CHAPTER

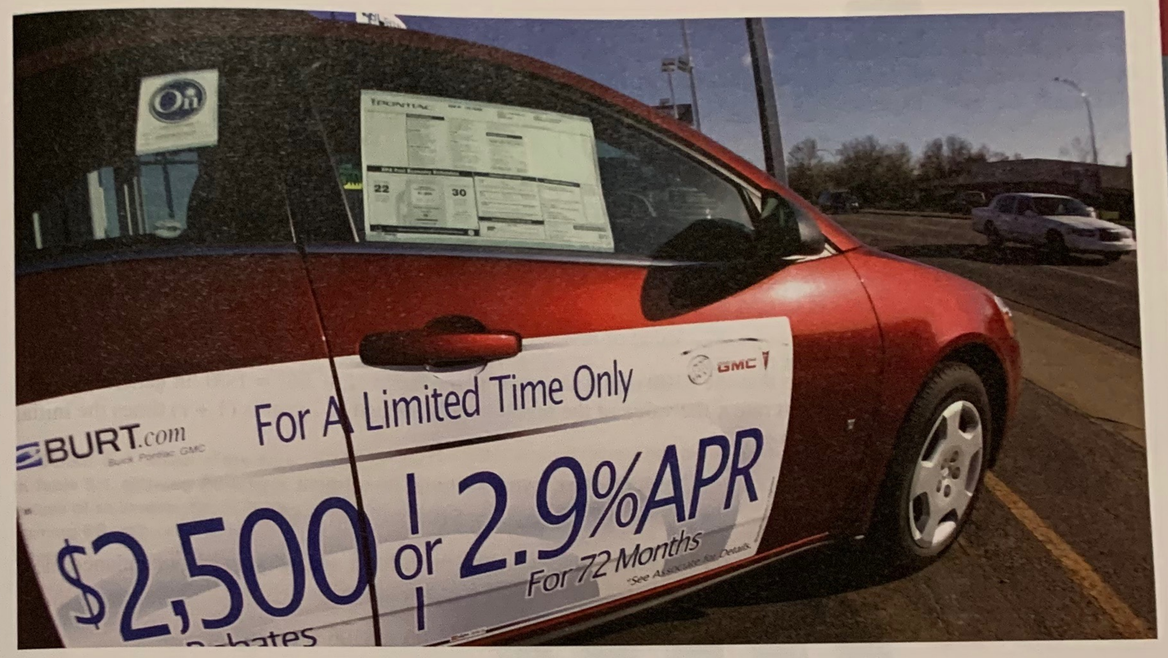
5 The Time Value of Money

LEARNING OBJECTIVES

After studying this chapter, you should be able to:

- 5-1 Calculate the future value of money that is invested at a particular interest rate.
- 5-2 Calculate the present value of a future payment.
- 5-3 Calculate present and future values of a level stream of cash payments.
- 5-4 Compare interest rates quoted over different time intervals—for example, monthly versus annual rates.
- 5-5 Understand the difference between real and nominal cash flows and between real and nominal interest rates.

RELATED WEBSITES FOR THIS CHAPTER CAN BE FOUND IN CONNECT.



Time affects the value of a dollar. © *David Zalubowski/AP Images*

Companies invest in lots of things. Some are *tangible assets*—that is, assets you can kick, like factories, machinery, and offices. Others are *intangible assets,* such as patents or trademarks. In each case the company lays out some money now in the hope of receiving even more money later.

Individuals also make investments. For example, your college education may cost you $30,000 per year. That is an investment you hope will pay off in the form of a higher salary later in life. You are sowing now and expecting to reap later.

Companies pay for their investments by raising money and, in the process, assuming liabilities. For example, they may borrow money from a bank and promise to repay it with interest later. You also may have financed your investment in a college education by borrowing money that you plan to pay back out of that fat salary.

All these financial decisions require comparisons of cash payments at different dates. Will your future salary be sufficient to justify the current expenditure on college tuition? How much will you have to repay the bank if you borrow to finance your education?

In this chapter, we take the first steps toward understanding the time value of money, that is, the relationship between the values of dollars today and dollars in the future. We start by looking at how funds invested at a specific interest rate will grow over time. We next ask how much you would need to invest today to produce a specified future sum of money, and we describe some shortcuts for working out the value of a series of cash payments. Then we consider how inflation affects these financial calculations.

There is nothing complicated about the calculations, but if they are to become second nature, you should read the chapter thoroughly, work carefully through the examples (we have provided plenty), and make sure you tackle the self-test questions. We are asking you to make an investment now in return for a payoff later. One of the payoffs is that you will understand what is going on behind the screen when you value cash flows using a spreadsheet program or a financial calculator. We show how to use spreadsheets and financial calculators later in the chapter.

For simplicity, almost every example in this chapter is set out in dollars, but the concepts and calculations are identical in euros, yen, tugrik, or drams.[1]

[1] The tugrik is the currency of Mongolia, and the dram is the currency of Armenia.

