17 Sensitivity Analysis

17.1 INTRODUCTION

Expect that knowledge uncertainty will never be reduced to zero. Know that natural variability will never disappear. These two simple facts mean that the risk characterization from your risk assessment is, at best, an informed estimate. At worst, it may be well-intentioned speculation. What assessors and managers both need to know is that the outputs of a risk assessment are conditional answers based on the data and data gaps, on the assumptions and estimation tools, and on the techniques and methodologies used to arrive at the answers. They also need some idea of where the assessment answers lie on the continuum between the best estimate and well-intentioned speculation.

A simplistic schematic of a decision situation is shown in Figure 17.1. A risk assessment has several inputs that include knowledge, data, policy, and information in many forms. Data gaps and other forms of uncertainty are ubiquitous characteristics of these inputs.

Assumptions made to help address uncertain inputs find their way into the model and affect model outputs, e.g., risk characterizations and the like, which, in turn, influence the risk management decision. Any methodologies used to address the knowledge uncertainty and natural variability in inputs will result in a similar chain of events. Likewise, assumptions made about the model's structure will influence outputs and decisions.

Sensitivity analysis is the study of how the variation* in a risk assessment output can be apportioned, qualitatively or quantitatively, to different sources. Complex risk assessments may have dozens of input and output variables that are linked by a system of equations and calculations. Risk assessors need to consider how sensitive a model's output, a risk characterization, or other important assessment outputs are to changes or estimation errors that might occur in model inputs, model parameters, assumptions, scenarios, and the functional forms of models. This information must then be effectively conveyed to risk managers so they can explicitly consider its significance for their decision making.

Some risk assessment outputs and the decisions that rely on them may be sensitive to changes in assumptions and input values. However, it is not always immediately obvious which assumptions and uncertainties most affect outputs, conclusions, and decisions. The purpose of sensitivity analysis is to systematically find this out.

A good sensitivity analysis increases the assessor's and manager's confidence in the risk assessment model and its predictions. It provides a better understanding of how model outputs respond to changes in the inputs. Because risk assessments

* Variation includes the effects of natural variability and knowledge uncertainty.

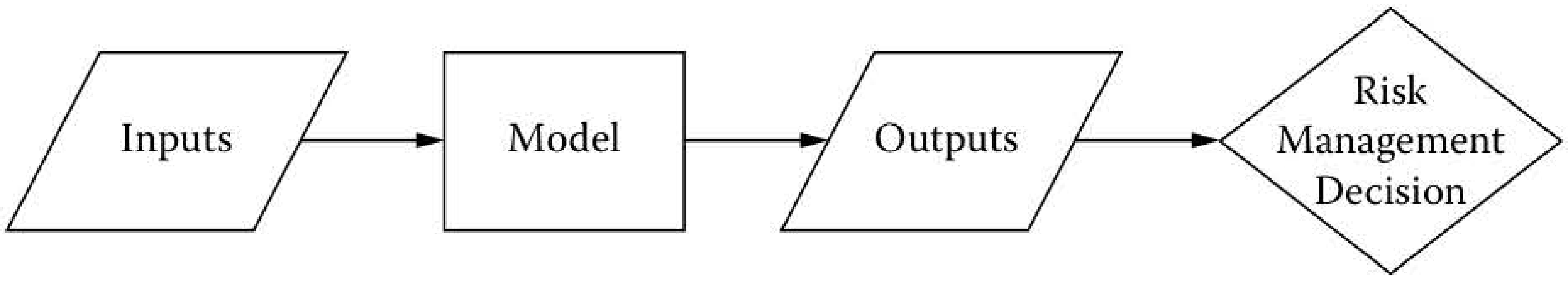

FIGURE 17.1 Decision model schematic.

can be qualitative or quantitative, sensitivity analysis can likewise be qualitative or quantitative. In a qualitative sensitivity analysis, the assessor identifies the uncertainties affecting the assessment and forms a judgment of their relative importance. A quantitative sensitivity analysis quantifies the variation in model outputs that is caused by specific model inputs and the model structure.

Some sensitivity analysis should be an integral component of every risk assessment. This is the point in a risk management activity when the risk analysis team focuses intentionally on better understanding the things we do not know and their importance for decision making. The results of the sensitivity analysis will provide insight into the importance of different sources of uncertainty.

Sensitivity analysis has at times been called what-if analysis. It can be used to answer questions like those below, which build on questions first provided by Mokhtari and Frey (2005).

- How might the decision be changed by increases or decreases in selected input values?
- What is the range of values a parameter can assume in a function without changing the decision?
- By how much must a value change to lead to an alternative best decision?
- How sensitive is our output to forecast error or other changes in inputs?
- Which inputs contribute most to the variation in the output?
- Which inputs are most responsible for the best (or worst) outcomes of the output?
- What is the rank order of importance among the model inputs?
- Are there two or more inputs to which the output has similar sensitivity, or is it possible to clearly distinguish and discriminate among the inputs with respect to their importance?
- Might changes in our decisions/actions improve our outputs?
- Does the model respond appropriately to changes in assumptions and inputs?

This chapter is divided into sections that address qualitative and quantitative approaches to sensitivity analysis. Material that could have been included in both of the sections will be addressed in the quantitative sensitivity analysis discussion. The most common sensitivity analysis methods are relatively simple techniques. There are others that are quite complex. The quantitative sensitivity analysis discussion summarizes methods from across this continuum. Examples are provided for the simpler methods; references are provided for the more complex ones.

17.2 QUALITATIVE SENSITIVITY ANALYSIS

Risk analysis is a framework for making decisions under uncertainty. Because your uncertainty cannot be eliminated, it is critically important to explore the importance of the uncertainty that remains prior to making a decision. A qualitative sensitivity analysis characterizes the uncertainty and its potential significance to decision making in nonnumerical ways. This is done to aid risk managers who need to make decisions in the face of this uncertainty. Quantitative sensitivity analysis is almost always preferred when it is feasible. Qualitative sensitivity analysis, being more subjective, is generally less reliable. At a minimum, qualitative sensitivity analysis provides a greater degree of confidence in assessment outputs and management decisions based on them for having identified and considered critical data gaps and other sources of uncertainty.

> Qualitative sensitivity analysis has been defined quite differently in some of the literature. Pianosi et al. (2016) say that in qualitative sensitivity analysis, sensitivity is assessed qualitatively by visual inspection of model predictions or by specific visualization tools like, for instance, tornado plots, scatter (or dotty) plots or representations of the posterior distributions of the input factors. In the view adopted in this text, this would more appropriately describe a form of exploratory quantitative analysis. The language is messy.

A basic methodology for qualitative sensitivity analysis includes:

- Identifying specific sources of uncertainty
- Ascertaining the sources of instrumental uncertainty
- Qualitatively characterizing the instrumental uncertainty

A reasonable objective for qualitative sensitivity analysis is to identify the sources of uncertainty that exert the most influence on the risk assessment outputs.

17.2.1 Identifying Specific Sources of Uncertainty

Identifying specific sources of uncertainty often begins with an acknowledgment that uncertainty exists. This is not always as easy as it seems it should be (see sidebar). A significant number of organizations still want to know "the number," and there is no incentive for acknowledging uncertainty in such a culture.

Once we are ready to acknowledge that uncertainty exists, we must recognize it when it is present. Knowledge is a relative commodity, and when we are used to working with relatively little data, for example, a lot of data can sometimes mask the uncertainty that remains. The assessor must be able to separate what is known from what is unknown in a decision problem.

We need to be able to identify and point to an input, assumption, scenario, or model and say this is uncertain. When we have done that, it is helpful to say why it is uncertain, i.e., identify what the cause of the uncertainty is. Next, it is useful to say how uncertain the cause of the uncertainty is and why the uncertainty is important. The initial goal is to honestly identify the things we do not know. A useful output from this step is a list of inputs recognized as uncertain, by type of input. The next step is to figure out which of them matter most.

THE CHANNEL BOTTOM

In an early proof of concept risk assessment for the U.S. Army Corps of Engineers an experienced engineer was questioned about a point estimate of the percentage of rock in a channel bottom. This is an important determinant of the cost of channel deepening. Offered the opportunity to bound this estimate he refused, insisting that he had better information from sample borings and more data than he had ever had in a long and successful career. He insisted there was no uncertainty about this value and was offended that we might think otherwise. His estimate turned out to be almost less than half of the actual rock content. Costs quickly doubled and he is now a proponent of risk assessment.

17.2.2 Ascertaining the Sources of Instrumental Uncertainty

An instrumental uncertainty is one that could affect the decision that is made or that could affect the outcome of a decision. Candidate instrumental uncertainties include those things that can affect model outputs, risk characterizations, conclusions, answers to the risk manager's questions, or other important decision criteria.

The best place to begin to identify the most important sources of uncertainty may be by considering what people say is important. Use the relevant theory, read the professional literature, talk to an expert, reason it through for yourself, and listen to what people are saying. Respect the wisdom of crowds and experts!

You need not be concerned about every uncertainty in your risk assessment. Using the extent of uncertainty may be a poor gage of the importance of an uncertainty. Focus first on identifying relevant uncertainties and then on identifying relevant and instrumental uncertainties. See Chapter 2 to review those concepts.

If you have used a mental or other model, look at the structure of the model. It reflects the extent to which a physical system or phenomenon is understood. Is there any aspect or element of the model that has more influence on its outputs than any other? It is almost invariably the case that some model elements have a disproportionate influence on model outputs. If so, these elements may be potential sources of instrumental uncertainties.

Which inputs can you control or affect? It is generally wise to pay extra attention to those things you can control, or at least affect. Inputs that people say are important, that are influential in your model, and that you can affect may well be relevant uncertainties. Take that list of uncertain inputs you prepared and put an asterisk next to each one that you suspect may be instrumental. Now it is time to characterize that uncertainty in ways that risk managers can understand and consider in decision making.

17.2.3 Qualitatively Characterizing the Uncertainty

The World Health Organization (2008) offers a useful scheme for qualitatively characterizing the uncertainty in your assessment, assuming that you know which outputs are instrumental to decision making. Although developed for use with chemical exposure assessments, it has general applicability. To qualitatively identify the most critical uncertainty, think of the process as locating each relevant uncertain item in the three-dimensional space shown in Figure 17.2. Values near the origin are the least uncertain. A low, medium, or high score is determined by the assessor for each of the axes. The most significant uncertainties are those with the highest scores for all three dimensions. The axes may be differentially weighted or not. Following the WHO process, level of uncertainty is more important than appraisal of the knowledge base, which is more important than subjectivity of choices. Explicit weights need not be assigned because the characterization is a sequential one, as seen in the hypothetical example in Table 17.1.

The level of uncertainty dimension expresses the degree of severity of the uncertainty, from the assessor's perspective. A scale ranging from low to high has been suggested, as shown in Figure 17.3 (WHO 2008).

A low score means that a large change in the source of uncertainty would have a small effect on the results. Inputs scored as medium would have proportional effects on outputs. A high score implies that a small change in the source of the uncertainty would have a large effect on the results. As qualitative inputs are scored in this dimension, it is helpful to indicate whether the uncertainty is reducible or irreducible (see Table 17.1) so that risk managers better understand their options for addressing the level of uncertainty.

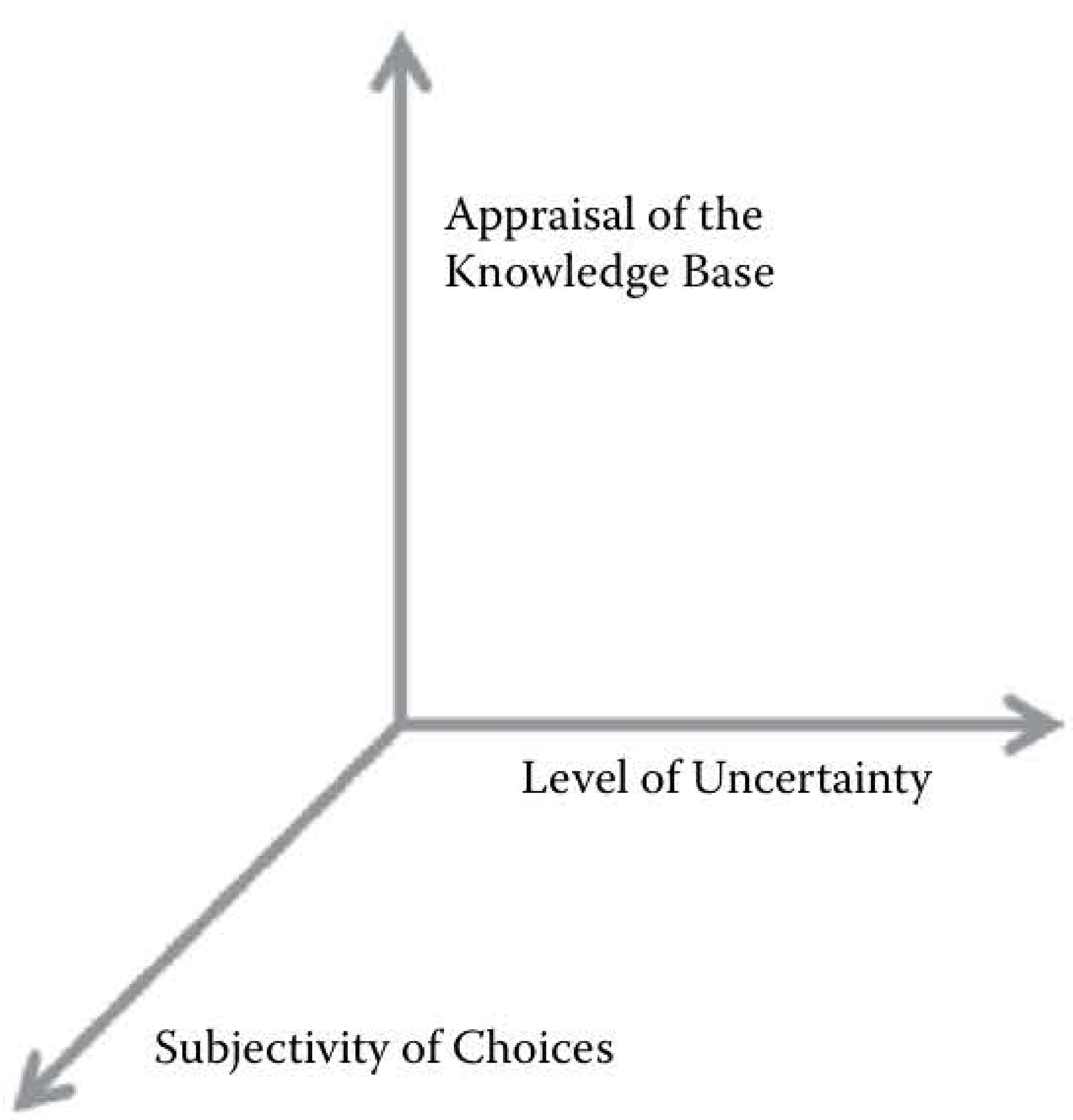

FIGURE 17.2 Three-dimensional space of qualitative risk assessment. (World Health Organization 2008.)

