CHAPTER 13

Inventory Management

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LEARNING OBJECTIVES

After reading this chapter, you will be able to:

- Explain why companies keep inventory and how advances in IT have impacted inventory decisions.
- Discuss the key elements and costs of inventory, and the relationship between inventory costs and customer service.
- Contrast continuous and periodic inventory systems, and classify inventory according to the ABC system.
- Utilize basic inventory models to calculate order quantity and related measures, as well as the annual cost of inventory.
- Determine the appropriate reorder point in a continuous inventory system based on a target service level.
- Calculate the order quantity for a periodic inventory system.

World-class Inventory Management at Apple

From 2008 to 2014 Apple ranked first every year in the Gartner Supply Chain Top 25 ranking, and in 2015 it was placed (with Proctor & Gamble) in a new "Supply Chain Masters" category that recognizes a company's sustained supply chain leadership. A primary reason for Apple's supply chain success and this ultimate special categorization is how it manages its inventory. Accurate demand forecasting and elimi-nating excess inventory is absolutely critical in the computer-related industry. Keeping little inventory on hand is important because of inventory (warehouse) costs and new products from competitors. A sudden new product announcement or a new innovation can immediately reduce the value of products in inventory. New products cannibalize old ones very quickly and the inventory for technology like smartphones, tablets, and laptops depreciates very rapidly, losing an estimated 1% to 2% of value each week. Apple CEO, Tim Cook, has stated that "inventory is fundamentally evil. You kind of want to manage it like you're in the dairy business. If it gets past its freshness date, you have a problem."

Apple manages its inventory much better than competitors like Samsung, Dell, and Motorola, with a higher number of inventory turns (i.e., how many times a company's inventory can be sold and replaced over a specific period of time). In general, a fundamental part of Apple's inventory management planning process is to sell everything it makes, with no waste from unsold inventory. Apple operates its supply chain by purchasing components and materials from various suppliers, has them shipped to assembly plants primarily in China, and then ships the finished products directly to their customers (via UPS and FedEx) who purchase them online. Apple ships products to other distributors and

their own retail stores from its central warehouse and call center in Elk Grove, California. When Tim Cook joined Apple in 1998 (the same year Steve Jobs rejoined the company) he immediately went to work remaking the supply chain by slashing inventory from a month's worth to six days, shutting down 10 of 19 warehouses to limit overstocking, and reducing component suppliers from 100 to 24 to make them compete for Apple's business. By 2012 Apple was turning its inventory every 5 days compared to Samsung's 21 days. Apple is better able to manage its inventory because it forecasts demand better than its competitors by having fewer product SKUs to keep track of, and because its products tend to have a longer life cycle. The company has also been remarkably adept at predicting (and sometimes initiating) the kind of technologies that will be in demand in the future, which limits imitations, and which also allows them to place orders with suppliers farther in advance, thus reducing costs and closing out competitors' orders.

In this chapter we will learn about some of the different inventory models, techniques, and technologies companies use to determine the lowest-cost amount of inventory to order and keep on hand, which is one of the primary objectives of supply chain management.

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Source: Based on Clara Lu, "Apple Had the Best Supply Chain in the World for the Last Four years – Here is What You Can Learn from It," http://www.tradegecko.com (August 26, 2014)

The objective of inventory management has been to keep enough inventory to meet customer demand and also be cost effective. However, inventory has not always been perceived as an area to control cost. Traditionally, companies maintained "generous" inventory levels to meet long-term customer demand because there were fewer competitors and products in a generally sheltered market environment. In the current global business environment, with more competitors and highly diverse markets in which new products and new product features are rapidly and continually introduced, the cost of inventory has increased due in part to quicker product obsolescence. At the same time, companies are continuously seeking to lower costs so they can provide a better product at a "lower" price.

Inventory is an obvious candidate for cost reduction. It is estimated that U.S. companies carry over a half trillion dollars in inventory spread out along their supply chains. It is also estimated that the average holding cost of inventory in the United States is between 18% and 35% of the total value of the inventory. That means if a company has $10 million worth of products in inventory, and the cost of holding the inventory (including insurance, obsolescence, depreciation, interest, opportunity costs, storage costs, and so on) is approximately 30% or $3 million; if inventory could be reduced by half, to $5 million, then $1.5 million would be saved, a significant cost reduction.

The high cost of inventory has motivated companies to focus on efficient supply chain management and quality management. They believe that inventory can be significantly reduced by reducing uncertainty at various points along the supply chain. In many cases, uncertainty is created by poor quality on the part of the company or its suppliers or both. This can be in the form of variations in delivery times, uncertain production schedules caused by late deliveries, or large numbers of defects that require higher levels of production or service than what should be necessary, large fluctuations in customer demand, or poor forecasts of customer demand.

With efficient supply chain management, products or services are moved from one stage in the supply chain to the next according to a system of constant communication between customers and suppliers. Items are replaced as they are diminished without maintaining larger buffer stocks of inventory at each stage to compensate for late deliveries, inefficient service, poor quality, or uncertain demand. An efficient, well-coordinated supply chain reduces or eliminates these types of uncertainty so that this type of system will work.

Some companies maintain in-process, buffer inventories between production stages to offset irregularities and problems and keep the supply chain flowing smoothly. Quality-oriented companies consider large buffer inventories to be a costly crutch that masks problems and inefficiency primarily caused by poor quality. Adherents of quality management believe that inventory should be minimized. However, this works primarily for a production or manufacturing process. For the retailer who sells finished goods directly to the consumer or the supplier who sells parts or materials to the manufacturer, inventory is a necessity. Few shoe stores, discount stores, or department stores can stay in business with only one or two items on their shelves or racks. For these supply chains, the traditional inventory decisions of how much

to order and when to order continue to be important. In addition, the traditional approaches to inventory management are still widely used by most companies.

In this chapter we review the basic elements of traditional inventory management and discuss several of the more popular models and techniques for making cost-effective inventory decisions. These decisions are basically *how much to order* and *when to order* to replenish inventory to an optimal level.

The Role of Inventory in Supply Chain Management

A company employs an inventory strategy for many reasons. The main reason is holding inventories of finished goods to meet customer demand for a product, especially in a retail operation. However, customer demand can also be a secretary going to a storage closet to get a printer cartridge or paper, or a carpenter getting a board or nails from a storage shed.

Since demand is usually not known with certainty, it is not possible to produce exactly the amount demanded. An additional amount of inventory, called safety, or buffer, stocks, is kept on hand to meet variations in product demand. In the *bullwhip effect* (which we have discussed previously in our chapters on supply chain and forecasting), demand information is distorted as it moves away from the end-use customer. This uncertainty about demand back upstream in the supply chain causes distributors, manufacturers, and suppliers to stock increasingly higher safety stock inventories to compensate.

Additional stocks of inventories are sometimes built up to meet demand that is seasonal or cyclical. Companies will continue to produce items when demand is low to meet high seasonal demand for which their production capacity is insufficient. For example, toy manufacturers produce large inventories during the summer and fall to meet anticipated demand during the holiday season. Doing so enables them to maintain a relatively smooth supply chain flow throughout the year. They would not normally have the production capacity or logistical support to produce enough to meet all of the holiday demand during that season. In the same way retailers might find it necessary to keep large stocks of inventory on their shelves to meet peak seasonal demand, or for display purposes to attract buyers.

At the other end of the supply chain from finished goods inventory, a company might keep large stocks of parts and material inventory to meet variations in supplier deliveries. Inventory provides independence from vendors that a company does not have direct control over. Inventories of raw materials and purchased parts are kept on hand so that the production process will not be delayed as a result of missed or late deliveries or shortages from a supplier.

A company will purchase large amounts of inventory to take advantage of price discounts, as a hedge against anticipated price increases in the future, or because it can get a lower price by purchasing in volume. Walmart stores have been known to purchase a manufacturer's entire stock of soap powder or other retail item because they can get a very low price, which they subsequently pass on to their customers. Companies purchase large stocks of low-priced items when a supplier liquidates. In some cases, large orders will be made simply because the cost of ordering may be very high, and it is more cost-effective to have higher inventories than to order frequently.

Many companies find it necessary to maintain buffer inventories at different stages of their production process to provide independence between stages and to avoid work stoppages or delays. Inventories are kept between stages in the manufacturing process so that production can continue smoothly if there are temporary machine breakdowns or other work stoppages. Similarly, a stock of finished parts or products allows customer demand to be met in the event of a work stoppage or problem with transportation or distribution.

