.000000001 or 10^{-9}, so the wavelength of red light is in the region of 700 thousand millionths of a metre.

Where might this lead in a conversation between the physics teacher and the mathematics teacher? There's certainly the possibility of discussing how students might learn to understand very large and very small numbers. There's also the possibility of discussing how students might develop an understanding of direct and indirect proportional relationships described by simple y = mx type equations, or even simple trigonometry functions. Such conversations could lead to mathematics lessons in which such topics were introduced or revised in the context of waves and the simple wave equation.

Measuring particle size

An interesting part of teaching the particle theory of matter is to engage pupils in the measurement of the actual size of particles. The 'penny dropping' moment when pupils realise just how small particles are is worth striving for.

Practical Physics, developed by the Institute of Physics and the Nuffield Foundation, has a useful experiment to estimate the size of a molecule using an oil film.[1] The diameter of a tiny droplet of olive oil is measured and the droplet placed on a water surface covered with lycopodium powder. The drop is spread out to form a circle and it is assumed the circle is one molecule thick. The basis for this assumption is that the oil molecule has a hydrophilic end that is attracted into the water and a hydrophobic chain that is repelled by the water and stands up out of the water. The diameter of the circle is measured. (See Figure 5.4)

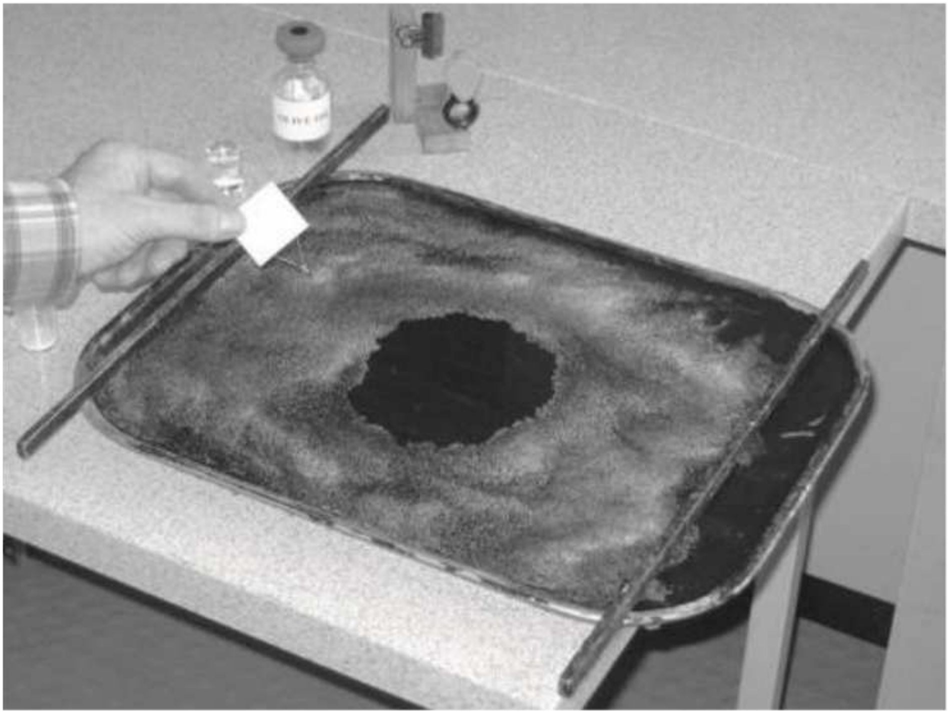

FIGURE 5.4 The Practical Physics oil drop experiment.

Mathematical thinking is required to turn the measurements into an estimate of molecular size.

- The volume of the oil drop is proportional to the diameter cubed.
- The oil drop spreads out to form a cylinder one molecule thick.
- The volume of the cylinder is given by area of the circle times its depth.
- The area of the circle is proportional to the diameter squared.
- If the diameter of the circle is D, and the diameter of the oil drop of d, and the length of the molecule is l, then $d^3 = D^2 \times l$.
- A typical diameter of the initial oil drop would be 0.5 mm.
- A typical diameter of the film would be 250 mm.

Hence

$$0.5 \times 0.5 \times 0.5 = 250 \times 250 \times \text{length of the molecule (in mm)}$$
$$\frac{0.5 \times 0.5 \times 0.5}{250 \times 250} = \text{length of molecule (in mm)}$$

Using a calculator:

$$0.000002 = \text{length of molecule (in mm)}$$
$$0.000000002 = \text{length of molecule (in m)}$$

Using standard notation this becomes:

$$2 \times 10^{-9}\text{m} = \text{approximate length of the oil molecule}$$

There are approximately 12 atoms in the olive oil chain the size of an atom is given by $2 \times 10^{-9} \div 12$.

Hence the approximate size of an atom is 1.7×10^{-10} m

This can be written as 0.00000000017 m
$\approx$ 0.0000000002 m
or 0.2 nanometres

The above is of course an approximation, as the precise formulae for the volume of the initial drop and the cylinder have not been used in order to keep the calculation relatively simple. The result is however of the correct order of magnitude, the size of a carbon atom being 0.7×10^{-10} m

So what sort of conversation might the mathematics teacher have with the physics teacher about this experiment? A discussion on accuracy of measurement and the impact of inaccuracies on the estimate of molecule size is a possibility. The formulae for the volumes of spheres and cylinders might feature along with the possible effect of the approximations in the calculations shown above. The arithmetic involved in the calculations, the use of power of ten notation, and the prefix nano meaning one billionth. With so much varied mathematics embedded in the activity it is unlikely that the mathematics teacher can use the experiment to introduce these topics in the mathematics curriculum. However, discussing the experiment would provide an interesting revision exercise across a wide range of topics and could be used as an assessment for learning exercise. Discussion of the results of the experiment and how to derive an estimate for molecular size would reveal where pupils were comfortably confident and where they were experiencing difficulties. Let us now move on to biology.