TABLE 17.1
Sample Summary of the Evaluation of Uncertainty in a Qualitative Risk Assessment

		Characteristics of Uncertainty		
Sources of Uncertainty		**Level of Uncertainty**	**Appraisal of Knowledge Base**	**Subjectivity of Choices**
Scenario		Medium	Low	Medium
Model	Conceptual	High	Medium	Medium
	Mathematical	NA	NA	NA
Inputs				
Input 1 (irreducible)		Low	Low	Low
Input 2 (reducible)		High	Medium	Low
Input 3 (reducible)		Medium	Medium	High

The appraisal of the knowledge base is based on accuracy, reliability, plausibility, scientific backing, and robustness, all seen on the vertical dimension axis of Figure 17.4 (WHO 2008). It is intended to rate the adequacy of the state of knowledge about the input. A low uncertainty in the knowledge-base rating suggests the qualities listed in the first column. The other columns illustrate the two other possible ratings of medium and large.

The final dimension along which to rate an uncertain input is the subjectivity of the choice the assessor had in making an assumption about this input. The aspects of the choice include the choice space, intersubjectivity among peers and among stakeholders, influence of situational limitations (e.g., money, tools, and time) on choices, sensitivity of choices to the analysts' interests, and the influence of choices on results. Descriptions for each aspect can be read across the vertically arranged rows in Figure 17.5 (WHO 2006).

The WHO method proceeds to rank sources of uncertainty by aspects; first, ranking sources of uncertainty by level of uncertainty, then appraisal of the knowledge base, and finally by subjectivity of choices. A hypothetical qualitative summary evaluation of the uncertainty in an assessment is shown in Table 17.1. Imagine that

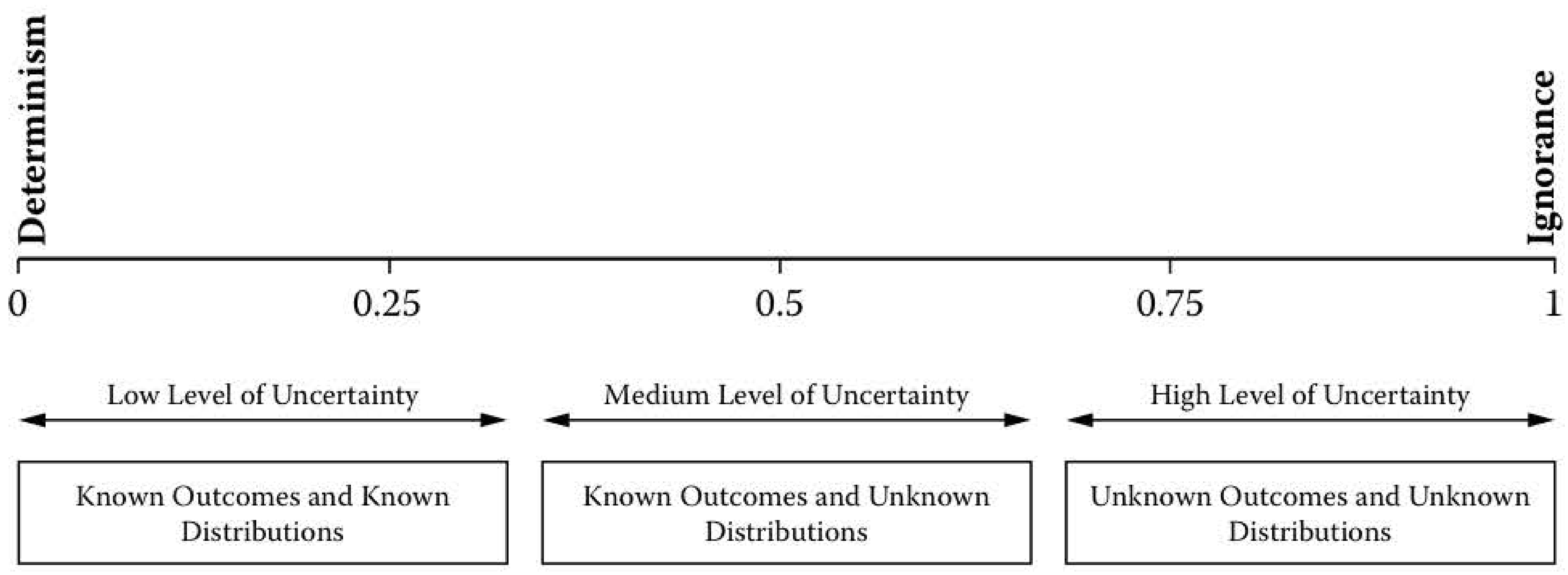

FIGURE 17.3 Rating the level of uncertainty. (World Health Organization 2008.)

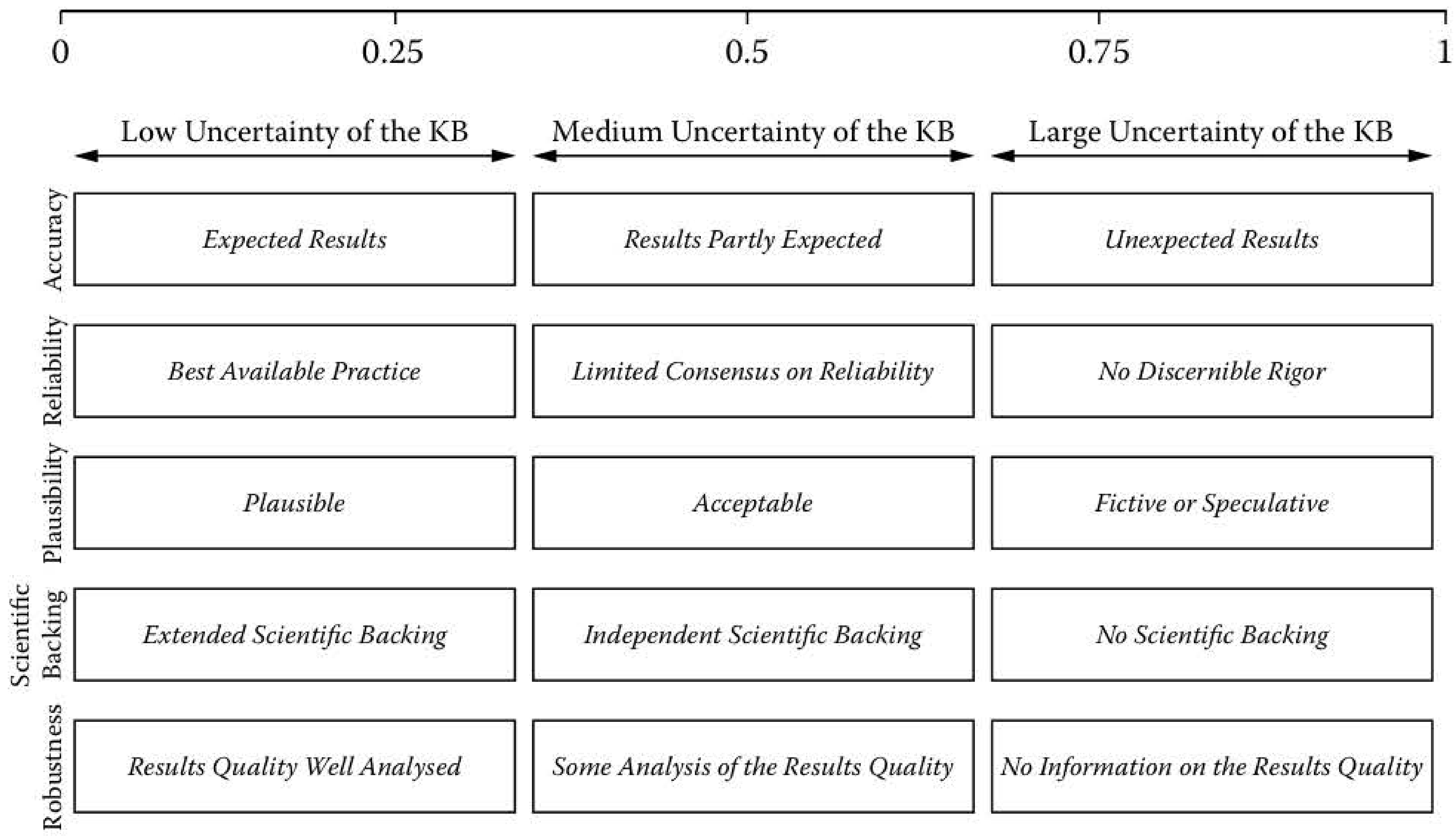

FIGURE 17.4 Rating the appraisal of the knowledge base. (World Health Organization 2006.)

the preceding qualitative sensitivity analysis steps have identified the base scenario, the conceptual models, and three of its inputs as potential sources of instrumental uncertainty. If we follow the WHO rationale and consider the three dimensions of uncertainty as sequential screening tools, we would find the model and input 2 of most interest because each scored high on the first screening criterion. Subsequent screening criteria are used to refine the sorting process. Both the model and input 2 are rated medium on the knowledge-base criterion. Moving to the subjectivity-of-choices criterion identifies the conceptual model as the more significant uncertainty

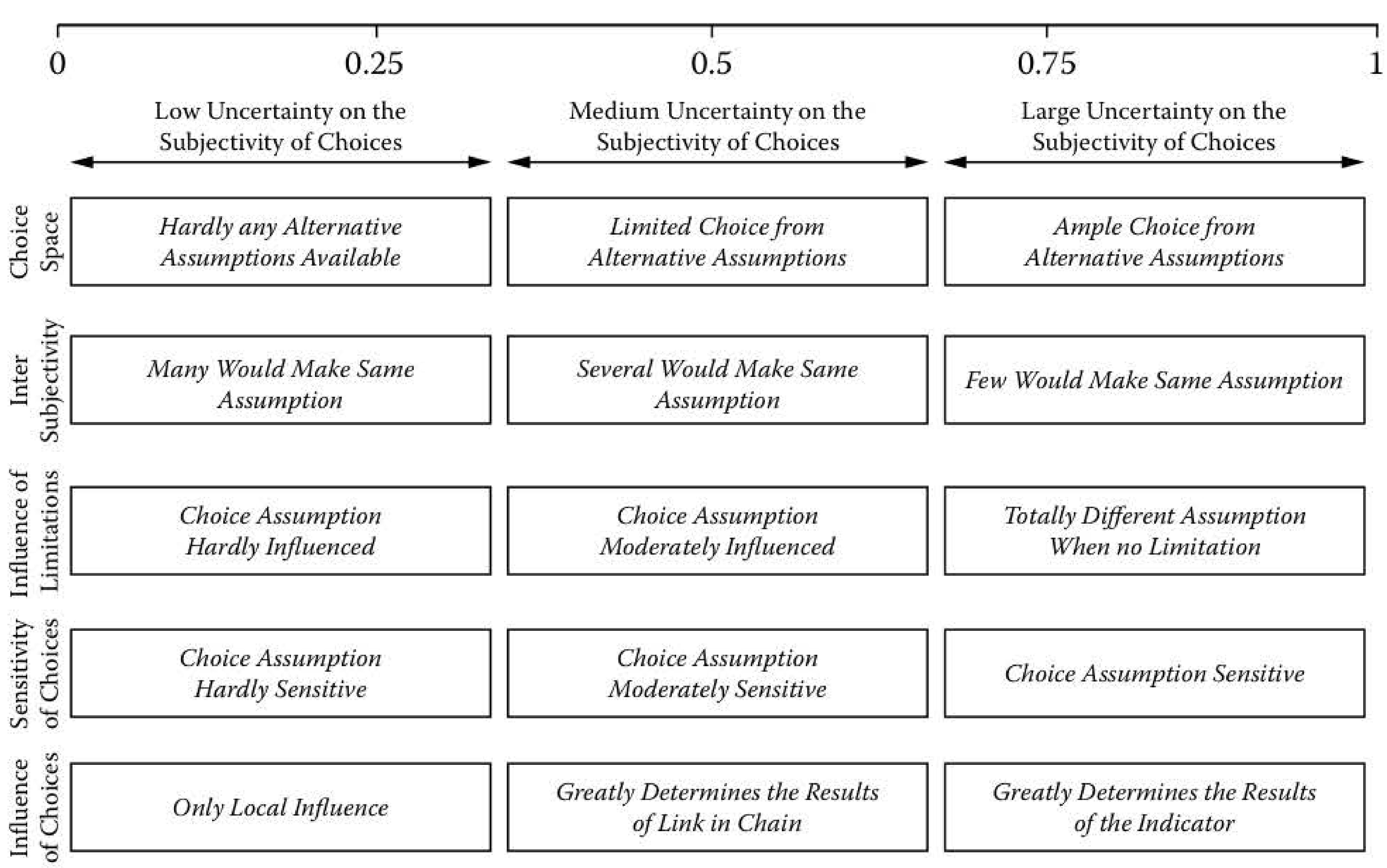

FIGURE 17.5 Rating the subjectivity of choice. (World Health Organization 2006.)

in this case. Between the two medium levels of uncertainty for the scenario and input 3, input 3 is rated as more significant based on an appraisal of the knowledge base.

