Chapter # 4.

Fall, 2019

Section 4.1: Sample Space and Probability.

Basic Concepts

A **probability experiment** is a chance process that leads to well-defined results called **outcomes.**

An outcome is the result of a single trial of a probability experiment.

Each repetition of the experiment we call a **trial.**

A sample space is the set of all possible outcomes of a probability experiment.

Example # 1: Give a sample space for each of the following experiments:

Experiment	Sample space
Answering a true/false question	
Roll a die (6 sides)	
Toss a coin one time	
The grade received in a course (Give two ways in which the space might be forms)	1st way: 2nd way:
Toss a coin two times	

A tree diagram is a device consisting of a line segments emanating from a starting point and also from the outcome points. It is used to determine all possible outcomes of a probability experiment.

Example # 2: Find the sample space for the gender of the children if a family has three children. Use a tree diagram.

Example # 3: A box contains a $1 bill, a $5 bill, and a $10 bill. A bill is selected at random, and it is **not replaced**; then a second bill is selected at random. Draw a tree diagram and determine the sample space.

Example 4: Three balls numbered 1 through 3 are placed in a box. A ball is selected at random, and its number is noted; then it is **replaced**. A second ball is selected at random, and its number is noted. Draw a tree diagram and determine the sample space.

An event is subset of a sample space.

An event with **one outcome** is called **a simple event.**

An event with **more than one outcome** is called **a compound event.**

Example # 5: The experiment is to draw one number from the numbers 1 through 10. List all outcomes for the following events:

Event	Outcomes	Number of outcomes	Probability
Draw an even number			
Draw an odd number			
Draw a prime number			
Draw a multiple of 4			
Draw a multiple of 7			
Draw a number that is greater than 12			
Draw a natural number that is less than 11			

Example # 6: (Use the result of Example 2). A family has three children. List all outcomes for the following events:

Event	Outcomes	Number of outcomes	Probability
Exactly 2 girls			
At least 2 girls			
Less than 2 boys			
More than 2 boys			

Classical probability

Classical probability assumes that all outcomes in the sample space are equally likely to occur.

Formula for Classical Probability

The probability of any event E is

$$\frac{\text{Number of outcomes in E}}{\text{Total number of outcomes in the sample space}}$$

This probability is denoted by $P(E) = \dfrac{n(E)}{n(S)}$

Probability Rules:

1. The probability of any event E is a number (either a fraction or decimal) between and including 0 and 1. This is denoted by $0 \le P(E) \le 1$.
2. If an event E cannot occur (the event contains no members in the sample space), its probability is 0.
3. If an event E is certain, then the probability of E is 1.
4. The sum of the probabilities of all outcomes in the sample space is 1.

Rounding Rule for Probability

Probabilities should be expressed as reduced fractions or rounded to two or three decimal places. When probability of an event is an extremely small decimal, it is permissible to round to the first nonzero digit after the point.

Example # 7: If a die is rolled one time, find these probabilities:

a) Getting a number a 2

b) Getting an odd number

c) Getting a number greater than 6

d) Getting a 2 or an odd number

e) Getting a number less than 7

f) Getting an even number greater than 3

g) Getting a prime or odd number

h) Getting a prime number less than 7

Example # 8: A couple has three children. Find each probability:

a) All boys

b) All boys or all girls

c) Exactly 2 girls or exactly 2 boys

d) At least one child of each gender

e) All girls or exactly one boy

f) Exactly one child of each gender

Empirical Probability

Empirical probability relies on actual experience to determine the likelihood (probability) of outcomes.

Formula for Empirical Probability

Given a frequency distribution, the probability of an event being in a given class is

$$P(E) = \frac{\text{Frequency for the class}}{\text{Total frequencies in the distribution}} = \frac{f}{n}$$

This probability is denoted by $P(E) = \frac{n(E)}{n(S)}$

Example # 9: The owner of The Smoke House founds that 800 people bought hamburgers as follows:

Kind of Burger	Frequency	Relative frequency (Probability)
Miniburger	140	
Burger	345	
Big Burger	315	
Total		

Find the probability of a customer purchasing each kind of hamburger.

Law of Large Numbers

As an experiment is repeated more and more times, the relative frequency obtained approaches the actual probability.