Estimating population size

Deciding on the number of particular animals or plants in a particular location is a challenge for professional biologists. In most situations it is impossible to count the actual number of flora or fauna present so experimental procedures for estimating the population from a small sample have been devised. Pupils are introduced to these procedures in most biology courses and these estimation procedures are underpinned by mathematical understanding.

Using quadrats

This procedure is relatively simple and involves using a metal or wooden frame called a quadrat. It is usually used to count plants but can be used for slow moving insects. The most basic approach is as follows:

a) Placing the quadrat on the habitat under investigation.
b) Counting the number of a particular plant present inside the quadrat.
c) Estimating how many quadrats are needed to cover the habitat.
d) Multiplying the answer to c) by the answer to b) to estimate the number of the particular species in the habitat.

It is possible that the number of plants within the quadrat will vary from place to place in the habitat. A habitat could be variable in terms of abiotic factors, e.g., light and shade, exposure to wind, availability of water and minerals and biotic factors involving competition from other organisms. Hence the procedure often involves placing several quadrats at different positions in the habitat in order to improve the estimate and avoid bias. Part of a typical examination question might include the results of such an experiment as follows:

quadrat	*number of dandelions*
1st	5
2nd	1
3rd	0
4th	2

- Each quadrat has an area of $0.25m^2$
- The total area of the habitat, a playing field, is 20,000 m^2

Q: Estimate the total number of dandelion plants in the playing field.

If the candidate realises that the sum of the area of the four quadrats is $1m^2$ the task becomes simple. Simply add the number of dandelions in each quadrat to find the number of dandelions in $1m^2$ and then multiply the result by 20,000. Surprisingly teachers report that many pupils find this sort of question confusing and use inappropriate arithmetical techniques – multiplying the areas and dandelion numbers instead of adding them giving 10 dandelions in .0039 m^2; averaging the number of dandelion in a quadrat and then miscalculating the number of 0.25 m^2 in the field. It is as if the requirement to 'be mathematical' presses a panic button. A conversation between the biology teacher and the mathematics teacher to explore why pupils tried such apparently illogical approaches would be a start to overcoming pupil's poor responses. A further step would be for a quadrat exercise to be undertaken as part of a mathematics lesson and the results then considered in a biology lesson.

Using mark, release, recapture

Mark, release, recapture is a common approach to estimating the size of an animal population in a habitat. A portion of the population is captured, marked, and released. Later, another portion is captured and the number of marked individuals within the sample is counted. This method assumes that the study population is 'closed'. In other words, the two visits to the study area are close enough in time so that no individuals die, are born, move into the study area (immigrate) or move out of the study area (emigrate) between visits. It is usual in biology texts book to simply provide the following formula which allows the population of the particular animal to be estimated.

$$N = \frac{MC}{R}$$

Where

N = Estimate of total population size
M = Total number of animals captured and marked on the first visit
C = Total number of animals captured on the second visit
R = Number of animals captured on the first visit that were then recaptured on the second visit

I must admit that I did not find the formula easy to understand intuitively. I wasn't sure why it worked. Some teachers try to provide insight by talking pupils through a situation in which 10 labelled ladybirds are released in a greenhouse and on recapture of 10 ladybirds some time later only one is marked indicating that the ladybird population in the greenhouse is 100. However, I do wonder whether this insight is sufficient to provide genuine understanding. Then I found out how the formula was derived.

The method assumes that in the second sample, the proportion of marked individuals that are caught (R ÷ M) should equal the proportion of the total population that is caught (C/N).

This can be written

$$\frac{R}{M} = \frac{C}{N}$$

This can be rearranged to give

$$N = \frac{MC}{R}$$

This is the formula often given to the pupils without explanation. I wonder whether it is worth pupils understanding where the formula comes from i.e., how it is derived, to better understand the logic behind the technique. Before attempting to do this with a class it would be wise for the biology teacher to have a conversation with the mathematics teacher. The result of this conversation might well be that the mathematics teacher would be happy to use the derivation of the formula as a means of teaching or reinforcing algebraic manipulation. Once the formula had been derived in mathematics, pupils could ideally use it in actual investigations of populations of small creatures in a local environment, e.g., wood lice or ground beetles. If this were not possible then second hand data from some original investigations could be used. Either way the pupils would be in a stronger position to understand the mathematical basis of the experimental procedure. Before moving on, let us look at another example in biology.

Probability and genetics

Some diseases are genetic. They are passed from the parents to their children. For example, causes and probability of a child having the genetic disorder cystic fibrosis is taught in most biology courses. The explanation requires pupils to understand several concepts: dominant and recessive alleles, faulty alleles, the production of two sets of alleles by each parent, and the combination of alleles during sexual reproduction, some of which give rise to the disease, some of which don't. These concepts are embedded in explanatory diagrams like the one shown in Figure 5.5.

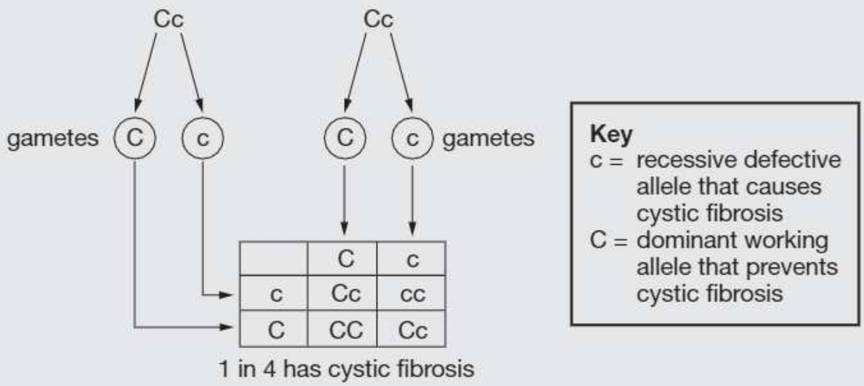

FIGURE 5.5 An explanatory diagram to show the probability of inheriting cystic fibrosis. Adapted from Williams (2006)

So if both parents carry the recessive defective allele (c) and the dominant working allele (C) there is a one in four chance that their child will inherit the disease because only one in four of the possible combinations gives two recessive defective alleles.

