Chapter 5

Adjustable and Floating Rate Mortgage Loans

In the preceding chapter, we discussed fixed interest rate mortgage (FRM) instruments, giving particular attention to payment patterns. We saw how payment structures and amortization rates can be negotiated.

This chapter deals with a variety of mortgage loans with interest rates that are tied to a market rate, or index, and may change with market conditions. When used to finance single family residences, these loans are called **adjustable rate mortgages—ARMS.** When used to finance commercial properties, these mortgages frequently are called **floating rate loans.** Because both types of mortgages are very similar, we will not differentiate between them. In our discussion, we will use the term ARM more frequently. These instruments differ from fixed interest rate mortgages (FRMs) in that they are designed to adjust in one or more ways to changes in economic conditions. Rather than making mortgages with fixed rates of interest over long periods of time, these mortgages provide an alternative method of financing through which lenders and borrowers *share the risk* of interest rate changes, or **interest rate risk.** This enables lenders to match changes in interest costs with changes in interest revenue more effectively and thus provide borrowers with potentially lower financing costs.

In this chapter, we begin with a general discussion of the price level adjusted mortgage (PLAM). Although not widely used, the PLAM illustrates many of the problems that must be considered by lenders and borrowers in financial decision making. We then consider ARMs and issues relative to how they are "priced." As a part of the analysis of ARMs, we investigate the effects of limitations on (1) interest rate changes, (2) payment increases, and (3) negative amortization and the resultant effects on ARM loan yields. We also consider how these mortgages should be priced relative to FRMs and other ARMs made on different loan terms. At the conclusion of the chapter, we consider the shared appreciation mortgage (SAM), whose repayment terms are partially based on appreciation in property values.

One major issue faced by lenders and borrowers with FRMs is that the *interest rate is fixed on the date of origination and remains fixed until the loan is repaid.* Hence, from the day of origination, lenders are underwriting the risk of any significant changes in the implicit components of mortgage interest rates, that is, the real rate of interest (r), the risk premium (p), and the premium for expected inflation (f). To the extent that lenders underestimate any or all of these components at the time of mortgage origination, they will incur a financial loss. For example, assume that a fully amortizing mortgage loan for $60,000 is made for 30 years at 10 percent interest. This mortgage would require monthly payments of about $527 (rounded). Should such a loan be made, it must follow that the consensus of lenders at the time the loan is made is that a 10 percent rate of interest is sufficiently high to compensate them for all forms of risk bearing expected to occur over the time that the loan is expected to be outstanding. If over that time, one or more of the components of the mortgage interest rate (i) are significantly higher than was anticipated at the time of origination, lenders will suffer a loss.[1] If, for example, lenders make an inaccurate prediction of inflation and unanticipated inflation occurs, warranting a 12 percent interest rate instead of 10 percent, the magnitude of the loss to the lender over a 30-year period is determined as follows:

Solution:	Function:
i = 12%/12	PV (i, n, PMT, FV)
n = 360	
PMT = $527	
FV = 0	
Solve for PV = $51,190	

The loss would be equal to $60,000 − $51,190 = $8,810. Hence, in this case, a 2 percent rate of **unanticipated inflation** would result in a financial loss of $8,810, or 14.7 percent of the loan amount. Based on this example, it should be easy to see the relationship between *interest rate risk* and potential losses to lenders. That there is always some additional risk because of the *uncertainty* about expected levels of each of the components of i is one of the reasons why a risk premium, p, is demanded by lenders. To the extent that this uncertainty about future levels of r and f increases, p will also increase, and vice versa.[2]

It should be noted that *losses incurred by lenders result in gains to borrowers.* Of course, one could argue that if interest rates declined, then lenders would gain. However, when this occurs, borrowers usually try to refinance their loans. This pattern implies that with fixed interest rate lending, risk bearing may not be "symmetric," or evenly balanced; that is, lenders bear the risk of loss when interest rates increase, which may not be equally offset by gains if interest rates decline because borrowers can usually prepay loans and will do so when interest rates decline. This problem has motivated lenders and borrowers to use interest rate caps and floors, interest rate swaps, "lockout," and/or prepayment prohibitions and/or penalties. These features and other options will be discussed in various chapters throughout this book.

[1] There are many reasons why lenders may inaccurately predict the components of i over the expected repayment period. Monetary growth may expand or contract, causing changes in the rate of inflation (f). General economic activity may expand (contract), resulting in a change in the general level of investment and employment, thereby affecting real interest rates and default risk (r and p).

[2] The reader should realize that there will always be some likelihood that expected levels of r and f will not always be accurate because of *unanticipated* changes. During some time periods, when economic conditions are stable, the uncertainty in these estimates is likely to be less, whereas in other periods, uncertainty may be greater. Hence, the *uncertainty* of these estimates is what causes interest rate risk and, in turn, larger or smaller risk premiums.

The Price Level Adjusted Mortgage (PLAM)

One *concept* that has been discussed as a remedy for the uncertainty problem for lenders is the **price level adjusted mortgage (PLAM)**. Recalling from the discussion in the previous chapter on the determinants of mortgage interest rates, i (where r = expected real rate of interest; p = risk premium; and f = expected inflation), we displayed the following equation:

$$i = r + p + f$$

We also indicated that perhaps the most difficult variable in the equation to predict was a premium for expected inflation (f). To help reduce interest rate risk, or the uncertainty of inflation and its effect on interest rates, it has been suggested that lenders should *originate* mortgages at interest rates that reflect expectations of the real interest rate plus a risk premium for the likelihood of loss due to default on a given mortgage loan, or $r + p$.

After estimating initial values for r and p, the PLAM loan balance would be adjusted up or down by a *price index*. Payments would then be based on a new loan balance, adjusted for inflation. This would shift the risk of changes in market interest rates brought about by inflation (f) to borrowers and relieve lenders of the difficult task of forecasting future interest movements when originating loans. The lender would still bear the risk of any unanticipated change in r or p.[3]

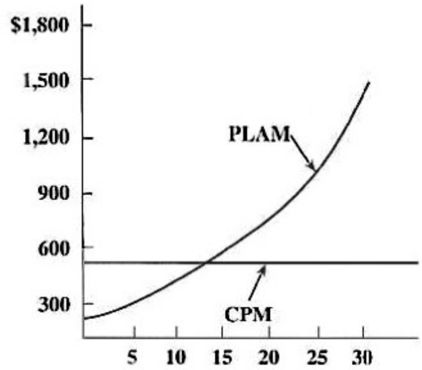
PLAM: Payment Mechanics

An example of a PLAM loan would have payments based on a rate of interest consisting only of expectations for r and p for an expected maturity period. Payments would be adjusted periodically, based on the indexed value of the *mortgage balance* for the remaining loan term. To illustrate, assume that a mortgage is made for $60,000 for 30 years at an interest rate of 4 percent, or a lender's estimate of $r + p$ The lender and borrower may agree that the loan balance will be indexed to the consumer price index (CPI) and adjusted annually. Initial monthly payments would be based on $60,000 at 4 percent for 30 years, or approximately $286. After one year, the loan balance, based on a 30-year amortization schedule for the 4 percent interest rate, would be about $58,943. If it is assumed that the CPI increased by 6 percent during the first year of the loan, the *loan balance* at the end of year 1 would become $58,943 (1.06) or $62,480. This balance would be repaid over 29 remaining years. Monthly payments, beginning in the second year, would be based on the higher-indexed loan balance of $62,480 at the same 4 percent interest rate for 29 years or $304 per month. This process would continue each year thereafter: (1) computing the loan balance using an amortization schedule based on a 4 percent interest rate for the remaining term, (2) increasing the balance by the change in the CPI during the next year, and (3) computing the new payment over the remaining loan term.

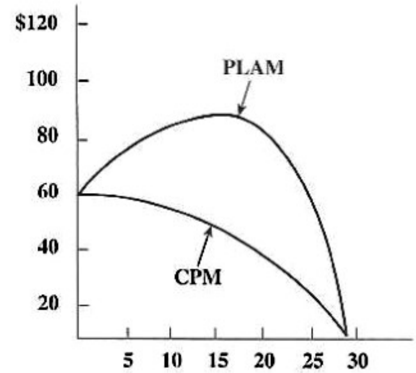
Assuming inflation continued at an annual rate of 6 percent for the remaining loan term, Exhibit 5–1 shows the nominal payment and loan balance pattern every year for the PLAM loan. There are many patterns that should be pointed out in Exhibit 5–1. Note that

[3] Although we are treating each of these variables making up i as independent and additive, they may not be independent and may well interact with one another. For example, the risk premium (p) is partially dependent on the likelihood that a borrower's income and wealth will rise or fall, which may depend on changes in the economy and, hence, the underlying real rate of interest (r). Changes in income would affect the likelihood of default on a loan because of payments rising relative to income (which may rise or fall) or the loan balance exceeding the market value of the house. Similarly, we do not fully understand the relationship between inflation (f) and real growth (r) and the possible interaction between them. Hence, the reader should be aware that we are dealing with these influences in a conceptual way to illustrate the importance of each component, but we do not mean to imply that the specification of i is this simplistic.

EXHIBIT 5–1
Payments and Loan Balance Patterns, $60,000 PLAM, 4%, Inflation = 6% per Year, versus $60,000 CPM, 10% Interest, 30 Years



Panel A: Monthly payments



Panel B: Outstanding loan balances ($000s)

the PLAM payments shown in panel A increase at approximately the same rate as the change in the price level, or 6 percent over the life of the loan. This increase in payments continues over the life of the loan even though loan amortization begins to occur as the number of remaining years to maturity declines (see panel B). This pattern of rising payments occurs (1) because of the effect of the increasing price index on the loan balance and (2) because each succeeding year's payment is computed over a shorter remaining loan term.[4] It is also interesting to compare the payments on this PLAM to a $60,000, constant payment FRM made at 10 percent for 30 years. Payments on the FRM would be approximately $527, as compared to the initial PLAM payment of $286. Thus, it would appear that many more households could qualify to purchase housing with PLAMs when compared to CPMs.