5.4 Present value of t-year annuity $= C\left[\frac{1}{r} - \frac{1}{r(1+r)^t}\right]$

5.5 Future value (FV) of annuity of \$1 a year = present value of annuity of \$1 a year $\times (1+r)^t$

$$= \left[\frac{1}{r} - \frac{1}{r(1+r)^t}\right] \times (1+r)^t$$

$$= \frac{(1+r)^t - 1}{r}$$

5.6 Present value of annuity due = present value of ordinary annuity $\times (1+r)$

5.7 Future value of annuity due = future value of ordinary annuity $\times (1+r)$

5.8 $1 + \text{real interest rate} = \frac{1 + \text{nominal interest rate}}{1 + \text{inflation rate}}$

5.9 Real interest rate $\approx$ nominal interest rate $-$ inflation rate

QUESTIONS AND PROBLEMS

connect

1. **Compound Interest.** Old Time Savings Bank pays 4% interest on its savings accounts. If you deposit \$1,000 in the bank and leave it there: (*LO5-1*)
   a. How much interest will you earn in the first year?
   b. How much interest will you earn in the second year?
   c. How much interest will you earn in the 10th year?
2. **Compound Interest.** New Savings Bank pays 4% interest on its deposits. If you deposit \$1,000 in the bank and leave it there, will it take more or less than 25 years for your money to double? You should be able to answer this without a calculator or interest rate tables. (*LO5-1*)
3. **Compound Interest.** Suppose that the value of an investment in the stock market has increased at an average compound rate of about 5% since 1900. It is now 2016. (*LO5-1*)
   a. If your great grandfather invested \$1,000 in 1900, how much would that investment be worth today?
   b. If an investment in 1900 has grown to \$1 million, how much was invested in 1900?
4. **Future Values.** Compute the future value of a \$100 cash flow for the following combinations of rates and times. (*LO5-1*)
   a. $r = 8\%$, $t = 10$ years
   b. $r = 8\%$, $t = 20$ years
   c. $r = 4\%$, $t = 10$ years
   d. $r = 4\%$, $t = 20$ years
5. **Future Values.** You deposit \$1,000 in your bank account. (*LO5-1*)
   a. If the bank pays 4% simple interest, how much will you accumulate in your account after 10 years?
   b. How much will you accumulate if the bank pays compound interest?
6. **Future Values.** If you earn 6% per year on your bank account, how long will it take an account with \$100 to double to \$200? (*LO5-1*)
7. **Future Values.** In 1880 five aboriginal trackers were each promised the equivalent of 100 Australian dollars for helping to capture the notorious outlaw Ned Kelley. In 1993 the granddaughters of two of the trackers claimed that this reward had not been paid. The Victorian prime minister stated that if this was true, the government would be happy to pay the \$100. However, the granddaughters also claimed that they were entitled to compound interest. (*LO5-1*)
   a. How much was each granddaughter entitled to if the interest rate was 4%?
   b. How much was each entitled to if the interest rate was 8%?

8. **Future Values.** How long will it take for \$400 to grow to \$1,000 at the following interest rates? (*LO5-1*)
   a. 4%
   b. 8%
   c. 16%
9. **Future Values.** You invest \$1,000 today and expect to sell your investment for \$2,000 in 10 years. (*LO5-1*)
   a. Is this a good deal if the interest rate is 6%?
   b. What if the interest rate is 10%?
10. **Future Values.** Your wealthy uncle established a \$1,000 bank account for you when you were born. For the first 8 years of your life, the interest rate earned on the account was 6%. Since then, rates have been only 4%. Now you are 21 years old and ready to cash in. How much is in your account? (*LO5-1*)
11. **Present Values.** You can buy property today for \$3 million and sell it in 5 years for \$4 million. (You earn no rental income on the property.) (*LO5-2*)
    a. If the interest rate is 8%, what is the present value of the sales price?
    b. Is the property investment attractive to you?
    c. Would your answer to part (b) change if you also could earn \$200,000 per-year rent on the property? The rent is paid at the end of each year.
12. **Present Values.** Compute the present value of a \$100 cash flow for the following combinations of discount rates and times. (*LO5-2*)
    a. $r = 8\%$, $t = 10$ years
    b. $r = 8\%$, $t = 20$ years
    c. $r = 4\%$, $t = 10$ years
    d. $r = 4\%$, $t = 20$ years
13. **Present Values.** You will require \$700 in 5 years. If you earn 5% interest on your funds, how much will you need to invest today in order to reach your savings goal? (*LO5-2*)
14. **Present Values.** What is the present value of the following cash-flow stream if the interest rate is 6%? (*LO5-2*)

Year	Cash Flow
1	\$200
2	400
3	300

15. **Calculating the Interest Rate.** Find the interest rate implied by the following combinations of present and future values. (*LO5-2*)

Present Value	Years	Future Value
\$400	11	\$684
183	4	249
300	7	300

16. **Calculating the Interest Rate.** Find the annual interest rate. (*LO5-2*)

Present Value	Future Value	Time Period
\$100	\$115.76	3 years
200	262.16	4
100	110.41	5

17. **Calculating the Interest Rate.** A zero-coupon bond that will pay \$1,000 in 10 years is selling today for \$422.41. What interest rate does the bond offer? (*LO5-2*)

18. **Present Values.** A factory costs $400,000. You forecast that it will produce cash inflows of $120,000 in year 1, $180,000 in year 2, and $300,000 in year 3. The discount rate is 12%. (*LO5-2*)
    a. What is the value of the factory?
    b. Is the factory a good investment?