Parents whose genetic make up is Cc are known as carriers as they can pass on the defective allele but do not themselves have the condition. It is possible to test an embryo in the womb to discover if it has two defective alleles in which case the child would suffer from cystic fibrosis. It is now possible for parents to have their DNA tested to discover if they are carriers. But even if they find they *are* carriers, the chance that their child will be born with cystic fibrosis is only one in four. And probability has no memory. Hence if a couple who are carriers have three children, all of whom are healthy, the chance of their next child having cystic fibrosis would still be one in four. And a couple who were carriers might have just one child and that child could have cystic fibrosis even though the chance of that child having the disease is one in four. Without some understanding of probability it seems likely that pupils could become confused with regard to the factors effecting parent's decision making. Hence a conversation with the mathematics teacher might not go amiss. Indeed the mathematics teacher might consider using recessive/dominant allele combination as a way of teaching probability. A difficulty with this as an introductory approach might be that the science terminology gets in the way of understanding the probability. If this occurs, then it might be preferable for the mathematics teacher to use this as revision exercise in probability once pupils are familiar with the science. Now let us move on to design & technology.

Choosing materials

Choosing which material to use for the components of a design is always a challenge. In most of the products designed and made by pupils in schools the choice is inevitably limited. Often the choices made are based on a combination of precedent – what others have used when they have designed similar products – and availability – what the school has in stock or can afford to purchase. This experience, whilst defendable on grounds of practicality, does not engage pupils with serious thinking about material choice with regard to matching the required physical characteristics with those of available materials.

Questions concerning both strength (will the part break?) and stiffness (how much will the part deform?) are important, as poor choice of material will lead to poor product performance. Investigations into the properties of materials can give pupils insight into the behaviour of materials. The results of such investigations can be presented graphically and the interpretation of such graphs requires mathematical thinking. The simplified stress versus strain graph for a metal under tension shown in Figure 5.6 provides a good example. The behaviour of the metal in the linear part of the graph shows elastic behaviour. In this part of the curve the metal will stretch under load and when the load is removed return to its original size. In the non-linear part of the curve the metal deforms but when the load is removed the metal stays permanently deformed. The metal becomes thinner in the final downward part of the curve (known as 'necking') and eventually breaks when the loading exceeds the strength of the metal.

Conversations between the design & technology teacher and the mathematics teacher are essential here if pupil interpretation of such graphs is to be sound. Indeed it might be possible to go further than a conversation. The mathematics teacher could use stress-strain graphs for different materials as a means of teaching 'interpretation of graphs' and the understanding of compound measures – stress has units of force/area, strain has no units as it is a ratio of extension: original length. The stress/strain ratio (slope of the line in the linear region) gives Young's modulus for the material and is a measure of the materials elasticity. Because strain has no units, the units for Young's modulus are the same as the units for stress. Hence the units for elasticity have the same units as strength – a cause of confusion for many pupils. Of course the above consideration is an oversimplification of the thinking required to decide on which material to use for a component but it does open the way for the design & technology teacher to ask questions, such as how strong does it need to be? How stiff does it need to be? How will your design decisions about form and material ensure that the component is strong enough and stiff enough? When pupils become intrigued by such questions and how they may be answered they are beginning to appreciate STEM as an holistic approach to designing.

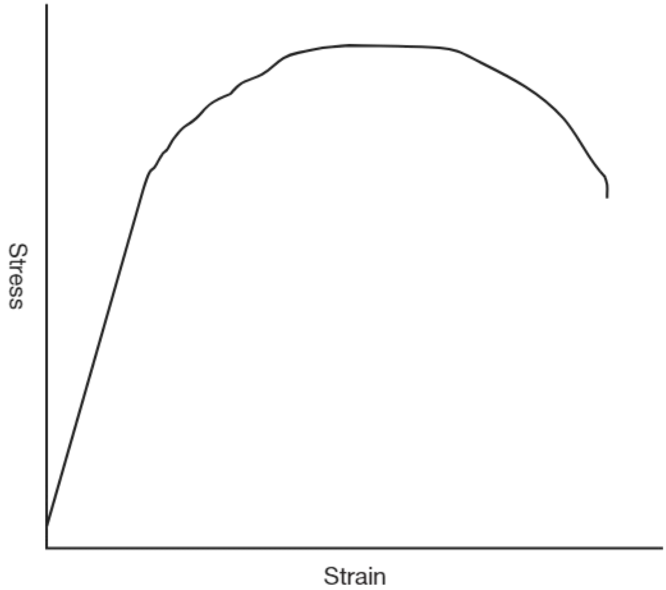

FIGURE 5.6 Simple stress strain graph for ductile metal.

Designing mechanisms

There is no shortage of mathematics embedded in the design of mechanical systems but some design & technology teachers have questioned the contexts into which such designing is embedded. Their position is summarised by the question 'Just how many pupils in the twenty-first century really want to make a mechanical toy or point of sale device?' They also argue that technically the results are generally unsophisticated and use technology from the nineteenth if not eighteenth centuries – which should not be the hallmark of modern technological learning. Whilst I have some sympathy with this argument, I am reminded of a conversation I had with a friend and colleague who trained as a mechanical engineer. 'You know David,' he said, 'four bar linkages are bloody amazing!'