Example # 10: The Law of Large Numbers has been tested for the coin tossing problem. The Comte de Buffon (1701 – 1788) , Karl Pearson (1857 – 1936), and John Kerrich, a prisoner of war during World War II , each tossed a coin many times. The results of their efforts were as follows:

	Tosses	Heads	Relative Frequency
Buffon	4,040	2,048	0.5069
Pearson	12,000	6,019	0.5016
Pearson	24,000	12,012	0.5005
Kerrich	10,000	5,067	0.5067

Subjective Probability

Subjective probability uses a probability value based on an educated guess or estimate, employing opinions and inexact information.

Example # 11:

a) A physician might say that, on the basis of his diagnosis, there is a 30% chance the patient will need an operation.

b) A seismologist might say there is an 80% chance that an earthquake will occur in certain area.

Complimentary Event

The **compliment** of an event E is the set of outcomes in the sample space that are not included in the outcomes of event E. The complement of E is denoted $\overline{E}$ (read " E bar").

Rule for Complimentary Events

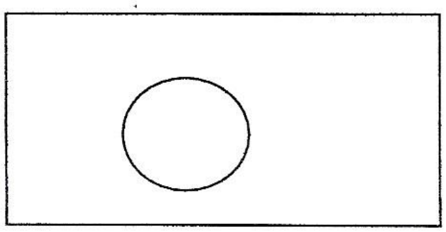
$$P(\overline{E})=1-P(E) \quad \text{or} \quad P(E)=1-P(\overline{E}) \quad \text{or} \quad P(E)+P(\overline{E})=1$$

Example # 12: The probability that Mary can work a problem is 70%. Find the probability that Mary cannot work the problem.

Example # 13: In 2004, 57.2% of all enrolled college students were females. Choose one enrolled student at random. What is the probability that the student was a male?

A Visual Model of Probability

Probabilities can be represented pictorially by **Venn diagrams.**
The area inside the circle represent the probability of event E, that is P(E). The area inside the rectangle represents the probability of all the events in the sample space P(S) = 1.



Section 4.2: Addition Rules for Probability.

Two events are **mutually exclusive events** if they cannot occur at the same time (they have no outcomes in common).

Example # 14: Determine whether these events are mutually exclusive.

a) Select any course: It is a calculus course and it is an English course.

b) Select any course: It is a calculus course and it is a mathematics course.

Addition Rule 1

When two events A and B are mutually exclusive, the probability that A and B will occur is $P(A \text{ or } B) = P(A) + P(B)$

Example # 15: In a fish tank, there are 24 goldfish, 2 angel fish, and 5 guppies. If a fish is selected at random, find the probability that it is a goldfish or an angel fish.

Addition Rule 2

When two events A and B are not mutually exclusive, the probability that A and B will occur is $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

Example # 16: In a particular school with 400 students, 110 play football, 80 play basketball, and 30 play both. What is the probability that randomly selected student plays football or basketball?

Example # 17: In a statistics class there are 18 juniors and 10 seniors; 6 of the seniors are females, and 12 of the juniors are males. If the student is selected at random, find the probability of selecting the following:

a) A junior or a female

b) A senior or a female

c) A junior or a senior

d) A female

e) A junior female

f) A junior female or a senior male

g) A male or a junior female

	Juniors	Seniors	Total
Male			
Female			
Total			

Example # 18: The frequency distribution shown here illustrates the number of medical tests conducted on 30 randomly selected emergency patients.

Number of tests performed	Number of patients
0	12
1	8
2	2
3	3
4 or more	5
Total	

If the patient is selected at random, find these probabilities:

a) The patient has had exactly 2 tests done.

b) The patient has had at least 2 tests done.

c) The patient has had at most 3 tests done.

d) The patient has had 3 or fewer tests done.

e) The patient has had 1 or 2 tests done.

f) The patient has had at least 1 test done.

g) The patient has had at most 1 test done.

Section 4.3: The Multiplication Rules and Conditional Probability.

Two events A and B are **independent events** if the fact that A occurs does not affect the probability of B occurring.

When the outcome or occurrence of the first event affects the outcome or occurrence of the second event in such way that the probability is changed, the events are said to be **dependent events.**

Example # 19: Determine whether these events are dependent or independent:

a) Tossing a coin and drawing a card from a deck.

b) Drawing a ball from an urn, **not replacing it**, and then drawing a second ball.

c) Drawing a ball from an urn, **replacing it**, and then drawing a second ball.