Once the uncertainties have been identified in this or another manner, it remains for the assessor and the manager working together to subjectively evaluate the significance of these findings for the purposes of risk management. The risk manager should explicitly consider the results of such a sensitivity analysis and how changes in input 2 or the model might affect the decision or its outcome. His decision could include additional research to minimize the reducible uncertainty. Progressing to a quantitative risk assessment may be an option for some situations. Systematically exploring the impact of alternative assumptions or ranges of values on the decision to be made is, hopefully, an obvious response to uncertainty in a qualitative assessment.

17.2.4 Vary the Key Assumptions

A sensitivity analysis structured as described in the previous section is not going to be done for every qualitative assessment. However, doing no sensitivity analysis is unacceptable. The absolute, bare minimum sensitivity analysis is to carefully and systematically explore and vary the assumptions that underlie the assessment's instrumental uncertainty. When we separate what we know from what we do not know during a risk assessment, it is necessary to find effective means for dealing with that which we do not know. Making assumptions about uncertain values is one of the most common and expedient ways of addressing uncertainty.

We make two basic kinds of assumptions. Those we know we are making, i.e., explicit assumptions, and those we do not know we are making, i.e., implicit assumptions. Explicit assumptions should be documented, written down, and preserved for the attention of assessors and managers and other interested parties. Implicit assumptions are often not recognized by the assessors or managers. Peer review by multidisciplinary reviewers can be an effective technique for detecting implicit assumptions. It often takes someone outside one's own discipline, specialty, or organization to recognize the assumptions we make as the common practice of our fields and organizations.

To the extent that we use assumptions to address uncertainty and to enable ourselves to move forward with our assessments, we should routinely test the sensitivity of our assessment outputs to those assumptions. The simplest way to do this is to first make a list of the key assumptions used in a risk assessment. Next, explore what happens as you drop or challenge each assumption individually and replace it with alternative assumptions. Do your outputs change? Do the answers to the risk manager's questions change? Might any of these changes affect the risk management decision? If so, that information needs to be conveyed to the risk managers. For example, imagine an assessment of the risks of a nuclear power plant site. Suppose we have assumed sea level rise of one foot or less in the next 100 years. If it is more than that, this site will be surrounded by water year-round. This is information that needs to be carefully communicated to risk managers. We have assumed sea level rise will pose no hazard, but that is a remaining uncertainty.

It can sometimes be instructive to explore what happens as you drop/change your assumptions in combinations. Assumption dropping is a very effective kind of qualitative sensitivity analysis that can also be useful in quantitative assessments. When the assumptions do make a difference to your assessment outputs and answers that could affect decision making, you have identified an instrumental uncertainty. Risk managers should respond accordingly either by reducing the uncertainty through additional data collection or research or by having the risk assessors address the assumptions more thoroughly in their assessment, perhaps with probabilistic risk assessment methods. The final obvious alternative for risk managers is to take the knowledge of that sensitivity into account when making a decision.

17.3 QUANTITATIVE SENSITIVITY ANALYSIS

There are four classes of quantitative sensitivity analysis tools. These are scenario, mathematical, statistical, and graphical analysis. In the discussion that follows, example methods from each class are described. The more popular methods are illustrated by reference to a simple cost estimating example.* Two of these methods will rely on different examples that are better suited to the methods demonstrated.

17.3.1 Scenario Analysis

Analyzing the sensitivity of assessment results to the scenarios we use is called scenario analysis. The base-case scenario for most risk assessments is either an existing condition or the "without" additional risk management options (RMOs) condition. In the simplest scenarios, model inputs are entered as point estimates that represent the "best guess" as to the true but unknown value of each input. Rather than to alter these inputs one or two at a time, scenario analysis alters the entire scenario. If we think of scenarios as the stories we tell about risks, these alternative scenarios include different plot lines such as best case, worst case, common practice, the most likely case, a locally focused scenario, a new policy scenario, and so on.

The alternative scenarios can vary markedly in their structure and details. When a new scenario is considered as a starting point for the assessment it can sometimes lead to a markedly different characterization of the risk. Existing, as-planned, failure and improvement scenarios are all subject to scenario sensitivity analysis. It is worth noting that scenario analysis does not require the construction of structurally different scenarios. The alternative scenarios may simply comprise alternative sets of input values within an unchanging scenario structure. The goal is to focus on the structural elements or inputs of a scenario that give rise to instrumental uncertainty and to vary them to explore the sensitivity of risk characterizations and answers to the risk manager's questions to these plausible different scenarios. If they could alter the decision or the outcome of a decision, risk managers must carefully consider this information.

One of the most commonly used alternative scenarios is the worst-case scenario. If we can identify and live with the worst-case scenario, a common and sometimes

* Cost-estimating examples are used in this chapter and the next one. Although they may not spring first to mind when thinking about risks, they have the advantage of being easily understood and accessible to readers from every discipline. The risk of a cost overrun or under-estimating the cost of an action can be significant. As such they provide a friendly context for learning the concepts.

wrong presumption is that we can live with lesser scenarios as well. Consequently, assessors will sometimes try to envision the most extreme negative set of input values possible.

Engineers and public health officials, to name just two professions, are biased toward designing systems conservatively to try to minimize or eliminate the chance of adverse outcomes. The engineering profession in particular has regarded conservatism in design as the accumulated wisdom of centuries of experience that has taught that the conditions of the real world are not always predictable and it makes good sense to provide some margin of error for unforeseen events. This drive toward conservatism has led to the widespread propagation and use of worst-case scenarios.

There is no real formal definition of a worst-case scenario. It is simply that future in which everything that can reasonably go wrong does go wrong. If the worst-case scenario yields an acceptable result, decision makers in the past have often assumed there is no need to manage a risk. On the other hand, worst-case scenarios that result in unacceptable consequences often have lead decision makers to take precautions to preclude the worst-case scenario from occurring.

Despite its widespread usage, the worst-case scenario is not without its problems. First among these is that introducing this conservatism into an analysis focuses policy and possibly resources on what is often a deliberately unrealistic scenario. This is precisely what risk assessment is designed to prevent. Second, given any worst-case scenario, an even worse case can, paradoxically, still be defined. Third, the likelihood of a worst-case scenario may be so small as to lead to the waste of efforts, materials, and other resources in attempts to reduce it. Fourth, there is an almost hypnotic appeal to think that if we have covered the worst-case, we have covered everything. Failure in the better-than-worst-case world is still possible and is often overlooked with a worst-case orientation to risk management. Nonetheless, worst-case scenarios are likely to remain useful and popular failure scenarios to investigate. In general, though, it is not wise to make policy based on worst-case scenario analysis.

Other scenarios can be evaluated for the benefit of risk assessors and managers alike. An optimistic or best case (if everything that can break our way does) scenario may be of interest to risk-taking risk managers, for example. Different stakeholders may proffer alternative views of the future that should be explicitly considered in scenario analysis. The risk assessors may develop a range of scenarios to reflect their concerns about the key uncertainties in a risk assessment. What the team assumes about climate change, sea level rise, geopolitical events, natural disasters, technological advancements and the like could have important ramifications for decision making. The most likely scenario is often uncertain and any number of scenarios can be developed. Deterministic scenario analysis is not an uncommon sensitivity analysis approach.

When assessors use scenario analysis, the idea is to compare the different situations to identify differences in important model outputs. Differences that make a difference for decision making are important, and assessors and risk managers must become aware of them in a sensitivity analysis. Risk managers then must decide how much to weigh the range of potential outcomes that could result from the residual uncertainty in the decision-making process.

17.3.2 Mathematical Methods for Sensitivity Analysis

Mathematical methods rely on calculating outputs for a range of input values or for different combinations of input values. These methods differ from statistical methods, which rely on simulations in which inputs are represented by probability distributions. Although mathematical methods do not describe variance in outputs due to variance in inputs as statistical methods do, they are still useful in estimating the impact of a range of input values on model outputs. In addition, mathematical methods can help identify the most important inputs, and they are useful for verifying models. Sensitivity of model outputs to individual inputs or groups of inputs can be explored by various means (Frey and Patil 2002).

Several sensitivity analysis methods presented in the pages that follow use the cost estimate example seen in Table 17.2. These are the costs of dredging a navigation channel and using the material to create wetlands by placing it behind geotube barriers

TABLE 17.2
Cost Estimate for Channel Dredging

A Description	B Quantity	C Unit	D Price	E Amount
Lands and damages	0	LS	$...	$...
Relocations				
Lower 20 pipeline, 653+00	425	LF	$730.00	$310,250
Remove 8″ pipeline, 678+00	1,000	LF	$50.00	$50,000
Total—Relocations				*$360,250*
Fish and Wildlife Facilities (Mitigation)				
Oyster reef creation	0	ACR	$...	$...
Total—Fish and Wildlife Facilities (Mitigation)				*$...*
Navigation, Ports and Harbors				
Mobe and demobe	1	LS	$500,000.00	$500,000
Pipeline dredging, Reach 1	576,107.00	CY	$2.78	$1,601,577
Pipeline dredging, Reach 2	1,022,769.00	CY	$2.60	$2,659,199
Pipeline dredging, Reach 3A	1,182,813.00	CY	$3.16	$3,737,689
Pipeline dredging, Reach 3B	736,713.00	CY	$2.76	$2,033,328
Scour pad, Reach 1	17,550	SY	25.69	$450,860
Geotubes, 30′, Reach 1	1,400	LF	$188.52	$263,928
Geotubes, 45′, Reach 1	4,912	LF	$222.18	$1,091,348
Scour pad, Reach 3	38,750	SY	$25.69	$995,488
Geotubes, 45′, Reach 3	13,940	LF	$222.18	$3,097,189
Total—Navigation, Ports and Harbors				*$16,430,606*
Subtotal				*$16,790,856*
Engineering and Design	8%			$1,343,268
Construction Management	6%			$1,007,451
Total project cost				*$19,141,576*

Abbreviations: LF = linear feet; LS = lump sum; CY = cubic yards; SY = square yards; ACR = acres.

adjacent to the shoreline. Note that row and column locations are provided. This may be helpful for interpreting some of the tables and figures that follow. The first column (A) describes the basic work item, column (B) provides the quantity in the units of column (C). The price (D) is per unit. The amount (E) is the product of the quantity and unit price. The subtotal sums the total amounts that precede it in the table. Of this total, 8% and 6% are calculated to cover, respectively, advanced engineering and design (AED) as well as construction management (CM). They are added to the subtotal to obtain the total project cost, $19.1 million, the only output of interest for this example.

The numerical values shown in the table are the best point estimates of the cost estimators. For the mathematical methods, the model is treated as a deterministic one. Once we progress to the use of statistical sensitivity measures, these point estimates are replaced by probability distributions and a Monte Carlo process is then used to simulate results. Assume that there is knowledge uncertainty and natural variability "sprinkled" among the various inputs required to estimate the total project cost.

17.3.2.1 Nominal Range Sensitivity

Nominal range sensitivity is a mathematical method used for deterministic, rather than probabilistic, models. It is usually used to identify the most important input(s) (Cullen and Frey 1999), and this can be useful for setting research and data collection priorities.

Nominal range sensitivity analysis is also known as one-at-a-time analysis (OAATA). It works by evaluating the effect of changes in an individual input on an output variable. This local sensitivity is conceptually equivalent to a partial derivative. Although simple, this technique has at least two key shortcomings: (1) it does not account for simultaneous variation of multiple model inputs; and (2) it does not account for any nonlinearities in the model that create interactions among the inputs. Nonetheless, for simple linear models, OAATA can be instructive.

The selected input may be changed incrementally or allowed to vary across its entire range of plausible values while holding all other input values constant, usually at their nominal, mean, representative, or base-case values. This is sometimes done to identify threshold values that result in significant changes in model outputs. It can also be used to identify which uncertain variables may have the greatest impact on outputs of interest in the risk assessment. Alternatively, a plausible range of variation, say ±20%, may be applied to one or more inputs. Morgan and Henrion (1990) call the change in the model output due to a unit change in the input the sensitivity or swing weight of the model for the chosen input variable. This sensitivity analysis can be repeated for as many input variables as desired.

LIMITATIONS OF ONE-AT-A-TIME ANALYSIS

Do not automatically equate the magnitude of an uncertain variable with its influence. Consider two random variables expressed by the following uniform distributions: $A = U(10^7,10^8)$, $B = U(2,6)$. Some might be tempted to assume A is more influential because of its sheer magnitude.

It is essential to know the structure of your model and nonlinearities can change everything you think you know about sensitive variables. For example, consider a new variable, C, that is a function of A and B. If $C = A + B$; A dominates. If $C = A^B$; B dominates.

Dependence and branching in a model can also create flaws with the logic of OAATA. Consider this example:

If $X<50$ then
$Y = Z + 1$
Else
$Y = Z^{100}$

What value will you set X equal to when you investigate the sensitivity of Y and Z?

Bearing in mind that the sensitivity can be described in terms of a change in output for a unit or any given percentage change in the input variable value, the direction or sign of the change is especially useful to note. Nominal range sensitivity analysis works best with linear models where rank orders can be easily established based on this measure of sensitivity. In nonlinear models, output sensitivities may depend on interactions with other inputs that may not always be obvious and therefore cannot be placed in a rank order.