This comment made me wonder about the intrinsic interest there might be in some mechanisms and that a more purist approach might pay dividends. What if one were to consider a mechanism as just an item of intrigue and did not worry too much about a context for use? I realise that this flies in the face of the conventional wisdom that the context for designing is of paramount importance and provides significant motivation for the pupil, but when I read a little more about four bar linkages I became convinced that this almost reactionary idea might have some worth. I found out about Grashof's rule. Franz Grashof was a distinguished nineteenth century German engineer whom in 1883, came to the conclusion that with regard to four bar linkages:

> If the total length of the shortest and longest bars is equal to or shorter than the lengths of the remaining two bars, then the shortest link can make complete revolutions.
>
> (Hartenburg and Denavit, *Kinematic Synthesis of Linkages*, 1964, p. 77)

This rule struck me as a having great mathematical potential and the possibility of simple practical work involving card strips and split pin paper fasteners. In a delightful book, *Mathematics Meets Technology* the author Brian Bolt describes three cases in which a four bar linkage obeys Grashof's rule: the crank and rocker, the double crank mechanism and the double rocker mechanism (see Figure 5.7). I can

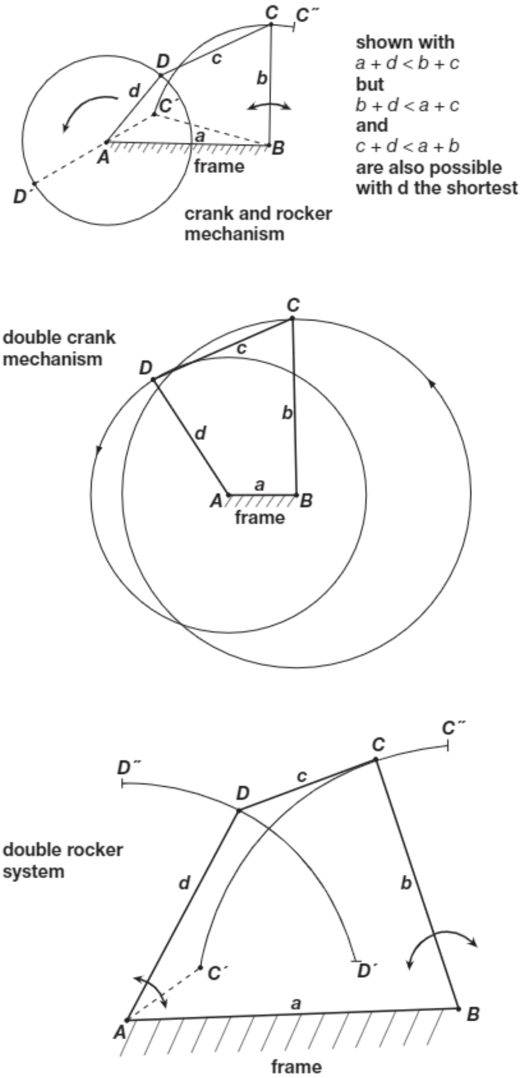

FIGURE 5.7 Three possible arrangements of the four bar linkage.
Adapted from Bolt (1991)

envisage mathematics lessons in which the teacher takes four bar linkages as a topic for practical and theoretical investigation with a view to pupils formulating Grashof's rule for themselves. If this investigation took place at a time when pupils were being asked to learn about mechanisms in design & technology and develop products which used mechanical systems this would provide them with a new mechanism to consider and a mathematical way of considering its design.

Considering robots

Simple robots feature in many design & technology curricula and an important aspect of working with robots is exploring the different ways they can be programmed to perform their various functions. This will involve both mathematical and computational thinking and will be considered in Chapter 9. The physical design of robots, as opposed to their programming, can involve significant mathematics.

Consider three basic types of robots: Cartesian robots, cylindrical robots and spherical robots – shown in Figure 5.8. The working envelopes of the robots are an important design feature. This describes the space in which the end effector of the robot can operate. In the case of the Cartesian robot, the position of the end effector at any one point will be described by three numbers – the x coordinate, the y coordinate and the z coordinate. The design of the robot governs the possible sizes of these coordinates and hence the size and shape of the working envelope, which is a cuboid. In the case of the cylindrical robot the radial arm can extend and retract horizontally, rotate about a vertical axis and the entire arm can be raised and lowered. The position of the end effector at any one point is described by its distance along the horizontal axis, (r), the angle it has rotated about the vertical axis (θ) and the distance it has moved along the vertical axis (z). The design of the robot governs the possible sizes of these cylindrical coordinates and hence the size and shape of the working envelope, which is cylindrical. In the case of the spherical robot the arm can extend along its length (r), rotate about a vertical axis (θ)) and can be rotated about a horizontal axis (φ) to elevate it above or below the horizontal. The design of the robot governs the possible sizes of these spherical coordinates and hence the size and shape of the working envelope, which is a spherical. The design & technology teacher can engage pupils with the design of these different types of robot quite easily with the use of construction kits such as Lego or Fisher Technic and the mathematics teacher can give such work a significant mathematical dimension by introducing pupils to three different ways of defining points in three-dimensional space.

There are many places in which robots are being used in society. Robots are finding their way into homes as domestic cleaning machines, into hospitals to perform surgery, in care homes for the elderly, in military operations such as bomb disposal, in search and rescue, in autonomous transport, in teaching, in manufacturing and in data collection. There will almost certainly be mathematical dimensions to their design in these different situations.