19. **Present Values.** The 2-year discount factor is .92. What is the present value of $1 to be received in year 2? What is the present value of $2,000? (*LO5-2*)

20. **Annuities.** A famous quarterback just signed a $15 million contract providing $3 million a year for 5 years. A less famous receiver signed a $14 million 5-year contract providing $4 million now and $2 million a year for 5 years. The interest rate is 10%. Who is better paid? (*LO5-3*)

21. **Annuities.** Would you rather receive $1,000 a year for 10 years or $800 a year for 15 years if the interest rate is 5%? What if the interest rate is 20%? (*LO5-3*)

22. **Perpetuities.** A local bank advertises the following deal: "Pay us $100 a year for 10 years and then we will pay you (or your beneficiaries) $100 a year forever." Is this a good deal if the interest rate is 6%? (*LO5-3*)

23. **Perpetuities.** A local bank will pay you $100 a year for your lifetime if you deposit $2,500 in the bank today. If you plan to live forever, what interest rate is the bank paying? (*LO5-3*)

24. **Perpetuities.** A property will provide $10,000 a year forever. If its value is $125,000, what must be the discount rate? (*LO5-3*)

25. **Perpetuities.** British government 4% perpetuities pay £4 interest each year forever. Another bond, 2.5% perpetuities, pays £2.50 a year forever. (*LO5-3*)
    a. What is the value of 4% perpetuities if the long-term interest rate is 6%?
    b. What is the value of 2.5% perpetuities?

26. **Annuities.** (*LO5-3*)
    a. What is the present value of a 3-year annuity of $100 if the discount rate is 6%?
    b. What is the present value of the annuity in part (a) if you have to wait 2 years instead of 1 year for the first payment?

27. **Annuities.** Professor's Annuity Corp. offers a lifetime annuity to retiring professors. For a payment of $80,000 at age 65, the firm will pay the retiring professor $600 a month until death. (*LO5-3*)
    a. If the professor's remaining life expectancy is 20 years, what is the monthly interest rate on this annuity?
    b. What is the effective annual interest rate?
    c. If the monthly interest rate is .5%, what monthly annuity payment can the firm offer to the retiring professor?

28. **Annuities.** You want to buy a new car, but you can make an initial payment of only $2,000 and can afford monthly payments of at most $400. (*LO5-3*)
    a. If the APR on auto loans is 12% and you finance the purchase over 48 months, what is the maximum price you can pay for the car?
    b. How much can you afford if you finance the purchase over 60 months?

29. **Annuities.** You can buy a car that is advertised for $24,000 on the following terms: (a) pay $24,000 and receive a $2,000 rebate from the manufacturer; (b) pay $500 a month for 4 years for total payments of $24,000, implying zero percent financing. Which is the better deal if the interest rate is 1% per month? (*LO5-3*)

30. **Annuities.** You have just borrowed $100,000 to buy a condo. You will repay the loan in equal monthly payments of $804.62 over the next 30 years. (*LO5-4*)
    a. What monthly interest rate are you paying on the loan?
    b. What is the APR?
    c. What is the effective annual rate on that loan?
    d. What rate is the lender more likely to quote on the loan?

31. **Future Value of Annuities.** I now have $20,000 in the bank earning interest of .5% per month. I need $30,000 to make a down payment on a house. I can save an additional $100 per month. How long will it take me to accumulate the $30,000? (*LO5-3*)

32. **Real Annuities.** A retiree wants level consumption in real terms over a 30-year retirement. If the inflation rate equals the interest rate she earns on her $450,000 of savings, how much can she spend in real terms each year over the rest of her life? (*LO5-3*)

33. **Delayed Annuities.** Suppose that you will receive annual payments of $10,000 for a period of 10 years. The first payment will be made 4 years from now. If the interest rate is 5%, what is the present value of this stream of payments? (*LO5-3*)

34. **Annuity Due.** Your landscaping company can lease a truck for $8,000 a year (paid at year-end) for 6 years. It can instead buy the truck for $40,000. The truck will be valueless after 6 years. The interest rate your company can earn on its funds is 7%. (*LO5-3*)
    a. What is the present value of the cost of leasing?
    b. Is it cheaper to buy or lease?
    c. What is the present value of the cost of leasing if the lease payments are an annuity due, so the first payment comes immediately?
    d. Is it now cheaper to buy or lease?

35. **Annuity Due.** Recall that an annuity due is like an ordinary annuity except that the first payment is made immediately instead of at the end of the first period. (*LO5-3*)
    a. Why is the present value of an annuity due equal to $(1 + r)$ times the present value of an ordinary annuity?
    b. Why is the future value of an annuity due equal to $(1 + r)$ times the future value of an ordinary annuity?

36. **Annuity Due.** A store offers two payment plans. Under the installment plan, you pay 25% down and 25% of the purchase price in each of the next 3 years. If you pay the entire bill immediately, you can take a 10% discount from the purchase price. (*LO5-3*)
    a. Which is a better deal if you can borrow or lend funds at a 5% interest rate?
    b. How will your answer change if the payments on the 4-year installment plan do not start for a full year?

37. **Annuity Due.** (*LO5-3*)
    a. If you borrow $1,000 and agree to repay the loan in five equal annual payments at an interest rate of 12%, what will your payment be?
    b. What will your payment be if you make the first payment on the loan immediately instead of at the end of the first year?

