American University Of The Middle East

Regular Expressions & Finite State Automata (FSA)

Examples of Alphabet Σ:

1. English Language: $\Sigma_E = \{a, b, c, d, e, f, g, h, \ldots, x, y, z\}$
2. Digits Language: $\Sigma_D = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$
3. Binary Language: $\Sigma_B = \{0, 1\}$
4. Season Language: $\Sigma_S = \{Winter, Spring, Summer, Fall\}$

Examples of Alphabet Σ:

1. $\Sigma_B \cup \Sigma_S = \{Winter, Spring, Summer, Fall, 0, 1\}$
2. $\Sigma_B \cap \Sigma_S = \emptyset$
3. $\Sigma_B - \Sigma_S = \Sigma_B$
4. $\Sigma_S - \Sigma_B = \Sigma_S$
5. $\Sigma_S \times \Sigma_B = \{Winter0, Winter1, Spring0, Spring1,$ $Summer0, Summer1, Fall0, Fall1\}$

Regular Expressions

1. A regular expression is how to describe language strings in a mathematical way
2. Languages could have infinite number of strings that belong to it and that is why it is impossible to be able to mention all strings that belong to this language.
3. That is why regular expressions are very important because they turn an infinite set into a single expression.
4. An example to a regular expression is $(0|1)^*$ which represents all binary strings. Note that binary strings are infinite since the number of characters is not defined.

1. $a^* = \{\lambda, a, aa, aaa, aaaa, \ldots\}$
2. $(ab)^* = \{\lambda, ab, abab, ababab, abababab, \ldots\}$
3. $(a|b)^* = \{\lambda, a, b, aa, ab, ba, bb \ldots\}$

$$a^+ \equiv aa^*$$

1. $a^+ = \{a, aa, aaa, aaaa, \ldots\}$
2. $(ab)^+ = \{ab, abab, ababab, abababab, \ldots\}$
3. $(a|b)^+ = \{a, b, aa, ab, ba, bb \ldots\}$

a^* means repeating a **zero or more times**

a^+ means repeating a **one or more times**

Example 1: Find the language that the following regular expressions represent:

$$\mathbb{R} = a^*b^*$$

The expression states that we have 0 or more **a** followed by 0 or more **b** :

$$\mathbb{L} = \{\lambda, a, aa, aaa, \ldots, b, bb, bbb, \ldots ab, aab, abb, \ldots\}$$

Example 2: Find the language that the following regular expressions represent:

$$\mathbb{R} = a^*|b^+$$

The expression states that we have either 0 or more **a** or 1 or more **b** :

$$\mathbb{L} = \{\lambda, a, aa, aaa, \dots, b, bb, bbb, \dots\}$$

Example 3: Find the language that the following regular expressions represent:

$$\mathbb{R} = a^+b^+$$

The expression states that we have 1 or more **a** followed by 1 or more **b** :

$$\mathbb{L} = \{ab, aab, abb, \dots\}$$

Example 4: Find the language that the following regular expressions represent:

$$\mathbb{R} = (a|bc)^*$$

The expression states that we have any combination of 0 or more **a** or 0 or more **bc** or both:

$$\mathbb{L} = \{\lambda, a, aa, aaa, \ldots, bc, bcbc \ldots abc, aabc, aabcbc, \ldots\}$$

Example 5: Find the language that the following regular expressions represent:

$$\mathbb{R} = ((ab)|b)^+$$

The expression states that we have 1 or more **ab** or 1 or more **b** or any combination :

$$\mathbb{L} = \{\lambda, ab, abab, ababab, \ldots, b, bb, bbb, \ldots abb, bab, \ldots\}$$

Example 6: Find the language that the following regular expressions represent:

$$\mathbb{R} = (a|b|c)^*$$

The expression states that we have 0 or more **a** or 0 or more **b** or 0 or more **c** or any combination:.

$$\mathbb{L} = \{\lambda, a, aa, \ldots, b, bb, \ldots, c, cc, \ldots, ab, abb, aab, aac, acc, bc, cb, ca, \ldots\}$$

Example 7: Find the language that the following regular expressions represent:

$$\mathbb{R} = (abc|(ba)^*)^+$$

The expression states that we have 1 or more **abc** or 0 or more **ba** :

$$\mathbb{L} = \{\lambda, ba, baba, abc, abcabc, abcba, baabc, babaabc, abcabcba, \ldots\}$$