This method is easy to use. It is most reliable: when assessors have a good idea of the plausible range of input values; in linear models when the effect of a one-unit change in an input does not depend on the starting value of the input; and when there is no significant interaction of the chosen input with other input variables. When interactions are possible among variables, this method is likely to produce an inadequate description of the range of possible input values and, therefore, output responses.

To illustrate this technique, consider the quantity of pipeline dredge material in reach 1 from the cost estimation case study in (cell B20) Table 17.2. What happens to total costs if that quantity increases 20%? Table 17.3 shows the effect of a 20% increase in the reach 1 dredging quantity on total project costs. Only the reach 1 dredging cost line is shown as it is the only change in the model inputs.

A 20% increase in this one input causes total project cost to rise by $365,000, a 1.9% increase in total project cost. Note that the simple nature of this example renders the one-unit change in the input (from 576,107 to 576,108 CY) a trivial exercise. Every cubic yard costs the same $2.78 in the model. The $320,000 change increases to $365,000 to reflect allowances for AED and CM.

The input changes investigated may be percentages of a value, like the 20% increase used here, or specific values of interest can be chosen. For example, we might ask what happens to total project costs if the dredging quantity in reach 1 rises to 750,000 CY. This process can be repeated for as many individual inputs as desired.

TABLE 17.3
One-At-A-Time Analysis Example for Dredging Cost Estimate

Description	Quantity	Unit	Unit Price	Amount	Change
Original Estimate					
Pipeline Dredging, Reach 1	576,107	CY	$2.78	$1,601,577	NA
Total Project Cost				$19,141,576	NA
20% Increase in Reach 1 Quantity					
Pipeline Dredging, Reach 1	691,328	CY	$2.78	$1,921,893	$320,316
Total Project Cost				$19,506,736	$365,160

It is easy to see that two or more inputs could be varied simultaneously in a similar fashion. This is a simple method, but it is tedious.

Palisade Corporation's TopRank 7.5 (see Appendix A for details on the use of this software) can be used to conduct both a one-way what-if analysis and a multiway what-if analysis. This is done by allowing the assessor to vary a fixed-point estimate of an input by some plus or minus percentage. To demonstrate this technique, vary every variable input in the model (excluding the percentages for engineering design and construction management and the number of mobilization and demobilizations of equipment, cell B19) by $\pm 20\%$.

Allowing each input to change one at a time, first by its minimum and then by its maximum value, we see which inputs have the greatest potential impact on total project cost in Table 17.4. Only the top ten inputs are shown. The table identifies the input by name and location in the model. This name is chosen automatically by the software. The first input is the quantity of pipeline dredging material in reach 3a. It is found in cell B22 of the model. When it assumes its minimum value of 946,250 CY, total project cost is $18,289,383, a decrease of 4.45%. When the maximum of 1,419,375 CY is substituted into the model, costs rise to $19,993,769, an increase of 4.45%. The table shows the top ten most influential inputs. The software output evaluates every input one-at-a-time, so that if you are interested in the effect of a specific input you can find it readily. A graphic display of this result is shown under the discussion of graphic sensitivity techniques.

It is often common practice to build models initially using plausible values or point estimates of expected values. In a complex model there could be dozens or even hundreds of inputs. Many models lack the transparent simplicity of our example. When that is the case, the risk assessor may not want to go to the time and trouble to bound each input or to specify a probability distribution to use for each model input if it has little or no effect on the model output(s) of interest. A one-way what-if analysis (equivalent to OAATA) will quickly identify those inputs that the assessor ought to focus attention on when considering the effects of uncertainty.

Presuming no more data than the values shown in Table 17.2, the OAATA summarized in Table 17.4 provides the assessor with a clear identification of the inputs with the most significant impact on total cost. Table 17.5 goes one step further

TABLE 17.4
Top Ten Inputs from a One-Way What-If Analysis for the Total Project Cost Output

			Minimum			Maximum		
			Output			Output		
Rank	Input Name	Cell	Value ($)	Change (%)	Input Value	Value ($)	Change (%)	Input Value
1	Pipeline Dredging, Reach 3A/ Quantity (B22)	B22	18,289,383	−4.45	946250.4	19,993,769	4.45	1419375.6
2	CY/Price (D22)	D22	18,289,383	−4.45	2.528	19,993,769	4.45	3.792
3	Geotubes, 45′, Reach 3/Quantity (B28)	B28	18,435,417	−3.69	11152	19,847,735	3.69	16728
4	LF/Price (D28)	D28	18,435,417	−3.69	177.744	19,847,735	3.69	266.616
5	Pipeline Dredging, Reach 2/Quantity (B21)	B21	18,535,279	−3.17	818215.2	19,747,874	3.17	1227322.8
6	CY/Price (D21)	D21	18,535,279	−3.17	2.08	19,747,874	3.17	3.12
7	Pipeline Dredging, Reach 3B/ Quantity (B23)	B23	18,677,977	−2.42	589370.4	19,605,175	2.42	884055.6
8	CY/Price (D23)	D23	18,677,977	−2.42	2.208	19,605,175	2.42	3.312
9	Pipeline Dredging, Reach 1/Quantity (B20)	B20	18,776,416	−1.91	460885.6	19,506,736	1.91	691328.4
10	CY/Price (D20)	D20	18,776,416	−1.91	2.224	19,506,736	1.91	3.336

Note: Top ten inputs ranked by percent change.

by showing additional percentage changes in the model inputs. The ranking of inputs is identical to that seen in Table 17.4. Here you see that the software enables you to examine a change of ±20% and ±10% in the same analysis. These expanded results support spider plots, which are discussed in section 17.3.4.3.

Analyses like those seen in the tables can be prepared for a variety of ranges of change in the input values. They need not be ±20% and the number of increments can vary from the five used for this example. In a linear model this produces a reliable indication of those inputs to which the model output will be most sensitive. If your assessment model has multiple outputs, a separate OAATA must be performed for each output. These results are a valuable guide for handling uncertainty in subsequent iterations of the model. This kind of analysis helps the risk assessor understand which uncertainties have the greatest influence on output quantities of interest. With this information the assessor can prioritize data gaps and future research needs.

TABLE 17.5
An Expanded One-Way What-If Analysis for the Total Project Cost Output

			Input Variation			Output Variation		
Input Name	Cell	Step	Value	Change	Change (%)	Value ($)	Change ($)	Change (%)
Pipeline Dredging, Reach 3A/ Quantity (B22)	B22	1	946250.4	−236562.6	−20.00	18,289,383	(852.193)	−4.45
		2	1064531.7	−118281.3	−10.00	18,715,479	(426.097)	−2.23
		3	1182813	0	0.00	19,141,576	(0)	0.00
		4	1301094.3	118281.3	10.00	19,567,673	426,097	2.23
		5	1419375.6	236562.6	20.00	19,993,769	852,193	4.45
CY/Price (D22)	D22	1	2.528	−0.632	−20.00	18,289,383	(852.193)	−4.45
		2	2.844	−0.316	−10.00	18,715,479	(426.097)	−2.23
		3	3.16	0	0.00	19,141,576	(0)	0.00
		4	3.476	0.316	10.00	19,567,673	426,097	2.23
		5	3.792	0.632	20.00	19,993,769	852,193	4.45
Geotubes, 45; Reach 3/Quantity (B28)	B28	1	11152	−2788	−20.00	18,435,417	(706,159)	−3.69
		2	12546	−1394	−10.00	18,788,496	(353,080)	−1.84
		3	13940	0	0.00	19,141,576	(0)	0.00
		4	15334	1394	10.00	19,494,646	353,080	1.84
		5	16728	2788	20.00	19,847,735	706,159	3.69
LF/Price (D28)	D28	1	177.744	−44.436	−20.00	18,435,417	(706,159)	−3.69
		2	199.962	−22.218	−10.00	18,788,496	(353,080)	−1.84
		3	222.18	0	0.00	19,141,576	(0)	0.00
		4	244.398	22.218	10.00	19,494,646	353,080	1.84
		5	266.616	44.436	20.00	19,847,735	706,159	3.69
Pipeline Dredging, Reach 2/Quantity (B21)	B21	1	818215.2	−204553.8	−20.00	18,535,279	(606,297)	−3.17
		2	920492.1	−102276.9	−10.00	18,838,427	(303,149)	−1.58
		3	1022769	0	0.00	19,141,576	(0)	0.00
		4	1125045.9	102276.9	10.00	19,444,725	303,149	1.58
		5	1227322.8	204553.8	20.00	19,747,874	606,297	3.17
CY/Price (D21)	D21	1	2.08	−0.52	−20.00	18,535,279	(606,297)	−3.17
		2	2.34	−0.26	−10.00	18,838,427	(303,149)	−1.58
		3	2.6	0	0.00	19,141,576	(0)	0.00
		4	2.86	0.26	10.00	19,444,725	303,149	1.58
		5	3.12	0.52	20.00	19,747,874	606,927	3.17
Pipeline Dredging, Reach 3B/ Quantity (B23)	B23	1	589370.4	−147342.6	−20.00	18,677,977	(463,599)	−2.42
		2	663041.7	−73671.3	−10.00	18,909,777	(231,799)	−1.21
		3	736713	0	0.00	19,141,576	(0)	0.00
		4	810384.3	73671.3	10.00	19,373,375	231,799	1.21
		5	884055.6	147342.6	20.00	19,605,175	463,599	2.42

(*Continued*)

TABLE 17.5 ***Continued***

An Expanded One-Way What-If Analysis for the Total Project Cost Output

			Input Variation			Output Variation		
Input Name	**Cell**	**Step**	**Value**	**Change**	**Change (%)**	**Value ($)**	**Change ($)**	**Change (%)**
CY/Price (D23)	D23	1	2.208	−0.552	−20.00	18,677,977	(463,599)	−2.42
		2	2.484	−0.276	−10.00	18,909,777	(231,799)	−1.21
		3	2.76	0	0.00	19,141,576	(0)	0.00
		4	3.036	0.276	10.00	19,373,375	231,799	1.21
		5	3.312	0.552	20.00	19,605,175	463,599	2.42
Pipeline Dredging, Reach 1/Quantity (B20)	B20	1	460885.6	−115221.4	−20.00	18,776,416	(365,160)	−1.91
		2	518496.3	−57610.7	−10.00	18,958,996	(182,580)	−0.95
		3	576107	0	0.00	19,141,576	(0)	0.00
		4	633717.7	57610.7	10.00	19,324,156	182,580	0.95
		5	691328.4	115221.4	20.00	19,506,736	365,160	1.91
CY/Price (D20)	D20	1	2.224	−0.556	−20.00	18,776,416	(365,160)	−1.91
		2	2.502	−0.278	−10.00	18,958,996	(182,580)	−0.95
		3	2.78	0	0.00	19,141,576	(0)	0.00
		4	3.058	0.278	10.00	19,324,156	182,580	0.95
		5	3.336	0.556	20.00	19,506,736	365,160	1.91

As we have cautioned, there may be times when it does not make sense to vary one input at a time. The desired changes in a model can always be made manually, but this is very limiting, especially when the assessor would like to explore the sensitivity of model outputs to the many different inputs. TopRank offers the capability to conduct multi-way what-if analysis. As with the one-way analysis, the assessor can choose a percentage by which to vary the inputs, identify which variables to consider in a multiway analysis, and how many to consider at a time. An examination of all possible combinations of two-at-a-time variable inputs is summarized for the cost estimate in Table 17.6. This shows the top five input combinations from the model.

The report shown provides the results of an analysis of the extremes, i.e., when both inputs assume their minimum values and when both inputs assume their maximum values. It should come as no surprise that the ranking of paired variables mirrors the results of the OAATA. Detailed outputs are provided by TopRank for the multi-way what-if analysis, so that additional pairs of inputs and additional combinations of values (%'s) can be explored.

17.3.2.2 Difference in Log-Odds Ratio (ΔLOR)

A specific application of the nominal range sensitivity methodology is the log-odds ratio method. This mathematical methodology can be used when the output of interest is a probability. That means our cost example does not lend itself to this method, so we will briefly switch to a different example. First, we need to understand the method.

TABLE 17.6
Top Five Inputs from a Two-at-a-Time Multiway What-If Analysis for Total Project Cost

		Output Variation			
		Minimum		Maximum	
Rank	Multi-Way Name	Value ($)	Change (%)	Value ($)	Change (%)
1	Pipeline Dredging, Reach 3A/ Quantity [B22] CY/Price [D22]	$17,607,628	−9.79%	$21,016,401	9.79%
2	Pipeline Dredging, Reach 3A/ Quantity [B22] Geotubes, 45′, Reach 3/Quantity [B28]	$17,583,224	−8.14%	$20,699,928	8.14%
3	Pipeline Dredging, Reach 3A/ Quantity [B22] LF/Price [D28]	$17,583,224	−8.14%	$20,699,928	8.14%
4	CY/Price [D22] Geotubes, 45′, Reach 3/Quantity [B28]	$17,583,224	−8.14%	$20,699,928	8.14%
5	CY/Price [D22] LF/Price [D28]	$17,583,224	−8.14%	$20,699,928	8.14%

Note: Top five inputs ranked by percent change.

The odds ratio or odds of an event is simply $\frac{P(A)}{1-P(A)}$ or, in words, it is the ratio of the probability that the event occurs to the probability that the event does not occur (Gordis 1996). The log of the odds ratio or logit simply takes the log of the odds ratio, i.e., logit $= \log \frac{P(A)}{1-P(A)}$.