38. **Annuity Due.** The $40 million lottery payment that you have just won actually pays $2 million per year for 20 years. The interest rate is 8%. (*LO5-3*)
    a. If the first payment comes in 1 year, what is the present value of the winnings?
    b. What is the present value if the first payment comes immediately?

39. **Amortizing Loan.** You take out a 30-year $100,000 mortgage loan with an APR of 6% and monthly payments. In 12 years you decide to sell your house and pay off the mortgage. What is the principal balance on the loan? (*LO5-3*)

40. **Amortizing Loan.** Consider a 4-year amortizing loan. You borrow $1,000 initially and repay it in four equal annual year-end payments. (*LO5-3*)
    a. If the interest rate is 8%, what is the annual payment?
    b. Fill in the following table, which shows how much of each payment is interest versus principal repayment (that is, amortization) and the outstanding balance on the loan at each date.

Time	Loan Balance ($)	Year-End Interest Due on Loan Balance ($)	Total Year-End Payment ($)	Amortization of Loan ($)
0	1,000	____	____	____
1	____	____	____	____
2	____	____	____	____
3	____	____	____	____
4	0	0		

41. **Retirement Savings.** A couple will retire in 50 years; they plan to spend about $30,000 a year in retirement, which should last about 25 years. They believe that they can earn 8% interest on retirement savings. (*LO5-3*)
    a. If they make annual payments into a savings plan, how much will they need to save each year? Assume the first payment comes in 1 year.
    b. How would the answer to part (a) change if the couple also realize that in 20 years they will need to spend $60,000 on their child's college education?
42. **Retirement Savings.** You believe you will need to have saved $500,000 by the time you retire in 40 years in order to live comfortably. If the interest rate is 6% per year, how much must you save each year to meet your retirement goal? (*LO5-3*)
43. **Retirement Savings.** You believe you will need to have saved $500,000 by the time you retire in 40 years in order to live comfortably. You also believe that you will inherit $100,000 in 10 years. If the interest rate is 6% per year, how much must you save each year to meet your retirement goal? (*LO5-3*)
44. **Retirement Savings.** You believe you will spend $40,000 a year for 20 years once you retire in 40 years. If the interest rate is 6% per year, how much must you save each year until retirement to meet your retirement goal? (*LO5-3*)
45. **Retirement Savings.** A couple thinking about retirement decide to put aside $3,000 each year in a savings plan that earns 8% interest. In 5 years they will receive a gift of $10,000 that also can be invested. (*LO5-3*)
    a. How much money will they have accumulated 30 years from now?
    b. If their goal is to retire with $800,000 of savings, how much extra do they need to save every year?
46. **Perpetuities and Effective Interest Rate.** What is the value of a perpetuity that pays $100 every 3 months forever? The interest rate quoted on an APR basis is 6%. (*LO5-3*)
47. **Amortizing Loans and Inflation.** Suppose you take out a $100,000, 20-year mortgage loan to buy a condo. The interest rate on the loan is 6%. To keep things simple, we will assume you make payments on the loan annually at the end of each year. (*LO5-3*)
    a. What is your annual payment on the loan?
    b. Construct a mortgage amortization table in Excel similar to Table 5.5 in which you compute the interest payment each year, the amortization of the loan, and the loan balance each year. (Allow the interest rate to be an input that the user of the spreadsheet can enter and change.)
    c. What fraction of your initial loan payment is interest?
    d. What fraction of your initial loan payment is amortization?
    e. What fraction of the loan has been paid off after 10 years (halfway through the life of the loan)?
    f. If the inflation rate is 2%, what is the real value of the first (year-end) payment?
    g. If the inflation rate is 2%, what is the real value of the last (year-end) payment?
    h. Now assume the inflation rate is 8% and the real interest rate on the loan is unchanged. What must be the new nominal interest rate?
    i. Recompute the amortization table. What is the real value of the first (year-end) payment in this high-inflation scenario?
    j. What is the real value of the last payment in this high-inflation scenario?
48. **Mortgage with Points.** Home loans typically involve "points," which are fees charged by the lender. Each point charged means that the borrower must pay 1% of the loan amount as a fee. For example, if the loan is for $100,000 and 2 points are charged, the loan repayment schedule is calculated on a $100,000 loan but the net amount the borrower receives is only $98,000. Assume the interest rate is 1% per month. What is the effective annual interest rate charged on such a loan, assuming loan repayment occurs over 360 months? (*LO5-4*)
49. **Effective Interest Rate.** A store will give you a 3% discount on the cost of your purchase if you pay cash today. Otherwise, you will be billed the full price with payment due in 1 month. What is the implicit borrowing rate being paid by customers who choose to defer payment for the month? (*LO5-4*)
50. **Effective Interest Rate.** You've borrowed $4,248.68 and agreed to pay back the loan with monthly payments of $200. If the interest rate is 12% stated as an APR, how long will it take you to pay back the loan? What is the effective annual rate on the loan? (*LO5-4*)