The PLAM is not without problems, however. Panel B in Exhibit 5–1 shows that the loan balance on the PLAM increases to about 155 percent of the original loan amount, or from $60,000 to approximately $93,000, after 15 years. Housing is only one of many components making up the CPI. Hence, should prices of other goods represented in the CPI increase faster than house prices, or if house prices declined, indexing loan balances to the CPI could result in loan balances increasing faster than property values. When this occurs, borrowers have an incentive to default. This possibility would place a considerable burden on lenders because now, instead of dealing with inflation and fixed interest rate loans, they would have to establish adequate down payment levels for all borrowers, forecast future housing prices, and be assured that the value of the property that serves as collateral for the mortgage would always be greater than the outstanding loan balance. Hence, it is questionable whether the CPI is the proper index to use when adjusting PLAM balances.

A second problem with PLAMs has to do with the relationship between mortgage payments and borrower incomes. It would appear that the tilt problem, discussed in the appendix to Chapter 4, would be greatly reduced, because payments would be matched more closely with borrower incomes. However, this assumes that both the CPI, which is used to index the PLAM, and borrower incomes change in the same way. A desired ratio

[4] The reader should realize that the process of adjustments occurring at the end of each year can be viewed as an annual series of new mortgage loan originations. As such, payments may be modified based on different rates of interest or maturities, with the outstanding loan balance always representing the new amount being borrowed. Hence, it is possible for changes in interest rates or maturities to be renegotiated or varied by the lender and borrower at any time to moderate or increase monthly payments.

between mortgage payments and borrower incomes may be easy to maintain as long as incomes keep pace with increases in the CPI. Over the long run, this relationship may be possible as increases in income and mortgage payments "balance out." However, if inflation increases sharply, it is not likely that borrower incomes would increase at the same rate in the short run. During such periods, the payment burden may increase and households may find it more difficult to make mortgage payments. Because of this possibility and the need to develop a desired relationship to mortgage payments, lenders would have to estimate future income for households in different occupational categories and the relationship of that income to inflation. The problems of rising loan balances and payments just discussed make estimating the risk premium (p) that lenders must charge extremely difficult.

A third problem with PLAMs is that the price level chosen for indexation is usually measured on a historical, or ex post facto, basis. In other words, the index is based on data collected in the *previous period* but published currently. In as much as mortgage payments are to be made in the future, historic prices may not be an accurate indication of future prices. To illustrate, the change in the CPI may have been 10 percent during the past year (published currently). This figure would be used to index the outstanding mortgage balance, which will determine payments during the *next* year. If the rate of increase in the CPI subsequently slowed to 2 percent during the next year, it is easy to see that mortgage payments would be rising at a faster rate (10%) than current prices (2%) and, perhaps, faster than borrower incomes. Although borrower incomes may have increased by 10 percent in the previous year, the lag between realization of income in one period and higher payments in the next still presents a problem. This lag problem could become even more distorted in our example if the CPI were to decline and then increase. For this reason, many observers believe that if the PLAM programs were adopted extensively, the time intervals between payment adjustment periods would have to be shortened considerably. This time is called an *adjustment interval*.

In spite of the practical problems with implementing PLAMs, many PLAM features form the framework for understanding adjustable rate mortgages (ARMs). We now turn our attention to ARMs, which will be our focus for the remainder of this chapter.

ARMs and Floating Rate Loans: An Overview

Rather than using changes in the price level as a mechanism to adjust mortgage interest rates and payments, lenders are choosing a variety of mortgages with *interest rates* that are *indexed to other market interest rates*. By choosing indexes based on interest rates rather than on a price index, lenders partially avoid having to estimate real interest rates and risk premiums for the entire period that loans are expected to be outstanding. With ARMs, lenders are, in effect, making a loan, with terms that are updated to current interest rate levels at the end of each adjustment period. By using an interest rate index instead of an ex post measure of inflation based on the CPI or any other price index, lenders earn expected yields based on *expected future values* for r, p, and f over a future period of time. Because interest rates are a reflection of lender and borrower expectations of r, p, and f over specific future periods of time, revisions in ARM payments are always based on future expectations. Therefore, by tying the terms of a mortgage to an index of interest rates, payments are updated frequently. Hence, an ARM provides for adjustments that are more timely for lenders than a PLAM because values for r, p, and f are revised at *specific* time intervals to reflect market expectations of future values for *each* component of i *between adjustment dates*. For example, the value for f, or expected inflation, is based on an estimate of *future* prices rather than a past measure as exemplified in the CPI or other price indexes. Similarly, values for r and p are based on the market's current assessment of risks in the prospective economic environment between adjustment dates.

Payment Basics Illustrated

We can begin to illustrate ARM mechanics with a simple example. An ARM for $60,000 with an *initial* interest rate of 10 percent is originated with a term of 30 years, but its payments are to be reset at the end of one year based on an interest rate determined by a specified index at that time. Based on these initial loan terms, monthly payments would be approximately $527 per month for the first year, and the balance at the end of the year would be $59,666. If the market index were to rise at the end of one year and change the interest rate on the ARM to 12 percent, payments would be determined based on the outstanding loan balance for 29 years as follows:

STEP A:	Function:
Solution: Find loan balance at end of year 1:	*FV* (*PV*, *n*, *i*, *PMT*)
$n = 12$	
$PMT = \$527$	
$PV = \$60{,}000$	
$i = 10\%/12$	
Solve for $FV = \$59{,}666$	
STEP B:	
Solution: Find new payments at 12%:	Function:
$n = 29 \times 12 = 348$	*PMT* (*n*, *i*, *FV*, *PV*)
$i = 12\%/12 = 1\%$	
$PV = \$59{,}666$	
$FV = 0$	
Solve for *PMT* = $616	

Hence, the new 12 percent interest rate on the ARM at the end of the first year is an updated estimate of the components of *i* for the *coming year,* and payments will increase from $527 to $616 per month.

At least three general observations should be made concerning our simple example.

1. The use of ARMs does not completely eliminate the possibility of lenders realizing losses because of *interest rate risk.* In our first example, the yield to the lender on the ARM during the first year was 10 percent. If market rates move to 12 percent *the day after* the ARM is originated, the lender would sustain a 2 percent loss for the remaining 12-month period during year 1. Obviously, this loss would be eliminated if the adjustment period was reduced to one day, or the loss could be reduced to the extent the adjustment period was less than one year.
2. The longer the payment **adjustment interval,** the greater the interest rate risk to the lender is. Hence, the expected yield to the lender on such a mortgage should be greater. This idea will be elaborated later in the chapter.
3. Finally, as the lender assumes *less* interest rate risk, the borrower incurs *more* interest rate risk, depending on the nature of the index chosen and the frequency of payment adjustments. This point can be appreciated if one compares an FRM, where the lender assumes that the full risk of future interest rate changes to an ARM with payments

adjusting freely with market conditions. Clearly, in the latter case the borrower would be assuming more interest rate risk and the lender less. Because the borrower assumes more risk, the *initial interest rate,* or *start rate,* on an ARM should generally be *less* than that on an FRM. Further, because the lender is shifting interest rate risk to the borrower, the lender should also expect, at the time of loan amortization, to earn a lower yield on an ARM over the term of the loan. These three factors are obviously considered by both lenders and borrowers as they negotiate such loans.

Other ARM Characteristics

The following list contains a description of some of the more important terms used when dealing with adjustable rate mortgages:

- *Index.* The **index** is the interest rate series (such as one from the list below) agreed on by both the borrower and the lender and over which the lender has no control. This index may be very short or long term in nature and will be used to reset the interest rate on an ARM on the reset date. Some commonly used indexes are:

  —Interest rates on one-year Treasury securities or Treasury indexes with a specific maturity (6 months, 10 years, etc.).

  —The average cost-of-funds index (COFI) for the 11th FHLB District.

  —The London Interbank Offered Rate (LIBOR) for various loan periods.

- *Margin.* A premium in addition to the index chosen for an ARM is known as the **margin,** or **spread.**
- *Composite rate.* The sum of the interest rate based on the index chosen plus the margin used to establish the new rate of interest on each reset date is called the **composite rate.** It can differ from the initial interest rate on the origination date.
- *Reset date.* The point in time when mortgage payments will be adjusted is called the **reset date.** This time period is usually six months or one year. However, it could be as long as every three to five years, or it could be as short as one month or less.
- *Negative amortization.* To the extent accrued interest in a given period exceeds the periodic payment, the difference may be compounded at current rates and added to the outstanding loan balance. When additions to the outstanding loan balance are allowed in the loan agreement, such amounts are referred to as **negative amortization** (see the discussion in the previous chapter).
- *Caps.* Maximum *increases* allowed in payments, interest rates, maturity extensions, and negative amortization (or loan balances) on reset dates are called **caps.**
- *Floors.* Maximum *reductions* in payments or interest rates on reset dates are called **floors.**
- *Assumability.* The ability of the borrower to allow a subsequent purchaser of a property to assume a loan under the existing terms is called **assumability.**
- *Discount points.* As with FRMs, these points or fees are also used with ARMs to increase the lender's yield.
- *Prepayment privilege.* Most *residential* borrowers usually have the option to prepay without penalty. However, because prepayment is a privilege and not a right, lenders may charge penalties if a loan is prepaid within a certain period of time.
- *Lockouts.* Many commercial mortgage loans prohibit repayment for a specified number of years. In these cases, should a borrower wish to prepay, a prepayment fee may have to be negotiated with the lender.

- *Conversion option.* The right of an ARM borrower to convert to an FRM. Depending on the agreement, this conversion option may be exercised by the borrower at will or only after a specific period of time. Lenders also may charge a fee for this option.

Clearly, many other combinations of the above provisions could be used to allocate interest rate risk between the lender and borrower.[5] Space does not allow for an in-depth analysis of all of these combinations. What we will provide, however, is a framework which should provide the necessary tools that can be used to analyze any given set of ARM or floating rate provisions.