With this definition ΔLOR can be defined as follows:

$$\Delta\text{LOR} = \log\left(\frac{\frac{P(A)}{1-P(A)}}{\frac{P(B)}{1-P(B)}}\right) \tag{17.1}$$

Which simplifies to:

$$\Delta\text{LOR} = \log\left(\frac{P(A)}{1-P(A)}\right) - \log\left(\frac{P(B)}{1-P(B)}\right) \tag{17.2}$$

Or

$$\Delta\text{LOR} = logit\, P(A) - logit\, P(B) \tag{17.3}$$

If we now define event B to be the original probability estimate and event A is the probability recalculated with changes in the sensitive input, then a positive ΔLOR means that a change in the selected input increases the probability of the event. A negative ΔLOR means that a change in the input decreases the probability of the event. The larger the ΔLOR value, the greater is the sensitivity of the probability output to changes in that input. ΔLOR is subject to many of the same limitations as the nominal range sensitivity when it comes to nonlinear models.

An example for a single variable is presented in the two event-tree models shown in Figure 17.6. This model represents a small boat harbor where shoaling has decreased the depth of the navigation channel. Vessels come into the harbor with oil and other commodities. Some vessels have drafts that exceed the controlling depth of the channel. This affects the probability of a marine casualty occurring. Casualties include groundings, allisions, and collisions.

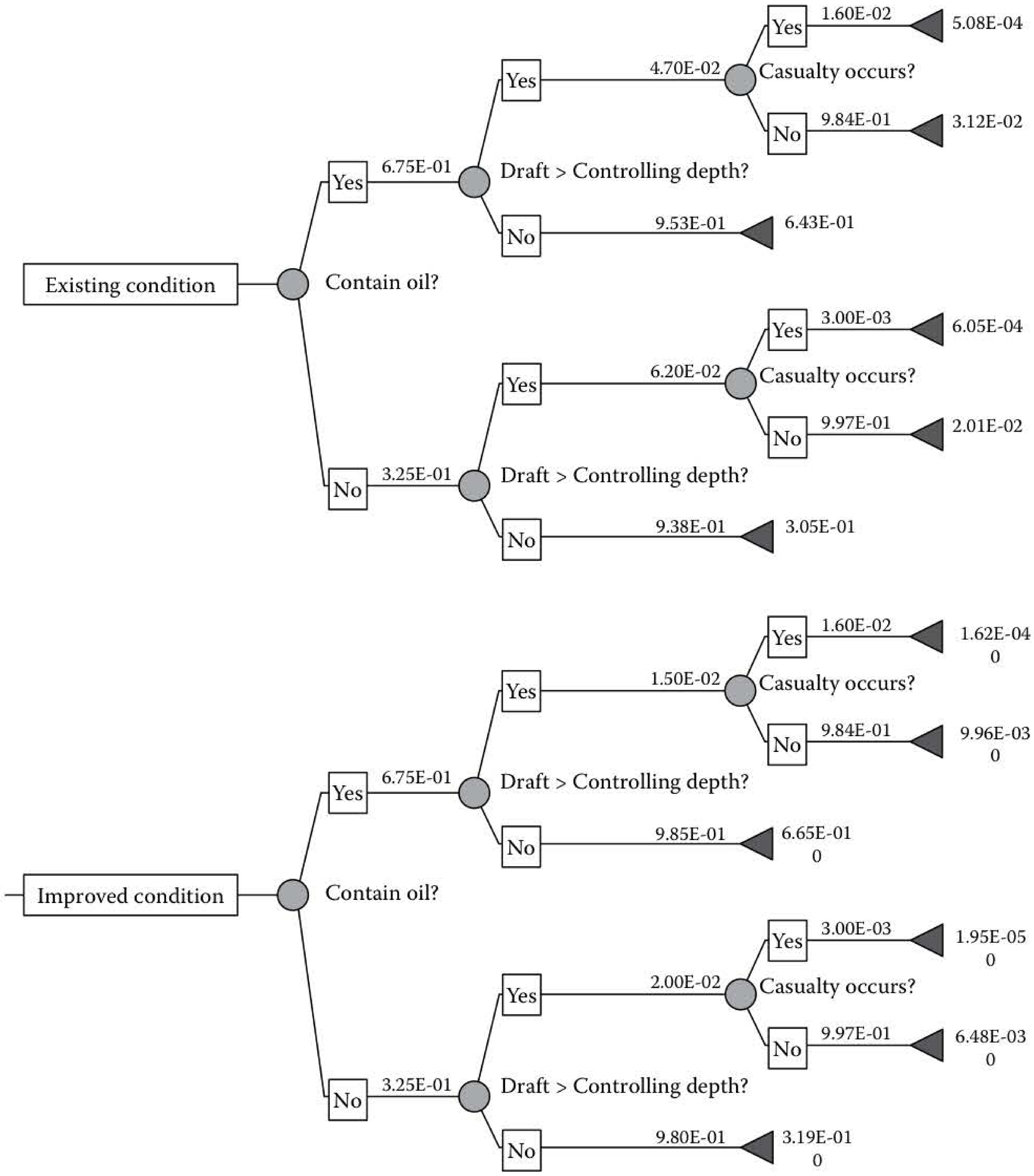

FIGURE 17.6 Example of event-tree models for the difference in log-odds ratio.

TABLE 17.7
Difference in Log-Odds Ratio Calculation

	Probability			
Condition	**Casualty**	**No Casualty**	**Ratio**	**Logit**
Existing	0.0006	0.9994	0.000568	–3.24537
Improved	0.0002	0.9998	0.000182	–3.74104

ΔLOR = *logit P(A)* – *logit P(B)* = −3.74 = (−3.25) = −0.5

The output of interest is the probability of a casualty. This is obtained by summing the relevant endpoint probabilities in each model. Note that there are two different scenarios here: an existing and an improved scenario. The improvement is considered to be maintenance dredging to increase the controlling dept of the channel. This alters several input values between the two scenarios, but it does not affect the possibility of a casualty.

Note that the model ends if the controlling depth of the channel is not a binding constraint. The assumption is not that these vessels have no casualties, but that the number and nature of the casualties will not be affected by dredging the channel. Note also that the only change captured in this model is the change in the probability that the controlling depths will be exceeded and the effect of this change on subsequent calculations in the models.

To calculate the ΔLOR = *logit* P(A) – *logit* P(B), the values shown in Table 17.7 are obtained from the model. The probabilities of a casualty are sums of two model endpoints. The ratio of P(Casualty)/P(No Casualty) is given in column 4. The log of these ratios is found in the last column. Subtracting the two produces the ΔLOR = −0.5. The negative value means that dredging the channel, thereby decreasing the probability of a vessel exceeding the controlling depth, reduces the probability of a casualty.

In this simple computational example there is no other input to change to produce another ΔLOR value to compare to this one. In a more complex problem or model, there might be multiple model inputs to change that would influence the output probability of interest. Finding the control variable that has the largest desirable effect on the probability output via the ΔLOR is potentially a significant help to the risk manager. The largest absolute value of the ΔLOR would identify the sensitive input with the greatest impact by this method.

17.3.2.3 Break-Even Analysis

This mathematical method applies the familiar concept of breaking even to sensitivity analysis. The notion, borrowed from economics, is that at the break-even point we are indifferent to producing or not producing a good or service. The important idea here is to look for a break-even/cutoff/threshold value for a parameter or decision variable, i.e., a point where something interesting happens, such as, something goes negative, turns good, turns bad, equals zero, and the like. In a decision context, the break-even point is any value where a decision could change.

Applied as a sensitivity method, break-even analysis requires one to find values of inputs that provide a model output for which the risk manager is interested in the so-called break-even point. Such a breakeven point could influence the choice between accepting the risk or managing a risk, choosing option A or option B, and so on. Alternatively, a threshold might indicate a point at which the risk manager has a strong preference for one course of action over another.

The input value or combination of input values for which the risk manager is indifferent is called the switch-over or break-even value. Once the break-even input value(s) is determined, the risk manager must judge whether the most likely input values will lie above or below these break-even values. If an input's range of uncertainty includes the break-even point, that input is instrumental for decision making. In other words, it will not be clear which is the best decision because the switch-over point may or may not be exceeded. In that case, additional research or efforts to reduce the uncertainty may be necessary for a more confident decision. Conversely, if the uncertainty about an input does not include the break-even point, a decision can be made more confidently.

Finding these break-even values is a unique endeavor for each risk assessment; there is no generalized technique applicable to all models. It can be difficult to find these values when the number of sensitive inputs increases. There is also no clear ranking to be obtained from this method. Frey and Patil (2002) provide an example of a patient choosing between a medication and an operation that produces an iso-risk line for the utility of the medication versus the probability of success for an operation. The application of this technique can be sophisticated.

We can simplify here to aid understanding by returning to the dredging cost estimate. Suppose this project was to be constructed under a government program with a $20-million budget cap. A modified application of this break-even method would be to calculate what the dredging quantity in reach 1, for example, would have to be before the threshold (break-even point) is passed. If dredging in that reach exceeds 846,972 cubic yards,* costs will exceed $20 million. If our uncertainty about this input value is, say, from 500,000 to 900,000 cubic yards, then this quantity alone is sufficient to exceed the budget constraint, and the project may not be able to proceed. Risk assessors may be motivated to estimate the probability that the dredging quantity will equal or exceed this amount.

Risk managers would be well advised to do all they can to improve the estimate of the required dredging in this reach. It is not difficult to imagine that this technique can become tedious when multiple inputs are considered. It also does not lend itself readily to considering multiple inputs without the use of iso-risk lines.

17.3.2.4 Automatic Differentiation Technique

The automatic differentiation (AD) technique is a mathematical method that relies on the use of partial derivatives of outputs with respect to small changes in sensitive inputs to calculate local sensitivities. It is usually reserved for larger models. Because

* Costs are $858,424 below the cutoff. At $3.17 a CY ($2.78 plus 14% for AED and CM), the dredging quantity would have to rise by $858,424/$3.17 = 270,865 CY to exceed the budget cap. The current estimate of 576,107 CY would rise by this amount.

these models are not always well-behaved systems of equations, differentiation often relies on numerical techniques that can be time consuming and difficult to calculate. In addition, they may yield inaccurate results. What makes mathematical AD techniques different is that they rely on precompilers that analyze the code of the model and then compile algorithms for computing first or higher order derivatives in an efficient and accurate manner.

This is a software-intensive technique that is not going to be available to most risk assessors. For more information on this technique, see the work of Bischof et al. (1992, 1994, 1996). Applications also can be found in the work of Carmichael, Sandu, and Potra (1997); Issac and Kapania (1997); Ozaki, Kimura, and Berz (1995); and others.

17.3.3 Statistical Methods for Sensitivity Analysis

17.3.3.1 Regression Analysis

Regression analysis is one of the more common sensitivity methods used because simple linear regression sensitivities have been built into some of the commercially available risk assessment software packages. This statistical method can be a useful probabilistic sensitivity analysis technique.

There are many standard econometric textbooks that explain regression analysis techniques. In best practice, the assessor will specify a functional form for the cause-and-effect relationship between the output (dependent variable) and the relevant inputs (independent variables) based on sound theory. In a probabilistic risk assessment, a random sample of values for these variables will be obtained through some probabilistic analysis like a Monte Carlo simulation.

Using data from the simulation for inputs and outputs, a multiple regression model of the form

$$Y_i = \beta_o + \beta_1 X_{1,i} + \ldots + \beta_m X_{m,i} + \varepsilon_i \tag{17.4}$$

is estimated. The $X_{m,i}$ are the inputs, where m indicates the number of the individual input variable and i indicates the ith input data point. The Y_i indicates the ith output data point.

The beta values, β_m, are regression coefficients. Because they are estimated from a random sample, they are themselves random variables. When the beta coefficients are statistically significantly different from zero they provide a measure of the effect of the particular input variable X_m on Y, all other input variables being held constant. In fact, the β_m value shows the effect of a one-unit change in X_m on the output variable, when all the other input variables in the regression equation have been accounted for. This makes regression coefficients similar to the nominal range sensitivity technique in terms of interpretation, although it is a more sophisticated technique that can account for some nonlinearities in the model, if they can be transformed to a linear form. It also can handle variation among multiple assessment inputs.

Input variables with beta regression coefficients that lack a statistically significant difference from zero relationship with the output variable are not sensitive. Those with significant regression coefficients indicate a sensitivity. The regression coefficients can be normalized over the [−1,1] interval to eliminate dimensional effects and to allow ranking based on the absolute value of the coefficient.

Some of the popular risk assessment software generates simple linear regressions as part of their sensitivity analysis package. A simple linear regression has only one independent (X) variable. The software will regress each input value against the same selected output using the ordinary least squares (OLS) estimation technique and rank them by normalized regression coefficients (β). If the output (dependent variable) is actually a function of several inputs (independent variables), this technique could violate one or more classical assumptions for OLS estimators (Studenmund 2006) and provide misleading results. Consequently, one should be wary of these built-in regression results.

Multiple regression techniques allow evaluation of the sensitivity of individual model inputs, taking into account the influence of other inputs on the output. The cost estimate model does not provide a very interesting example for this technique because it is so linear, so let us switch examples again for the moment. The example used for this demonstration is based on an FDA (2001) risk assessment entitled "The Human Health Impact of Fluoroquinolone Resistant *Campylobacter* Attributed to the Consumption of Chicken." Fluoroquinolone (FQ) drugs had been administered to some domestic U.S. poultry at subtherapeutic doses. There was some evidence to suggest this was causing an increase in antibiotic resistance of *Campylobacter* to this drug. The risk hypothesis suggested that some consumers who were made ill by FQ-resistant *Campylobacter* in chickens would seek medical care and subsequently be prescribed FQ drugs, which would be ineffective against their illness. The FDA conducted a risk assessment to learn how many people may have been affected in this way.