Example 8: Find the language that the following regular expressions represent:

$$\mathbb{R} = (ab|c|d)^*$$

The expression states that we have 0 or more **ab** or 0 or more **c** or 0 or more **d** or any combination:.

$$\mathbb{L} = \{\lambda, ab, c, d, abab, cc, dd, abc, abd, cab, dab, dc, \ldots\}$$

Example 9: Find the language that the following regular expressions represent:

$$\mathbb{R} = (a^* b^+ | c^*)^*$$

The expression states that we have 0 or more **a** followed by 1 or more **b** or 0 or more **c** and the outer star makes all combination possible.

$$\mathbb{L} = \{\lambda, b, ab, abb, aab, c, cab, abc, babc \ldots\}$$

Example 10: Find the language that the following regular expressions represent:

$$\mathbb{R} = ((a|b)^* c)^*$$

The expression states that we have 0 or more **a** or 0 or more **b** followed by a c :

$$\mathbb{L} = \{\lambda, c, ac, bc, abc, abababc, abcabc, \ldots\}$$

Example 11: For the following language, find the regular expression.

$$\mathbb{L} = \{\lambda, 0, 1, 01, 10, 11, \dots\}$$

The language above represents the binary language with an alphabet:

$$\Sigma = \{0, 1\}$$

. If you look at the strings you will find that every string is built of 0s or 1s with string length of 0 characters or more. The expression then is:

$$\mathbb{R} = (0|1)^*$$

Example 12: For the following language, find the regular expression.

$$\mathbb{L} = \{0, 1, 01, 10, 11, \ldots\}$$

The language above represents the binary language with an alphabet

$$\Sigma = \{0, 1\}$$

. If you look at the strings you will find that every string is built of 0s or 1s with string length 1 character or more. The expression then is:

$$\mathbb{R} = (0|1)^+$$

Example 13: For the following language, find the regular expression.

$$\mathbb{L} = \{0, 1, 10, 11, 100, 101, 110, 111 \ldots\}$$

The language above represents the binary language with an alphabet

$$\Sigma = \{0, 1\}$$

and strings that either a 0 or starts with a 1. The expression then is:

$$\mathbb{R} = 0|1(0|1)^*$$

Example 14: For the following language, find the regular expression.

$$\mathbb{L} = \{0, 00, 0000, 000000, \dots\}$$

The language above represents the binary language with an alphabet

$$\Sigma = \{0\}$$

and strings that either a 0 or have even number of zeros and it cannot be empty. The expression then is:

$$\mathbb{R} = 0|(00)^{+}$$

Example 15: For the following language, find the regular expression.

$$\mathbb{L} = \{0, 010, 01010, 0101010, \dots\}$$

The language above represents the binary language with an alphabet

$$\Sigma = \{0, 1\}$$

and strings that start with a 0 followed by 0 or more of 10. The expression then is:

$$\mathbb{R} = 0(10)^*$$

Example 16: For the following language, find the regular expression.

The language of binary even numbers

The language above represents the binary language with an alphabet

$$\Sigma = \{0, 1\}$$

and strings that represent binary even numbers. In other words, binary numbers that end with a 0:

$$\mathbb{R} = (1|0)^*0$$

Example 17: For the following language, find the regular expression.

The language of binary odd numbers

The language above represents the binary language with an alphabet

$$\Sigma = \{0, 1\}$$

and strings that represent binary odd numbers. In other words, binary numbers that end with a 1:

$$\mathbb{R} = (1|0)^*1$$

Example 18: For the following language, find the regular expression.

The language of binary numbers with strings that start with 1 and end with 1

The language above represents the binary language with an alphabet

$$\Sigma = \{0, 1\}$$

and strings that start with 1 and end with 1 :

$$\mathbb{R} = 1(1|0)^*1$$

Example 19: For the following language, find the regular expression.

<u>The language of binary numbers with strings that end with 2 zeros</u>

The language above represents the binary language with an alphabet

$$\Sigma = \{0, 1\}$$

and strings that end with 2 zeros :

$$\mathbb{R} = (1|0)^*00$$

Example 20: For the following language, find the regular expression.

The language of binary numbers with strings that contain at least 2 ones

The language above represents the binary language with an alphabet

$$\Sigma = \{0, 1\}$$

and strings that contain at least 2 ones :

$$\mathbb{R} = (1|0)^* 1 (1|0)^* 1 (1|0)^*$$

Example 21: For the following language, find the regular expression.