Information Technology and Inventory Management

As we pointed out in previous chapters, information technology (IT) has become an enabler for effective supply chain management. Traditionally inventory was owned by the buyer (as opposed to the supplier), it was kept at the buyer's location, and the buyer controlled how and when its inventory was replenished. However, in recent years these traditional aspects of

inventory management have changed, due in large part to advances in IT. Because of technology and software—including such IT tools as enterprises resource planning (ERP) systems (including forecasting software), barcodes, radio frequency identification (RFID), EDI, and point-of-sales data—companies can track and locate inventory throughout their supply chain, which enables them to locate inventory somewhere other than their own facility, and control it remotely or have someone else control it. These technologies have enabled modern supply chain management practices such as vendor managed inventory (VMI), continuous replenishment programs (CRP), supplier hubs, and outsourcing operations to third-party service providers (3PL). In these practices inventory can be located at the supplier's facility, at the buyer's, or somewhere in between. Unlike traditional practices, the supplier owns inventory until the buyer needs it and it is delivered, thus relieving the buyer of inventory costs; order sizes are reduced, deliveries (which the supplier pays for) are increased, and the buyer avoids maintaining storage facilities. However, for this to be effective the supplier must be able to minimize its own inventory costs and optimize its own supply chain, which can be achieved if the buyer shares end-use demand and sales data with its suppliers through IT. This enables suppliers to make replenishment decisions and provide inventory to the buyer, as it's needed. A recent supply chain management practice is for inventory to be located at “supplier hubs” that are usually at, or in very close proximity to, the buyer, and are often owned and operated by a 3PL provider, which shifts all responsibility and liability for inventory to the suppliers that share the hub. For a supplier hub to work, the supply chain members—buyers, suppliers, and 3PL providers—must share information through information technology. The 3PL provider uses information provided by the buyer and suppliers (including forecasts and sales data) to consolidate shipping among suppliers, plan and execute all logistics, connect and coordinate all supply chain members through an IT system, and operate the hub facility. Supplier hubs are being used successfully by such companies as Dell, Apple, Fiat, Hewlett-Packard, Nokia, Cisco, Sam's Club, Samsung, and Volkswagen.

Inventory and Quality Management in the Supply Chain

A company maintains inventory to meet its own demand and its customers' demand for items in the supply chain. The ability to meet effectively internal organizational demand or external customer demand in a timely, efficient manner is referred to as the *level of customer service*. A primary objective of supply chain management is to provide as high a level of customer service in terms of on-time delivery as possible. This is especially important in today's highly competitive business environment, where quality is such an important product characteristic. Customers for finished goods usually perceive quality service as availability of goods they want when they want them. (This is equally true of internal customers, such as company departments or employees.) To provide this level of quality customer service, the tendency is to maintain large stocks of all types of items. However, there is a cost associated with carrying items in inventory, which creates a cost tradeoff between the quality level of customer service and the cost of that service.

As the level of inventory increases to provide better customer service, inventory costs increase, whereas quality-related customer service costs, such as lost sales and loss of customers, decrease. The conventional approach to inventory management is to maintain a level of inventory that reflects a compromise between inventory costs and customer service. However, according to the contemporary “zero defects” philosophy of quality management, the long-term benefits of quality in terms of larger market share outweigh lower short-term production-related costs, such as inventory costs. Attempting to apply this philosophy to inventory management is

not simple because one way of competing in today's diverse business environment is to reduce prices through reduced inventory costs.

The Elements of Inventory Management

Inventory A stock of items kept to meet demand.

Inventory is a stock of items kept by an organization to meet internal or external customer demand. Virtually every type of organization maintains some form of inventory. Department stores and grocery stores carry inventories of all the retail products they sell; a nursery has

inventories of different plants, trees, and flowers; a rental-car agency has inventories of cars; and a major league baseball team maintains an inventory of players on its minor league teams. Even a family household maintains inventories of items such as food, clothing, medical supplies, and personal hygiene products.

Most people think of inventory as a final product waiting to be sold to a retail customer—a new car or a can of tomatoes. This is certainly one of its most important uses. However, especially in a manufacturing firm, inventory can take on forms besides finished goods, including:

- Raw materials
- Purchased parts and supplies
- Partially complete work in process (WIP)
- Items being transported
- Tools and equipment

The purpose *of inventory management* is to determine the amount of inventory to keep in stock —how much to order and when to replenish, or order. In this chapter we describe several different inventory systems and techniques for making these determinations.

Demand

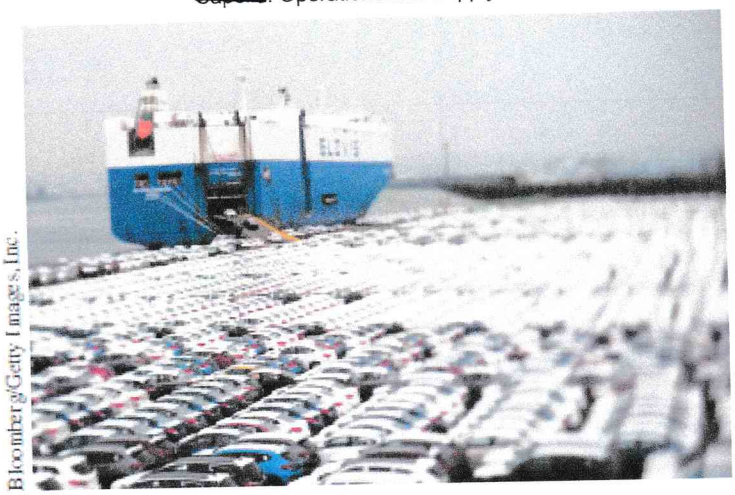
The starting point for the management of inventory is customer demand. Inventory exists to meet customer demand. Customers can be inside the organization, such as a machine operator waiting for a part or partially completed product to work on. Customers can also be outside the organization—for example, an individual purchasing groceries or a new DVD player. In either case, an essential determinant of effective inventory management is an accurate forecast of demand. For this reason the topics of forecasting (Chapter 12) and inventory management are directly interrelated.

Dependent demand Items are used internally to produce a final product.

Independent demand Items are final products demanded by external customers.

In general, the demand for items in inventory is either dependent or independent. **Dependent demand** items are typically component parts or materials used in the process of producing a final product. If an automobile company plans to produce 1000 new cars, then it will need 5000 wheels and tires (including spares). The demand for wheels is dependent on the production of cars—the demand for one item depends on demand for another item (see photo).

Cars, retail items, grocery products, and office supplies are examples of independent demand items. **Independent demand** items are final or finished products that are not a function of, or dependent on, internal production activity. Independent demand is usually determined by external market conditions and, thus, is beyond the direct control of the organization. In this chapter we focus on the management of inventory for independent demand items.



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These offloaded cars at a port are an example of independent demand, as are appliances, computers, and houses. The tires on these cars are an example of a dependent demand item.

Inventory Costs

Three basic costs are associated with inventory: carrying, or holding, costs; ordering costs; and shortage costs.

Carrying costs The costs of holding an item in inventory.

Carrying (or holding) costs are the costs of holding items in inventory. Annual inventory carrying costs in the United States are estimated to be over $400 billion. These costs vary with the level of inventory in stock and occasionally with the length of time an item is held. That is, the greater the level of inventory over a period of time, the higher the carrying costs. In general, any cost that grows linearly with the number of units in stock is a carrying cost. Carrying costs can include the following items:

- Facility storage (rent, depreciation, power, heat, cooling, lighting, security, refrigeration, taxes, insurance, etc.)
- Material handling (equipment)
- Labor
- Record keeping
- Borrowing to purchase inventory (interest on loans, taxes, insurance)
- Product deterioration, spoilage, breakage, obsolescence, pilferage

Carrying costs are normally specified in one of two ways. The usual way is to assign total carrying costs, determined by summing all the individual costs just mentioned, on a per-unit basis per time period, such as a month or year. In this form, carrying costs are commonly expressed as a per-unit dollar amount on an annual basis; for example, $10 per unit per year. Alternatively, carrying costs are sometimes expressed as a percentage of the value of an item or as a percentage of average inventory value. It is generally estimated that carrying costs range from 10% to 40% of the value of a manufactured item.

Ordering costs The costs of replenishing inventory.

Ordering costs are the costs associated with replenishing the stock of inventory being held. These are normally expressed as a dollar amount per order and are independent of the order size. Annual ordering costs vary with the number of orders made—as the number of orders increases, the ordering cost increases. In general, any cost that increases linearly with the number of orders is an ordering cost. Costs incurred each time an order is made can include requisition and purchase orders, transportation and shipping, receiving, inspection, handling, and accounting and auditing costs.

Ordering costs react inversely to carrying costs. As the size of orders increases, fewer orders are required, reducing ordering costs. However, ordering larger amounts results in higher inventory levels and higher carrying costs. In general, as the order size increases, ordering costs decrease and carrying costs increase.

Shortage costs Temporary or permanent loss of sales when demand cannot be met.

Shortage costs, also referred to as *stockout costs*, occur when customer demand cannot be met because of insufficient inventory. If these shortages result in a permanent loss of sales, shortage costs include the loss of profits. Shortages can also cause customer dissatisfaction and a loss of goodwill that can result in a permanent loss of customers and future sales. Some studies have shown that approximately 8% of shoppers will not find the product they want to purchase in stock, which will ultimately result in total lost sales of about 3%.

In some instances, the inability to meet customer demand or lateness in meeting demand results in penalties in the form of price discounts or rebates. When demand is internal, a shortage can cause work stoppages in the production process and create delays, resulting in downtime costs and the cost of lost production (including indirect and direct production costs).

Costs resulting from lost sales because demand cannot be met are more difficult to determine than carrying or ordering costs. Therefore, shortage costs are frequently subjective estimates and sometimes an educated guess.

Shortages occur because carrying inventory is costly. As a result, shortage costs have an inverse relationship to carrying costs—as the amount of inventory on hand increases, the carrying cost increases, whereas shortage costs decrease.

The objective of inventory management is to employ an inventory control system that will indicate how much should be ordered and when orders should take place so that the sum of the three inventory costs just described will be minimized.

Inventory Control Systems

An inventory system controls the level of inventory by determining how much to order (the level of replenishment) and when to order. There are two basic types of inventory systems: a *continuous* (*or fixed-order-quantity*) *system* and a *periodic* (or *fixed-time-period*) *system*. In a continuous system, an order is placed for the same constant amount whenever the inventory on hand decreases to a certain level, whereas in a periodic system, an order is placed for a variable amount after specific regular intervals.