Five input variables are used to estimate the number of people affected by this problem. They are the independent variables in the regression and are listed in Table 17.8. A sample of $n = 500$ values for each input and the output was obtained through a Monte Carlo simulation using the FDA model. A multiple regression was run with this sample using the number of people with *campylobacteriosis* seeking the care of a doctor and being administered FQ drugs for the dependent variable. The result in Table 17.8 shows all independent variables, but the proportion of sick people seeking care from a doctor have effects that are statistically significantly different from zero. The beta coefficients have not been normalized over the [−1,+1] interval because all inputs have the same metric units, so the usual coefficients provide a ranking.

TABLE 17.8
Regression Results

Regression	Coefficient	p-Value
Constant	–25,523	<0.0001
Proportion of *Campylobacter* cases associated with chicken	171	<0.0001
Proportion of FQ resistant *Campylobacter* infections from chicken	538	<0.0001
Proportion sick seeking care	129	0.1164
Proportion treated with antibiotic	91	0.0029
Proportion receiving FQ treatment	189	<0.0001

Table 17.8 shows that a 1% increase in the proportion of FQ-resistant *Campylobacter* infections from chicken has the potential to affect an average of 538 people, with all other inputs held constant. This makes it the most sensitive input of those investigated.

Using simple (one independent variable) linear regression can produce less reliable results. In addition, the built-in software features may fail to select important intermediate calculations as inputs. In general, these software features identify inputs as those spreadsheet cells that have been assigned a probability distribution. We will return to this simple regression technique and the cost estimation example when we consider graphical sensitivity analysis techniques.

17.3.3.2 Correlation

Correlation is a statistical method used to determine whether two variables (an input and an output) tend to move together. If above-mean values of one variable are associated with above-mean values of the other, and vice versa, there is a positive correlation. A negative correlation results when above-mean values of one variable are associated with below-mean values of another variable.

The Pearson correlation coefficient is a numerical value between −1 and 1. The sign of the coefficient indicates whether the association is a positive one or a negative one. The size of the coefficient indicates the strength of the association. The top row of Figure 17.7 shows negative correlations with coefficients of −1, −0.5, and −0.25. The bottom row shows positive correlations of 0.8, 0.5, and 0.25. Note that as the absolute value approaches 1, the cloud of points approaches a straight line.

Data from a simulation that produces a sample of input and output values can be used to calculate correlation coefficients. This is a feature on some simulation software. Correlation is not causation, and correlation coefficients are best understood when accompanied by a scatter plot like those in Figure 17.7. In general, inputs that are highly correlated with the output are of potential interest in a sensitivity analysis.

For a correlation example, let us return to our cost estimate example. A 10,000-iteration simulation of costs produced the characterization of the possible costs of dredging the waterway and creating wetlands with the dredged material seen in Figure 17.8.

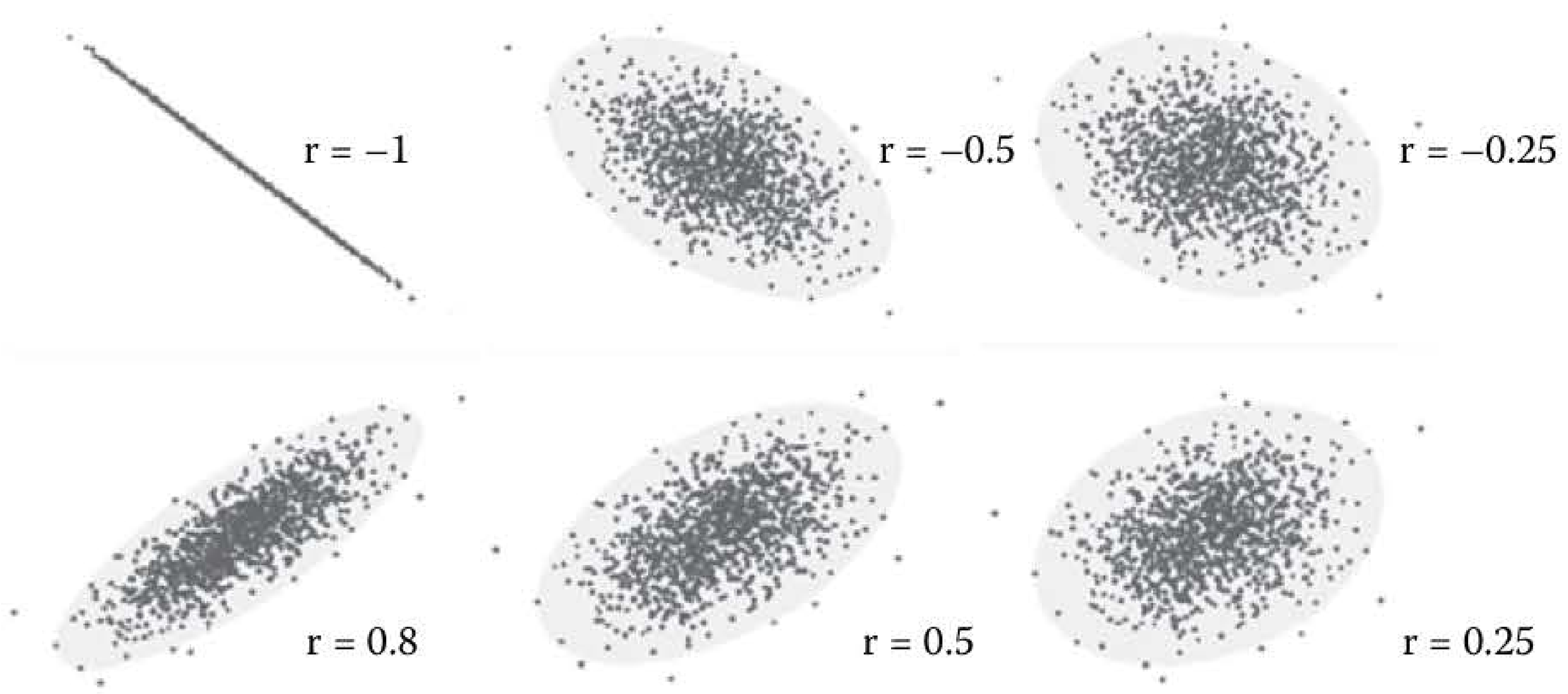

FIGURE 17.7 Selected scatter plots and Pearson correlation coefficient examples.

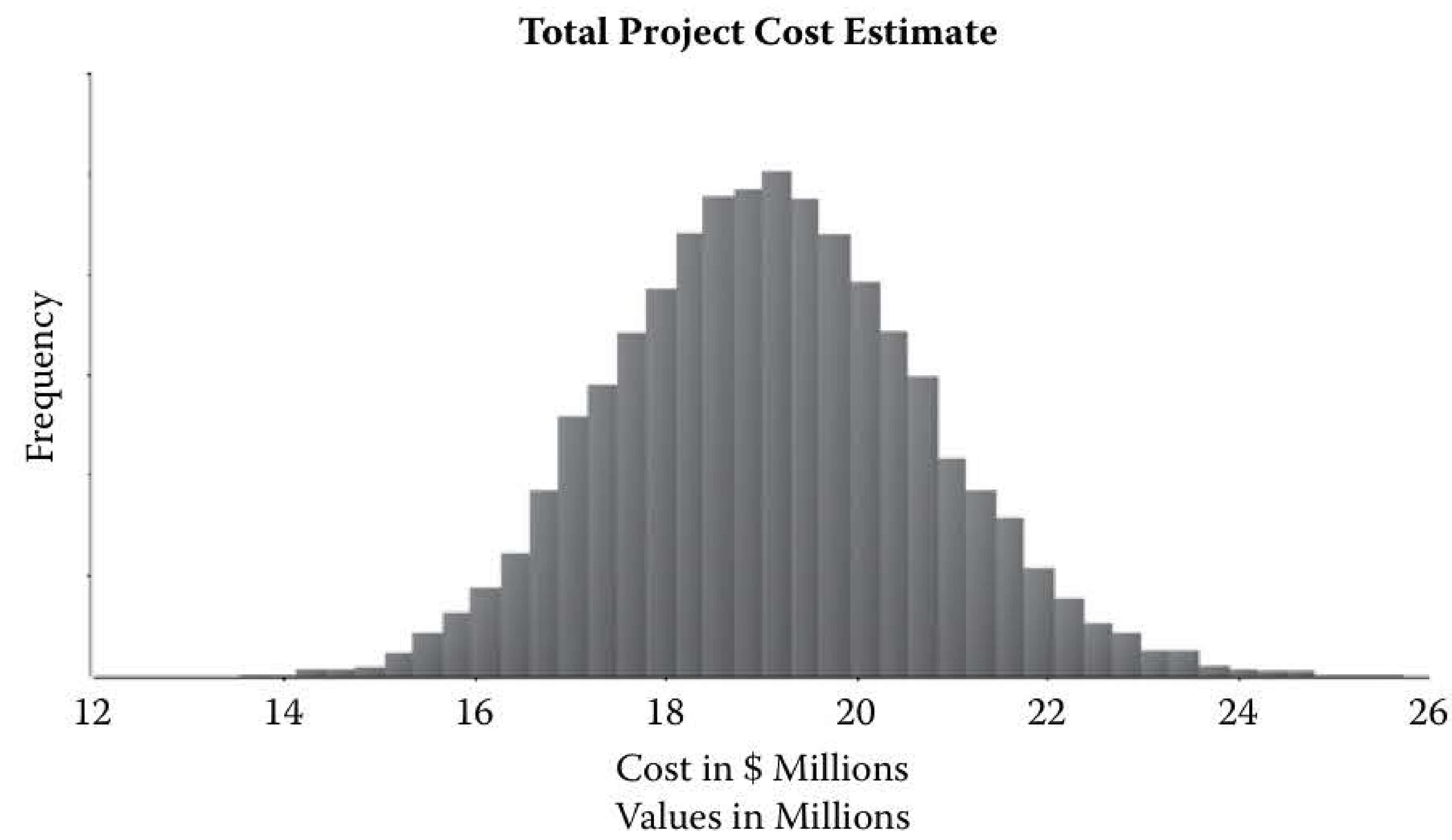

FIGURE 17.8 Distribution of dredging cost estimates.

The simulation had 20 variable inputs and a single output. @RISK 7.5's built-in simulation sensitivity produced Spearman rank correlation coefficients for each input-output pair as shown in Table 17.9.

This technique indicates that the total project cost is most highly correlated to the price of 45-foot diameter geotubes in reach 1, followed by the dredging quantity to be removed from reach 3A. Thus, any efforts to reduce uncertainty in these variables would produce a more reliable cost estimate. This sensitivity analysis can be equally useful at times in identifying variables that are of no special concern. Variables with low correlation coefficients can be treated as point estimates with no loss of fidelity. You need not develop probability distribution estimates for every uncertain input.

TABLE 17.9
Importance Analysis of Cost Estimate Inputs Using Spearman Rank Correlation Coefficient

Rank Four Output	Cell	Name	Spearman Correlation Coeff.
1	E26	Price of Reach 1 45′ Geotube	0.529
2	C22	Quantity Pipeline Dredging, Reach 3A	0.406
3	E24	Price of Reach 1 Scour Pad	0.4
4	C21	Quantity Pipeline Dredging, Reach 2	0.325
5	E22	Price per CY Reach 3A	0.297
6	E21	Price per CY Reach 2	0.208
7	C20	Quantity Pipeline Dredging, Reach 1	0.164
8	E23	Price per CY Reach 3B	0.145
9	E20	Price per CY Reach 1	0.133
10	C23	Quantity Pipeline Dredging, Reach 3B	0.091

The cruel irony is that you may not know this until after you have done so and conducted this kind of sensitivity analysis.

17.3.3.3 Analysis of Variance

Analysis of variance (ANOVA) is another statistical method that is a model-independent probabilistic sensitivity analysis method. It is used to determine if there is a statistical association between an output and one or more inputs. Unlike regression analysis, ANOVA requires no assumption about the functional form of the relationships between inputs and the outputs.

Inputs are called "factors." Values of quantitative factors and categories of qualitative variables are called factor levels. The output is called a "response variable." Single-factor and multifactor ANOVA are options for the analyst. ANOVA is used to determine if values of the output vary in a statistically significant manner associated with variation in values for one or more inputs. If variation in the output is not statistically significant with respect to the input(s), then it is considered random. There are somewhat stringent assumptions required to validate the ANOVA process. If these assumptions are violated, corrective measures must be taken to address the problem. ANOVA is a technique preferred by certain disciplines. It is a topic that can be found in most standard statistics texts. Warner (2008) has an especially detailed and helpful treatment of the topic.

17.3.3.4 Response-Surface Method (RSM)

The response-surface method (RSM) is a statistical technique used to estimate the relationship between a response variable (output) and one or more explanatory inputs. It is a complex method. Think of it as the graph of a surface in n-space that identifies curvatures in this space by accounting for second-order effects that enable assessors to observe the effect on the output given selected effects in one or more inputs.

It is best to limit the number of inputs so as to limit the size of n-space. Therefore, RSM is best used after other sensitivity screening methods have identified the most important inputs. Frey and Patil (2002) suggest Monte Carlo simulation methods can be used to generate multiple values of each model input and the corresponding output, and then a least squares regression method is used to fit a standardized first- or second-order equation to the data obtained from the original model. If the classic assumptions of least squares regression are not satisfied, other techniques such as rank-based or nonparametric approaches should be used (Khuri and Cornell 1987, Vidmar and McKean 1996).

A response surface can be linear or nonlinear. Think of it as a "model of a model." Once generated, it is often easier to conduct sensitivity analysis of the response surface than it is of the original model. The sensitivity analysis of the response-surface analysis is often simpler and faster to execute than sensitivity analysis of the original model. This means that computationally intensive sensitivity analysis methods, such as Mutual Information Index (Finn 1993) or others, may be more readily applied to the response surface than to the original model. Applications of the RSM can be found in Gardiner and Gettinby (1998); Moskowitz (1997); Hopperstad et al. (1999); Williams, Varahramyan, and Maszara (1999); and others. This is not a technique that many risk assessors are likely to use due to its complexity.