Variations: ARM and Floating Rate Loans

3/1, 5/1, and 7/1 "Hybrid Loans"

Another frequently used category of ARMs is sometimes referred to as *hybrid* ARMs. This category combines elements of FRMs for periods of three, five, or seven years, after which interest rates are *reset* and the loan becomes an ARM. Subsequent payments are usually reset every year for the remaining maturity period. For example, a 3/1 hybrid would mean a three-year fixed rate after which the interest rate would become adjustable, tied to an index, *and would be reset each year thereafter.* ARM payments on such a loan, with a 30-year maturity and an initial rate of 6 percent, would begin with fully amortizing monthly payments like an FRM made for a period of 30 years. For the first three years, payments would be

Solution:	Function:
Years 1–3:	*PMT* (*PV*, *i*, *n*, *FV*)

$PV = -\$100{,}000$

$i = 6\%/12$

$n = 360$

$FV = 0$

Solve for $PMT = \$599.55$

[5] In *residential lending*, there are three additional terms that are commonly negotiated by borrowers and lenders. These include:

- *Expected start rate.* The expected start rate is based on the index chosen plus the margin on the loan closing date. Because the lender is assuming less interest rate risk than the borrower, it will almost always be lower than rates on FRMs.
- *Actual start rate, or initial interest rate.* The actual start rate is determined by competitive market conditions among lenders at the time ARM loan commitments are made. It may be the same as or lower than the expected start rate. When the initial, or actual, start rate is *lower* than the expected start rate, it is sometimes referred to as a "teaser rate."
- *Teaser rate.* When the actual start rate is very low when compared to the expected start rate, it is usually an indication that lenders are actively competing and are willing to offer a lower initial rate of interest (teaser rate) in order to attract borrowers. One additional issue that borrowers must determine is whether they are getting a true discount or whether the lower rate of interest will be deferred interest which will be added to the loan balance.

On the reset date at the end of three years, the reset rate would be based on the prevailing index and margin agreed on by the borrower and the lender. For example, at the end of three years, if the ARM rate has risen to 6.5 percent, payments for the first year thereafter (year 4) would be based on the balance at the end of year 3, or $96,084.

Solution: Function:
Beginning of year 4 through year 30 $PMT\ (PV, i, n, FV)$

$PV = -\$96{,}084$

$i = 6.5\%/12$

$n = 324$

$FV = 0$

Solve for $PMT = \$629.88$

In this example, for the first three years, payments would be the same as a $100,000 FRM made at 6 percent for 30 years, or $599.55. At the end of three years, the loan balance would be $96,084, the interest rate would be reset at 6.5 percent (assumption), and payments during year 4 would become $629.88. Payments would be recalculated on the reset date every year thereafter.

As discussed above, possible variations on a hybrid loan could consist of the payments in years 1 to 3 being interest only, with payments beginning in year 4 being based on the ARM index at the end of year 3. Because payments would be interest only, the loan balance beginning in year 4 would remain at $100,000. Payments beginning in year 4 would be recalculated based on the prevailing interest rate plus amortization of loan principal over 27 years. Payments would then be recalculated on each reset date at the beginning of each of the remaining 27 years.

Interest-Only ARM and Floating Rate Loans

Many floating rate loans are used to finance commercial properties. The **interest-only ARM** or floating rate loan requires that the borrower pay interest monthly. Interest payments vary in accordance with the underlying index and margin. For example, a $100,000 loan with an initial rate of 6 percent for 30 years and a one-year reset period would require monthly interest-only payments as follows:

$$\begin{aligned}\text{Monthly payment} &= \text{Loan amount} \cdot (\text{Interest rate} \div 12)\\ &= \$100{,}000 \cdot (.06 \div 12)\\ &= \$500\end{aligned}$$

If, at the beginning of the next year (the reset date), the index has increased and the new interest rate becomes 8 percent, the new monthly payment becomes:

$$\begin{aligned}&= \$100{,}000 \cdot (.08 \div 12)\\ &= \$666.67\end{aligned}$$

Note that in both cases, payments are interest only and do not include any amortization of principal. Now, consider the event that at the end of year 1, the borrower begins to make

Importance of the "Teaser Rate" on ARMs—Residential Loans

Concept Box 5.1

As has been pointed out, interest rates and monthly payments on ARMs are reset after expiration of a specific time interval (commonly one year). As has also been pointed out, the interest rate on the reset date will be based on the index chosen by the borrower plus a fixed margin. However, it is important to understand what is referred to as the *teaser rate*.

The *initial rate of interest* on an ARM is effective on the day that the ARM *closes* and is used to calculate monthly payments during the *first year*. To illustrate: On the date of closing the ARM loan, if one year treasuries are 4 percent and the margin is 2 percent, it would be logical to assume that the expected initial, or start, rate used to compute mortgage payments would be 6 percent. However, the initial rate *does not necessarily* have to be 6 percent. This is because at the point of origination, lenders are free to compete with other lenders by offering borrowers different initial interest rate points, other financing fees, and the like. For this reason, when the initial interest rate quoted on an ARM is *below* the prevailing market index rate plus the margin, it is sometimes referred to as a "teaser rate." Lenders use teaser rates to attract borrowers and compete for business with other lenders. However, the teaser rate will usually prevail only from the date of loan origination until the *first reset date*. On that date, the teaser rate ceases to exist and the interest rate and payments will be calculated based on the prevailing index plus the margin. Interest rates and payments will be reset each year thereafter based on the index plus margin for the remaining life of the loan.

In our example, if we assume that when the ARM is closed (with interest at the prevailing market rate of 6%), a teaser rate of 1.5 percent is offered by the lender, monthly payments for the *first year* would be

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Solution: Payments Based on Teaser Rate:	Function:
$PV = -\$100{,}000$	$PMT\ (PV, i, n, FV)$
$i = 1.5\%/12$	
$n = 30 \times 12$	
$FV = 0$	
Solve for $PMT = \$345.12$	

This payment is far lower than would be the case if the loan was originated at 6 percent interest. For example, "interest-only" payments would be $500 at 6 percent interest. Or, if the payment was based on a fully amortizing 30-year-loan schedule, it would be $607.32.

THE TEASER RATE AND THE ACCRUAL RATE

Recall in the above example that the teaser rate was set at 1.5 percent when the prevailing rate on one-year treasuries (4%) plus the margin would indicate that prevailing interest rates, or the expected start rate, should be in the range of 6 percent. It is also possible that the loan agreement may specify that during the first year, interest will *accrue* at 6 percent even though payments may be based on the teaser rate of 1.5 percent. If this is the case, the difference between the teaser rate and the accrual rate will be included in the loan balance with interest. Therefore, on the reset date, payments will be based on the outstanding loan balance which will have this difference, plus interest, added to the loan balance (negative amortization).

STEP 1:

Solution: Loan Balance EOY 1:

Function: $FV(PMT, i, n, PV)$

$PMT = \$345.12$

$i = 6\%$

$n = 12$ mos.

$PV = -\$100{,}000$

Solve for $FV = \$101{,}910.53$ (balance)

STEP 2:

Solution: New Payment on Reset Date at 6.5% Interest:

Function: $PMT(PV, i, n, FV)$

$PV = -\$101{,}910.53$

$i = 6.5\%$

$n = 29$ years

$FV = 0$

Solve for $PMT = \$651.43$

Note that if the loan agreement specifies that the difference between what the initial rate on the ARM would have been if payments were based on the market rate of 6 percent (the accrual rate) and the teaser rate of 1.5 percent will be accrued in the loan balance, then the loan balance on the reset date ($101,910.53) will be greater than the initial amount of the loan ($100,000.00) because of negative amortization. (This addition to the loan balance may be calculated as the difference in monthly interest accrued at 6 percent, or $500, less the amount paid, $345.12, or $154.88 compounded monthly at 6%.) This results in $1,910.53 being added to the loan balance. This provision may become even more important if the reset date is scheduled for two, three, or more years after the origination date. In this event, the loan balance may increase by an even greater amount because of more accrued interest (negative amortization).

PAYMENT SHOCK

In the above example we determined that when the ARM is originated at a 1.5 percent teaser rate, monthly payments are $345.12. As shown in our example, at the beginning of year 2, payments could increase to $651.43. This amounts to over an 80 percent increase in monthly payments. An increase of this magnitude is sometimes referred to as **payment shock.** This can occur if large increases in monthly payments occur on the reset date. This can happen when the index to which the ARM interest rate is tied has increased considerably. Payment shock can be even more serious for ARMs originated at teaser rates. Depending on the borrower's income, other assets, and the value of the property on the reset date, this shock could result in financial difficulty and force the borrower to default. *For this reason, use of a very low teaser rate to qualify a borrower based on their current ability to make monthly payments may create future problems when it is time for interest rates to be reset.*

payments which will fully amortize the loan balance over the remaining 29 years. Now, instead of "interest-only" payments of $666.67, the new payments become

Solution:	Function:
$PV = -\$100{,}000$	$PMT\ (PV, n, i, FV)$
$n = 29 \times 12 = 348$	
$i = 8\%$	
$FV = 0$	
Solve for $PMT = \$739.95$	

The payment at the beginning of year 2 would be $739.95, of which $73.27 would be applied to reduce principal. When monthly compound interest is applied to $73.27 at 8 percent, the loan amortization for the year is $912.20. (This can be seen by calculating $PMT = \$73.27$, $n = 12i = 8\%$, and $FV = \$912.20$.) When this amount is subtracted from $100,000, it produces a loan balance at the end of year 2 of $99,087.79. It should be pointed out that with a floating rate loan (ARM), lenders and borrowers may choose to begin a full or partial amortization schedule at specified times. For example, at the end of year 3, "interest-only" monthly payments must stop and all further payments must begin to include principal so as to fully or partially amortize the loan balance at maturity.