Continuous Inventory Systems

Continuous inventory system A constant amount is ordered when inventory declines to a predetermined level.

In a **continuous inventory system** (also referred to as a *perpetual system* and a *fixed-order-quantity system*), a continual record of the inventory level for every item is maintained. Whenever the inventory on hand decreases to a predetermined level, referred to as the *reorder point*, a new order is placed to replenish the stock of inventory. The order that is placed is for a fixed amount that minimizes the total inventory costs. This amount, called the *economic order quantity*, is discussed in greater detail later.

A positive feature of a continuous system is that the inventory level is continuously monitored, so management always knows the inventory status. This is advantageous for critical items such as replacement parts or raw materials and supplies. However, maintaining a continual record of the amount of inventory on hand can also be costly.

A simple example of a continuous inventory system is a ledger-style checkbook that many of us use on a daily basis. Our checkbook comes with 300 checks; after the 200th check has been used (and there are 100 left), there is an order form for a new batch of checks. This form, when turned in at the bank, initiates an order for a new batch of 300 checks. Many office inventory systems use *reorder cards* that are placed within stacks of stationery or at the bottom of a case of pens or paper clips to signal when a new order should be placed. If you look behind the items on a hanging rack in a Kmart store, there will be a card indicating it is time to place an order for the item for an amount indicated on the card.

Continuous inventory systems often incorporate information technology tools to improve the speed and accuracy of data entry. A familiar example is the computerized checkout system with a laser scanner used by many supermarkets and retail stores. The laser scanner reads the universal product code (UPC), or bar code, from the product package; the transaction is instantly recorded and the inventory level updated. Such a system is not only quick and accurate, it also provides management with continuously updated information on the status of inventory levels. Many manufacturing companies' suppliers and distributors also use bar code systems and handheld laser scanners to inventory materials, supplies, equipment, in-process parts, and finished goods, as shown in the photo.

Clerkenwell Images/Stockphoto

To consumers the most familiar type of bar code scanner is used with cash registers at retail stores, where the bar code is a single line with 11 digits, the first 6 identifying a manufacturer and the last 5 assigned to a specific product by the manufacturer. This employee is using a portable hand-held bar code scanner to scan a bar code for inventory control. In addition to identifying the product, it can indicate where a product came from, where it is supposed to go, and how the product should be handled in transit.

Periodic Inventory Systems

Periodic inventory system An order is placed for a variable amount after a fixed passage of time.

In a **periodic inventory system** (also referred to as a *fixed-time-period system* or a *periodic review system*), the inventory on hand is counted at specific time intervals—for example, every week or at the end of each month. After the inventory in stock is determined, an order is placed for an amount that will bring inventory back up to a desired level. In this system, the inventory level is not monitored at all during the time interval between orders, so it has the advantage of little or no required record keeping. The disadvantage is less direct control. This typically results in larger inventory levels for a periodic inventory system than in a continuous system to guard against unexpected stockouts early in the fixed period. Such a system also requires that a new order quantity be determined each time a periodic order is made.

An example of a periodic inventory system is a college or university bookstore. Textbooks are normally ordered according to a periodic system, wherein a count of textbooks in stock (for every course) is made after the first few weeks of a semester or quarter. An order for new

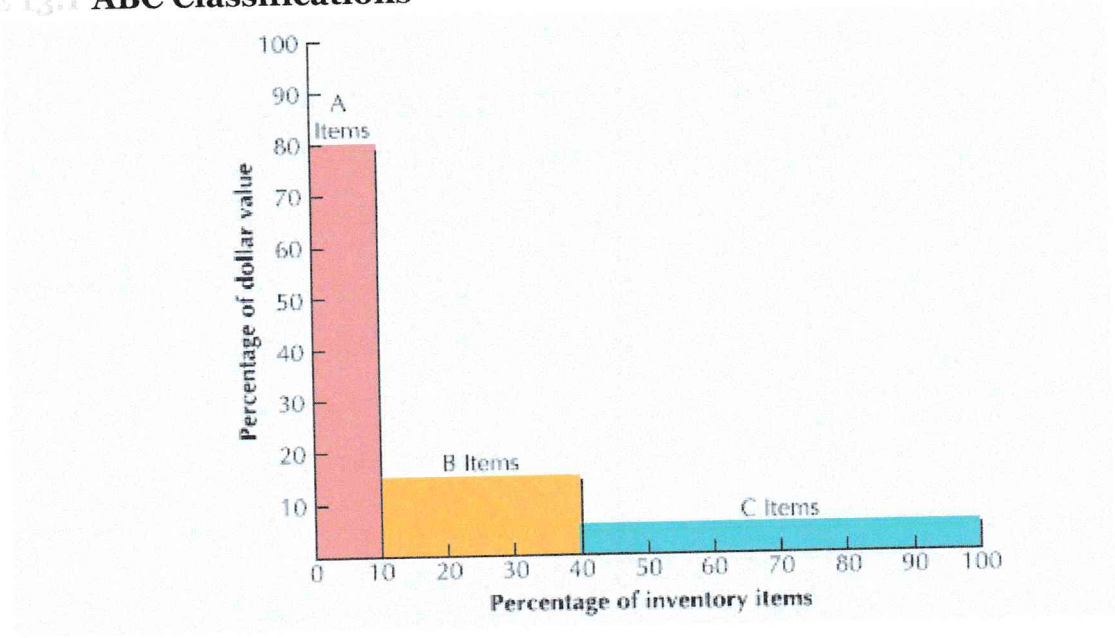
textbooks for the next semester is then made according to estimated course enrollments for the next term (i.e., demand) and the amount remaining in stock. Smaller retail stores, drugstores, grocery stores, and offices sometimes use periodic systems—the stock level is checked every week or month, often by a vendor, to see how much should be ordered.

The ABC Classification System

ABC system An inventory classification system in which a small percentage of (A) items account for most of the inventory value.

The **ABC system** is a method for classifying inventory according to several criteria, including its dollar value to the firm. Typically, thousands of independent demand items are held in inventory by a company, especially in manufacturing, but a small percentage is of such a high dollar value to warrant close inventory control. In general, about 5% to 15% of all inventory items account for 70% to 80% of the total dollar value of inventory. These are classified as *A*, or *Class A*, items. *B* items represent approximately 30% of total inventory units but only about 15% of total inventory dollar value. *C* items generally account for 50% to 60% of all inventory units but represent only 5% to 10% of total dollar value. For example, a discount store such as Walmart normally stocks a relatively small number of televisions, a somewhat larger number of bicycles or sets of sheets, and hundreds of boxes of soap powder, bottles of shampoo, and AA batteries. Figure 13.1 shows the approximate ABC classes.

FIGURE 13.1 ABC Classifications



In ABC analysis each class of inventory requires different levels of inventory monitoring and control—the higher the value of the inventory, the tighter the control. Class A items should experience tight inventory control; B and C require more relaxed (perhaps minimal)

attention. However, the original rationale for ABC analysis was that continuous inventory monitoring was expensive and not justified for many items. The wide use of bar code scanners may have eroded that reasoning. At least for larger companies, bar codes have made continuous monitoring cheap enough to use for all item classes.

The first step in ABC analysis is to classify all inventory items as A, B, or C. Each item is assigned a dollar value, which is computed by multiplying the dollar cost of one unit by the annual demand for that item. All items are then ranked according to their annual dollar value, with, for example, the top 10% classified as A items, the next 30% as B items, and the last 60% as C items. These classifications will not be exact, but they have been found to be close to the actual occurrence in firms with remarkable frequency.

The next step is to determine the level of inventory control for each classification. Class A items require tight inventory control because they represent such a large percentage of the total dollar value of inventory. These inventory levels should be as low as possible, and safety stocks minimized. This requires accurate demand forecasts and detailed record keeping. The appropriate inventory control system and inventory modeling procedure to determine order quantity should be applied. In addition, close attention should be given to purchasing policies and procedures if the inventory items are acquired from outside the firm. B and C items require less stringent inventory control. Since carrying costs are usually lower for C items, higher inventory levels can sometimes be maintained with larger safety stocks. It may not be necessary to control C items beyond simple observation. In general, A items frequently require a continuous control system, where the inventory level is continuously monitored; a periodic review system with less monitoring will suffice for C items.

Although cost is the predominant reason for inventory classification, other factors such as scarcity of parts or difficulty of supply may also be reasons for giving items a higher priority. For example, long lead times for some parts might be a problem for a company in Australia ordering from Europe, thus requiring a higher-priority classification for those parts.

EXAMPLE 13.1 | ABC System Classification

The maintenance department for a small manufacturing firm has responsibility for maintaining an inventory of spare parts for the machinery it services. The parts inventory, unit cost, and annual usage are as follows:

PART	UNIT COST	ANNUAL USAGE
1	$ 60	90
2	350	40
3	30	130
4	80	60
5	30	100
6	20	180
7	10	170

PART	UNIT COST	ANNUAL USAGE
8	320	50
9	510	60
10	20	120

The department manager wants to classify the inventory parts according to the ABC system to determine which stocks of parts should most closely be monitored.

Solution:

First rank the items according to their total value and also compute each item's percentage of total value and quantity.