51. **Effective Interest Rate.** You invest $1,000 at a 6% annual interest rate, stated as an APR. Interest is compounded monthly. How much will you have in 1 year? In 1.5 years? (*LO5-4*)
52. **Effective Interest Rate.** If a bank pays 6% interest with continuous compounding, what is the effective annual rate? (*LO5-4*)
53. **Effective Interest Rate.** In a discount interest loan, you pay the interest payment up front. For example, if a 1-year loan is stated as $10,000 and the interest rate is 10%, the borrower "pays" .10 × $10,000 = $1,000 immediately, thereby receiving net funds of $9,000 and repaying $10,000 in a year. (*LO5-4*)
    a. What is the effective interest rate on this loan?
    b. What is the effective annual rate on a 1-year loan with an interest rate quoted on a discount basis of 20%?
54. **Effective Interest Rate.** Banks sometimes quote interest rates in the form of "add-on interest." In this case, if a 1-year loan is quoted with a 20% interest rate and you borrow $1,000, then you pay back $1,200. But you make these payments in monthly installments of $100 each. (*LO5-4*)
    a. What is the true APR on this loan?
    b. What is the effective annual rate on the loan?
55. **Effective Interest Rate.** You borrow $1,000 from the bank and agree to repay the loan over the next year in 12 equal monthly payments of $90. However, the bank also charges you a loan initiation fee of $20, which is taken out of the initial proceeds of the loan. What is the effective annual interest rate on the loan, taking account of the impact of the initiation fee? (*LO5-4*)
56. **Effective Interest Rate.** First National Bank pays 6.2% interest compounded semiannually. Second National Bank pays 6% interest compounded monthly. Which bank offers the higher effective annual interest rate? (*LO5-4*)
57. **Loan Payments.** You take out an $8,000 car loan that calls for 48 monthly payments starting after 1 month at an APR of 10%. (*LO5-4*)
    a. What is your monthly payment?
    b. What is the effective annual interest rate on the loan?
    c. Now assume the payments are made in four annual year-end installments. What annual payment would have the same present value as the monthly payment you calculated?
58. **Continuous Compounding.** How much will $100 grow to if invested at a continuously compounded interest rate of 10% for 8 years? What if it is invested for 10 years at 8%? (*LO5-4*)
59. **Effective Interest Rate.** Find the effective annual interest rate for each case. (*LO5-4*)

APR	Compounding Period
12%	1 month
8	3
10	6

60. **Effective Interest Rate.** Find the APR (the stated interest rate) for each case. (*LO5-4*)

Effective Annual Interest Rate	Compounding Period
10.00%	1 month
6.09	6
8.24	3

61. **Effective Interest Rate.** Lenny Loanshark charges "1 point" per week (i.e., 1% per week) on his loans. What APR must he report to consumers? Assume exactly 52 weeks in a year. What is the effective annual rate? (*LO5-4*)
62. **Effective Interest Rate.** Suppose you can borrow money at 8.6% per year (APR) compounded semiannually or 8.4% per year (APR) compounded monthly. Which is the better deal? (*LO5-4*)
63. **Effective Interest Rate.** If you take out an $8,000 car loan that calls for 48 monthly payments of $240 each, what is the APR of the loan? What is the effective annual interest rate on the loan? (*LO5-4*)

64. **Real versus Nominal Perpetuities.** If the interest rate is 6% per year, how long will it take for your money to quadruple in value? If the inflation rate is 4% per year, what will be the change in the purchasing power of your money over this period? (*LO5-5*)

65. **Inflation.** In April 2016 a pound of apples cost $1.41, while oranges cost $1.05. Four years earlier the price of apples was only $1.20 a pound and that of oranges was $.91 a pound. (*LO5-5*)
    a. What was the annual compound rate of growth in the price of apples?
    b. What was the annual compound rate of growth in the price of oranges?
    c. If the same rates of growth persist in the future, what will be the price of apples in 2030?
    d. What about the price of oranges?

66. **Real versus Nominal Dollars.** An engineer in 1950 was earning $6,000 a year. In 2015 she earned $80,000 a year. However, on average, prices in 2015 were higher than in 1950. What was her real income in 2015 in terms of constant 1950 dollars? Use the data in Table 5.8. (*LO5-5*)

67. **Real versus Nominal Dollars.** Your consulting firm will produce cash flows of $100,000 this year, and you expect cash flow thereafter to keep pace with any increase in the general level of prices. The interest rate currently is 6%, and you anticipate inflation of about 2%. (*LO5-5*)
    a. What is the present value of your firm's cash flows for years 1 through 5?
    b. How would your answer to part (a) change if you anticipated no growth in cash flow?

68. **Real versus Nominal Rates.** If investors are to earn a 3% real interest rate, what nominal interest rate must they earn if the inflation rate is: (a) zero? (b) 4%? (c) 6%? (*LO5-5*)

69. **Real versus Nominal Rates.** If investors receive a 6% interest rate on their bank deposits, what real interest rate will they earn if the inflation rate over the year is: (a) zero? (b) 3%? (c) 6%? (*LO5-5*)

70. **Real versus Nominal Annuities.** Good news: You will almost certainly be a millionaire by the time you retire in 50 years. Bad news: The inflation rate over your lifetime will average about 3%. (*LO5-5*)
    a. What will be the real value of $1 million by the time you retire in terms of today's dollars?
    b. What real annuity (in today's dollars) will $1 million support if the real interest rate at retirement is 2% and the annuity must last for 20 years?