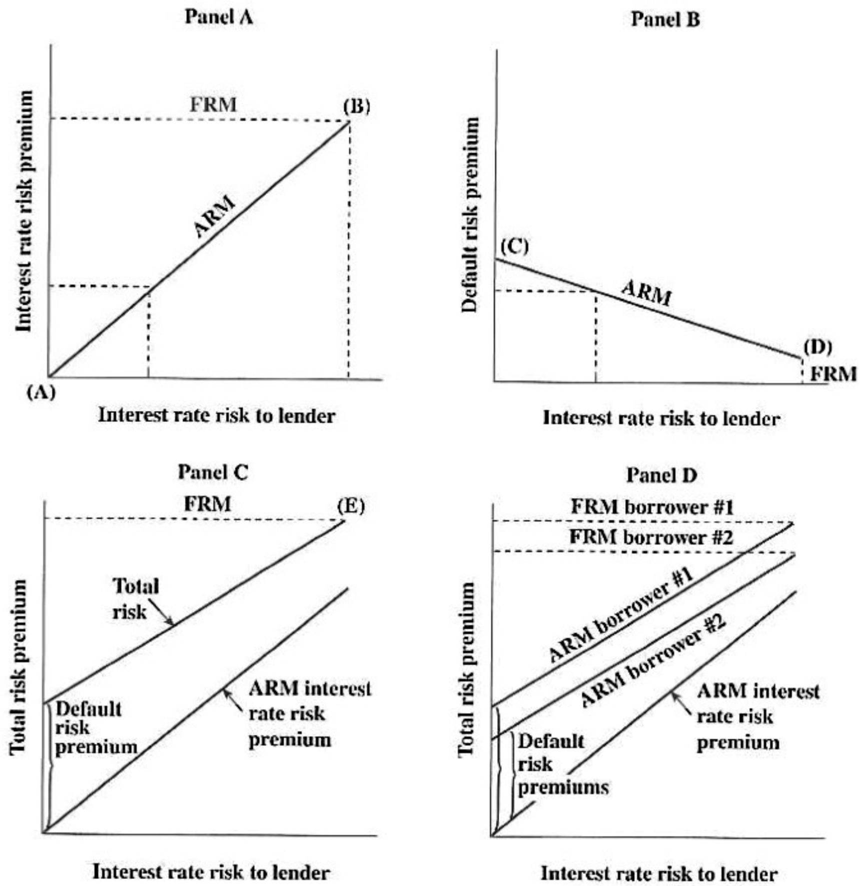
Risk Premiums, Interest Rate Risk, and Default Risk

It is very difficult to determine how expected yields will vary among ARMs containing different repayment characteristics. However, for any given class of borrowers, the expected yield (cost) of borrowing with an ARM generally depends on the ARM provisions described in detail earlier: (1) the initial interest rate, (2) the index to which the interest rate is tied, (3) the margin, or spread, over the index chosen for a given ARM, (4) discount points charged at origination, (5) the frequency of payment adjustments, and (6) the inclusion of caps or floors on the interest rate, payments, or loan balances. The loan amount and each of the six characteristics listed will determine the cash outflow or amount loaned, expected monthly payments, and the expected loan balance for an expected time period from which an expected yield (internal rate of return) can be computed. In addition to understanding how each of the above relationships is likely to affect the expected yield (or cost of borrowing), further complications include understanding how combinations of these terms may *interact* over time and possibly amplify or reduce *default risk* to the lender.

While much has been said about benefits to lenders from shifting interest rate risk to borrowers, there are added risks that lenders must assume with ARMs. The combination of the six characteristics also affects default risk either (1) by interfering with the ability of the borrower to make mortgage payments or (2) by increasing the loan balance so that it is too high in relation to the value of the house, assuming that negative amortization is allowed. While we discuss lender underwriting standards used to gauge default risk in more detail in a later chapter, we also want to stress the importance of default risk in our present discussion.

A useful way to approach the relationships between interest rate risk and default risk for an individual and lender is to examine panel A of Exhibit 5–2. The exhibit shows the risk premium (p) demanded by the lender on the vertical axis and interest rate risk assumed by the lender on the horizontal axis. Looking at line A–B in the exhibit, we see that as more interest rate risk is assumed by the lender (less by the borrower), the lender will demand a higher risk premium. Hence, the interest rate risk curve is positively sloped. In the extreme, if the lender assumes all interest rate risk (point B), this would be equivalent to the amount

EXHIBIT 5–2
The Relationship between Interest Rate Risk, Default Risk, and Risk Premiums



of interest rate risk assumed with an FRM. Note that when the lender assumes no interest rate risk, the borrower is assuming all interest rate risk. This is represented by the intersection of the interest rate risk line at the origin of the diagram (point A).

When interest rate risk assumed by the borrower increases (as would be the case with an ARM with *no cap* on payments or the interest rate), *default risk* assumed by the lender increases (see panel B). Default risk is greatest at point C, where there are no restrictions on ARM interest rates or payments, because the borrower faces a greater likelihood that unanticipated changes in interest rates may cause significant increases in payments ("payment shock") relative to income. Hence, the likelihood of default is greater when the borrower assumes all interest rate risk. However, as more interest rate risk is assumed by the lender, we should note in panel B that risk of borrower default declines because payment shock to the borrower is restricted when caps on payments or interest rates are used. In essence, by assuming more interest rate risk, the *lender* absorbs more shock, thereby reducing *borrower* default risk. This pattern is exhibited in panel B where the default risk curve is shown as negatively related to the risk premium demanded by the lender as interest rate risk to the lender increases. However, the level of default risk never declines below the risk assumed by the lender on a fixed rate mortgage (point D), which is also coincident with the lender's assumption of all interest rate risk (point B in panel A).

The total risk curve (see panel C of Exhibit 5–2) establishes the risk premium demanded by the lender for both risks (interest rate risk and default risk) assumed under various ARM terms. The total amount of risk assumed by the lender corresponds to various combinations of ARM terms, ranging from the assumption of all interest rate risk by the borrower (panel A, point A) to the assumption of all interest rate risk by the lender (panel A, point B).

coupled with the amount of default risk incurred by the lender given different levels of interest rate risk (panel B). Hence, panel C shows the total risk premium the lender should earn, given levels of interest rate risk and default risk that correspond to various levels of interest rate risk. However, the total risk premium should not exceed the total premium that would be earned on an FRM (panel C, point E).

Panel D in Exhibit 5–2 shows the relationship between total risk and the risk premium demanded by the lender for *different borrowers*. Note that the amount of *interest rate risk* remains the same for each borrower; however, *default risk* differs. Hence, the premium charged by the lender on ARMs will vary, depending on the amount of default risk being assumed for each borrower, (1) or (2), and how that default risk interacts with expected changes in interest rate risk. We should point out that many other non-interest-rate factors can cause default such as loss of employment, divorce, and so on. We focus here only on how default risk changes with fluctuations in interest rates.

Exhibit 5–2 is appropriate only for specific borrowers, however. To date, the exact relationship between default risk and interest rate risk for many different classes of borrowers has not been studied extensively. Hence, you cannot generalize the example shown to all borrowers and lenders in the mortgage market. However, it is safe to say that ARM loans will only be made to an individual borrower as long as the expected benefits to the lender from shifting interest rate risk exceed the potential default losses. Similarly, as long as a given borrower is willing to undertake interest rate risk in exchange for paying a lower risk premium to a lender, the ARM will be acceptable to the borrower.

While Exhibit 5–2 graphically portrays the risk/return trade-off faced by lenders and borrowers, ARM terms may be structured in many ways to provide a trade-off between interest rate and default risk that is satisfactory to both. These terms could include many possible combinations of initial interest rates, margin, points, the index chosen, frequency of payment adjustments, caps on payments, and so on. We will explore various combinations of ARM terms later in this chapter.

From the above discussion, then, mortgage lending (borrowing) can be viewed as a process of *pricing risk*, with the expected yield being the return received (paid) by lenders (borrowers) for making loans with terms under which lenders and borrowers bear various amounts of risk. The terms utilized in construction of an ARM (e.g., initial rate, index, adjustment period, caps) are simply the "tools" at the disposal of borrowers and lenders to negotiate and allocate the amounts of interest rate and default risk being shared.[6]

Expected Yield Relationships and Interest Rate Risk

While the contracting process used by lenders and borrowers to allocate risk is a complicated one, there are some general relationships regarding interest rate risk and yields that can be employed in this process. The following general relationships regarding interest rate risk may be useful when comparing ARMs with FRMs and comparing ARMs containing different loan provisions with one another. The relationships focus on the effects of interest rate risk on ARM yields, given the conclusion that an ARM will never be made unless the expected benefit to a lender from shifting interest rate risk to a borrower exceeds expected losses from default risk. Proceeding with this assumption when evaluating ARM terms, interest rate risk, and expected yields to lenders, we should consider the following relationships:

1. At the time of origination, the *expected* yield on an ARM should be less than the *expected* yield on an FRM, to the extent that benefits to lenders from shifting interest

[6] If risk could be quantified, or reduced to some unit quantity such as dollars, an agreement could be devised that would specify exactly how much risk was being shared. However, because risk is an abstract concept, this is not possible. This is the reason why borrowers and lenders include various provisions in contracts to share risk under any set of unknown future economic conditions.

rate risk exceed increases in default risk to borrowers. Otherwise, the borrower and lender will always prefer an FRM. Coincident with the lower expected ARM yield, the *initial interest rate* on an ARM will *usually* be less than that of an FRM.[7]

2. Adjustable rate mortgages tied to short-term indexes are generally riskier to borrowers than ARMs tied to long-term indexes because the former are generally more variable than long-term interest rates. Therefore, the more risk-averse ARM borrowers will generally prefer ARMs tied to a longer-term index and they should be willing to pay more (a higher risk premium and expected yield to the lender). Less risk-averse borrowers will prefer a shorter-term index and will expect to pay less for taking additional interest rate risk. Borrowers who prefer no interest rate risk will choose a fixed rate mortgage and will pay the highest total risk premium to the lender.
3. Coincidentally with (2), ARMs with shorter time intervals between adjustments in payments are generally riskier to borrowers than those with longer time periods because, although an ARM may be tied to a short-term index, the adjustment period may not coincide with the index. For example, an ARM may be adjusted every *three* years based on the value of the *one-year* index at the time of adjustment. Hence, the more frequent the adjustment interval, the lower the interest rate risk to lenders because ARM payments will reflect current market conditions irrespective of the index chosen. Borrowers preferring no adjustment in payments will choose FRMs.
4. To the extent ARMs contain maximum caps on interest rate adjustments, the interest rate risk incurred by borrowers will be lower. Hence, the expected yield realized by lenders should be higher than if no restrictions were present. The expected yield will vary with the size of the limitations. When floors are used, the risk to the borrower is greater because of the limit placed on the decline in the interest rate used to compute ARM payments in any given year. Borrowers who prefer certainty in payments and interest rates will choose an FRM, which will always provide the highest *expected* yield to the lender.
5. If an ARM has negative amortization due to a payment cap, then the effect of changes in interest rates will not materially reduce interest rate risk to borrowers or the expected yield to lenders because any interest forgone because of limitations or caps will be deferred and become a part of the loan balance. Any amounts of negative amortization will also accrue compound interest and must be eventually paid by borrowers.