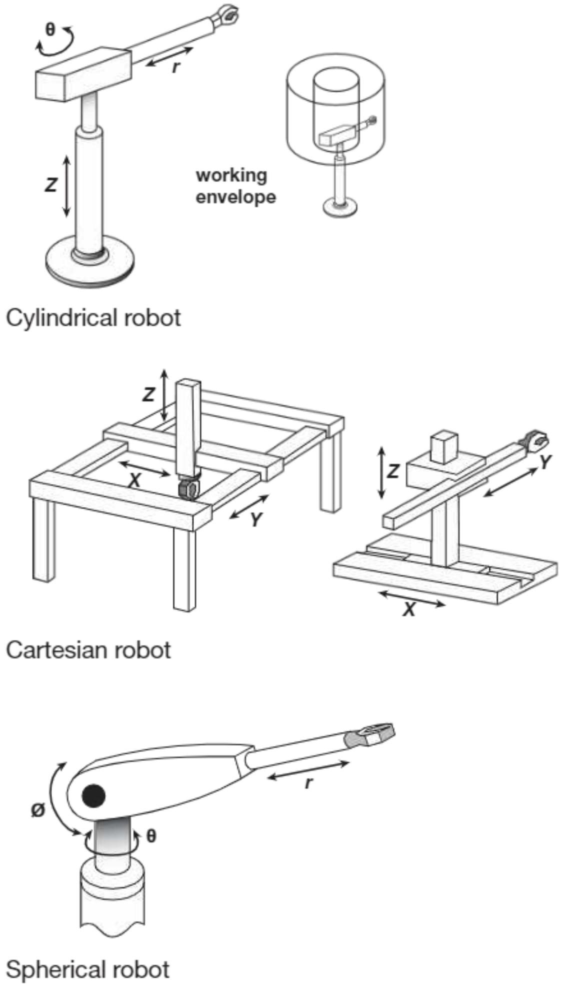

FIGURE 5.8 Three types of robot arms: Cartesian, cylindrical and spherical.
Adapted from Bolt (1991)

Surface decoration

Surface decoration plays a large part in many textile courses. Repeat patterns of various sorts are one of the main ways of achieving surface decoration and, of course, the mathematics of symmetry underpins such pattern generation.

Starting with a simple geometric shape, a triangle or half circle perhaps it is a relatively simple exercise to use basic transformation operations such as translation, reflection and rotation to produce a variety of different patterns. More complex transformations such as glide translation and helical translation can be added to the mix of operations. Assigning colours as the result of particular sequence of transformations can add even more visual interest, e.g., every time a reflection is followed by a rotation the colour changes from red to blue. So it is possible for pupils to write algorithms of transformations to generate patterns. Traditionally, such algorithms can be applied to fabric by using block printing techniques with the block undergoing a particular set of transformations between the making of each print on the fabric. And it is possible to carry out this activity pattern generating activity on screen and then use sublimation printing to produce the patterned fabric. The basic unit of the pattern need not be confined to a simple geometric shape. Suitable shapes can be derived from natural form via observational drawing and simplification, abstract form by assembling a variety of curved and straight lines into an enclosed shape. However the shape is derived, the way it can be used to produce a repeat pattern can be developed using transformations. School mathematics courses often include an introduction to symmetry and transformation geometry. The generation of patterns for use as surface decoration in a textiles component of a design & technology programme provides an interesting context for the application this mathematics. Hence a conversation between the mathematics teacher and textiles teacher would seem to be in order. The motivation for learning the somewhat abstract ideas of symmetry transformations can be enhanced if the teaching by the mathematics teacher acknowledges explicitly with the class that they will be able to use the learning in their textiles lessons. Indeed the development of the algorithms to produce patterns in the mathematics lesson can be seen as an essential first step in the overall textiles task of designing and making a patterned fabric. It is of course important that the ultimate use for the patterned fabric is considered so that the pattern is appropriate for the garment or furnishing that is being designed. An interesting possibility is that the mathematics teacher, having introduced the pupils to pattern design using transformations, consults with the class as they develop patterned textile products. In this way the teacher can use the students' efforts in design & technology to assess their understanding of transformations and where appropriate intervene to help pupils overcome misunderstandings.

Developing 3D textile structures

School mathematics courses deal with 3D geometry and this often requires pupils to understand the structure of polyhedrons. The simplest is the regular tetrahedron, consisting of four triangular faces and this polyhedron is often used as the basis for constructing juggling balls. The regular dodecahedron consists of twelve regular pentagonal faces. This polyhedron is often used as the basis for children's soft toys giving an almost spherical shape which can be used in mobiles, pram toys and squashy footballs. With slightly stiffer fabric polyhedron or part-polyhedron can be used as the basis for lampshades. The mathematics of polyhedron is intriguing. Regular polyhedron obey Euler's theorem which can be written

$$V + F = E + 2$$

Where

V = the number of vertices
F = the number of faces
E = the number of edges

In the case of a cube

$$V = 8, F = 6, E = 12$$

In the case of an octahedron

$$V = 6, F = 8, E = 12$$

Each regular polyhedron has a dual polyhedron which is also regular. Duals are created by using the midpoints of the faces in the original polyhedron to form the vertices of the dual polyhedron. Hence the dual of a cube is a regular octahedron as shown here in Figure 5.9.

A polyhedron can of course be opened out flat to form nets. These nets can form the basis for templates for designing and making the items noted above. So, as with teaching transformations, the motivation for learning the somewhat abstract ideas of 3D geometry can be enhanced if the mathematics teaching acknowledges explicitly with the class that they will be able to use their learning in textiles lessons. Designing nets is an inherently mathematical process and asking students how many different nets they can find for a cube or cuboid encourages mathematical thinking and problem solving. Hence the basis for the conversation between the mathematics teacher and the textiles teacher might consider using the mathematics lessons on 3D geometry to provide the pupils with the design tools to develop appropriate forms for products. And again, the mathematics teacher could visit design & technology lessons to observe pupils using their knowledge of 3D geometry as a useful means of assessment. If this proves unrealistic then it should be possible for the design & technology teacher to report back to the mathematics teacher on the extent to which the pupils were able to use their understanding of 3D geometry to make design decisions. Alternatively, pupils might use an e-portfolio to share examples of 3D geometry work in textiles with their mathematics teacher.