71. **Inflation.** In the summer of 2007, Zimbabwe's official inflation rate was about 110% per month. What was the annual inflation rate? (*LO5-5*)

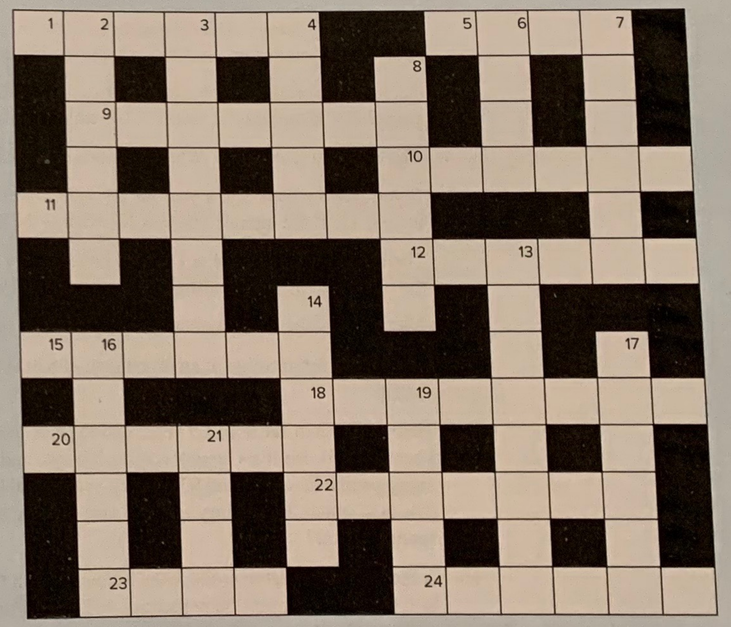
72. **Real versus Nominal Annuities.** You plan to retire in 30 years and want to accumulate enough by then to provide yourself with $30,000 a year for 15 years. (*LO5-5*)
    a. If the interest rate is 10%, how much must you accumulate by the time you retire?
    b. How much must you save each year until retirement in order to finance your retirement consumption?
    c. Now you remember that the annual inflation rate is 4%. If a loaf of bread costs $1 today, what will it cost by the time you retire?
    d. You really want to consume $30,000 a year in real dollars during retirement and wish to save an equal real amount each year until then. What is the real amount of savings that you need to accumulate by the time you retire?
    e. Calculate the required preretirement real annual savings necessary to meet your consumption goals.
    f. What is the nominal value of the amount you need to save during the first year? (Assume the savings are put aside at the end of each year.)
    g. What is the nominal value of the amount you need to save during the 30th year?

73. **Retirement and Inflation.** A couple will retire in 50 years; they plan to spend about $30,000 a year (in current dollars) in retirement, which should last about 25 years. They believe that they can earn 8% interest on retirement savings. The inflation rate over the next 75 years is expected to average 5%. (*LO5-5*)
    a. What is the real annual savings the couple must set aside?
    b. How much do they need to save in nominal terms in the first year?
    c. How much do they need to save in nominal terms in the last year?
    d. What will be their nominal expenditures in the first year of retirement?
    e. What will be their nominal expenditures in the last year of retirement?

74. **Real versus Nominal Cash Flows.** (*LO5-5*)
    a. It is 2016, you've just graduated college, and you are contemplating your lifetime budget. You think your general living expenses will average around $50,000 a year. For the next 8 years, you will rent an apartment for $16,000 a year. After that, you will want to buy a house that should cost around $250,000. In addition, you will need to buy a new car roughly once every 10 years, costing around $30,000 each. In 25 years, you will have to put aside around $150,000 to put a child through college, and in 30 years you'll need to do the same for another child. In 50 years, you will retire and will need to have accumulated enough savings to support roughly 20 years of retirement spending of around $35,000 a year on top of your Social Security benefits. The interest rate is 5% per year. What average salary will you need to earn to support this lifetime consumption plan?
    b. Whoops! You just realized that the inflation rate over your lifetime is likely to average about 3% per year, and you need to redo your calculations. As a rough cut, it seems reasonable to assume that all relevant prices and wages will increase at around the rate of inflation. What is your new estimate of the required salary (in today's dollars)?

75. **Real versus Nominal Rates.** You will receive $100 from a savings bond in 3 years. The nominal interest rate is 8%. (*LO5-5*)
    a. What is the present value of the proceeds from the bond?
    b. If the inflation rate over the next few years is expected to be 3%, what will the real value of the $100 payoff be in terms of today's dollars?
    c. What is the real interest rate?
    d. Show that the real payoff from the bond [from part (b)] discounted at the real interest rate [from part (c)] gives the same present value for the bond as you found in part (a).

76. **Time Value of Money.** Solve the following crossword. (Round your final answers to the nearest dollar, but do not round intermediate calculations.)



ACROSS

1. A rich uncle has promised to pay you $3,160 at the end of each of the next 25 years. If you invest this money at 4.9%, how much will you have at the end of the 25 years?

5. Hepsibah Sloop put $10,000 under his mattress 20 years ago. Since then, inflation has averaged 1%. What is the real value of his savings?