More Complex Features

In the preceding section, we described some general relationships regarding ARM loan terms, risk bearing, and what lenders (borrowers) should expect to yield (pay) over the life of the ARM contract, or repayment period. We must point out, however, that lenders and borrowers also negotiate certain *initial* loan provisions that (1) will be known at the point of origination and (2) will affect expected yields. Once the index frequency of payment adjustments, rates of payments, and negative amortization have been negotiated, the magnitude of the effect on lenders and borrowers will be determined solely by future market conditions. However, the initial terms on ARMs, or the loan amount, maturity, initial interest rate, margin, and discount points, are quantifiable and can be negotiated with complete certainty at the time the loan is made. These initial loan terms will reflect the net effect of (1) the amount of interest rate risk assumed by the lender as determined by the

[7] Although the initial interest rate on an ARM should generally be less than that of an FRM, in cases where short-term interest rates are greater than long-term rates and an ARM is tied to a short-term rate, it is possible that the initial rate on an ARM may be greater than an initial interest rate on an FRM. However, the *expected yield* on an ARM should be lower because yields are computed to maturity, which includes expected future interest rate patterns.

EXHIBIT 5–3
Comparison of Hypothetical Loan Terms

Contents	ARM I	ARM II	ARM III	FRM
(*a*) Initial interest rate, or start rate	8%	9%	11%	14%
(*b*) Loan maturity	30	30	30	30
(c) Maturity of instruments making up index	1 year	1 year	1 year	—
(*d*) Percent margin above index	2%	2%	2%	—
(e) Adjustment interval, or reset date	1 year	1 year	1 year	—
(f) Points	2%	2%	2%	2%
(*g*) Payment cap	None	7.5%	—	—
(*h*) Interest rate cap	None	None	2%, 5%*	—
(*i*) Negative amortization	—	Yes	—	—

*2 percent maximum annual increase, 5 percent total increase over the loan term.

index chosen, adjustment period, any caps or negative amortization and (2) the amount of default risk assumed by the lender as determined by the amount of interest rate risk shifted to a specific borrower. Exhibit 5–3 contains a summary of hypothetical loan terms being quoted on three ARMs and one FRM.

A careful review of these loans reveals considerable differences in terms. We note that the initial interest rate for ARM I is 8 percent, for ARM II it is 9 percent, and for ARM III it is 11 percent, while the fixed interest rate mortgage (FRM) is quoted at 14 percent. Why is this?

A quick review of the terms for ARM I shows that it has the same terms (*b*) to (*f*) as ARMs II and III; however, characteristics (*g*) to (*i*) reveal that future payments and interest rates are *unrestricted* since there are no caps on payments or interest rates. These terms may now be compared with ARM II, which has a cap of 7.5 percent between any adjustment period plus a provision for negative amortization. ARM III has an interest rate cap of 2 percent between adjustment periods and 5 percent over the life of the loan. When all three ARMs are compared, it is clear that the borrower is assuming more interest rate risk with ARM I than with any of the other ARMs. Hence, the expected yield on ARM I to the lender should be *less*, when compared with other ARMs, for an otherwise qualified borrower (i.e., a borrower with an acceptable level of default risk under all three ARM choices).

Because the *expected yield* should be *less* for ARM I, the *initial interest rate* will also generally be *lower* than each of the initial rates shown for the other ARM alternatives. Given that all ARMs are tied to the same index and have the same margin and discount points, the only way to "price" ARM I to achieve a lower expected yield is to *reduce* the initial interest rate relative to the other ARMs. ARM I should also have the largest discount, or spread, relative to the interest rate on the FRM. This would be expected because the borrower is bearing all interest rate risk; hence, the lender should expect to earn a lower risk premium and therefore a lower *yield* on ARM I when compared with the FRM (again, default risk is assumed to be acceptable for this borrower if ARM I is made).

Using a lower initial rate as an inducement for borrowers to accept more interest rate risk and unrestricted payments in the future is obviously only one of many combinations of terms that may be used to differentiate ARM I from ARMs II and III and from the FRM. For example, the lender could keep the initial rate on ARM I the same as that offered on ARM II, but reduce the margin on ARM I or charge fewer discount points, or both. Other terms, such as the choice of index, payment adjustment intervals, and so on, could also be varied with these three terms to accomplish the same objectives.

Moving to ARMs II and III in Exhibit 5–3, we note that both have initial interest rates that are greater than the initial rate on ARM I. The interest rate on ARM II is greater than that of ARM I because ARM II has a cap on payments that reduces payment uncertainty for the borrower.

When ARM III is compared to ARMs I and II, the interest rate risk assumed by the lender is clearly greater because payments are limited by interest rate caps. In this case, should market interest rates rise, the interest rate cap would restrict interest payments and not allow the lender to recover any lost interest. When compared with ARMs I and II, ARM III provides that more interest rate risk will be borne by the lender. Hence, it should be originated at a higher initial rate of interest.

Important note should be taken of other possibilities in Exhibit 5–3. If other terms, such as the index and adjustment interval, were to be changed, we would also expect changes in the initial loan terms. Suppose that in ARM I an index based on securities with longer maturities were to be chosen or payment intervals were longer than those shown. We would then expect either or all of the initial rate, index, or points to *increase* because of lower interest rate risk to the borrower; the risk is less because indexes tied to securities based on longer maturities are not as volatile as those based on shorter maturities. Obviously, the same would hold true for the other ARMs if a longer-term index and payment interval were used. Indeed, if such changes were made to the other ARMs, they would become more like an FRM. If longer-term indexes and lower caps were used on ARMs II and III, interest rate risk bearing would become greater for the lender; hence, the expected yield earned by the lender should approach that of an FRM as of the date of origination.

ARM Payment Mechanics

To illustrate how payment adjustments and loan balances are determined over the term for the ARMs in Exhibit 5–3, consider the example of a loan amount of $60,000 with a term of 30 years. We assume that the ARM interest rate will be adjusted annually. Hence, the first adjustment will occur at the beginning of the second year. At that time, the composite rate on the loan will be determined by the index of one-year U.S. Treasury securities, plus a 2 percent margin. If we assume (1) that the index of one-year Treasury securities takes on a pattern of 10, 13, 15, and 10 percent for the *next* four years, based on forward rates in existence at the time each ARM is originated and (2) that monthly payment and interest rate adjustments are made annually, what would payment adjustments, loan balances, and expected yields be for an ARM with these assumed characteristics?

No Caps or Limitations on Payments or Interest Rates

The first case to consider is ARM I, where payments are unrestricted or allowed to move up or down with the index without limit. What would be the payment pattern on such an ARM given that the expected distribution of future interest rates actually occurred? This unrestricted case, where no limitations apply to payments or interest, is straightforward to deal with.

The first four columns of Exhibit 5–4 contain the data needed for our computations. Note that we assume that the initial interest rate is 8 percent for the first year, but after the first year the index *plus* the 2 percent margin establish what the payment will be. From the

EXHIBIT 5–4
Summary Data and Results: ARM I (Unrestricted Case)

(1)	(2)		(3)		(4)	(5)	(6)
Year	Index	+	Margin	=	Interest Rate	Payments	Balance†
1					8%*	$440.26	$59,499
2	10%		2%		12	614.25	59,255
3	13		2		15	752.27	59,106
4	15		2		17	846.21	58,990
5	10		2		12	617.60	58,639

*Initial rate.
†Rounded.

EXHIBIT 5–5
Determination of Payment Limits (Negative Amortization: ARM II, with Payment Cap = 7.5 Percent Annually)

(1) Beginning of Year	(2) Balance (Rounded)	(3) Uncapped Payment	(4) Payment Capped at 7.5 Percent
1	$60,000	$482.77	$482.77
2	59,590	615.18	518.98
3	60,566	768.91	557.90
4	63,128	903.79	599.74
5	66,952	700.96	644.72

(5) Monthly Interest Rate	(6) Monthly Interest (5) × (2)	(7) Monthly Amortization (4) − (6)	(8) Compounded Monthly Rate (End Year) from (5)	(9) Annual Amortization (7) × (8)
.09 ÷ 12	$450.00	$32.77	12.507596	$409.87
.12 ÷ 12	595.90	(76.92)	12.682503	(975.54)
.15 ÷ 12	757.08	(199.18)	12.860378	(2,561.53)
.17 ÷ 12	894.31	(294.57)	12.980582	(3,823.69)
.12 ÷ 12	669.52	(24.80)	12.682503	(314.53)

Payments in the third year of the ARM are determined by again establishing whether uncapped payments would increase by more than 7.5 percent. To determine this, we find that the loan balance, which includes the previous year's negative amortization, is $59,590 + $975.54 = $60,566 (rounded). The *unrestricted* interest rate of 15 percent for the remaining 336 months is used to compute the uncapped payment. Uncapped payments based on the unrestricted rate of 15 percent would be $768.91. This is a 48 percent increase from $518.98; hence, the payment will again be capped at a 7.5 percent increase, and negative amortization will be computed on the interest shortfall, compounded at 15 percent monthly, and added to the loan balance. This process is repeated for each adjustment interval over the life of the loan.[9] Actual loan balances with payments capped at 7.5 percent are shown in Exhibit 5–6.