The language of binary numbers with strings that contain contain 110

The language above represents the binary language with an alphabet

$$\Sigma = \{0, 1\}$$

and strings that contain 110 :

$$\mathbb{R} = (1|0)^*110(1|0)^*$$

Example 22: For the following language, find the regular expression.

The language of binary numbers with strings that have at least 3 chars 3^{rd} is 0

The language above represents the binary language with an alphabet

$$\Sigma = \{0, 1\}$$

and strings that have at least 3 chars, 3^{rd} is 0 :

$$\mathbb{R} = (0|1)(0|1)0(0|1)^*$$

Example 23: For the following language, find the regular expression.

The language of binary numbers with strings with length that is a multiple of 3

The language above represents the binary language with an alphabet

$$\Sigma = \{0, 1\}$$

and strings with length that is a multiple of 3 :

$$\mathbb{R} = ((0|1)|(0|1)|(0|1))^*$$

Example 24: For the following language, find the regular expression.

The language of binary numbers with strings having the number of 0s a multiple of 3

The language above represents the binary language with an alphabet

$$\Sigma = \{0, 1\}$$

and strings with number of 0s a multiple of 3 :

$$\mathbb{R} = 1^*|(1^*01^*01^*01^*)^*$$

Example 25: For the following language, find the regular expression.

The language of binary numbers with strings that start and end with the same character

The language above represents the binary language with an alphabet

$$\Sigma = \{0, 1\}$$

and strings that start and end with the same character :

$$\mathbb{R} = (1(0|1)^*1)|(0(0|1)^*0)$$

Example 26: For the following language, find the regular expression.

The language of binary numbers with strings that start with 0 and has odd length

The language above represents the binary language with an alphabet

$$\Sigma = \{0, 1\}$$

and strings that start with 0 and has odd length :

$$\mathbb{R} = 0((0|1)(0|1))^*$$

Example 27: For the following language, find the regular expression.

The language of binary numbers with strings that start with 1 and has even length

Example 27: The answer is

The language above represents the binary language with an alphabet

$$\Sigma = \{0, 1\}$$

and strings that start with 1 and has even length :

$$\mathbb{R} = 1(0|1)((0|1)(0|1))^*$$

Example 28: For the following language, find the regular expression.

The language of binary numbers with strings that has odd length

Example 28: The answer is.

The language above represents the binary language with an alphabet

$$\Sigma = \{0, 1\}$$

and strings that has odd length :

$$\mathbb{R} = (1|0)((0|1)(0|1))^*$$

What is a regular expression again?

If we have the alphabet $\Sigma = \{a, b, c\}$ then

1.
   1. a is a regular expression
   2. b is a regular expression
   3. c is a regular expression
2. λ is a regular expression
3. if we have two regular expressions $\mathbb{R}_1$ and $\mathbb{R}_2$ then
   1. $\mathbb{R}_1\mathbb{R}_2$ is a regular expression
   2. $\mathbb{R}_1 + \mathbb{R}_2$ is a regular

What is a regular expression again?

If we have the regular expressions $\mathbb{R}_1, \mathbb{R}_2$ then

1. if $\mathbb{R}_3 = \mathbb{R}_1 + \mathbb{R}_2$ then, $\mathbb{R}_3$ a regular expression
2. if $\mathbb{R}_3 = \mathbb{R}_1\mathbb{R}_2$ then, $\mathbb{R}_3$ a regular expression
3. if $\mathbb{R}_3 = \mathbb{R}_1^*$ then, $\mathbb{R}_3$ a regular expression

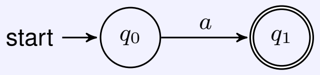
$\mathbb{R}_1 + \mathbb{R}_2$ is the same as $\mathbb{R}_1|\mathbb{R}_2$

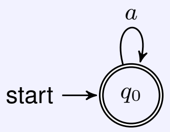
If we have the regular expressions $\mathbb{R}_1$, prove that $\mathbb{R}_1^+$ is a regular expressions.

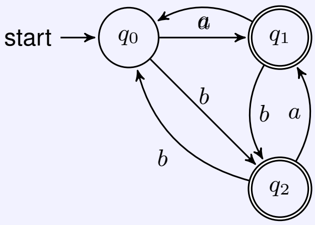
Finite **S**tate **A**utomata (FSA)

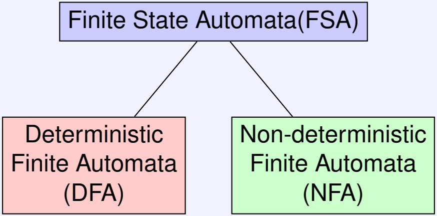
Finite State Automata (**FSA**)

A finite state automata is a **directed graph** that is built out of states and transitions.