9. A 20-year annuity has a present value of $42,419,233. If the interest rate is 15%, what is the annual cash flow?
10. A Treasury bond pays $1 million at the end of 20 years. What is its present value if the interest rate is 4%?
11. A project is forecast to produce a safe cash flow of $800,000 a year for 20 years. If the cost of capital is 4%, what is the present value of the project?
12. If you invest $100,000 today, how much will your savings be worth at the end of 9 years if the interest rate is 10%?
15. An annuity due with 10 annual payments has a present value of 1,244,353. If the interest rate is 15%, what is the annual payment?
18. Investment in a Dustinbourne grinder is expected to produce cash flows over the following 7 years of $7 million, $8 million, $9 million, $10 million, $9 million, $8 million, and $7 million. What is the present value of the project if the cost of capital is 5%?
20. You invest $95,525 for 20 years at 11.7%. Inflation over the same period is 4% a year. What is the real value of your savings at the end of that period?
22. You invest $1 million for 7 years at 5% and then for a further 7 years at 8%. How much will you have at the end of this time?
23. The discount factor is .8, and the present value of a cash flow is $2,703. What is the cash flow?
24. A deferred annuity makes four equal payments of $129,987 a year starting at the end of year 8. If the interest rate is 12%, what is its present value?

DOWN

2. The winning lottery ticket promises to make 30 payments of $30,000 a year starting immediately. If the interest rate is 5.8%, what is the present value of these payments?
3. An investment in consol bonds promises to provide an interest payment of $2.5 million a year in perpetuity. If the interest rate is 3.3%, what is the value of this investment?
4. The discount factor is .9. What is the present value of $63,269?
6. If you pay the bank $50 a year for 10 years, it promises to pay you $100 a year forever starting in year 11. If the interest rate is 4.2%, what is the value of this proposal to you?
7. The nominal interest rate is 13% and inflation is 4%. If you invest $100,000 today, what will be the real value of your investment at the end of 20 years?
8. What is the value of a payment of $2,125,000 in year 30 if the interest rate is 3.8%?
13. What is the future value of an investment of $20 million at the end of 11 years if the interest rate is 9%?
14. Henry Hub will retire in 25 years and expects to spend a further 20 years in retirement (i.e., years 26–45). He lives an extravagant lifestyle and has drawn up a detailed spreadsheet which suggests that he will spend $256,880 a year in real terms in retirement. If the real interest rate is 3%, how much does Henry need to save each year in real terms until retirement to attain his spending goal?
16. A factory is forecast to produce the following cash flows: year 1, $6,516; year 2, $7,000; year 3, $11,400; year 4 onward in perpetuity, $12,000. If the cost of capital is 6%, what is the factory's present value?
17. What is the future value of $189,956 at the end of 7 years if the interest rate is 11%?
19. Natasha Petrov has savings of $78,780. If she invests the full amount in a bank at an interest rate of 4%, how much will she have after 4 years?
21. You plan to save $1,000 a year in each of years 1 through 5. If the interest rate is 8%, how much will you have saved by the end of year 5?

WEB EXERCISES

1. In Example 5.10, we showed you how to work out mortgage payments. Log on to the personal finance page of **www.bankrate.com** and find the mortgage payment calculator. Assume a 20-year mortgage loan of $100,000 and an interest rate (APR) of 12%. What is the amount of the monthly payment? Check that you get the same answer when using the annuity formula. Now look at how much of the first month's payment goes to reduce the size of the mortgage. How much of the payment by the 10th year? Can you explain why the figure changes? If the interest rate doubles, would you expect the mortgage payment to double? Check whether you are right.
2. You can find data on the consumer price index (CPI) on the Bureau of Labor Statistics website, **www.bls.gov/cpi/home.htm**. Tables of historical data can be formatted to provide either levels of the index or changes in the index (i.e., the rate of inflation). Construct a table of annual inflation rates since 1913. When did the USA last experience a year of deflation (i.e., falling prices)? Find the inflation rate in the latest year. Now log on to **www.bloomberg.com**, and on the first page find a measure of the short-term interest rate (e.g., the 2-year rate). Use the recent level of inflation to calculate the *real* interest rate. Consider the case of Herbert Protheroe, who in 1920 was an eligible bachelor with an income of $2,000 a year. What is that equivalent to today?

SOLUTIONS TO SELF-TEST QUESTIONS

5.1 Value after 5 years would have been $24 \times (1.05)^5 = \$30.63$; after 50 years, $24 \times (1.05)^{50} = \$275.22$.

5.2 Call g the annual growth rate of transistors over the 44-year period between 1971 and 2015. Then

$$2{,}250 \times (1+g)^{44} = 5{,}500{,}000{,}000$$
$$(1+g)^{44} = 2{,}444{,}444$$
$$1+g = 2{,}444{,}444^{1/44} = 1.40$$

So the actual growth rate was $g = .40$, or 40%, not quite as high as Moore's prediction, but not so shabby either.

5.3 Multiply the €1,000 payment by the 3-year discount factor:

$$\text{PV} = €1{,}000 \times \frac{1}{(1.01)^3} = €970.59$$

5.4

Gift at Year	Present Value
1	$10{,}000/(1.07) = \$9{,}345.79$
2	$10{,}000/(1.07)^2 = 8{,}734.39$
3	$10{,}000/(1.07)^3 = 8{,}162.98$
4	$10{,}000/(1.07)^4 = 7{,}628.95$
	33,872.11

Gift at Year	Future Value
1	$10{,}000 \times (1.07)^3 = \$12{,}250.43$
2	$10{,}000 \times (1.07)^2 = 11{,}449$
3	$10{,}000 \times (1.07) = 10{,}700$
4	$10{,}000 = 10{,}000$
	$44,399.43

5.5 The rate is 4/48 = .0833, about 8.3%.

5.6 The 4-year discount factor is $1/(1.08)^4 = .7350$. The 4-year annuity factor is $[1/.08 - 1/(.08 \times 1.08^4)] = 3.3121$. This is the difference between the present value of a $1 perpetuity starting next year and the present value of a $1 perpetuity starting in year 5:

$$\text{PV (perpetuity starting next year)} = \frac{1}{.08} = 12.50$$

$$-\text{PV (perpetuity starting in year 5)} = \frac{1}{.08} \times \frac{1}{(1.08)^4} = 9.1879$$

$$= \text{PV (4-year annuity)} \qquad = 12.50 - 9.1879 = 3.3121$$

which matches the annuity factor.