Multiplication Rule 1

When two events A and B are **independent,** the probability of both occurring is
$P(A \text{ and } B) = P(A) \cdot P(B)$

Example # 20: A coin is flipped and a die is rolled. Find the probability of getting a head on the coin and a 4 on the die.

Example # 21: An urn contains 5 red balls and 3 white balls. A ball is selected and its color noted. Then it is replaced. A second ball is selected and its color noted. Find the probability of each of these.

a) Selecting two red balls.

b) Selecting two white balls.

c) Selecting 1 red ball and then 1 white ball.

The conditional probability of event B in relationship to an event A is the probability that event B occurs after event A has already occurred.

Multiplication Rule 2

When two events A and B are dependent, the probability of both occurring is
$P(A \text{ and } B) = P(A) \cdot P(B \mid A)$

Example # 22: An urn contains 5 red balls and 3 white balls. A ball is selected without replacement. Then a second ball is selected. Find the probability of each of these.

a) Selecting two red balls.

b) Selecting two white balls.

c) Selecting 1 red ball and then 1 white ball.

Formula for Conditional Probability

The probability that the second event B occurs given that the first event A has occurred can be found by the following formula:

$$P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)}$$

Example # 23: At a large university, the probability that a student takes calculus and is on the dean's list is 0.042. The probability that the student is on the dean's list is 0.21. Find the probability that the student is taking calculus, given that he or she is on the dean's list.

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Example # 24: A Gift Basket Store had the following premade gift baskets containing the following combinations in stock.

	Cookies	Mugs	Candy	Total
Coffee	20	13	10	
Tea	12	10	12	
Total				

Choose 1 basket at random. Find the probability that it contains:

a)Coffee or candy

b)Tea given that it contains mugs

c)Mugs given that it contains tea

d)Tea and cookies

e)Coffee and mugs or tea and candy

Example # 25: Urn 1 contains 5 red balls and 3 black balls. Urn 2 contains 3 red balls and 1 black ball. Urn 3 contains 4 red balls and 2 black balls. If an urn is selected at random and a ball is drawn, find the probability it will be red.

Example # 26: In 2006, 86% of U.S. households had cable TV. Choose 3 households at random. Find the probability that

a)None of the 3 households had cable TV

b)All 3 households had cable TV

c)At least 1 out of 3 households had cable TV

Example # 27: A coin is tossed 3 times. Find the probability of getting at least 1 tail.

Example # 28: A coin is tossed 5 times. Find the probability of getting at least 1 tail.

Example # 29: A medication is 75% effective against a bacterial infection. Find the probability that if 12 people take the medication, at least 1 person's infection will not improved.

Section 4.4: Counting Rules

The Fundamental Counting Rule

In a sequence of n events in which the first one has k_1 possibilities and the second event has k_2 possibilities and the third event has k_2 possibilities and so forth , the total number of possibilities of the sequence will be $k_1 \cdot k_2 \cdot k_3 \cdots \cdot k_n$

Example # 30: A particular cell phone company offers 4 models of phones, each in 6 different color and each available with any one of 5 calling plans. How many combinations are possible?

Example # 31: a) How many 5-digits zip codes are possible if digits can be repeated?

b) How many 5-digits zip codes are possible if digits cannot be repeated?

Permutations

A permutation is an arrangement of n objects in a specific order.

Permutation Rule

The number of permutations of r objects selecting from n objects is denoted by $_nP_r$ and is given by the formula

$$_nP_r = \frac{n!}{(n-r)!}$$

Example #32: Ten students each submit one essay for competition. In how many ways can the first, second, and third prizes be awarded?

Combinations

A selection of distinct objects without regard to order is called **combination.**

Combination Rule

The number of combinations of r objects selecting from n objects is denoted by $_nC_r$ and is given by the formula

$$_nC_r = \frac{n!}{(n-r)!r!}$$

Example # 33: A student has 7 books on his deck. In how many different ways can he select a set of 3?

Example # 34: a) In how many ways a committee of 4 be selected from a group of 10 people?

b) In how many ways can a slate of officers consisting of a president, vice-president, and secretary be selected from a group of 10 people?

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