17.3.3.5 Fourier-Amplitude Sensitivity Test

The Fourier amplitude sensitivity test (FAST) is a statistical method that can be used for both uncertainty and sensitivity analysis (Cukier et al. 1973, 1975, 1978). It is not for beginners. FAST is used to estimate the contribution of individual inputs to the variance of the output. It is independent of any assumptions about model structure. Assessors can study the effect of single or multiple inputs using FAST.

Frey and Patil (2002) describe the method as relying on a transformation function used to convert values of each model input to values along a search curve. The transformation specifies a frequency for each input, and using Fourier coefficients, the variance of the output is evaluated. The contribution of each input observation (x_i) to the total variance is also calculated based on the Fourier coefficients, fundamental frequency, and higher harmonics of the frequency, as explained by Cukier et al. (1975). The ratio of the contribution of each input to the output variance and the total variance of the output can be calculated and used to rank the inputs (Saltelli, Chan, and Scott 2000).

The model needs to be evaluated at enough points in the input parameter space that numerical integration can be used to determine the Fourier coefficients (Saltelli et al. 2000). For applications, see Lu and Mohanty (2001), Helton et al. (2000), and Rodriguez-Camino and Avissar (1998).

17.3.3.6 Mutual Information Index

The mutual information index (MII) is a statistical sensitivity analysis method that produces a measure of the information about the output provided by a specific input. The MII is based on conditional probabilistic analysis. The magnitude of the MII for different inputs can be compared to determine which inputs provide useful information about the output. This is a computationally intensive method typically used for models with dichotomous outputs.

Frey and Patil (2002) describe MII as typically involving three steps: (1) generating an overall confidence measure of the output value; (2) obtaining a conditional confidence measure for a given value of an input; and (3) calculating sensitivity indices (Critchfield and Willard 1986a, 1986b). The cumulative distribution function (CDF) of the output is used to estimate the overall confidence in the output, where confidence is the probability of the dichotomous outcome of interest. The conditional confidence is estimated by holding one input constant at some value and varying all other inputs. The new resulting CDF of the output is a measure of the assessor's confidence in the output conditioned on the particular value of the input used to generate it.

The mutual information between two random variables is the amount of information about a variable that is provided by the other variable (Jelinek 1970). In other words, it is a quantity that measures the mutual dependence of the two variables. A description of the calculation of the MII is found in Frey and Patil (2002).

Critchfield and Willard (1986a, 1986b) devised and demonstrated the application of the MII method using a decision-tree model. MII includes a more direct measure of the probabilistic relatedness of two random variables than correlation coefficients, and it can account for the joint effects of all inputs when evaluating sensitivities of an input. It is computationally complex and difficult to apply. It is not one of the more commonly applied sensitivity analysis methods.

17.3.4 Graphical Methods for Sensitivity Analysis

17.3.4.1 Scatter Plots

Scatter plots, shown earlier in Figure 17.7, can be used to visually assess the influence of individual inputs on an output. A Monte Carlo simulation, for example, can generate many input-output pairs, which when plotted can reveal potentially sensitive associations between these variables. Linear or nonlinear patterns, which may be observed, could depict potential dependencies between an input and an output.

There need to be enough points to show a pattern but not so many as to obscure the variability in the scatter. Pattern detection methods can be applied to help identify relationships between inputs and outputs (Kleijnen and Helton 1999, Shortencarier and Helton 1999). Applications are found in Sobell et al. (1982); Rossier, Wade, and Murphy (2001); Hagent (1992); Fujimoto (1998); Helton et al. (2000); and others.

When you have completed a probabilistic risk assessment, it can be helpful to plot the scatter plot for each input-output relationship as a first step in the sensitivity analysis of your statistical sample. It is a useful screening mechanism, but it can become quite tedious if the model has large numbers of inputs and outputs. When a pattern is evident, more rigorous sensitivity analysis is warranted.

For an example, consider the cost estimate again. Earlier, the Spearman rank correlation coefficient identified the price of 45-foot diameter geotube as a significant source of variation in the output. A plot of 500 data points from the 10,000-iteration simulation in Figure 17.9 shows a clear positive relationship between geotube cost/ft. and total project cost. Note that this correlation coefficient is a Pearson correlation and that it differs from the Spearman value often produced by commercial software packages.

17.3.4.2 Tornado Plots

Tornado graphs are a variation of a bar graph. They are used to show the relative sensitivity of an output to uncertain (or variable) inputs. They normally display a vertical zero-point axis when sensitivity is measured by a correlation or normalized regression coefficient. Bars extending to the right of this axis indicate a positive relationship with the selected output; bars extending to the left of the zero axis indicate a negative relationship.

The length of the bar indicates the relative strength of the positive or negative relationship. It is often measured as a normalized regression coefficient, when the sensitivity is based on a normalized simple linear regression between an individual input and the selected output, or it may be measured by a correlation (Spearman or Pearson) coefficient.

The example shown in Figure 17.10 begins with the distribution of total project costs. There is about a $12-million range in the cost estimate outputs. To learn what contributes most to this variation, a regression analysis or correlation sensitivity method must first be completed and its results plotted, as shown in the bottom graph of Figure 17.10. This tornado plot is based on normalized beta coefficients from a sequence of simple linear regressions. There are no inputs that decrease cost.

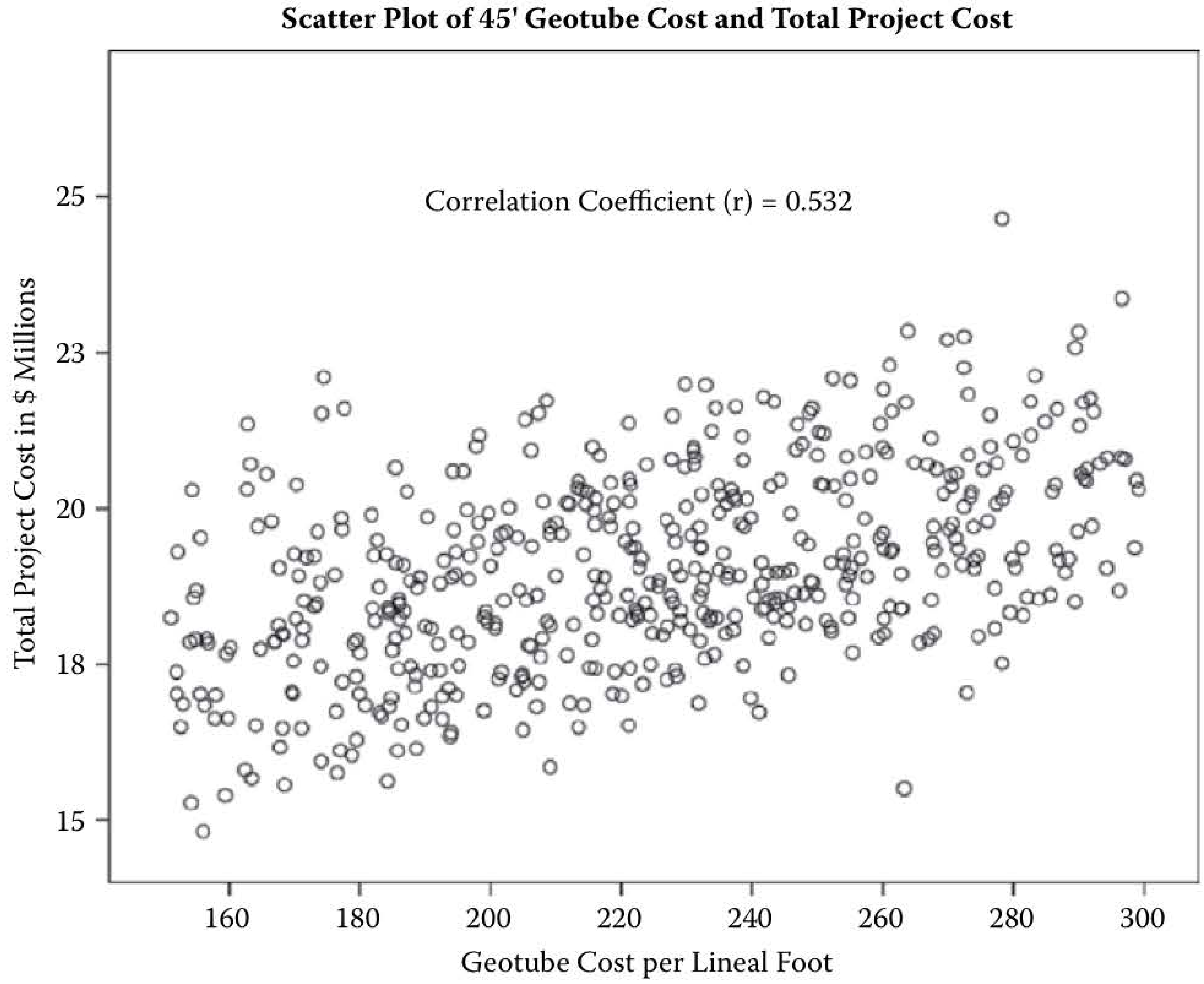

FIGURE 17.9 Scatterplot of total project cost and the price of a 45-foot-diameter geotube.

In this instance, the price of 45-foot geotubes is at the top of the list.* A tornado chart can be prepared for each model output of interest to risk managers. This statistical sensitivity analysis, which is based on data from a Monte Carlo process, yields different results than our initial OAATA did, as we should expect. These are all tools that require professional judgment. The price of 45-foot geotubes was not the most significant input during the OAATA analysis, although it was near the top. Had an assessor decided not to pursue the quantification of the uncertainty around that value, it would have been an error. We need to take into account not only the structure of the model, which OAATA did reasonably well, but also the magnitude of the uncertainty. As it turned out, the cost estimators were less certain about the price of the large geotubes than about most other inputs, i.e., the input varied by more than ±20%.

Let us return, briefly, to our OAATA analysis and consider another form of the tornado graph. Figure 17.11 shows a tornado graph prepared using a nominal range sensitivity (one-way what-if analysis) based on point estimates rather than a Monte Carlo process using probability distributions for the inputs.

* If you are curious why these results are different from the OAATA results, recall that that is a mathematical method based on point estimates for each input. This is a statistical one that is based on the expression of each input value as a probability distribution. The actual uncertainty among the inputs is more variable than the ±20% used for each input in the OAATA.

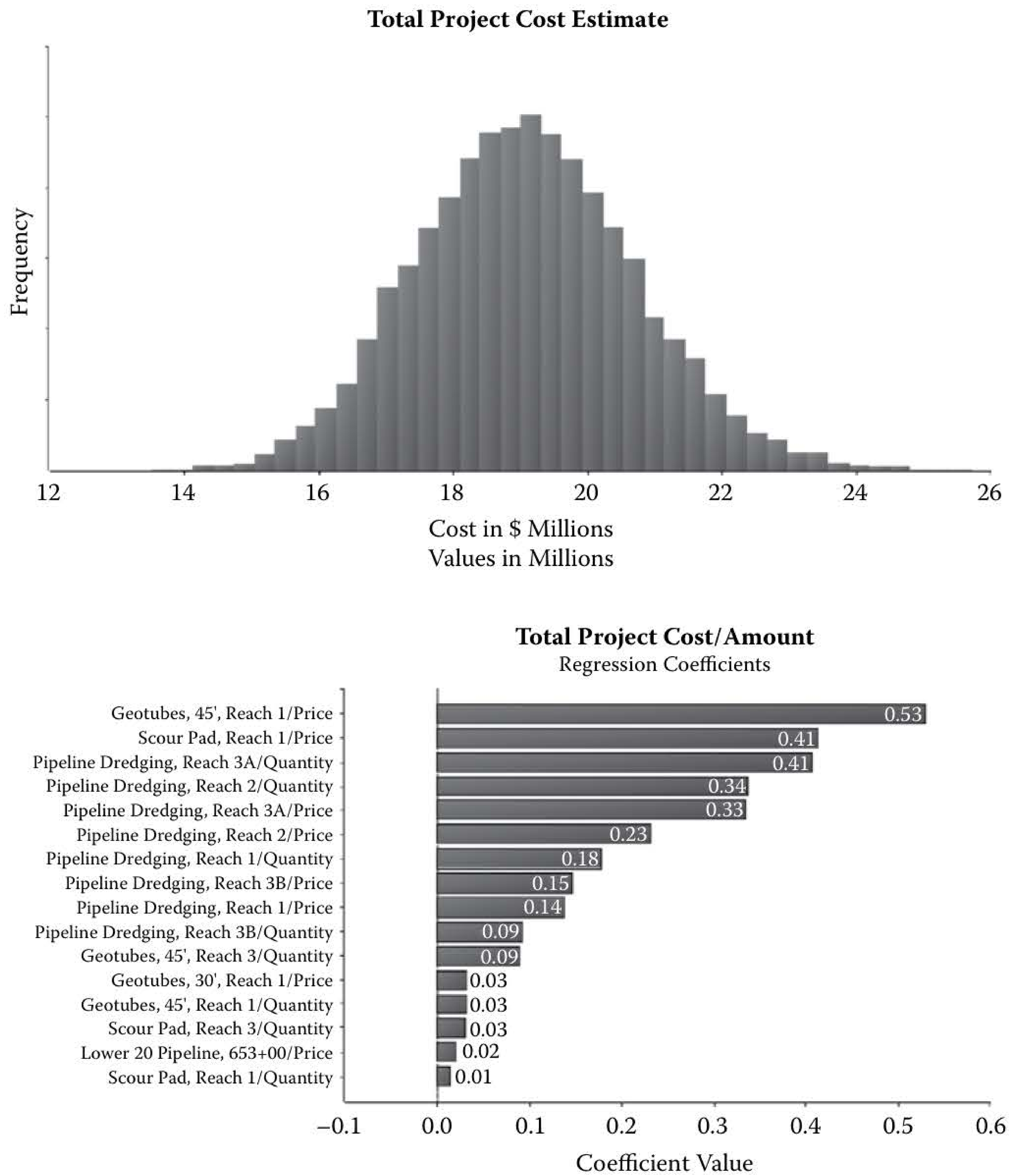

FIGURE 17.10 Total project cost distribution with a regression-based tornado chart.