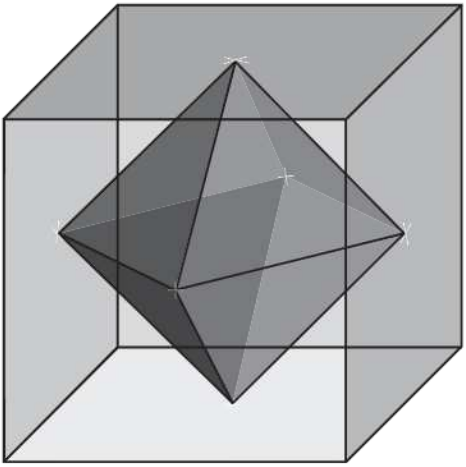

FIGURE 5.9 Duality of the cube and the octahedron.

Considering the energy content of food

Concerns about the levels of obesity in the population surface at regular intervals in the United Kingdom and the USA. One such example of this is the Foresight report, Tackling Obesities – Future Choices (2007), which is in little doubt as to the significance of the problem and the difficulty in developing a solution.

Sir David King, Chief Scientific Adviser to the UK government, captures this well in the foreword to the Foresight report.

> The project's findings challenge the simple portrayal of obesity as an issue of personal willpower – eating too much and doing too little. Although, at the heart of the problem there is an imbalance between energy intake and energy expenditure, the physical and psychological drivers inherent in human biology mean that the vast majority of us are predisposed to gaining weight. It's not surprising that the median body mass index in the UK is now above that considered to be in the 'healthy' range. We evolved in a world of relative food scarcity and hard physical work – obesity is one of the penalties of the modern world, where energy-dense food is abundant and labour-saving technologies abound.
>
> Creating an environment that better suits our biology and supports us in developing and sustaining healthy eating and activity habits is a challenge for society and for policymakers. It's not simply a health issue, nor a matter of individual choice. The current and likely future scale of the obesity problem is daunting, but the encouraging findings are that there is considerable scope to align policies to tackle climate change and sustainability, for example, with policies for public health.
>
> (Department for Business, Innovation & Skills, 2007, p. 1)

Foresight also commissioned the production of an educational resource, 'Take Shape', for use in secondary schools to help pupils aged 14–16 years appreciate the complex nature and extent of the problem. Many school food technology programmes deliberately educate pupils about the nature of consumer products developed by the food industry with a view to informing individual's food choices. Although this deals with only a minor contribution to the overall obesogenic environment it is significant in that it empowers pupils and their families to make decisions about their personal eating. Understanding the extent to which the ingredients in a product might contribute to an obesogenic environment relies on significant scientific and mathematical understanding. From the science perspective there is the nature of the ingredients and their ability to act as energy dense food. From the mathematics perspective there is the quantification of the scientific perspective. Overall the total number of calories in a portion will be important and products that are high in sugar and fat will be energy dense and contribute large numbers of calories. Fats in the diet, depending on their nature, may also lead to arteriosclerosis. For example, let us consider the energy intake from breakfast cereal. Calculating the calorie intake from a week's consumption of breakfast cereals is not a trivial task. It requires the pupils to take information from the packaging and consider it in the light of a typical portion size and the number of days per week it is eaten. It would be instructive to compare different cereals. This would be an intricate arithmetic exercise requiring the ability to perform a sequence of calculations and comment on the significance of the results. Enabling pupils to use arithmetic sensibly is a requirement of many school mathematics courses and using it to compare food energy content in the context of the looming obesity crisis provides the basis for collaboration between the mathematics teacher and the food technology teacher. Note that this task is made less demanding in terms of arithmetical manipulation if the pupils used a spreadsheet. This would also enable consideration of the questions 'What if I ate this instead of that?'

Ending hunger

Foresight has also produced a report: *Foresight – The Future of Food and Farming* (2011). Chapter 6 of the report deals with the problem of hunger which is significant. The report notes:

> Today, there are an estimated 925 million people hungry, and perhaps an additional one billion who are not hungry in the usual sense but suffer from the 'hidden hunger' of not having enough vitamins and minerals. Hunger is the antithesis of human development. It is important for policy makers to take a broad view of the nature and causes of hunger and its many impacts, including the severe and long-lasting nature of the effects that hunger and undernutrition can cause, particularly in children.
>
> (Department for Business, Innovation & Skills, 2011, p. 116)

Throughout this chapter there is a wide range of graphical information depicting various features relevant to world hunger. All school mathematics courses teach the accurate interpretation of graphs. For many pupils this can become an abstract exercise of little interest unless the information encapsulated by the graphs are of some interest to them. Few pupils will be unmoved or disinterested in the plight of hungry people, and the opportunity to use graphs dealing with this important issue provides the basis for collaboration between the mathematics teacher and the food technology teacher. It would require some effort on the part of both teachers to analyse the contents of Chapter 6 of the report and extract a range of graphical material that told a coherent story about world hunger which pupils would only be able to understand by interpreting the graphs. However, this would provide an interesting opportunity to show how this aspect of mathematics is a useful communication tool and to provide a contrast to the situation in the UK and USA where virtually all people have more than enough to eat. And by including this in food technology it will be possible to begin to engage pupils with the complex global system of food production/consumption and help them consider the moral dimension of a world where there is much hunger and the role that food technologies might play in tackling this problem.

Having presented a range of examples concerning the teaching of mathematics in the light of STEM we will revisit the 'life mathematical' in school.