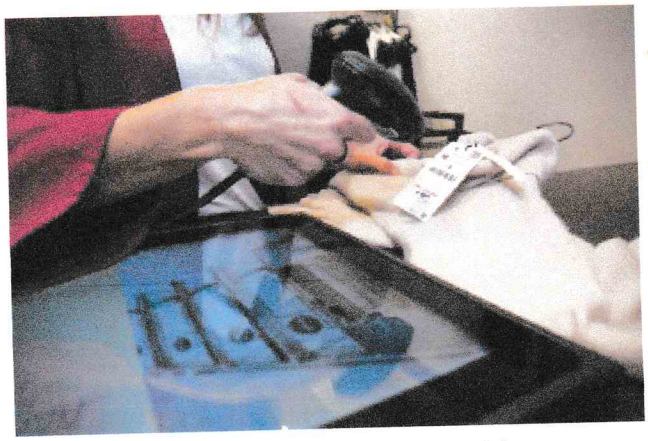
PART	TOTAL VALUE	% OF TOTAL VALUE	% OF TOTAL QUANTITY	% CUMULATIVE
9	$30,600	35.9	6.0	6.0
8	16,000	18.7	5.0	11.0
2	14,000	16.4	4.0	15.0
1	5,400	6.3	9.0	24.0
4	4,800	5.6	6.0	30.0
3	3,900	4.6	13.0	43.0
6	3,600	4.2	18.0	61.0
5	3,000	3.5	10.0	71.0
10	2,400	2.8	12.0	83.0
7	1,700	2.0	17.0	100.0
	$85,400			

Based on simple observation, it appears that the first three items form a group with the highest value, the next three items form a second group, and the last four items constitute a group. Thus, the ABC classification for these items is as follows:

CLASS	ITEMS	% OF TOTAL VALUE	% OF TOTAL QUANTITY
A	9, 8, 2	71.0	15.0
B	1, 4, 3	16.5	28.0
C	6, 5, 10, 7	12.5	57.0

Along the Supply Chain

Inventory Management at Zara



Angel Navarrete/Bloomberg/GettyImages, Inc.

Zara, a subsidiary of the Inditex Group, is one of the world's most successful apparel retailers with more than 2000 stores in 88 countries. It was ranked fifth in Gartner, Inc.'s 2015 Global Supply Chain Top 25, and one prominent reason is its unique approach to inventory management. Zara controls more of its supply chain than most retailers with most of its production in-house. It is renowned for its ability to deliver new clothes to stores quickly and in small batches. The company produces over 450 million items a year for its stores. Twice a week, at precise times, store managers order clothes, and twice a week, on schedule, new garments arrive. This results in frequent shipments and higher numbers of customer visits to the stores, creating a perception of shortages and uniqueness. This sense of scarcity and exclusiveness allows Zara to sell more items at full price. Zara gets 85 percent of the full price on its clothes, while the industry average is 60 to 70 percent. Unsold items account for less than 10 percent of its inventory stock, compared with an industry average of 17 to 20 percent.

Inventory optimization models help the company determine the quantity that should be delivered to every one of its retail stores in twice weekly shipments. The stock that is delivered is strictly limited, ensuring that each store only receives just what they need. This follows Zara's strategy of being exclusive while avoiding inventories of unpopular stock. If the design Zara hastily creates in an attempt to chase the latest fashion trend does not sell well, since the shipment is small, there's not a lot of unsold inventory to get rid of. Zara's inventory management models enable the store managers to communicate customer feedback to designers on what they are looking for, what they like, and what they dislike. Constant small design changes allows Zara to sell more items at full price; and fewer mark-downs means less inventory throughout the supply chain from raw materials to finished products.

Zara also makes extensive use of RFID technology for inventory management. RFID chips are inserted inside the slightly larger plastic security tags Zara attaches to each item, which protects the chips, allowing for their reuse after they are removed at checkout. The RFID chips and scanning devices require fewer Zara store employees to take inventory and about half the time, compared to scanning individual barcodes one at a time. Because of the time saved, store inventories can be conducted about every six weeks instead of every six months, which provides a more accurate picture of what styles are selling well and ones that are not. Also, each time an item is sold, data from its chip

triggers an instant order to the stockroom (as with JIT) for an identical item.

Using the Internet identify several other retail companies that are using RFID technology for inventory management.

Sources: Based on Christopher, Bjork, "Zara Builds its Business Around RFID," *The Wall Street Journal*, http://wsj.com (September 16, 2014); Clara Lu, "Zara's Secret to Retail Success — Its Supply Chain," *tradegecko*, https://www.tradegecko.com (December 4, 2014).

Economic Order Quantity Models

Economic order quantity (EOQ) The optimal order quantity that will minimize total inventory costs.

In a continuous, or fixed-order-quantity, system when inventory reaches a specific level, referred to as the *reorder point*, a fixed amount is ordered. The most widely used and traditional means for determining how much to order in a continuous system is the **economic order quantity (EOQ)** model, also referred to as the economic lot-size model. The earliest published derivation of the basic EOQ model formula in 1915 is credited to Ford Harris, an employee at Westinghouse.

The function of the EOQ model is to determine the optimal order size that minimizes total inventory costs. There are several variations of the EOQ model, depending on the assumptions made about the inventory system. We will describe two model versions: the basic EOQ model and the production quantity model.

The Basic EOQ Model

The *basic EOQ model* is a formula for determining the optimal order size that minimizes the sum of carrying costs and ordering costs. The model formula is derived under a set of simplifying and restrictive assumptions, as follows:

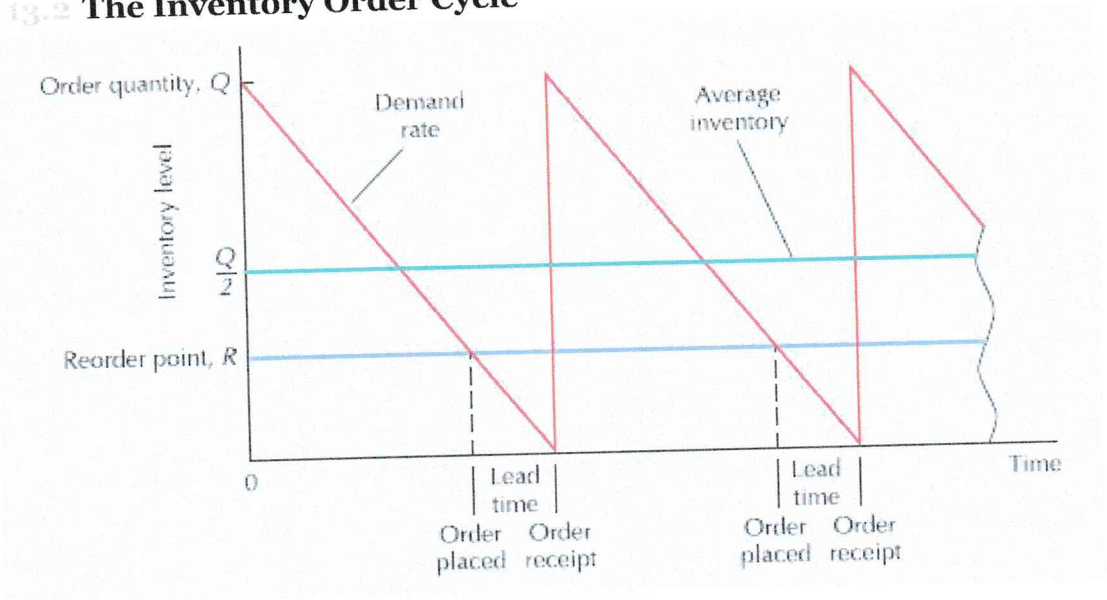
- Demand is known with certainty and is constant over time.
- No shortages are allowed.
- Lead time for the receipt of orders is constant.
- The order quantity is received all at once.

Order cycle The time between receipt of orders in an inventory cycle.

These basic model assumptions are reflected in **Figure 13.2**, which describes the continuous-inventory **order cycle** system inherent in the EOQ model. An order quantity, Q, is received and is used up over time at a constant rate. When the inventory level decreases to the reorder point, R, a new order is placed; a period of time, referred to as the *lead time*, is required for delivery. The order is received all at once just at the moment when demand depletes the entire stock of

inventory—the inventory level reaches 0—so there will be no shortages. This cycle is repeated continuously for the same order quantity, reorder point, and lead time.

FIGURE 13.2 The Inventory Order Cycle



As we mentioned, the economic order quantity is the order size that minimizes the sum of carrying costs and ordering costs. These two costs react inversely to each other. As the order size increases, fewer orders are required, causing the ordering cost to decline, whereas the average amount of inventory on hand will increase, resulting in an increase in carrying costs. Thus, in effect, the optimal order quantity represents a compromise between these two inversely related costs.

The total annual ordering cost is computed by multiplying the cost per order, designated as C_o, times the number of orders per year. Since annual demand, D, is assumed to be known and to be constant, the number of orders will be D/Q, where Q is the order size and

$$\text{Annual ordering cost} = \frac{C_o D}{Q}$$

The only variable in this equation is Q; both C_o and D are constant parameters. Thus, the relative magnitude of the ordering cost is dependent on the order size.

Total annual carrying cost is computed by multiplying the annual per-unit carrying cost, designated as C_c, by the average inventory level. The average inventory level is one-half of Q or $Q/2$, as shown in **Figure 13.2**.