Another observation regarding ARM II (see Exhibit 5–5) has to do with the increase in both the payment and loan balance during year 5 even though there is a significant decline in the interest index from 17 to 12 percent. This occurs because the loan balance has

EXHIBIT 5–6
ARM II: Loan Balances When Payments Are Capped at 7.5 Percent Annually (Negative Amortization Allowed)

Year	Index	Margin	Interest Rate	Beginning of Year Balances	Payments	Less: Annual Amortization	End of Year Loan Balances
1	—	—	9%*	$60,000	$482.77	409.87	$59,590
2	10%	2%	12	59,590	518.98	(975.54)	60,566
3	13	2	15	60,566	557.90	(2,561.53)	63,128
4	15	2	17	63,128	599.74	(3,823.69)	66,952
5	10	2	12	66,952	644.72	(314.53)	67,267

*Origination rate.

[9] ARMs with negative amortization provisions usually limit increases in the loan balance during the life of the loan because it is possible for the loan balance to increase to a level that exceeds the value of the property serving as security for the loan. Consequently, lenders and borrowers must agree that if a prespecified maximum is reached, the lender must either forgo further accumulation of interest in the loan balance or require that monthly payments be increased at that time.

increased, due to past negative amortization, to $66,952 at the end of year 4. Even though the interest rate declines to 12 percent, monthly interest will be $669.52, which is in excess of the maximum 7.5 percent increase from the $599.74 payment in the preceding year. Hence, payments would increase by 7.5 percent, even though interest rates have declined.

An *alternative* method that may be used to find loan balances for the ARM illustrated in Exhibits 5–5 and 5–6 is shown in the calculator sequence described below. Note in step 4 that even though payments due in year 2 have been increased by 7.5 percent to $518.98, they remain lower than the interest due which is $595.90 (the accrued rate of 12% ÷ 12, or 1% × $59,590.08). Furthermore, when the interest rate of 12 percent is used for *i* in the calculator sequence and the future value (*FV*) is solved for, the loan balance increases to $60,566 (rounded). When compared to the loan balance of $59,590 at the end of year 1, negative amortization of $975 (rounded) has occurred. This coincides with the amount shown in Exhibit 5–5 (column 9) during year 2 and consists of $76.92 per month compounded at a monthly rate of 12 percent(or 12% ÷ 12 = .01). The reader should read both Exhibits 5–5 and 5–6 and review the process involved to better understand the results from the alternative calculator solution shown below.

Alternative Solution for Determining Loan Balances and Negative Amortization for ARM in Exhibits 5–5 and 5–6:

STEP 1:

Solution: Determine Monthly Payments for Year (1):

$PV = \$60{,}000$

$n = 30 \times 12 = 360$

$i = 9\%/12 = .75\%$

$FV = 0$

Solve for $PMT = \$482.77$

Function: *PMT* (*PV*, *n*, *i*, *FV*)

STEP 2:

Solution: Determine Balance at the End of First Year:

$PV = \$60{,}000$ (from above)

$PMT = \$482.77$ (from above)

$i = 9\%/12 = .75\%$ (from above)

$n = 1 \times 12 = 12$

Solve for $FV = \$59{,}590.08$

Function: *FV* (*PV*, *PMT*, *i*, *n*)

STEP 3:

Solution: Determine Monthly Payments for Year (2):

$PMT = \$482.77 \times 1.075 = \$518.98$

STEP 4:

Solution: Determine Balance at the End of Second Year:

$PV = \$59{,}590.08$ (balance at end of first year calculated above)

$PMT = \$518.98$ (from above)

$i = 12\%/12 = 1\%$

$n = 1 \times 12 = 12$

Solve for $FV = \$60{,}565.61$

Function: *FV* (*PV*, *PMT*, *i*, *n*)

Repeat steps 2 through 4 for the remaining years.

beginning of year 2 through the beginning of year 5, the interest rates used to determine payments are 12, 15, 17, and 12, respectively, based on our assumptions. As previously pointed out, ARMs tied to the same index may vary with respect to the initial rate of interest, the margin, and, perhaps, discount points offered by lenders. These components are usually set by competitive conditions in the lending area and are the primary variables (along with caps or other restrictions) with which lenders compete when pricing loans. Lenders have no control over the index and, therefore, must rely on other components with which to compete when pricing the loan.

The payments column of Exhibit 5–4 is based on a series of relatively simple computations. They are carried out as though a new loan is originated at the end of each year based on a new rate of interest, as determined by the index plus the margin, applied to the outstanding loan balance.

The process of (1) computing the loan balance, based on the interest rate applicable during the year for which the balance is desired and (2) computing the new payment, based on any change in the index at the end of the appropriate adjustment interval, would continue after each adjustment interval over the remaining life of the loan.

STEP 1:

Solution: Determine First Year Payment: — Function: *PMT* (*n*, *i*, *PV*, *FV*)

$n = 30 \times 12 = 360$

$i = 8\%/12 = .66666\%$

$PV = \$60{,}000$

$FV = 0$

Solve for $PMT = -\$440.26$

STEP 2:

Solution: Determine First Year Mortgage Balance: — Function: *PMT* (*n*, *i*, *PMT*, *FV*)

$n = 29 \times 12 = 348$

$i = 8\%/12 = .66666\%$

$PMT = -\$440.26$

$FV = 0$

Solve for $PV = \$59{,}499$

STEP 3:

Solution: Determine Second Year Payment: — Function: *PMT* (*n*, *i*, *PV*, *FV*)

$n = 29 \times 12 = 348$

$i = 12\%/12 = 1\%$

$PV = \$59{,}499$

$FV = 0$

Solve for $PMT = -\$614.25$

STEP 4:

Solution: Determine Second Year Mortgage Balance: — Function: *PV* (*n*, *i*, *PMT*, *FV*)

$n = 28 \times 12 = 336$

$i = 12\%/12 = 1\%$

$PMT = -\$614.25$

$FV = 0$

Solve for $PV = \$59{,}255$

Looking again at Exhibit 5–4, we carry out the computations using the hypothetical interest rate pattern. Assuming no restrictions or caps on interest rates or payments, we see considerable variation in monthly payments. Depending on interest rate changes, payments increase by over 39.5 percent and decline by as much as 27 percent during the first five years. For borrowers who have a strong aversion to interest rate risk and the coincident variability in payments, the unrestricted ARM, tied to a short-term instrument, may not be desirable. One final pattern should be noted in Exhibit 5–4: regardless of the interest rate pattern chosen, the loan is amortizing. The rate of amortization will differ, however, depending on the rate of interest in effect at each adjustment interval.

The default risk associated with ARM I should also be clear from Exhibit 5–4. Note that although the initial payment level is low, the variation in payments over the five-year period is great. Clearly, for a borrower to take this risk, the lender must view the borrower's future income or present and future wealth as sufficient to cover significant changes in monthly payments.

Payment Caps and Negative Amortization

We now consider ARM II where the lender and borrower have agreed that to moderate possible interest rate fluctuations in the future, there will be a payment cap, or a maximum rate at which *payments* can increase between adjustment intervals. This maximum rate of increase will be 7.5 percent per year. In this case, however, any difference between payments and interest that should be earned, based on unrestricted changes in interest rates, will be *added* to the loan balance. As previously discussed, this type of ARM contains both a *payment cap* and *negative amortization.*

Because this ARM allows for a payment cap and negative amortization, the receipt of more cash flow is pushed further into the future than in the unrestricted case. Therefore, interest rate risk to the lender is somewhat greater than with ARM I, so we assume that the initial rate on the mortgage is quoted to be 9 percent, while the margin will remain at 2 percent. Exhibit 5–5 contains computations of the payment and loan balance patterns for the ARM just described. As shown in the exhibit, based on an unrestricted change in our hypothetical pattern of interest rates, monthly payments in the second year would be \$615.18, or 27.4 percent higher than the \$482.77 payment required during the first year. A payment of \$615.18 would obviously be greater than the 7.5 percent maximum allowable increase; hence, the payment would be capped at \$518.98, or 7.5 percent more than \$482.77. However, *because this ARM requires negative amortization,* the difference between interest charged during year 2, or 12 percent, and the amount actually paid will be added to the outstanding loan balance plus compound interest.

Negative amortization is computed by using the method shown for the graduated payment mortgage (GPM) in Chapter 4. Exhibit 5–5 contains a breakdown of interest and amortization for ARM II. Note that during the first year when loan payments are computed at 9 percent interest, monthly amortization occurs and the loan balance is reduced. After the first year, monthly payments must be computed *first* based on the unrestricted interest rate (column 3) to determine whether payments will increase at a rate greater than 7.5 percent. If uncapped payments would exceed 7.5 percent, then the payment cap (column 4) becomes operative and actual payments will be restricted to a 7.5 percent increase. The monthly interest that is *accruing* on the loan balance at the unrestricted rate is (.12 ÷ 12)\$59,590 = \$595.90 (column 6). However, the payment that will actually be made is \$518.98. The difference, \$76.92 (column 7), must be added to the loan balance with compound interest. Hence, the difference in year 2, \$76.92 per month, is compounded at 1 percent per month (column 8), resulting in an increase of \$975.54 in the loan balance.[8]

[8] Using a calculator: $PV = 0$, $PMT = \$76.92$, $i = 12\% \div 12$, $n = 12$: solve for $FV = \$975.54$.

Interest Rate Caps

The final case that we consider with ARMs is a common pattern in which interest rates are capped or limited (see Exhibit 5–7). In ARM III, the increase in interest rates is limited to 2 percent during any one adjustment interval (year in our example) and to a *total* of 5 percent over the life of the loan. If interest rates ever exceed these caps, payments are limited. Hence, the interest rate cap also acts as a payment cap because the maximum increase in interest rate determines the maximum increase in mortgage payments. This means that if the index plus the margin exceeds these caps, the lender will lose any amount of interest above the capped rates.[10] Exhibit 5–7 illustrates the payment mechanics of ARM III, where the interest rate quoted at origination, 11 percent, is higher than it is with ARMs I and II because the latter two have unrestricted interest rates, while ARM III has interest rate caps. Therefore, the lender is taking more interest rate risk with ARM III because of the possibility that the cap will be exceeded and interest will be lost. To compensate for this possibility, the lender will charge a higher initial interest rate and should expect to earn a higher expected yield.