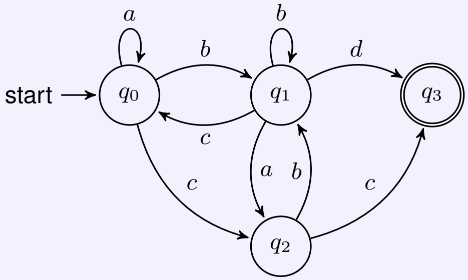

Finite State Automata(FSA)
Deterministic Finite Automata (DFA)
Non-deterministic Finite Automata (NFA)

Deterministic Finite State Automata **DFA**

$$\mathbb{M} = \langle \mathbb{Q}, \Delta, q_s, Q_a, \Sigma \rangle$$

Table 1: Transition Table.

	a	b	c	d
q_0	q_0	q_1	q_2	$\emptyset$
q_1	q_2	q_1	q_2	q_3
q_2	$\emptyset$	q_1	q_3	$\emptyset$
q_3	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$



This is Called **<u>D</u>**eterministic **<u>F</u>**inite State **<u>A</u>**utomata (**DFA**).. WHY?!

$$\mathbb{M} = \langle \mathbb{Q}, \Delta, q_s, Q_a, \Sigma \rangle$$

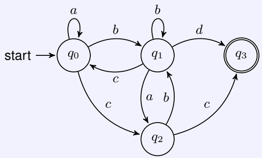
Δ :

1. $\delta(q_0, a) = q_0$
2. $\delta(q_0, b) = q_1$
3. $\delta(q_0, c) = q_2$
4. $\delta(q_0, d) = \emptyset$
5. $\delta(q_1, a) = q_2$
6. $\delta(q_1, b) = q_1$
7. $\delta(q_1, c) = q_0$
8. $\delta(q_1, d) = q_3$

Δ :

1. $\delta(q_2, a) = \emptyset$
2. $\delta(q_2, b) = q_1$
3. $\delta(q_2, c) = q_3$
4. $\delta(q_2, a) = \emptyset$
5. $\delta(q_3, a) = \emptyset$
6. $\delta(q_3, b) = \emptyset$
7. $\delta(q_3, c) = \emptyset$
8. $\delta(q_3, d) = \emptyset$

- $\mathbb{Q} = \{q_0, q_1, q_2, q_3\}$.
- $q_s = q_0$.
- $Q_a = \{q_3\}$.
- $\Sigma = \{a, b, c, d\}$.



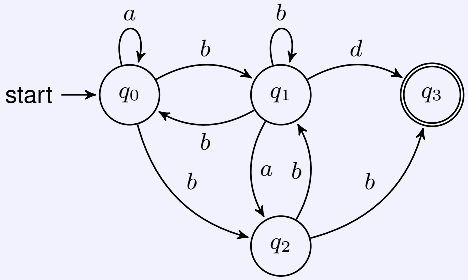
This is Called **D**eterministic **F**inite State **A**utomata (**DFA**).. WHY?!

non-deterministic Finite State Automata **NFA**

$$\mathbb{M} = \langle \mathbb{Q}, \Delta, q_s, Q_a, \Sigma \rangle$$

Table 2: Transition Table.

	a	b	d
q_0	q_0	$\{q_1, q_2\}$	$\emptyset$
q_1	q_2	$\{q_0, q_1\}$	q_3
q_2	$\emptyset$	$\{q_1, q_3\}$	$\emptyset$
q_3	$\emptyset$	$\emptyset$	$\emptyset$



This is Called **N**on-deterministic **F**inite State **A**utomata (**NFA**).. WHY?!

$$\mathbb{M} = \langle \mathbb{Q}, \Delta, q_s, Q_a, \Sigma \rangle$$

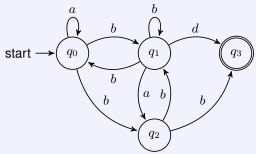
Δ :

1. $\delta(q_0, a) = q_0$
2. $\delta(q_0, b) = \{q_1.q_2\}$
3. $\delta(q_0, d) = \emptyset$
4. $\delta(q_1, a) = q_2$
5. $\delta(q_1, b) = \{q_0, q_1\}$
6. $\delta(q_1, d) = q_3$

Δ :

1. $\delta(q_2, a) = \emptyset$
2. $\delta(q_2, b) = \{q_1, q_3\}$
3. $\delta(q_2, d) = \emptyset$
4. $\delta(q_3, a) = \emptyset$
5. $\delta(q_3, b) = \emptyset$
6. $\delta(q_3, d) = \emptyset$

- $\mathbb{Q} = \{q_0, q_1, q_2, q_3\}$.
- $q_s = q_0$.
- $Q_a = \{q_3\}$.
- $\Sigma = \{a, b, d\}$.