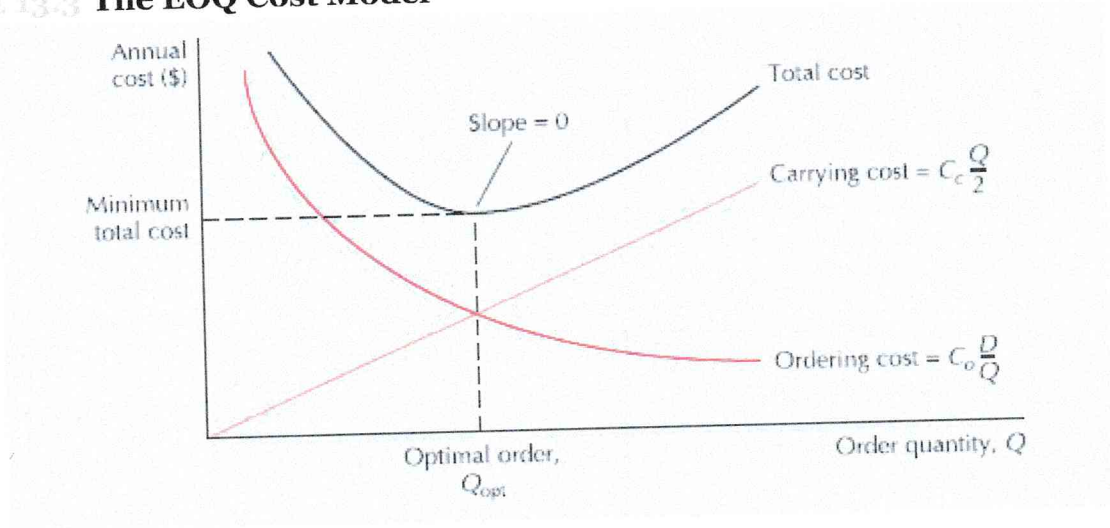
$$\text{Annual carrying cost} = \frac{C_c Q}{2}$$

The total annual inventory cost is the sum of the ordering and carrying costs:

$$\text{TC} = \frac{C_o D}{Q} + \frac{C_c Q}{2}$$

The graph in **Figure 13.3** shows the inverse relationship between ordering cost and carrying cost, resulting in a convex total cost curve.

FIGURE 13.3 The EOQ Cost Model



The optimal order quantity occurs at the point in **Figure 13.3** where the total cost curve is at a minimum, which coincides exactly with the point where the carrying cost curve intersects the ordering cost curve. This enables us to determine the optimal value of Q by equating the two cost functions and solving for Q:

$$\begin{aligned} \frac{C_o D}{Q} &= \frac{C_c Q}{2} \\ Q^2 &= \frac{2C_o D}{C_c} \\ Q_{\text{opt}} &= \sqrt{\frac{2C_o D}{C_c}} \end{aligned}$$

Alternatively, the optimal value of Q can be determined by differentiating the total cost curve with respect to Q, setting the resulting function equal to zero (the slope at the minimum point on the total cost curve), and solving for Q:

$$\begin{aligned} \text{TC} &= \frac{C_o D}{Q} + \frac{C_c Q}{2} \\ \frac{\partial \text{TC}}{\partial \text{Q}} &= -\frac{C_o D}{Q^2} + \frac{C_c}{2} \\ 0 &= -\frac{C_o D}{Q^2} + \frac{C_c}{2} \\ Q_{\text{opt}} &= \sqrt{\frac{2C_o D}{C_c}} \end{aligned}$$

The total minimum cost is determined by substituting the value for the optimal order size, Q_{opt}, into the total cost equation:

$$\text{TC}_{\text{min}} = \frac{C_o D}{Q_{\text{opt}}} + \frac{C_c Q_{\text{opt}}}{2}$$

EXAMPLE 13.2 | The Economic Order Quantity

The ePaint Store stocks paint in its warehouse and sells it online on its website. The store stocks several brands of paint; however, its biggest seller is Sharman-Wilson Ironcoat paint. The company wants to determine the optimal order size and total inventory cost for Ironcoat paint given an estimated annual demand of 10,000 gallons of paint, an annual carrying cost of $0.75 per gallon, and an ordering cost of $150 per order. It would also like to know the number of orders that will be made annually and the time between orders (i.e., the order cycle).

Solution:

$$\begin{aligned} C_c &= \$0.75 \text{ per gallon} \\ C_o &= \$150 \\ D &= 10{,}000 \text{ gallons} \end{aligned}$$

The optimal order size is

$$\begin{aligned} Q_{\text{opt}} &= \sqrt{\frac{2C_o D}{C_c}} \\ &= \sqrt{\frac{2(150)(10{,}000)}{(0.75)}} \\ &= 2000 \text{ gallons} \end{aligned}$$

The total annual inventory cost is determined by substituting Q_{opt} into the total cost formula:

$$\begin{aligned} \text{TC}_{\text{min}} &= \frac{C_o D}{Q_{\text{opt}}} + \frac{C_c Q_{\text{opt}}}{2} \\ &= \frac{(150)(10{,}000)}{2000} + \frac{(0.75)(2000)}{2} \\ &= \$750 + 750 \\ &= \$1500 \end{aligned}$$

The number of orders per year is computed as follows:

$$\begin{aligned}\text{Number of orders per year} &= \frac{D}{Q_{\text{opt}}} \\ &= \frac{10{,}000}{2000} \\ &= 5 \text{ orders per year}\end{aligned}$$

Given that the company processes orders 311 days annually (365 days minus 52 Sundays, Thanksgiving, and Christmas), the order cycle is

$$\begin{aligned}\text{Order cycle time} &= \frac{311 \text{ days}}{D/Q_{\text{opt}}} \\ &= \frac{311}{5} \\ &= 62.2 \text{ days}\end{aligned}$$

The optimal order quantity, determined in this example and in general, is an approximate value, since it is based on estimates of carrying and ordering costs as well as uncertain demand (although all of these parameters are treated as known, certain values in the EOQ model). In practice it is desirable to round the Q values off to some nearby pragmatic value. The precision of a decimal place is generally not necessary. In addition, because the optimal order quantity is computed from a square root, errors or variations in the cost parameters and demand tend to be dampened. For instance, in Example 13.2, if the order cost had actually been 30% higher, or $200, the resulting optimal order size would have varied only by a little under 10% (i.e., 2190 gallons instead of 2000 gallons). Variations in both inventory costs will tend to offset each other, since they have an inverse relationship. As a result, the EOQ model is relatively resilient to errors in the cost estimates and demand, or is *robust*, which has tended to enhance its popularity.

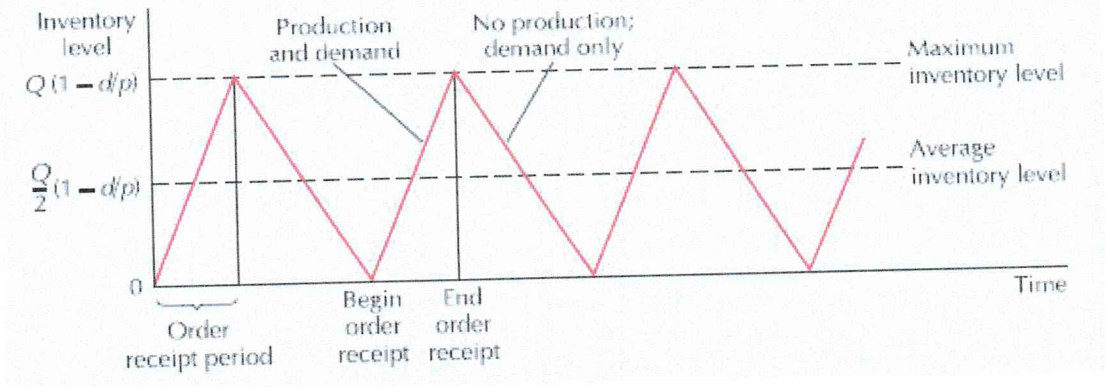
The Production Quantity Model

Production quantity model An inventory system in which an order is received gradually, as inventory is simultaneously being depleted.

A variation of the basic EOQ model is the **production quantity model**, also referred to as the *gradual usage* and *non-instantaneous receipt model*. In this EOQ model the assumption that orders are received all at once is relaxed. The order quantity is received gradually over time, and the inventory level is depleted at the same time it is being replenished. This situation is commonly found when the inventory user is also the producer, as in a manufacturing operation where a part is produced to use in a larger assembly. This situation also can occur when orders are delivered continuously over time or when a retailer is also the producer.

The noninstantaneous receipt model is shown graphically in **Figure 13.4**. The inventory level is gradually replenished as an order is received. In the basic EOQ model, average inventory was half the maximum inventory level, or $Q/2$, but in this model variation, the maximum inventory level is not simply Q; it is an amount somewhat lower than Q, adjusted for the fact the order quantity is depleted during the order receipt period.

FIGURE 13.4 The Production Quantity Model



In order to determine the average inventory level, we define the following parameters unique to this model:

p = daily rate at which the order is received over time, also known as the *production rate*

d = daily rate at which inventory is demanded

The demand rate cannot exceed the production rate, since we are still assuming that no shortages are possible, and, if $d = p$, there is no order size, since items are used as fast as they are produced. For this model the production rate must exceed the demand rate, or $p \geq d$.