The payment patterns shown in Exhibit 5–7 are determined from the loan balance established at the end of each adjustment interval. Payments are then computed based on the indicated rate of interest for the remaining term. Results of computations show that, compared with ARM I (the unrestricted case), payments on ARM III are higher initially, and then remain generally lower than payments on ARM I for the remaining term. Hence, borrowers would have to have more income to qualify for ARM III and default risk to the lender should be lower. The loan balances for both ARMs are about the same in year 5. ARM III payments begin at a higher level than those of ARM II, because of the higher initial rate of interest, and remain higher over the term of the loan. However, because of negative amortization, loan balances over time for ARM II are significantly higher than for ARM III.

Expected Yields on ARMs: A Comparison

In the preceding sections, we examined three kinds of ARMs with provisions commonly used in real estate lending. Other considerations are also important to lenders and borrowers. One important issue is the *yield* to lenders, or *cost* to borrowers, for each category of loan. Given the changes in interest rates, payments, and loan balances, it is not obvious what these yields (costs) will be.

EXHIBIT 5–7
Summary Data and Results: ARM III Interest Rates Capped at 2 Percent, 5 Percent (No Negative Amortization Allowed)

Year	Index + Margin	Capped Interest Rate	Payments	Balance
1		11%	$571.39	$59,730
2	12%	12	616.63	59,485
3	15	14	708.37	59,301
4	17	16	801.65	59,159
5	12	12	619.37	58,807

[10] In many cases, ARMs may contain floors as well as a cap. In our example, this would mean that a maximum reduction of 2 percent in the mortgage rate would be allowed, regardless of the decline in the index. These floors have limited effectiveness, however, because if a significant decline in the index occurs and the loan agreement allows for prepayment, borrowers may *refinance* with a new mortgage loan at a rate that is lower than the floor would allow.

Computing Yields on ARMs

To compare yields on ARMs, the yield (cost) to the lender (borrower) must be computed for each alternative by solving for the internal rate of return (IRR), or the rate of discount. This rate makes the present value of all expected mortgage payments and the loan balance in the year of repayment equal to the initial loan amount less discount points (or $58,800) for each alternative. To illustrate, consider the case of the *unrestricted* ARM I which is paid off in year 5. Using data from Exhibit 5–4, we compute the internal rate of return (or yield) as shown in Exhibit 5–8.

From the computations shown in Exhibit 5–8, we see that the solution is approximately 13.0 percent.[11] This means that even though the *initial* rate of interest was 8 percent and the forward rates of interest are expected to range from 8 to 17 percent over the five-year period, the *expected yield* (cost) is 13.0 percent. Hence, by computing the internal rate of return we have a result that can be compared among alternative ARMs.

Before comparing the results for each ARM considered, we examine the computational procedure used in Exhibit 5–8. Essentially, we are discounting a series of grouped cash flows. In the present case, we are dealing with five groups of monthly cash flows and a single receipt (the loan balance). Note that we discount each group of monthly cash flows by using the present value of a monthly annuity factor of 13 percent (column 3). However, this procedure gives us a present value for a *one-year group* of 12 monthly payments and does not take into account that the cash flows occurring from years 2 through 5 are not received during year 1. Hence, each of the grouped cash flows must be discounted again by the present value of $1 factor to recognize that the present value of each group of cash flows is not received at the same time. This is carried out in column 4. The loan balance, or $58,639, is then discounted as a lump sum.

Summary Observations: ARMs, Borrower, Lender, and Market Behavior

Recalling the graphic analysis of the risk premium and the relationship between interest rate and default risk in Exhibit 5–2, we now show how risk premiums demanded for ARMs I to III would fall on the total risk curve in Exhibit 5–9. This diagram basically indicates that in moving from ARMs I to III, interest rate risk to the lender increases. However, based on panel B in Exhibit 5–2, we recall that as interest rate risk increases to the lender,

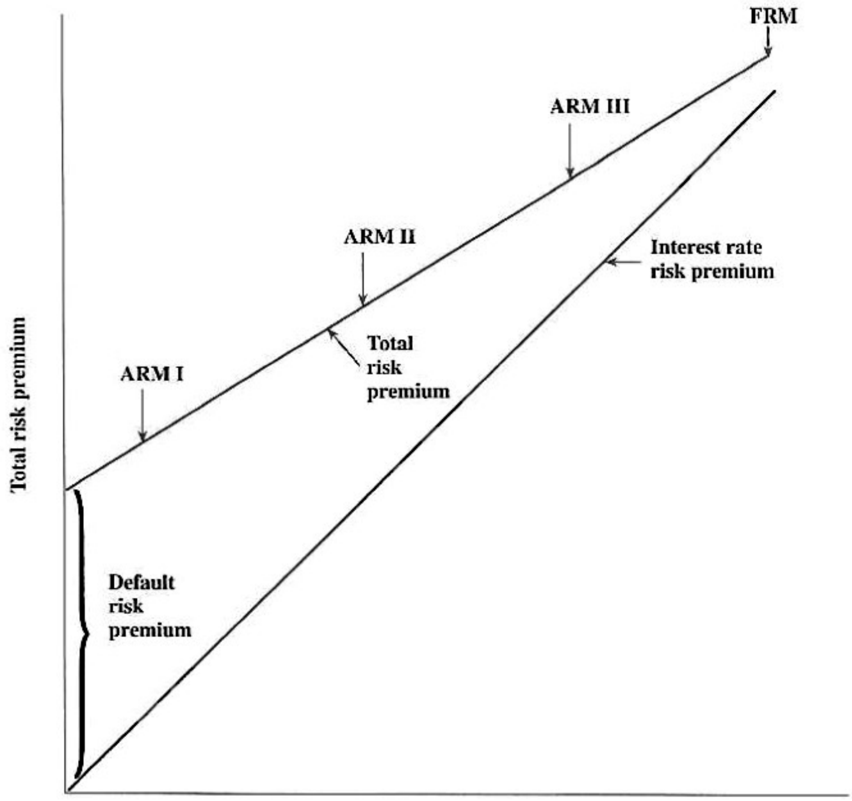
EXHIBIT 5–8
Computing the IRR for an Unrestricted ARM, Payoff at End of Year 5

(1) Year	(2) Monthly Payments		(3) PV Monthly Annuity 13%, 12 Months		(4) PV of $1 per Year, 13%, Years 1–5		(5) *PV*†
1	$440.26	×	11.196042	×	—	=	$ 4,929.39
2	614.25	×	11.196042	×	.878710	=	6,043.53
3	752.27	×	11.196042	×	.772130	=	6,503.22
4	846.21	×	11.196042	×	.678478	=	6,428.04
5	617.60	×	11.196042	×	.596185	=	4,122.43
5	$58,639.00	×	—	×	.523874	=	30,719.45
							$58,746.06*

*Desired *PV* = $58,800 : *IRR* approximately 13 percent.
†Rounded.

[11] Using a financial calculator yields a solution of 13.0 percent. Also see Concept Box 3.2 in Chapter 3 for solving for the IRR. We will rely on this result in our discussion.

EXHIBIT 5–9
Ranking ARMs Based on Total Risk



default risk for a specific borrower declines due to interest rate changes. Following the market rule that benefits to the lender (from shifting interest rate risk to the borrower) *must exceed* expected default risk for the ARM to be originated, we see in Exhibit 5–9 that the total risk premium, and hence the expected yield to the lender, increases as we move from ARMs I to III. All expected ARM yields remain below that of the FRM, as they should.

We also know that, in general, the initial interest rate and expected yield for all ARMs should be lower than that of an FRM on the day of origination. The extent to which the initial rate and expected yield on an ARM will be lower than that on an FRM or another ARM depends on the terms relative to payments, caps, and so on. Terms that are more unrestricted and shift more interest rate risk to the borrower will generally have initial interest rates that are discounted furthest from FRMs. They will also be discounted from ARMs containing caps on payment and interest rate increases. Hence, when a borrower is faced with selecting from a given set of ARMs with different terms and expectations of forward

Web App

Many lenders offer mortgage rate information online. Use a search engine like **www.yahoo.com** or **www.google.com** to find a lender offering an adjustable rate mortgage (ARM). Find out as many things as you can about how the mortgage works (e.g., What is the initial rate? What index is used for the adjustments? What is the margin over the index? How often does it adjust? What is the term of the loan? Are there any caps or floors on the loan? How does the rate on this ARM compare to the rate for a fixed rate mortgage? Which would you choose?)

interest rates, an expected yield must be calculated before a comparison among adjustable rate mortgages and a fixed rate mortgage can be accomplished. While there is no guarantee that the *expected* yield calculated at origination will be the actual yield or cost of funds over the term of the ARM, the expected yield represents the *best estimate* of the cost of an ARM based on the information available at the time of origination.

Conclusion

In this chapter, we have shown how mortgage loan terms can be modified to incorporate variable interest rates. Loans with adjustable interest rates become necessary from time to time, depending on the rate of economic expansion and expected rates of inflation. In many situations, when the expected rate of inflation accelerates and becomes more uncertain, questions arise as to whether borrowers or lenders will bear the risk of future interest rate changes. During these times, fixed interest rate lending becomes very costly to borrowers because fixed interest rates and mortgage payments increase at a greater rate than borrower incomes. This imbalance between loan payments and borrower incomes motivates both borrowers and lenders to seek ways to modify loan agreements so that real estate purchases can be financed at loan payment levels that are commensurate with current borrower incomes. Adjustable rate mortgage (ARM) loans provide one solution to the imbalance problem. Through a variety of options, including the benchmark index chosen for ARM interest rates, the volatility of the index, frequency of payment adjustment, annual and over-the-loan-life interest rate caps, and negative amortization and other features, lenders and borrowers can negotiate loans and payment structures that result in interest rate risk-sharing agreements that are satisfactory to all parties.