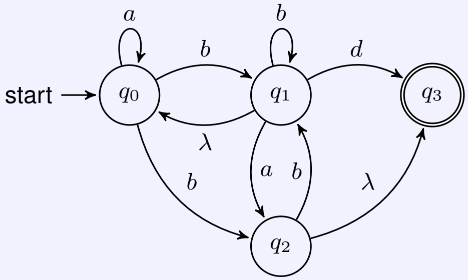


This is Called **N**on-deterministic **F**inite State **A**utomata (**NFA**).. WHY?!

$$\mathbb{M} = \langle \mathbb{Q}, \Delta, q_s, Q_a, \Sigma \rangle$$

Table 3: Transition Table.

	a	b	d	λ
q_0	q_0	$\{q_1, q_2\}$	$\emptyset$	q_0
q_1	q_2	q_1	q_3	q_0
q_2	$\emptyset$	q_1	$\emptyset$	q_3
q_3	$\emptyset$	$\emptyset$	$\emptyset$	q_3



This is Called **N**on-deterministic **F**inite State **A**utomata (**NFA**).. WHY?!

$$\mathbb{M} = \langle \mathbb{Q}, \Delta, q_s, Q_a, \Sigma \rangle$$

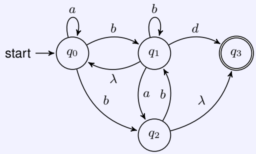
Δ :

1. $\delta(q_0, a) = q_0$
2. $\delta(q_0, b) = \{q_1 . q_2\}$
3. $\delta(q_0, d) = \emptyset$
4. $\delta(q_0, \lambda) = q_0$
5. $\delta(q_1, a) = q_2$
6. $\delta(q_1, b) = \{q_1\}$
7. $\delta(q_1, \lambda) = \{q_0\}$
8. $\delta(q_1, d) = q_3$

Δ :

1. $\delta(q_2, a) = \emptyset$
2. $\delta(q_2, b) = q_1$
3. $\delta(q_2, \lambda) = q_3$
4. $\delta(q_2, d) = \emptyset$
5. $\delta(q_3, a) = \emptyset$
6. $\delta(q_3, b) = \emptyset$
7. $\delta(q_3, d) = \emptyset$
8. $\delta(q_3, \lambda) = q_3$

- $\mathbb{Q} = \{q_0, q_1, q_2, q_3\}$.
- $q_s = q_0$.
- $Q_a = \{q_3\}$.
- $\Sigma = \{a, b, d\}$.

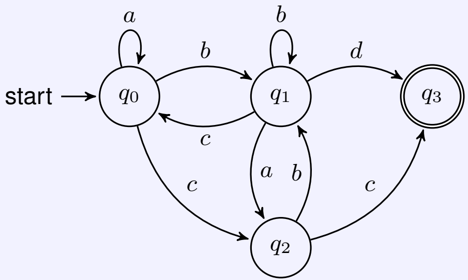


This is Called **N**on-deterministic **F**inite State **A**utomata (**NFA**).. WHY?!

String acceptance

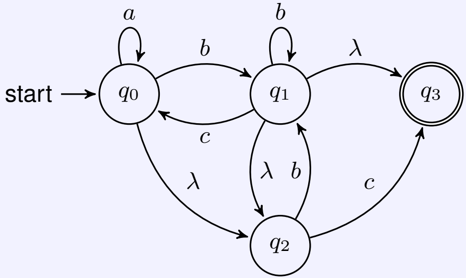
Which of the strings below is accepted by the given F.S.A

1. *abd*
2. *aaa*.
3. *bd*.
4. *bc*.
5. *bbbac*
6. *abbccbd*
7. *aaacbbbcabd*
8. *bbbac*
9. *babbbccc*
10. *babbbcccabbac*
11. *abbbab*
12. *cbbbbcc*
13. *cbababccbac*