Observing **Figure 13.4**, we see that the time required to finish receiving an order is the order quantity divided by the rate at which the order is received, or Q/p. For example, if the order size is 100 units and the production rate, p, is 20 units per day, the order will be received over five days. The amount of inventory that will be depleted or used up during this time period is determined by multiplying by the demand rate: $(Q/p)d$. For example, if it takes five days to receive the order and during this time inventory is depleted at the rate of two units per day, then 10 units are used. As a result, the maximum amount of

inventory on hand is the order size minus the amount depleted during the receipt period, computed as

$$\begin{aligned}\text{Maximum inventory level} &= Q - \frac{Q}{p}d \\ &= Q\left(1 - \frac{d}{p}\right)\end{aligned}$$

Since this is the maximum inventory level, the average inventory level is determined by dividing this amount by 2:

$$\begin{aligned}\text{Average inventory level} &= \frac{1}{2}\left[Q\left(1 - \frac{d}{p}\right)\right] \\ &= \frac{Q}{2}\left(1 - \frac{d}{p}\right)\end{aligned}$$

The total carrying cost using this function for average inventory is

$$\text{Total carrying cost} = \frac{C_c Q}{2}\left(1 - \frac{d}{p}\right)$$

In this case the ordering cost, C_o, is often the setup cost for production.

Thus, the total annual inventory cost is determined according to the following formula:

$$\text{TC} = \frac{C_o D}{Q} + \frac{C_c Q}{2}\left(1 - \frac{d}{p}\right)$$

Solving this function for the optimal value Q,

$$Q_{\text{opt}} = \sqrt{\frac{2C_o D}{C_c\left(1 - \frac{d}{p}\right)}}$$

EXAMPLE 13.3 | The Production Quantity Model

Assume that the ePaint Store has its own manufacturing facility in which it produces Ironcoat paint. The ordering cost, C_o, is the cost of setting up the production process to make paint. C_o = \$150. Recall that C_c = \$0.75 per gallon and D = 10,000 gallons per year. The manufacturing facility operates the same days the store is open (i.e., 311 days) and produces 150 gallons of paint per day. Determine the optimal order size, total inventory cost, the length of time to receive an order, the number of orders per year, and the maximum inventory level.

Solution:

$$
\begin{aligned}
C_o &= \$150 \\
C_c &= \$0.75 \text{ per gallons} \\
D &= 10{,}000 \text{ gallons} \\
d &= \frac{10{,}000}{311} = 32.2 \text{ gallons per day} \\
p &= 150 \text{ gallons per day}
\end{aligned}
$$

The optimal order size is determined as follows:

$$
\begin{aligned}
Q_{\text{opt}} &= \sqrt{\frac{2C_oD}{C_c\left(1-\frac{d}{p}\right)}} \\
&= \sqrt{\frac{2(150)(10{,}000)}{0.75\left(1-\frac{32.2}{150}\right)}} \\
&= 2256.8 \text{ gallons}
\end{aligned}
$$

Although an order of 2256.8 gallons should be rounded to 2257, we will use the 2256.8 to compute total cost.

This value is substituted into the following formula to determine total minimum annual inventory cost:

$$\begin{aligned} \text{TC}_{\text{min}} &= \frac{C_o D}{Q} + \frac{C_c Q}{2}\left(1 - \frac{d}{p}\right) \\ &= \frac{(150)(10{,}000)}{2256.8} + \frac{(0.75)(2256.8)}{2}\left(1 - \frac{32.2}{150}\right) \\ &= \$1329 \end{aligned}$$

The length of time to receive an order for this type of manufacturing operation is commonly called the length of the *production run*.

$$\begin{aligned} \text{Production run} &= \frac{Q}{p} \\ &= \frac{2256.8}{150} \\ &= 15.05 \text{ days per order} \end{aligned}$$

The number of orders per year is actually the number of production runs that will be made:

$$\begin{aligned} \text{Number of production runs (from orders)} &= \frac{D}{Q} \\ &= \frac{10{,}000}{2256.8} \\ &= 4.43 \text{ runs per year} \end{aligned}$$

Finally, the maximum inventory level is

$$\begin{aligned} \text{Maximum inventory level} &= Q\left(1 - \frac{d}{p}\right) \\ &= 2256.8\left(1 - \frac{32.2}{150}\right) \\ &= 1772 \text{ gallons} \end{aligned}$$

Thus, ePaint will need to set aside storage space sufficient to accommodate these 1772 gallons of paint.

Solution of EOQ Models with Excel

EOQ analysis can be done with Excel. The Excel solution screen for Example 13.2 is shown in Exhibit 13.1. The Excel screen for the production quantity model in Example 13.3 is shown in Exhibit 13.2.

EXHIBIT 13.1

Excel File

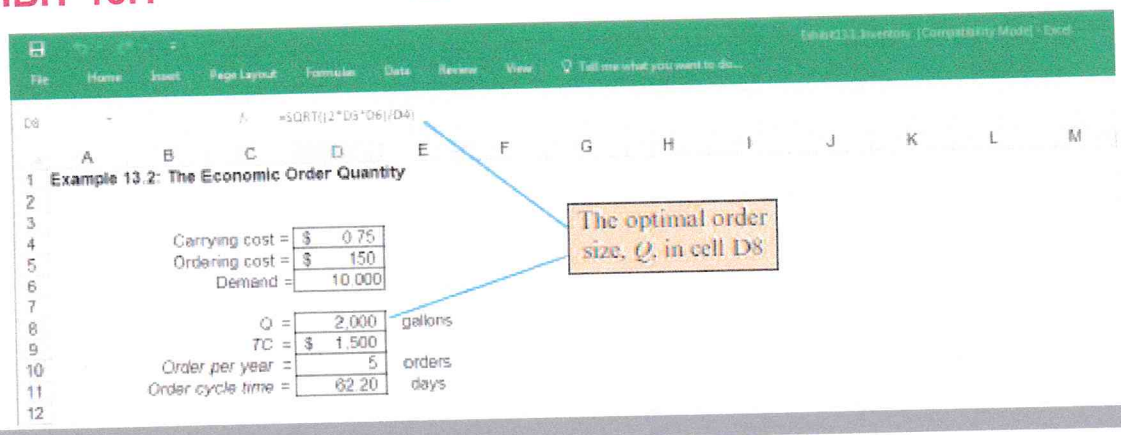
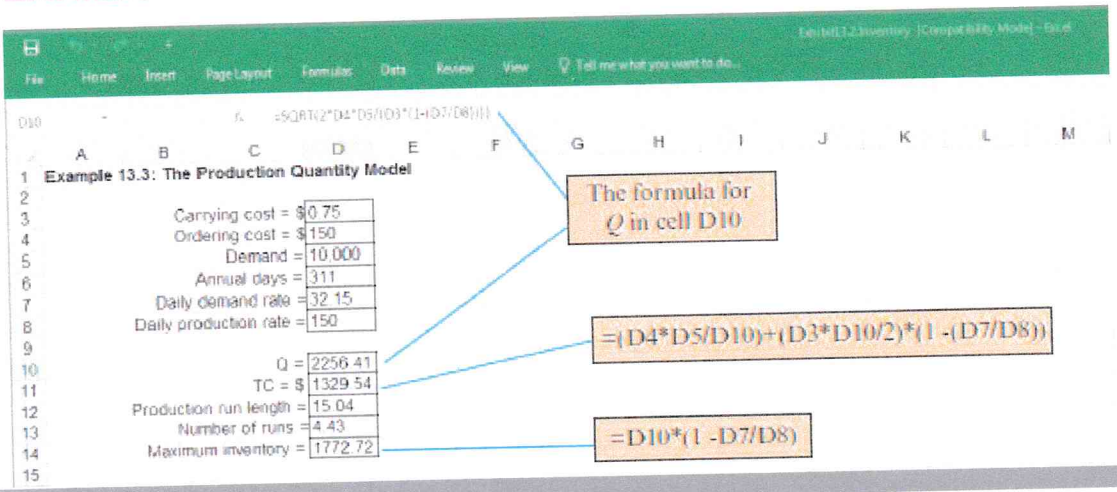