In this particular graph, the inputs on the left were allowed to vary by ±20%. The impact of this variation on total project costs is shown on the horizontal axis of the graph. In this graph, the impact is measured as a percentage. It could have just as easily been measured in actual costs, with the deterministic cost estimate as the center of the graph instead of 0%. The length of the bar indicates the amount of change the input caused to the output measured as a percentage. The input with the largest effects (and longest bar) is shown at the top, and those with less impact are shown below. The diminishing influence of less significant inputs produces the tornado shape. Note that geotube prices are listed third on the tornado chart.

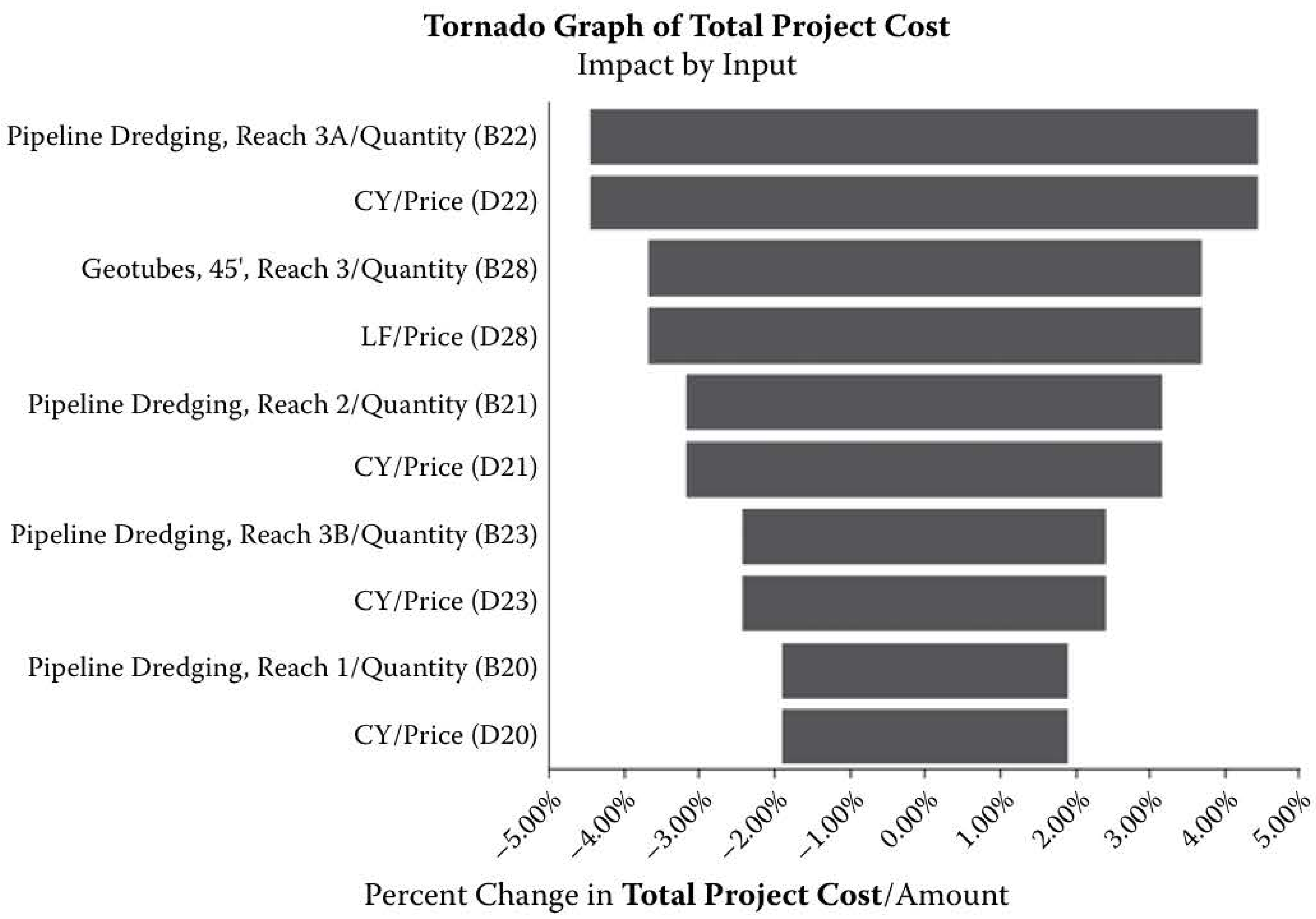

FIGURE 17.11 OAATA tornado graph for total project cost.

17.3.4.3 Spider Plot

Figure 17.12 shows a sensitivity graph. It shows the relationship between the quantity of dredged material in reach 3a and total project cost. The horizontal axis shows input changes of –20%, –10%, 0%, 10%, and 20%. The vertical axis shows the corresponding effect on total project costs. This is essentially the data from an analysis like that in Table 17.5.

A spider plot is a collection of multiple sensitivity graphs. The spider plot is an alternative way of visualizing effects of inputs on outputs that is useful for models with fixed-point inputs. It relies on the nominal range sensitivity method to generate data for the plot. Variation in input effects often creates a spiderlike spread of effects about the no-change point (0%, 0%). An example is shown in Figure 17.13.

For each input, the percentage change in its value from the fixed-point estimate (base case) is plotted on the x-axis, and the percentage change in the output is plotted on the y-axis. Lines that are higher to the right of the no-change (0%) point indicate inputs with a greater impact on the output under investigation. Conversely, lines that lie highest to the left of the no-change point have the least impact on the output. This figure is showing that a 20% change in the cost of dredging in reach 3a causes over a 4% change in total project cost, making it the most sensitive input in the spider plot. This is the same result reported numerically earlier in the chapter.

17.4 THE POINT

The point of doing sensitivity analysis does not end with the identification of your most sensitive inputs. Finding out what your most significant uncertainties are is an important part of a sensitivity analysis. The real purposes of a sensitivity analysis, however, are to help develop a plan for addressing the instrumental uncertainty to inform decision makers about the significance of uncertainty for decision making.

Nominal range sensitivity, for example, can be used early in a risk assessment to help identify those variables for which the most effort should be made to describe the uncertainty. It is not always necessary, as mentioned earlier, to enter every uncertain input as a probability distribution. Some of these techniques can suggest which variables to concentrate on.

When risk managers are presented with output distributions like the one in Figure 17.14 for costs, it is helpful to be able to explain why there is so much variation in the output upon which they will base their decisions. Furthermore, if we can present risk managers with options for further reducing the instrumental uncertainties, the sensitivity analysis adds even more value to the risk assessment. If we assume a budget cap on project cost of $20 million we see a 28.3% the cap will be exceeded. Sensitivity analysis can be used to identify the circumstances most likely to result in a cost over $20 million. Managers then can choose to reduce the uncertainty or to make a decision based on the available information.

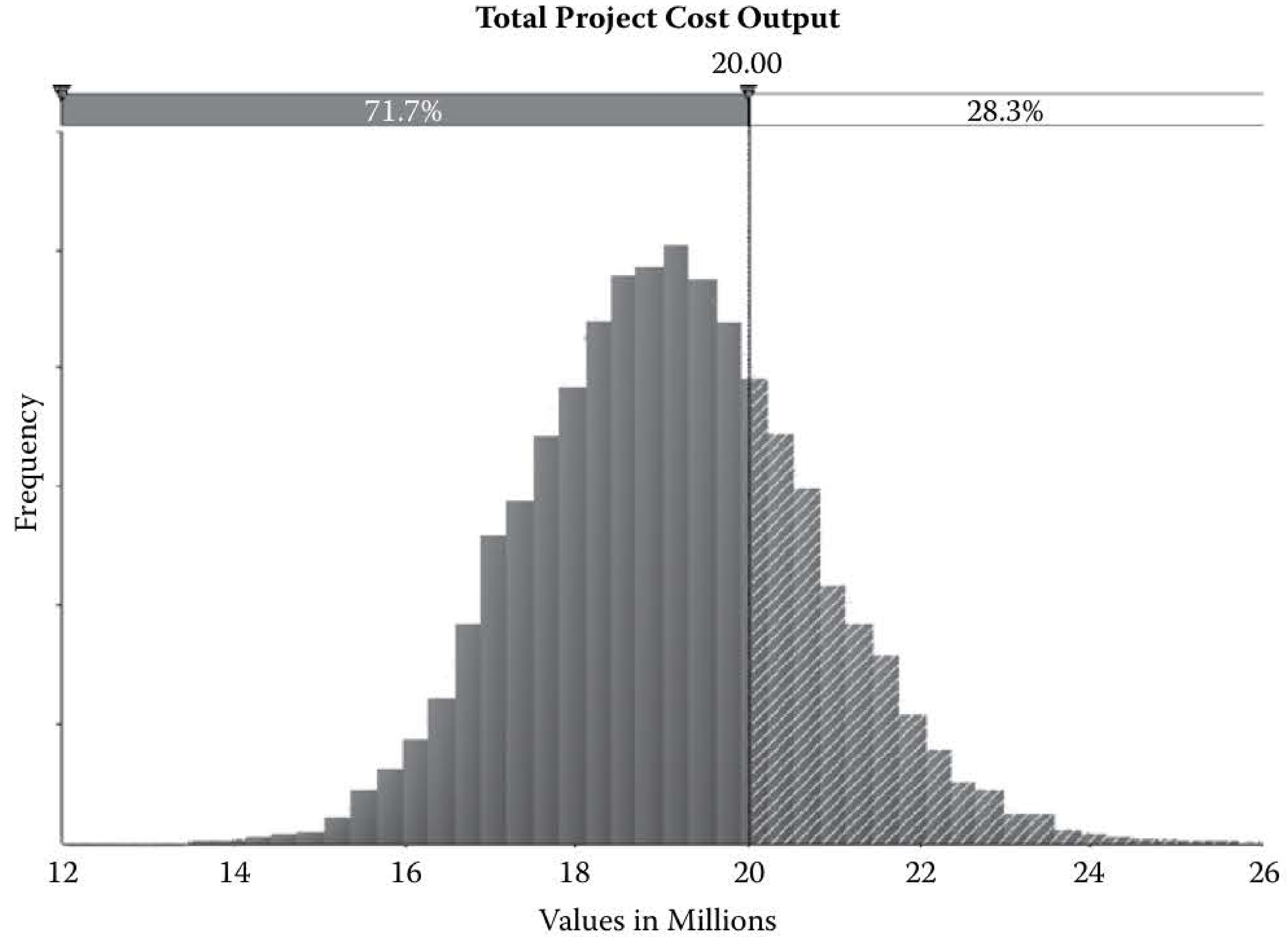

FIGURE 17.14 Distribution of total project cost estimates.

The best risk assessments will be subjected to sensitivity analysis that will discover the most important input variables. This helps everyone understand what contributes most to good and bad outcomes. Once these important variables have been identified, risk assessors should address the uncertainty systematically, by varying assumptions and examining their effects on outcomes, using one or more of the techniques here, or of course, by doing a probabilistic risk assessment. Good risk assessment fixes what can be fixed and addresses what can be addressed. If variation due to knowledge uncertainty can be reduced, risk managers may choose to do so. To the extent that the variation in output is attributable to natural variability, sensitivity analysis can help assessors describe that variability.

Risk managers need to understand the instrumental uncertainties that could influence their decisions. When a decision is sensitive to changes or uncertainties within the realm of possibility, then more precision and additional information may be required. In addition, they need to understand the potential quality of the risk management options they are considering. Thus, it is helpful, when conducting a sensitivity analysis, to identify sensitive inputs that are controllable, especially decision variables. When sensitivity analysis identifies those inputs with the greatest positive and negative effects on outputs as well as which of those we can influence, it adds value to risk assessment.

Consider a sensitivity analysis that suggests that the price of 45-foot geotubes is a significant source of output uncertainty. This could provide the impetus for an estimator to contact manufacturers of these tubes for a more reliable price quote. Sometimes sensitivity analysis can suggest new risk management options. For example, the uncertainty attending this cost estimate might be managed through an innovative futures contract arrangement. The responses to a good sensitivity analysis can influence both risk assessment and risk management.

17.5 SUMMARY AND LOOK FORWARD

Good risk assessment, whether qualitative or quantitative, must include some sensitivity analysis. Examining the effects of varying the assessor's assumptions should always be included in every sensitivity analysis. Qualitative sensitivity analysis has often been overlooked, and it should not be. A three-step process was suggested in this chapter. It includes identifying sources of uncertainty, identifying instrumental uncertainties and characterizing their effects on decision parameters.

Quantitative sensitivity analysis has a much larger toolbox available to it. Some of these tools can be quite sophisticated. My personal bias is to use the simplest quantitative technique you legitimately and usefully can. The purpose of sensitivity analysis is to support decision making that is better informed about the instrumental uncertainties in an assessment. Not only are complex techniques more difficult to execute properly, they are often much more difficult for others to understand. There will no doubt be circumstances when the most sophisticated and robust techniques are warranted by the goals of the risk management activity, but when a good job can be done with simpler methods, use the simpler methods.

By now we have discussed a good many sophisticated analytical techniques capable of generating a great deal of decision-critical information. The next question