CHAPTER 5½

The Monty Hall Problem

The "Monty Hall problem" is a famous probability-related conundrum faced by participants on the game show *Let's Make a Deal*, which premiered in the United States in 1963 and is still running in some markets around the world. (I remember watching the show whenever I was home sick from elementary school.) The program's gift to statisticians was described in the introduction. At the end of each day's show a contestant was invited to stand with host Monty Hall facing three big doors: Door no. 1, Door no. 2, and Door no. 3. Monty explained to the contestant that there was a highly desirable prize behind one of the doors and a goat behind the other two doors. The player chose one of the three doors and would get as a prize whatever was behind it. (I don't know if the participants actually got to keep the goat; for our purposes, assume that most players preferred the new car.)

The initial probability of winning was straightforward. There were two goats and one car. As the participant stood facing the doors with Monty, he or she had a 1 in 3 chance of choosing the door that would be opened to reveal the car. But as noted earlier, *Let's Make a Deal* had a twist, which is why the show and its host have been immortalized in the probability literature. After the contestant chose a door, Monty would open one of the two doors that the contestant *had not picked*, always

revealing a goat. At that point, Monty would ask the contestant if he would like to change his pick—to switch from the closed door that he had picked originally to the other remaining closed door.

For the sake of example, assume that the contestant has originally chosen Door no. 1. Monty would then open Door no. 3; a live goat would be standing there on a stage. Two doors would still be closed, nos. 1 and 2. If the valuable prize was behind no. 1, the contestant would win; if it was behind no. 2, he would lose. That's when Monty would turn to the player and ask whether he would like to change his mind and switch doors, from no. 1 to no. 2 in this case. Remember, both doors are still closed. The only new information the contestant has received is that a goat showed up behind one of the doors that he did not pick.

Should he switch?

Yes. The contestant has a $1/3$ chance of winning if he sticks with his initial choice and a $2/3$ chance of winning if he switches. If you don't believe me, read on.

I'll concede that this answer seems entirely unintuitive at first. It would appear that the contestant has a one-third chance of winning no matter what he does. There are three closed doors. At the beginning, each door has a one in three chance of holding the valuable prize. How could it matter whether he switches from one closed door to another?

The answer lies in the fact that Monty Hall knows what is behind each door. If the contestant picks Door no. 1 and there is a car behind it, then Monty can open either no. 2 or no. 3 to display a goat.

If the contestant picks Door no. 1 and the car is behind no. 2, then Monty opens no. 3.

If the contestant picks Door no. 1 and the car is behind no. 3, then Monty opens no. 2.

By switching after a door is opened, the contestant gets the benefit of choosing two doors rather than one. I will try to persuade you in three different ways that this analysis is correct.

The first is empirical. In 2008, *New York Times* columnist John Tierney wrote about the Monty Hall phenomenon.[1] The *Times* then constructed an interactive feature that allows you to play the game yourself, including the decision to switch or not. (There are even little goats and cars that pop out from behind the doors.) The game keeps track of your success when you switch doors after making your initial decision compared with when you do not. Try it yourself.* I paid one of my children to play the game 100 times, switching each time. I paid her brother to play the game 100 times without switching. The switcher won 72 times; the nonswitcher won 33 times. Both received two dollars for their efforts.

The data from episodes of *Let's Make a Deal* suggest the same thing. According to Leonard Mlodinow, author of *The Drunkard's Walk*, those contestants who switched their choice won about twice as often as those who did not.[2]

My second explanation gets at the intuition. Let's suppose the rules were modified slightly. Assume that the contestant begins by picking one of the three doors: no. 1, no. 2, or no. 3, just as the game is ordinarily played. But then, before any door is opened to reveal a goat, Monty says, "Would you like to give up your choice in exchange for *both of the other doors that you did not choose*?" So if you picked Door no. 1, you could ditch that door in exchange for what is behind no. 2 and no. 3. If you picked no. 3, you could switch to no. 1 and no. 2. And so on.

That would not be a particularly hard decision. Obviously you should give up one door in exchange for two, as it increases your chances of winning from $1/3$ to $2/3$. Here is the intriguing part: *That is exactly what Monty Hall allows you to do in the real game after he reveals the goat.* The fundamental insight is that if you were to choose two doors, one of them would always have a goat behind it anyway. When he opens a door to reveal a goat before asking if you'd like to switch, he's doing you a huge favor! He's saying (in effect), "There is a two-thirds chance that the car is behind one of the doors you didn't choose, and look, it's not that one!"

Think of it this way. Suppose you picked Door no. 1. Monty then

* You can play the game at http://www.nytimes.com/2008/04/08/science/08monty.html?_r=2&oref=slogin&oref=slogin.

offers you the option to take Doors 2 and 3 instead. You take the offer, giving up one door and getting two, meaning that you can reasonably expect to win the car $2/3$ of the time. At that point, what if Monty were to open Door no. 3—one of your doors—to reveal a goat? Should you feel less certain about your decision? Of course not. If the car were behind no. 3, he would have opened no. 2! *He's shown you nothing.*

When the game is played normally, Monty is really giving you a choice between the door you originally picked and the other two doors, only one of which could possibly have a car behind it. When he opens a door to reveal a goat, he's merely doing you the courtesy of showing you which of the other two doors does not have the car. You have the same probability of winning in both of the following scenarios:

1. Choosing Door no. 1, then agreeing to switch to Door no. 2 *and* Door no. 3 before any door is opened.
2. Choosing Door no. 1, then agreeing to switch to Door no. 2 after Monty reveals a goat behind Door no. 3 (or choosing no. 3 after he reveals a goat behind no. 2).

In both cases, switching gives you the benefit of two doors instead of one, and you can therefore double your chances of winning, from $1/3$ to $2/3$.

My third explanation is a more extreme version of the same basic intuition. Assume that Monty Hall offers you a choice from among 100 doors rather than just three. After you pick your door, say, no. 47, he opens 98 other doors with goats behind them. Now there are only two doors that remain closed, no. 47 (your original choice) and one other, say, no. 61. Should you switch?

Of course you should. There is a 99 percent chance that the car was behind one of the doors that you did not originally choose. Monty did you the favor of opening 98 of those doors that you did not choose, all of which he knew did not have the car behind them. There is only a 1 in 100 chance that your original pick was correct (no. 47). There is a 99 in 100 chance that your original pick was not correct. And if your original pick was not correct, then the car is sitting behind the other door, no. 61. If you want to win 99 times out of 100, you should switch to no. 61.

In short, if you ever find yourself as a contestant on *Let's Make a Deal*, you should definitely switch doors when Monty Hall (or his replacement) gives you the option. The more broadly applicable lesson is that your gut instinct on probability can sometimes steer you astray.