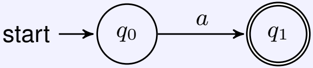


Which of the strings below is accepted by the given F.S.A

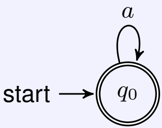
1. ab
2. aac.
3. aab.
4. b.
5. bb.
6. bbc
7. $abbc$ HARD ONE.
8. c
9. $aabbbcc$
10. $babbbaaabbbcbc$
11. $abbbca$ HARD ONE.
12. cab
13. $cbababccbac$



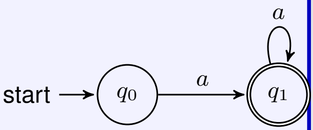
Regular Expressions to FSA and Vice Versa



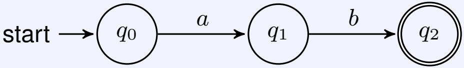
$\mathbb{R} = a$



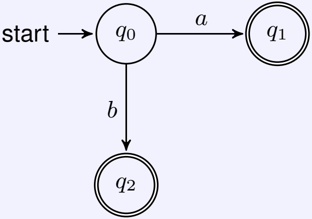
$\mathbb{R} = a^*$



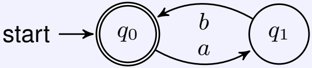
$\mathbb{R} = aa^* = a^+$



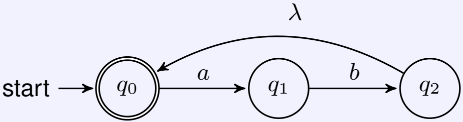
$\mathbb{R} = ab$



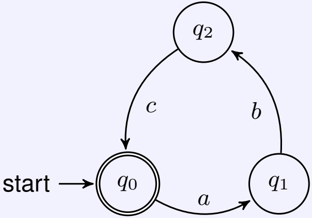
$\mathbb{R} = a + b = a|b$



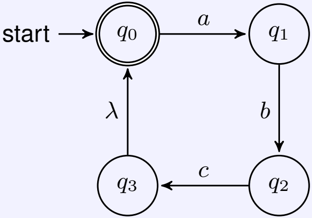
$\mathbb{R} = (ab)^*$



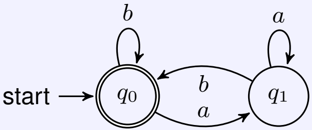
$\mathbb{R} = (ab)^*$



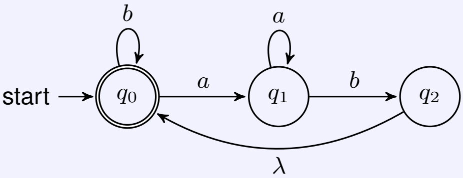
$\mathbb{R} = (abc)^*$



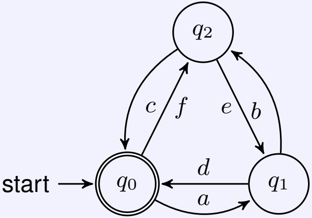
$\mathbb{R} = (abc)^*$



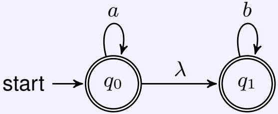
$\mathbb{R} = (b^*aa^*b)^*$



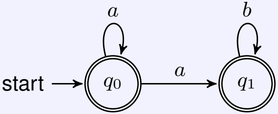
$\mathbb{R} = (b^*aa^*b)^*$



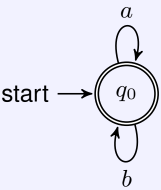
$$\mathbb{R} = (abc)^* + (fed)^* + (fc)^* + (febc)^* + (fedabc)^* + (abc)^* + (ad)^* + (abed)^* + (abcfed)^*$$