EXHIBIT 13.2 EXHIBIT 12.2

Excel File

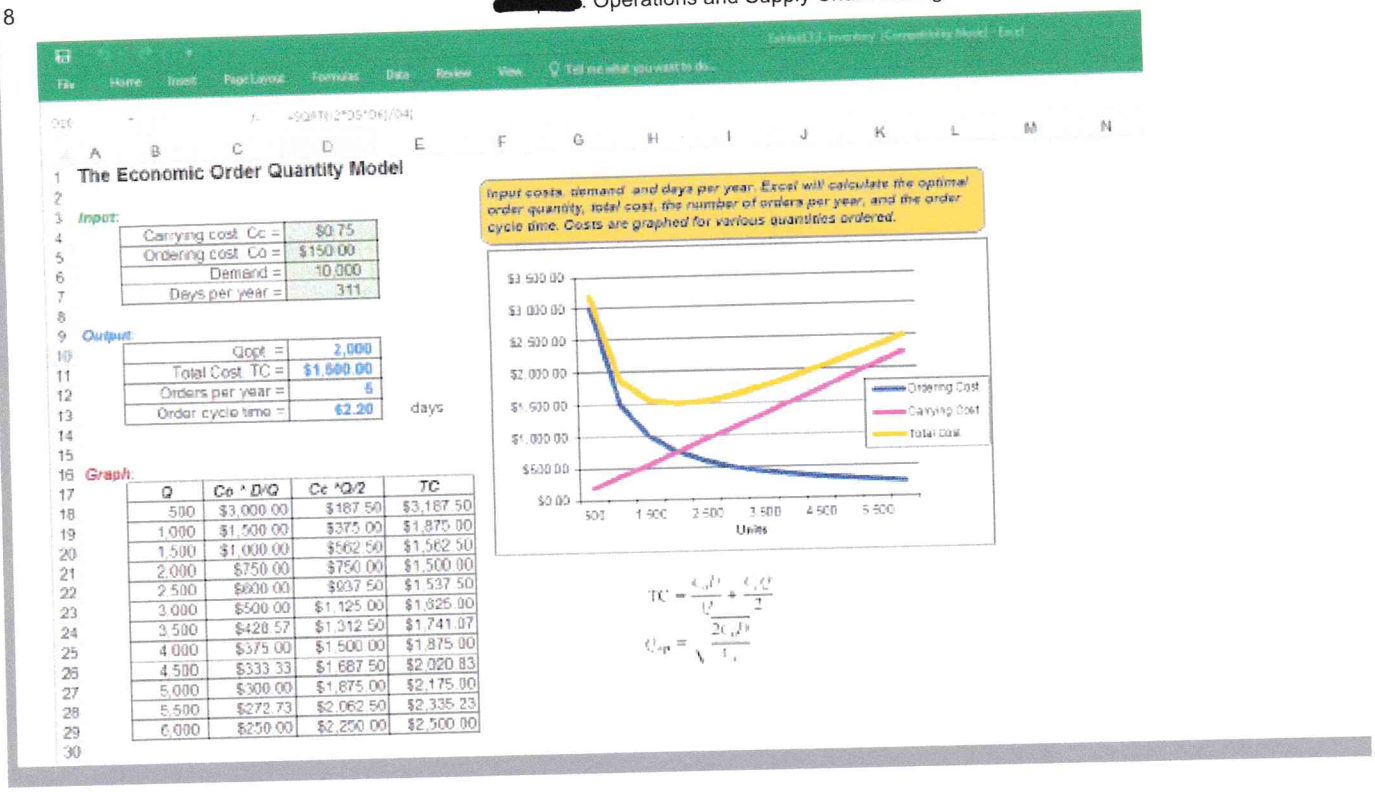


Solution of EOQ Models with OM Tools

OM tools has modules to solve all the various inventory models illustrated in this chapter, including the ABC model for Example 13.1, the EOQ and production quality models for Examples 13.2 and 13.3, the quantity discount model (Example 13.4), the recorder point model (Examples 13.5 and 13.6), and the fixed-period model (Example 13.7). As an example, Exhibit 13.3 shows the OM Tools spreadsheet for the EOQ Model in Example 13.2.

EXHIBIT 13.3

Excel File



The Economic Order Quantity Model

Input costs, demand and days per year. Excel will calculate the optimal order quantity, total cost, the number of orders per year, and the order cycle time. Costs are graphed for various quantities ordered.

Input:

Carrying cost, Cc =	$0.75
Ordering cost, Co =	$150.00
Demand =	10,000
Days per year =	311

Output:

Qopt =	2,000
Total Cost, TC =	$1,500.00
Orders per year =	5
Order cycle time =	62.20 days

Graph:

Q	Co * D/Q	Cc *Q/2	TC
500	$3,000.00	$187.50	$3,187.50
1,000	$1,500.00	$375.00	$1,875.00
1,500	$1,000.00	$562.50	$1,562.50
2,000	$750.00	$750.00	$1,500.00
2,500	$600.00	$937.50	$1,537.50
3,000	$500.00	$1,125.00	$1,625.00
3,500	$428.57	$1,312.50	$1,741.07
4,000	$375.00	$1,500.00	$1,875.00
4,500	$333.33	$1,687.50	$2,020.83
5,000	$300.00	$1,875.00	$2,175.00
5,500	$272.73	$2,062.50	$2,335.23
6,000	$250.00	$2,250.00	$2,500.00

$$TC = \frac{C_o D}{Q} + \frac{C_c Q}{2}$$

$$Q_{opt} = \sqrt{\frac{2 C_o D}{C_c}}$$

Along the Supply Chain

Vendor-Managed Inventory (VMI) at Dell

With annual revenues over $60 billion, Dell is the third largest personal computer vendor in the world behind Lenovo and HP, and is the number one supplier of PC monitors. From its start Dell has been an innovator in supply chain management, pioneering the "direct-to-customer" manufacturing model that delivers PCs to customers according to their individual specifications. To minimize the time between a PC order and its delivery Dell also employs a just-in-time (JIT) approach to manufacturing, which

requires that product manufacturing be relatively close to its customers, and that suppliers be relatively close to the manufacturer. Dell's suppliers keep inventory in small warehouses close to Dell's assembly plants and Dell withdraws inventory from its suppliers as needed (e.g., JIT), which reduces Dell's inventory at its assembly plants. Using this approach Dell minimizes its own inventory costs, which Dell is renowned for, and is critical in the computer-related industry where PC models and components depreciate very quickly. To complement their manufacturing approach Dell also was an early pioneer of vendor-managed inventory (VMI), which has become common in the electronics industry. VMI programs typically include a "percentage supply allocation (PSA)," which is a percentage of a component part's overall demand (multi-sourced from many suppliers) that is pre-negotiated with an individual supplier to provide. Dell's goals for its VMI program are to meet customer demand, minimize its own inventory, and meet the PSAs for all of its suppliers of a part. Using collaborative forecasting, Dell shares its demand forecasts with its suppliers who manage replenishments to Dell's assembly hubs based on Dell's general guideline that a supplier should carry an average on-hand inventory equal to the forecasted demand (provided by Dell) for the next ten days, i.e., ten days of inventory supply. Because of demand variability Dell might not always meet the PSAs for all suppliers or might require higher inventory levels to meet demand. To make sure that critical components are available so that it can meet customer demand, Dell will sometimes complement its VMI program by procuring inventory in advance of actual production.

Walmart is another large company that uses vendor-managed inventory. Discuss how Walmart's approach may be similar and different from Dell's VMI program.

Source: Based on Abhilasha Prakash Katariya, Sila Cetinkaya, and Eylem Tekin, "Cyclic Consumption and Replenishment Decisions for Vendor-Managed Inventory of Multisourced Parts in Dell's Supply Chain," Interfaces 44 (3) (May–June 2014), pp. 300–16.

Quantity Discounts

Quantity discount Given for specific higher order quantities.

A **quantity discount** is a price discount on an item if predetermined numbers of units are ordered. In the back of a magazine you might see an advertisement for a firm stating that it will produce a coffee mug (or hat) with a company or organizational logo on it, and the price will be $5 per mug if you purchase 100, $4 per mug if you purchase 200, or $3 per mug if you purchase 500 or more. Many manufacturing companies receive price discounts for ordering materials and supplies in high volume, and retail stores receive price discounts for ordering merchandise in large quantities.

The basic EOQ model can be used to determine the optimal order size with quantity discounts; however, the application of the model is slightly altered. The total inventory cost function must now include the purchase price of the item being ordered:

$$\text{TC} = \frac{C_o D}{Q} + \frac{C_c Q}{2} + PD$$

where

$$P = \text{per-unit price of the item}$$
$$D = \text{annual demand}$$

Purchase price was not considered as part of our basic EOQ formulation earlier because it had no impact on the optimal order size. In the preceding formula PD is a constant value that would not alter the basic shape of the total cost curve; that is, the minimum point on the cost curve would still be at the same location, corresponding to the same value of Q. Thus, the optimal order size is the same no matter what the purchase price is. However, when a discount price is available, it is associated with a specific order size, which may be different from the optimal order size, and the customer must evaluate the tradeoff between possibly higher carrying costs with the discount quantity versus EOQ cost. As a result, the purchase price does affect the order-size decision when a discount is available.

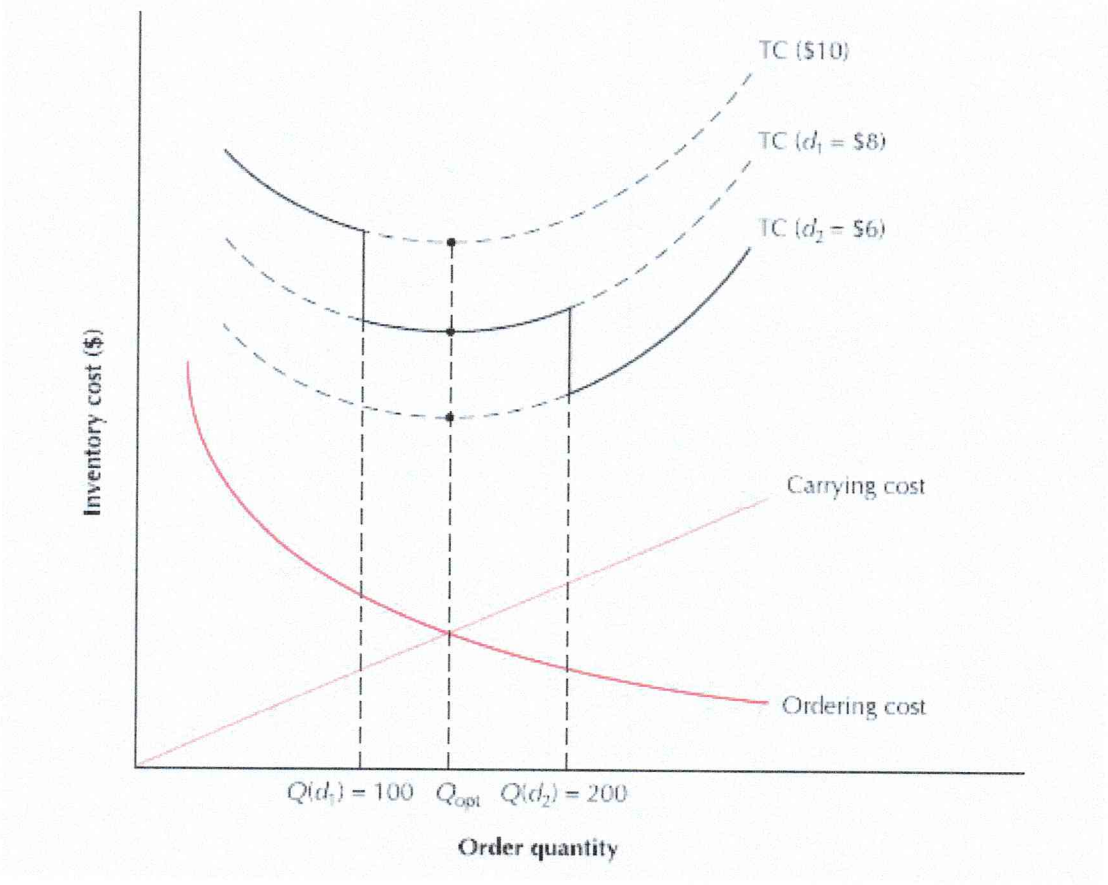
Quantity Discounts with Constant Carrying Cost