Revisiting the 'life mathematical' in school

Enabling success in international tests as a key requirement of the life mathematical in school has been questioned. In an article in the *New Scientist* (2013) MacGregor Campbell challenges the conventional wisdom that PISA and TIMMS test scores are important in gauging how well pupils will be in working effectively in a knowledge-based global economy. He argues that several researchers have shown that there is little if any correlation between test scores and measures of economic success such as per capita GDP, Growth Competitiveness Index and entrepreneurship. As a specific example, he notes that Japanese students have always been near the top of the TIMMS and as such you might expect such students to go on to drive a high-flying economy. Yet the Japanese economy stagnated throughout the 1990s and 2000s. He concludes that fixating on international tests as a way to promote the importance of mathematics (and science) is likely to prove counterproductive and that more emphasis should be placed on developing creativity and initiative. Being able to use mathematics fluently in subjects other than mathematics, as proposed and advocated in the above examples, will support the creative use of mathematics and help pupils show initiative in bringing mathematics to bear in learning both science and design & technology. It is of course unlikely that government ministers will ignore PISA and TIMMS rankings, but engaging pupils with the utility of mathematics as indicated is likely to develop a more positive attitude overcoming the 'hate it' disposition and leading to better overall mathematical confidence and attainment.

The life mathematical in school in England is now likely to be extended to include all pupils to the age of 18 as advocated in reports from the Nuffield Foundation and Carol Vorderman and Roger Porkess. But it will be important that such programmes do not provide more of the same diet that many pupils found so unpalatable. In December 2012, ACME responded to these imperatives by recommending that a new mathematical qualification, based on problem solving in realistic contexts, should be developed and introduced as part of wider A level (years 16–18) reforms. According to ACME this qualification would:

- be distinct from A level mathematics, with an emphasis on solving realistic problems, using a variety of mathematical approaches.
- give students confidence in using and applying pre-16 mathematics.
- be designed to be studied over two years.
- be designed so that it is not in competition with either AS or A level mathematics.

Many of those pupils who have enjoyed being taught mathematics in the light of STEM will continue their study of mathematics at AS and A levels but some will not. This is most likely to be the case for some pupils who choose to study design & technology. Yet as we have seen, being able to use mathematics is useful in a variety of design & technology contexts. Hence if this recommendation led to the development of such a qualification it could capitalise on an approach pre-16 in which pupils were taught mathematics in the light of STEM. Indeed, it is extremely likely that some of the problems which pupils tackled in this new course would be design & technology problems. Of course, assessment will be an issue here but the work being carried out by Ian Jones, described earlier, gives hope that it will be possible to develop assessment schemes that deal with and reward the mathematical problem-solving abilities that could be developed through this new qualification.

As we finish writing this chapter the Nuffield Foundation has published a report entitled 'Towards universal participation in post-16 mathematics: lessons from high performing countries'. Amongst the report's recommendations are that appropriate qualifications would focus on mathematical fluency, statistics and application of mathematics. Again, the accent on fluency and application indicates that previous experience of teaching mathematics in the light of STEM would pre-dispose young people to see the value of such a qualification

The NCETM report 'Developing Mathematics in Secondary Schools' noted that schools in which mathematics teaching is successful value highly the links between mathematics and other subjects in the curriculum. Clearly, the authors of this book value highly the links between mathematics and science and design & technology and perhaps it would not be going too far to argue that if a school deliberately forges such links then the mathematics teaching overall is likely to become more successful. Underpinning many of the activities that embody such links is the idea of the conversational classroom in which pupils actively discuss their approaches to using mathematics, the difficulties they are experiencing and how they overcome them. This was a key feature of the 'Increasing Competence and Confidence in Algebra and Multiplicative Structures' (ICCAMS) Project. This can be seen to mirror the conversations between mathematics teachers and teachers of other subjects. The importance of conversations cannot be underestimated in a school's life mathematical. One can see the failure to develop conceptual knowledge and the ability to solve problems, noted by the most recent Ofsted report into mathematics teaching, as being compounded by the classroom of certainty in which tentative attempts to understand and use difficult ideas through discussion find no place.

And what of the life mathematical in schools that embrace the Khan Academy's approach to teaching and learning mathematics? Will it be possible to extend the highly-focused approach to teaching particular aspects of mathematics to include a broader approach in which mathematics is used in the teaching and learning of other subjects? Proponents might argue that this is what teachers will do in the classroom with pupils who have learned particular mathematics online at home. It will be interesting to see if this actually happens. Another intriguing possibility is that the online activities in the Khan Academy are extended to engage with teaching mathematics in the light of STEM. This would be a departure from the current approach but given the enhanced funding now available it would not be impossible. And this development would to some extent meet the criticisms of those who consider that the Khan Academy approach denies pupils the opportunity to learn through conceptual struggle.

Conclusion

So what are we to make of this chapter? There is no doubt that in both the USA and the UK there are serious concerns about the response of many young people to the teaching and learning of mathematics. Perversely it might seem, those schools that acknowledge the conceptual struggle involved in learning mathematics and enable pupils to articulate this struggle have more success than schools which, with the best intentions, over-simplify and fragment the learning, denying pupils the opportunity to construct their own personal robust understanding and in the process gain significant mathematical skills. We advocate an approach that supports the importance of conceptual struggle, and suggest that one way to achieve this is to take the mathematics necessary for learning science and design & technology into the mathematics classroom.

We have developed the examples of teaching parts of the mathematics curriculum in this way to illustrate that it is both possible and worthwhile. If you are able to use these examples we will of course be delighted, but if you find them inappropriate then we believe that this should not deter you from developing ideas that will work for you in your situation. Our conversations with science and design & technology teachers have led us to the view that there is no shortage of possible examples that require the use of mathematics in those subjects. Our foray into presenting such examples is, of necessity, limited but we are convinced that conversations between mathematics teachers and those teaching science and design & technology will be able to identify many more examples, importantly, examples that will be successful in the individual circumstances of their particular schools. We have noted the importance of conversations in this endeavour: the initial and probably short conversations between teachers exploring possibilities; the subsequent more detailed and time-consuming conversations required to outline what might be done to respond to the possibilities and a consideration of the conversations to take place in the classroom between pupils as they learn mathematics through using it for other STEM subjects. It is this final set of conversations that will both enable and reveal the effectiveness of our suggested approach. Hence we suggest that you discuss with colleagues what sort of conversations you want to happen, how you might support such conversations and how you might monitor the conversations that do take place with the view to improving their effectiveness. In this way, we believe that you will be able to make a significant contribution to the life mathematical in your school, one which pupils will value and enjoy. Hence in teaching mathematics in the light of STEM our advice is don't neglect the importance of regular conversations with colleagues from science and design & technology.