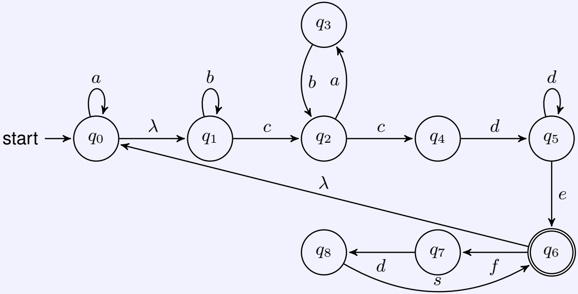
$\mathbb{R} = a^*b^*$



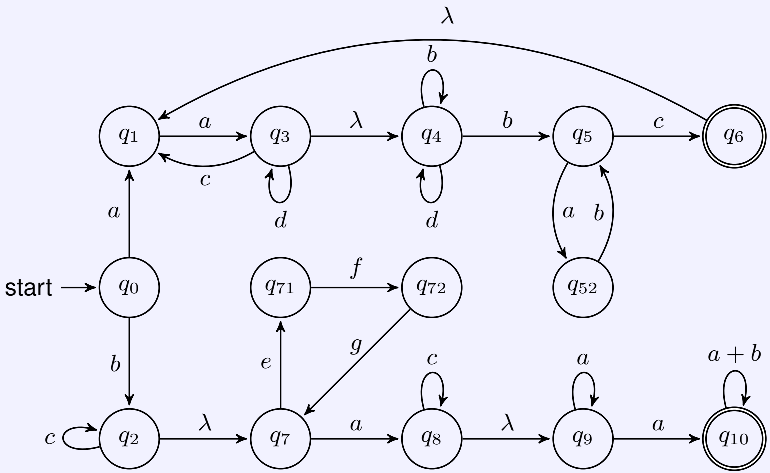
$\mathbb{R} = a^+b^*$



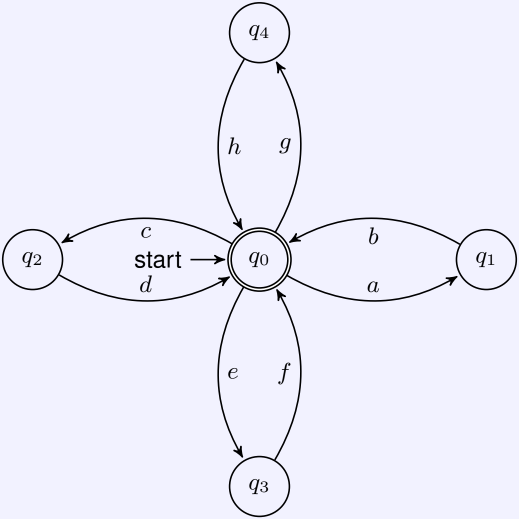
$\mathbb{R} = (a+b)^*$



$\mathbb{R} = (a^*b^*c(ab)^*cdd^*e(fds)^*)^*$

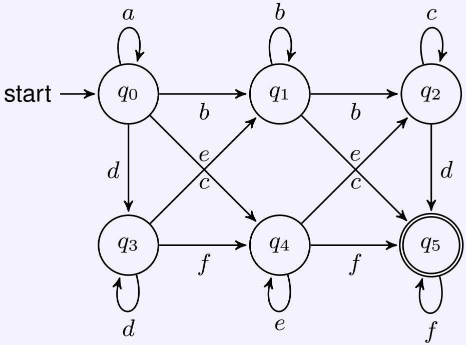


$$\mathbb{R} = a(a(d^*(ca)^*)^*(b+d)^*b(ab)^*c)^* + bc^*(efg)^*ac^*a^*a(a+b)^*$$



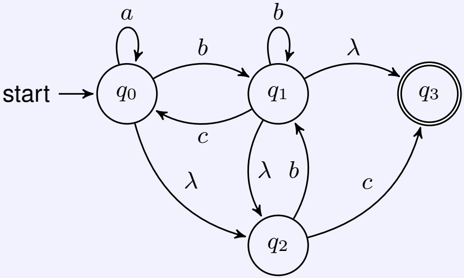
q_4
h
g
c
b
q_2
start
q_0
q_1
d
a
e
f
q_3

Find the regular expression for the machine below



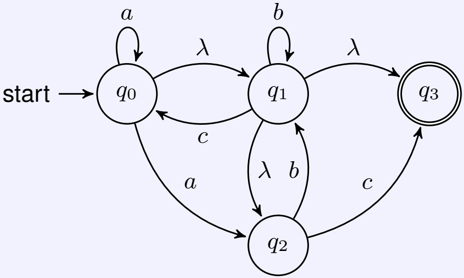
λ-closure

Find the $\lambda-$closure of the shown NFA below



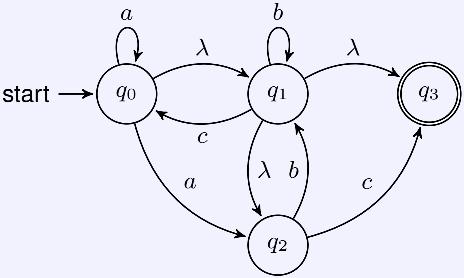
1. $\delta - closure(q_0) = \{q_0, q_2\}$
2. $\delta - closure(q_1) = \{q_1, q_2, q_3\}$