5.7 You will need the present value at 7% of a 20-year annuity of $55,000:

$$\text{Present value} = \text{annual spending} \times \text{annuity factor}$$

The annuity factor is $[1/.07 - 1/(.07 \times 1.07^{20})] = 10.5940$. Thus, you need $55,000 × 10.594 = $582,670.

5.8 Fifteen years means 180 months.

$$\text{Mortgage payment} = \frac{100{,}000}{\text{180-month annuity factor}}$$

$$= \frac{100{,}000}{83.32}$$

$$= \$1{,}200.17 \text{ per month}$$

$1,000 of the first payment is interest. The remainder, $200.17, is amortization.

5.9 We know that the future value of an annuity due is equal to the future value of an ordinary annuity × $(1 + r)$. Therefore, if you make the first of your 50 annual investments immediately, then by the end of the 50 years your retirement savings will be 10% higher, $550,000.

5.10 The quarterly rate is 8/4 = 2%. The effective annual rate is $(1.02)^4 - 1 = .0824$, or 8.24%.

5.11 The cost in dollars will increase by 5% each year, to a value of $\$5 \times (1.05)^{50} = \$57.34$. If the inflation rate is 10%, the cost will be $\$5 \times (1.10)^{50} = \$586.95$.

5.12 The CPI in 1980 was 3.452 times its value in 1950 (see Table 5.8). Therefore, purchases that cost $250 in 1950 would have cost $250 × 3.452 = $863 in 1980. The value of a 1980 salary of $30,000, expressed in real 1950 dollars, is $30,000 × (1/3.452) = $8,691.

5.13 a. If there's no inflation, real and nominal rates are equal at 8%. With 5% inflation, the real rate is (1.08/1.05) − 1 = .02857, a bit less than 3%.

b. If you want a 3% *real* interest rate, you need a 3% nominal rate if inflation is zero and an 8.15% rate if inflation is 5%. Note that 1.03 × 1.05 = 1.0815.

5.14 The present value is

$$\text{PV} = \frac{\$5{,}000}{1.08} = \$4{,}629.63$$

The real interest rate is 2.857% (see Self-Test 5.13a). The real cash payment is $5,000/1.05 = $4,761.90. Thus,

$$\text{PV} = \frac{\$4{,}761.90}{1.02857} = \$4{,}629.63$$

5.15 Calculate the real annuity. The real interest rate is 1.10/1.05 − 1 = .0476. We'll round to 4.8%. The real annuity is

$$\text{Annual payment} = \frac{\$3{,}000{,}000}{\text{30-year annuity factor}} = \frac{\$3{,}000{,}000}{\frac{1}{.048} - \frac{1}{.048(1.048)^{30}}} = \frac{\$3{,}000{,}000}{15.7292} = \$190{,}728$$

You can spend this much each year in dollars of constant purchasing power. The purchasing power of each dollar will decline at 5% per year, so you'll need to spend more in nominal dollars: $190,728 × 1.05 = $200,264 in the second year, $190,728 × 1.05^2 = $210,278 in the third year, and so on.

MINICASE

Old Alfred Road, who is well-known to drivers on the Maine Turnpike, has reached his 70th birthday and is ready to retire. Mr. Road has no formal training in finance but has saved his money and invested carefully.

Mr. Road owns his home—the mortgage is paid off—and does not want to move. He is a widower, and he wants to bequeath the house and any remaining assets to his daughter.

He has accumulated savings of $180,000, conservatively invested. The investments are yielding 9% interest. Mr. Road also has $12,000 in a savings account at 5% interest. He wants to keep the savings account intact for unexpected expenses or emergencies.

Mr. Road's basic living expenses now average about $1,500 per month, and he plans to spend $500 per month on travel and hobbies. To maintain this planned standard of living, he will have to rely on his investment portfolio. The interest from the portfolio is $16,200 per year (9% of $180,000), or $1,350 per month.

Mr. Road will also receive $750 per month in Social Security payments for the rest of his life. These payments are indexed for inflation. That is, they will be automatically increased in proportion to changes in the consumer price index.

Mr. Road's main concern is with inflation. The inflation rate has been below 3% recently, but a 3% rate is unusually low by historical standards. His Social Security payments will increase with inflation, but the interest on his investment portfolio will not.

What advice do you have for Mr. Road? Can he safely spend all the interest from his investment portfolio? How much could he withdraw at year-end from that portfolio if he wants to keep its real value intact?

Suppose Mr. Road will live for 20 more years and is willing to use up all of his investment portfolio over that period. He also wants his monthly spending to increase along with inflation over that period. In other words, he wants his monthly spending to stay the same in real terms. How much can he afford to spend per month?

Assume that the investment portfolio continues to yield a 9% rate of return and that the inflation rate will be 4%.