Note

1 Details of the oil drop experiment can be found at http://www.nuffieldfoundation.org/practical-physics/estimating-size-molecule-using-oilfilm

Background reading and references

ACME (2011) 'Mathematical Needs: The Mathematical Needs of Learners' report. Available at: www.acme-uk.org/media/7627/acme_theme_b_final.pdf (accessed 10 February 2014).

ACME (2012) 'Past 16 Mathematics: A Strategy for Improving Provision and Participation' report. Available at: www.nationalnumeracy.org.uk/resources/49/index.html (accessed 10 February 2014).

Bolt, B. (1991) *Mathematics Meets Technology*. Cambridge: Cambridge University Press.

Campbell, M. (2013) 'West vs. Asia Education Rankings are Misleading'. *New Scientist,* 217, (2898) 22–23.

Department for Business, Innovation & Skills (2007) 'Foresight – Tackling Obesities: Future Choices' report. Available at: www.bis.gov.uk/foresight/our-work/projects/published-projects/tackling-obesities/reports-and-publications (accessed 10 February 2014).

Department for Business, Innovation & Skills (2011) 'Foresight – The Future of Food and Farming'. Available at: www.bis.gov.uk/foresight/our-work/projects/published-projects/global-food-and-farming-futures/reports-and-publications (accessed 10 February 2014).

Duncan, A. (2009) Secretary Arne Duncan's Remarks at OECD's Release of the Program for International Student Assessment (PISA) 2009 Results. Available at: www.ed.gov/news/speeches/secretary-arne-duncans-remarks-oecds-release-program-international-student-assessment- (accessed 10 February 2014).

Gove, M. (2012) 'Education for Economic Success'. Speech to the Education World Forum. Available at: www.education.gov.uk/inthenews/speeches/a0072274/michael-gove-to-the-education-world-forum (accessed 10 February 2014).

Hodgen, J., Pepper, D., Sturman, L. and Ruddock, G. (2010) 'Is the UK an Outlier? An International Comparison of Upper Secondary Mathematics Education'. Nuffield Foundation Report. London: Nuffield Foundation. Available at: www.nuffieldfoundation.org/uk-outlier-upper-secondary-maths-education (accessed 10 February 2014).

Hodgen, J., Marks., R. and Pepper, D. (2013) 'Towards Universal Participation in Post-16 Mathematics: Lessons from High Performing Countries'. Nuffield Foundation Report. London: Nuffield Foundation. Available at: www.nuffieldfoundation.org/sites/default/files/files/Towards_universal_participation_in_post_16_maths_v_FINAL.pdf (accessed 10 February 2014).

ICCAMS Project. Available at: http://iccams-maths.org/ (accessed 10 February 2014).

John-Steiner, V. and Hersh, R. (2011) *Loving and Hating Mathematics: Challenging Myths of Mathematical Life*. Princeton, New Jersey: Princeton University Press.

Jones. I, Swann, M. and Pollitt, A. (in press) Assessing Mathematical Problem Solving Using Comparative Judgement. *International Journal of Science and Mathematics Education.*

Millar, R., Sang, D., Swinbank, E. and Tear, C. (2011) *21st Century Science: GCSE Physics*. Oxford: Oxford University Press.

NCETM (2009) 'Developing Mathematics in Secondary Schools' report. Available at: www.ncetm.org.uk/Default.aspx?page=41&module=file&fileid=651176 (accessed 10 February 2014).

Ofsted (2012) 'Mathematics: Made to Measure'. Ofsted report ref 110159. Available at: www.ofsted.gov.uk/resources/mathematics-made-measure (accessed 10 February 2014).

Pershan, M. (2012) 'What if Khan Academy was Made in Japan?' YouTube video. Available at: www.youtube.com/watch?feature=player_embedded&v=CHoXRvGTtAQ (accessed 10 February 2014).

PISA. Available at: www.oecd.org/pisa/ and www.oecd.org/pisa/pisaproducts/ 48852548.pdf (accessed 10 February 2014).

Porkess, R., Budd, C., Dunne, R. and Rahman-Hart, P. (2011) 'A World Class Mathematics Education for All Our Young People'. Developed by a task force chaired by Carol Vorderman and lead author Roger Porkess. Available at: www.tsm-resources.com/pdf/VordermanMathsReport.pdf (accessed 10 February 2014).

Russell, B. (1957) *The Study of Mathematics in Mysticism and Logic.* New York: Doubleday.

Strauss, V. (2012) 'Khan Academy: The Hype and the Reality'. Washington Post blog: The Answer Sheet. Available at: www.washingtonpost.com/blogs/answer-sheet/post/khan-academy-the-hype-and-the-reality/2012/07/22/gJQAuw4J3W_blog.html (accessed 10 February 2014).

TIMMS. Available at: http://nces.ed.gov/timss/index.asp and http://timssandpirls.bc.edu/ (accessed 10 February 2014).

Webley, K. (2012) 'Reboot the School', *Time*. Available at: http://content.time.com/time/magazine/article/0,9171,2118298,00.html (accessed 25 November 2013).

Williams, G. (2006) *Biology for You*. Cheltenham: Nelson Thornes.