Key Terms

actual start rate, *129*
adjustable rate mortgage (ARM), *122*
adjustment interval, *127*
assumability, *128*
caps, *128*
composite rate, *128*
expected start rate, *129*
floating rate loans, *122*
floors, *128*
index, *128*
interest-only ARM, *130*
interest rate risk, *122*
margin, *128*
negative amortization, *128*
payment shock, *132*
price level adjusted mortgage (PLAM), *124*
reset date, *128*
spread, *128*
teaser rate, *129*
unanticipated inflation, *123*

Useful Websites

www.hud.gov—Department of Housing and Urban Development.

www.va.gov—Veterans Association website.

www.freddiemac.com—Federal Home Loan Mortgage Corporation.

www.mbaa.org—Mortgage Bankers Association of America.

www.aba.com—America Banker Association.

www.pueblo.gsa.gov/cic_text/housing/handbook/handbook.htm—Provided by U.S. Federal Reserve's Office of Thrift Supervision. Answers frequently asked questions about adjustable rate mortgages, describes how ARMs work, and provides a means of comparing two different loans.

www.fanniemae.com/homebuyers/index.html—Provided by Fannie Mae. Information, tools, and resources for consumers on getting a mortgage, buying, or refinancing a home.

www.freddiemac.com/pmms/pmmsarm.htm—This is a good site for finding monthly average commitment rate and points on a one-year adjustable rate mortgage.

Questions

1. In the previous chapter, significant problems about the ability of borrowers to meet mortgage payments and the evolution of fixed interest rate mortgages with various payment patterns were discussed. Why did not this evolution address problems faced by lenders? What have lenders done in recent years to overcome these problems?
2. How do inflationary expectations influence interest rates on mortgage loans?

3. How does the price level adjusted mortgage (PLAM) address the problem of uncertainty in inflationary expectations? What are some of the practical limitations in implementing a PLAM program?
4. Why do adjustable rate mortgages (ARMs) seem to be a more suitable alternative for mortgage lending than PLAMs?
5. List each of the main terms likely to be negotiated in an ARM. What does pricing an ARM using these terms mean?
6. What is the difference between interest rate risk and default risk? How do combinations of terms in ARMs affect the allocation of risk between borrowers and lenders?
7. Which of the following two ARMs is likely to be priced higher, that is, offered with a higher initial interest rate? ARM A has a margin of 3 percent and is tied to a three-year index with payments adjustable every two years; payments cannot increase by more than 10 percent from the preceding period; the term is 30 years and no assumption or points will be allowed. ARM B has a margin of 3 percent and is tied to a one-year index with payments to be adjusted each year; payments cannot increase by more than 10 percent from the preceding period; the term is 30 years and no assumption or points are allowed.
8. What are forward rates of interest? How are they determined? What do they have to do with indexes used to adjust ARM payments?
9. Distinguish between the initial rate of interest and expected yield on an ARM. What is the general relationship between the two? How do they generally reflect ARM terms?
10. If an ARM is priced with an initial interest rate of 8 percent and a margin of 2 percent (when the ARM index is also 8% at origination) and a fixed rate mortgage (FRM) with constant payment is available at 11 percent, what does this imply about inflation and the forward rates in the yield curve at the time of origination? What is implied if an FRM were available at 10 percent? 12 percent?

Problems

1. A price level adjusted mortgage (PLAM) is made with the following terms:

   Amount = $95,000

   Initial interest rate = 4 percent

   Term = 30 years

   Points = 6 percent

   Payments to be reset at the beginning of each year.

   Assuming inflation is expected to increase at the rate of 6 percent per year for the next five years:

   a. Compute the payments at the beginning of each year (*BOY*).

   b. What is the loan balance at the end of the fifth year?

   c. What is the yield to the lender on such a mortgage?

2. A basic ARM is made for $200,000 at an initial interest rate of 6 percent for 30 years with an annual reset date. The borrower believes that the interest rate at the beginning of year (*BOY*) 2 will increase to 7 percent.

   a. Assuming that a fully amortizing loan is made, what will the monthly payments be during year 1?

   b. Based on (*a*) what will the loan balance be at the end of year (*EOY*) 1?

   c. Given that the interest rate is expected to be 7 percent at the beginning of year 2, what will the monthly payments be during year 2?

   d. What will be the loan balance at the *EOY* 2?

   e. What would be the monthly payments in year 1 if they are to be interest only?

3. A 3/1 ARM is made for $150,000 at 7 percent with a 30-year maturity.

   a. Assuming that fixed payments are to be made monthly for three years and that the loan is fully amortizing, what will be the monthly payments? What will be the loan balance after three years?

b. What would new payments be beginning in year 4 if the interest rate fell to 6 percent and the loan continued to be fully amortizing?

c. In (*a*) what would monthly payments be during year 1 if they were interest only? What would payments be beginning in year 4 if interest rates fell to 6 percent and the loan became fully amortizing?

4. An ARM for $100,000 is made at a time when the expected start rate is 5 percent. The loan will be made with a teaser rate of 2 percent for the first year, after which the rate will be reset. The loan is fully amortizing, has a maturity of 25 years, and payments will be made monthly.

   a. What will be the payments during the first year?

   b. Assuming that the reset rate is 6 percent at the beginning of year (*BOY*) 2, what will the payments be?

   c. By what percentage will the monthly payments increase?

   d. What if the reset date is three years after loan origination and the reset rate is 6 percent, what will the loan payments be beginning in year 4 through year 25?

5. An interest-only ARM is made for $200,000 for 30 years. The start rate is 5 percent and the borrower will (1) make monthly interest-only payments for three years. Payments thereafter must be sufficient to fully amortize the loan at maturity.

   a. If the borrower makes interest-only payments for three years, what will the payments be?

   b. Assume that at the end of year 3, the reset rate is 6 percent. The borrower must now make payments so as to fully amortize the loan. What will the payments be?

6. A borrower has been analyzing different adjustable rate mortgage (ARM) alternatives for the purchase of a property. The borrower anticipates owning the property for five years. The lender first offers a $150,000, 30-year fully amortizing ARM with the following terms:

   Initial interest rate = 6 percent

   Index = 1–year Treasuries

   Payments reset each year

   Margin = 2 percent

   Interest rate cap = None

   Payment cap = None

   Negative amortization = Not allowed

   Discount points = 2 percent

   Based on estimated forward rates, the index to which the ARM is tied is forecasted as follows: Beginning of year (*BOY*) 2 = 7 percent; *BOY* 3 = 8.5 percent; (*BOY*) 4 = 9.5 percent; (*BOY*) 5 = 11 percent.

   Compute the payments, loan balances, and yield for the unrestricted ARM for the five-year period.

7. An ARM is made for $150,000 for 30 years with the following terms:

   Initial interest rate = 7 percent

   Index = 1–year Treasuries

   Payments reset each year

   Margin = 2 percent

   Interest rate cap = None

   Payment cap = 5 percent increase in any year

   Discount points = 2 percent

   Fully amortizing; however, negative amortization allowed if payment cap reached

   Based on estimated forward rates, the index to which the ARM is tied is forecasted as follows: Beginning of year (*BOY*) 2 = 7 percent; (*BOY*) 3 = 8.5 percent; (*BOY*) 4 = 9.5 percent; (*BOY*) 5 = 11 percent.

   Compute the payments, loan balances, and yield for the ARM for the five-year period.

8. Assume that a lender offers a 30-year, $150,000 adjustable rate mortgage (ARM) with the following terms:

   Initial interest rate = 7.5 percent

   Index = one-year Treasuries

   Payments reset each year

   Margin = 2 percent

   Interest rate cap = 1 percent annually; 3 percent lifetime

   Discount points = 2 percent

   Fully amortizing; however, negative amortization allowed if interest rate caps reached

   Based on estimated forward rates, the index to which the ARM is tied is forecasted as follows: Beginning of year (*BOY*) 2 = 7 percent; (*BOY*) 3 = 8.5 percent; (*BOY*) 4 = 9.5 percent; (*EOY*) 5 = 11 percent.

   Compute the payments, loan balances, and yield for the ARM for the five-year period.

9. MakeNu Mortgage Company is offering a new mortgage instrument called the Stable Mortgage. This mortgage is composed of both a fixed rate and an adjustable rate component. Mrs. Maria Perez is interested in financing a property, which costs $100,000, and is to be financed by Stable Home Mortgages (SHM) on the following terms:

   a. The SHM requires a 5 percent down payment, costs the borrower 2 discount points, and allows 75 percent of the mortgage to be fixed and 25 percent to be adjustable. The fixed portion of the loan is for 30 years at an annual interest rate of 10.5 percent. Having neither an interest rate nor payment cap, the adjustable portion is also for 30 years with the following terms:

      Initial interest rate = 9 percent

      Index = one-year Treasuries

      Payments reset each year

      Margin = 2 percent

      Interest rate cap = None

      Payment cap = None

   The projected one-year U.S. Treasury-bill index, to which the ARM is tied, is as follows: (*BOY*) 2 = 10 percent; (*BOY*) 3 = 11 percent; (*BOY*) 4 = 8 percent; (*BOY*) 5 = 12 percent.

   Calculate Mrs. Perez's total monthly payments and end-of-year loan balances for the first five years. Calculate the lender's yield, assuming Mrs. Perez repays the loan after five years.

   b. Repeat part (*a*) under the assumption that the initial interest rate is 9.5 percent and there is an annual interest rate cap of 1 percent.

10. A floating rate mortgage loan is made for $100,000 for a 30-year period at an initial rate of 12 percent interest. However, the borrower and lender have negotiated a monthly payment of $800.

    a. What will be the loan balance at the end of year 1?

    b. What if the interest rate increases to 13 percent at the end of year 1? How much interest will be accrued as negative amortization in year 1 if the payment remains at $800? Year 5?

11. **Excel.** Refer to the "Ch5 ARM No Caps" tab in the Excel Workbook provided on the website. Suppose that the index goes to 18 percent in year 5. What is the effective cost of the unrestricted ARM?

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12. **Excel.** Refer to the "Ch5 ARM Int Cap" tab in the Excel Workbook provided on the website. Suppose that the index goes to 18 percent in year 5. What is the effective cost of this ARM? What cap affected the rate in year 5?

13. **Excel.** Refer to the "Ch5 ARM Pmt Cap" tab in the Excel Workbook provided on the website. Suppose that the index goes to 18 percent in year 5. What is the effective cost of the ARM? Does the payment cap keep the effective cost from rising?