Find the $\lambda-$closure of the shown NFA below

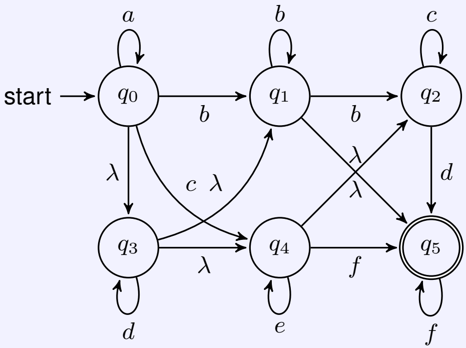


1. $\delta - closure(q_0) = \{q_0, q_1, q_2, q_3\}$
2. $\delta - closure(q_1) = \{q_1, q_2, q_3\}$

Find the λ−closure of the shown NFA below



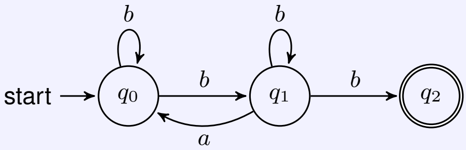
1. $\delta - closure(q_0) = \{q_0, q_1, q_2, q_3\}$
2. $\delta - closure(q_1) = \{q_1, q_2, q_3\}$



1. $\delta - closure(q_0) = \{q_0, q_1, , q_2, q_3, q_4, q_5\}$
2. $\delta - closure(q_1) = \{q_1, q_5\}$
3. $\delta - closure(q_3) = \{q_1, q_2, q_3, q_4, q_5\}$
4. $\delta - closure(q_4) = \{q_2, q_4\}$

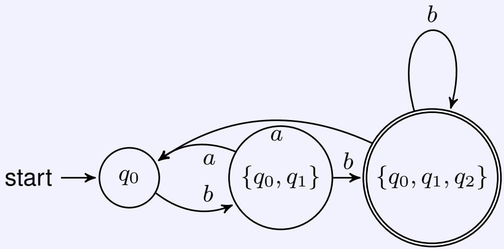
NFA to DFA

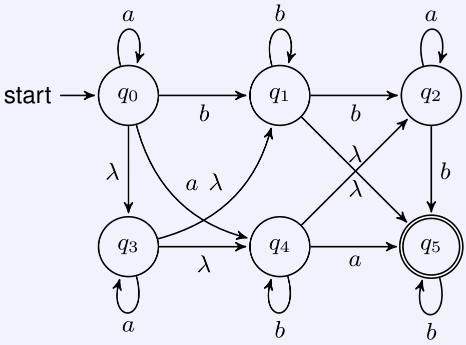
Convert the shown NFA to a DFA



1. $\delta(q_0, a) = \emptyset, \quad \delta(q_0, b) = \{q_0, q_1\}$
2. $\delta(q_1, a) = q_0, \quad \delta(q_1, b) = \{q_1, q_2\}$
3. $\delta(q_2, a) = \emptyset, \quad \delta(q_2, b) = \emptyset$

	a	b
q_0	$\emptyset$	$\{q_0, q_1\}$
$\{q_0, q_1\}$	q_0	$\{q_0, q_1, q_2\}$
$\{q_0, q_1, q_2\}$	q_0	$\{q_0, q_1, q_2\}$





1. $\delta - closure(q_0) = \{q_0, q_1, , q_2, q_3, q_4, q_5\}$
2. $\delta - closure(q_1) = \{q_1, q_5\}$
3. $\delta - closure(q_3) = \{q_1, q_2, q_3, q_4, q_5\}$
4. $\delta - closure(q_4) = \{q_2, q_4\}$

	a	b
q_0	$\{q_0, q_1, q_2, q_3, q_4, q_5\}$	$\{q_1, q_2, , q_4, q_5\}$
$\{q_0, q_1, q_2, q_3, q_4, q_5\}$	$\{q_0, q_1, q_2, q_3, q_4, q_5\}$	$\{q_1, q_2, q_4, q_5\}$
$\{q_1, q_2, q_4, q_5\}$	$\{q_2, q_5\}$	$\{q_1, q_2, q_4, q_5\}$
$\{q_2, q_5\}$	$\{q_2\}$	$\{q_5\}$
$\{q_2\}$	$\{q_2\}$	$\{q_5\}$
$\{q_5\}$	$\emptyset$	$\{q_5\}$

DRAW THE DFA


Thank you!

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