The EOQ cost model with constant carrying costs for a pricing schedule with two discounts, d_1 and d_2, is illustrated in **Figure 13.5** for the following discounts:

ORDER SIZE	PRICE
0–99	$10
100–99	$8(d_1)$
200+	$6(d_2)$

Notice in **Figure 13.5** that the optimal order size, Q_{opt}, is the same regardless of the discount price. Although the total cost curve decreases with each discount in price (i.e., d_1 and d_2), since ordering and carrying cost are constant, the optimal order size, Q_{opt}, does not change.

FIGURE 13.5 Quantity Discounts with Constant Carrying Cost



The graph in **Figure 13.5** reflects the composition of the total cost curve resulting from the discounts kicking in at two successively higher order quantities. The first segment of the total cost curve (with no discount) is valid only up to 99 units ordered. Beyond that quantity, the total cost curve (represented by the topmost dashed line) is meaningless because above 100 units there is a discount (d_1). Between 100 and 199 units the total cost drops down to the middle curve. This middle-level cost curve is valid only up to 199 units because at 200 units there is another, lower discount (d_2). So the total cost curve has two discrete steps, starting with the original total cost curve, dropping down to the next level cost curve for the first discount, and finally dropping to the third-level cost curve for the final discount.

Notice that the optimal order size, Q_{opt}, is feasible only for the middle level of the total cost curve, TC(d_1)—it does not coincide with the top level of the cost curve, TC, or the lowest level, TC(d_2). If the optimal EOQ order size had coincided with the lowest level of the total cost curve, TC(d_2), it would have been optimal order size for the entire discount price schedule. Since it does not coincide with the lowest level of the total cost curve, the total cost with Q_{opt} must be

compared to the lower-level total cost using $Q(d_2)$ to see which results in the minimum total cost. In this case the optimal order size is 200.

EXAMPLE 13.4 | A Quantity Discount with Constant Carrying Cost

Avtek, a distributor of audio and video equipment, wants to reduce a large stock of televisions. It has offered a local chain of stores a quantity discount pricing schedule, as follows:

QUANTITY	PRICE
1–49	$1400
50–89	1100
90+	900

The annual carrying cost for the stores for a TV is $190, the ordering cost is $2500, and annual demand for this particular model TV is estimated to be 200 units. The chain wants to determine if it should take advantage of this discount or order the basic EOQ order size.

Solution: First determine the optimal order size and total cost with the basic EOQ model.

$$\begin{aligned} C_o &= \$2500 \\ C_c &= \$190 \text{ per TV} \\ D &= 200 \text{ TVs per year} \\ Q_{\text{opt}} &= \sqrt{\frac{2C_oD}{C_c}} \\ &= \sqrt{\frac{2(2500)(200)}{190}} \\ &= 72.5 \text{ TVs} \end{aligned}$$

Although we will use $Q_{\text{opt}} = 72.5$ in the subsequent computations, realistically the order size would be 73 televisions. This order size is eligible for the first discount of $1100; therefore, this price is used to compute total cost:

$$\begin{aligned} \text{TC}_{\text{min}} &= \frac{C_oD}{Q_{\text{opt}}} + \frac{C_cQ_{\text{opt}}}{2} + PD \\ &= \frac{(2500)(200)}{72.5} + \frac{(190)(72.5)}{2} + (1100)\,(200) \\ \text{TC}_{\text{min}} &= \$233{,}784 \end{aligned}$$

Since there is a discount for a larger order size than 50 units (i.e., there is a lower cost curve), this total cost of $233,784 must be compared with total cost with an order size of 90 and a discounted price of $900:

$$\begin{aligned} \text{TC} &= \frac{C_oD}{Q} + \frac{C_cQ}{2} + PD \\ &= \frac{(2500)(200)}{90} + \frac{(190)(90)}{2} + (900)\,(200) \\ &= \$194{,}105 \end{aligned}$$

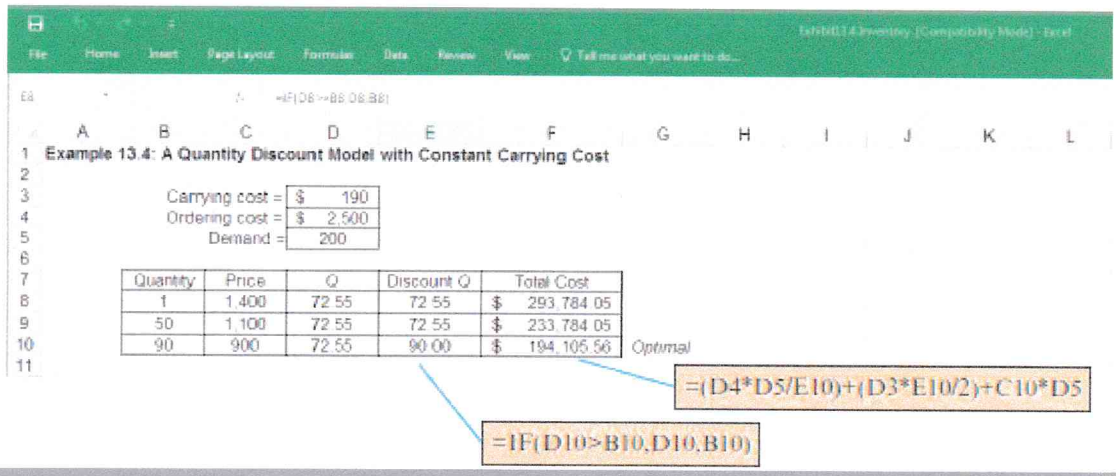
Since this total cost is lower ($194,105 < $233,784), the maximum discount price should be taken, and 90 units should be ordered. We know that there is no order size larger than 90 that would result in a lower cost, since the minimum point on this total cost curve has already been determined to be 73.

Quantity Discount Model Solution with Excel

It is also possible to use Excel to solve the quantity-discount model with constant carrying cost. Exhibit 13.4 shows the Excel solution screen for Example 13.4. Notice that the selection of the appropriate order size, Q, that results in the minimum total cost for each discount range is determined by the formulas embedded in cells E8, E9, and E10. For example, the formula for the first quantity-discount range, "1–49," is embedded in cell E8 and shown on the formula bar

at the top of the screen, "=IF(D8≥B8,D8,B8)." This means that if the discount order size in cell D8 (i.e., $Q = 72.55$) is greater than or equal to the quantity in cell B8 (i.e., 1), then the quantity in cell D8 (72.55) is selected; otherwise the amount in cell B8 is selected. The formulas in cells E9 and E10 are constructed similarly. The result is that the order quantity for the final discount range, $Q = 90$, is selected.

EXHIBIT 13.4 Excel File



Example 13.4: A Quantity Discount Model with Constant Carrying Cost

Carrying cost =	$ 190
Ordering cost =	$ 2,500
Demand =	200

Quantity	Price	Q	Discount Q	Total Cost	
1	1,400	72.55	72.55	$ 293,784.05	
50	1,100	72.55	72.55	$ 233,784.05	
90	900	72.55	90.00	$ 194,105.56	Optimal

=(D4*D5/E10)+(D3*E10/2)+C10*D5

=IF(D10>B10,D10,B10)

Reorder Point

Reorder point The level of inventory at which a new order should be placed.

In our description of the EOQ models in the previous sections, we addressed how much should be ordered. Now we will discuss the other aspect of inventory management, when to order. The determinant of when to order in a continuous inventory system is the **reorder point**, the inventory level at which a new order is placed.

The reorder point for our basic EOQ model with constant demand and a constant lead time to receive an order is equal to the amount demanded during the lead time,

$$R = dL$$

where

$$d = \text{demand rate per period (e.g., daily)}$$
$$L = \text{lead time}$$

EXAMPLE 13.5 | Reorder Point for the Basic EOQ Model

The ePaint Internet Store in Example 13.2 is open 311 days per year. If annual demand is 10,000 gallons of Ironcoat paint and the lead time to receive an order is 10 days, determine the reorder point for paint.

Solution:

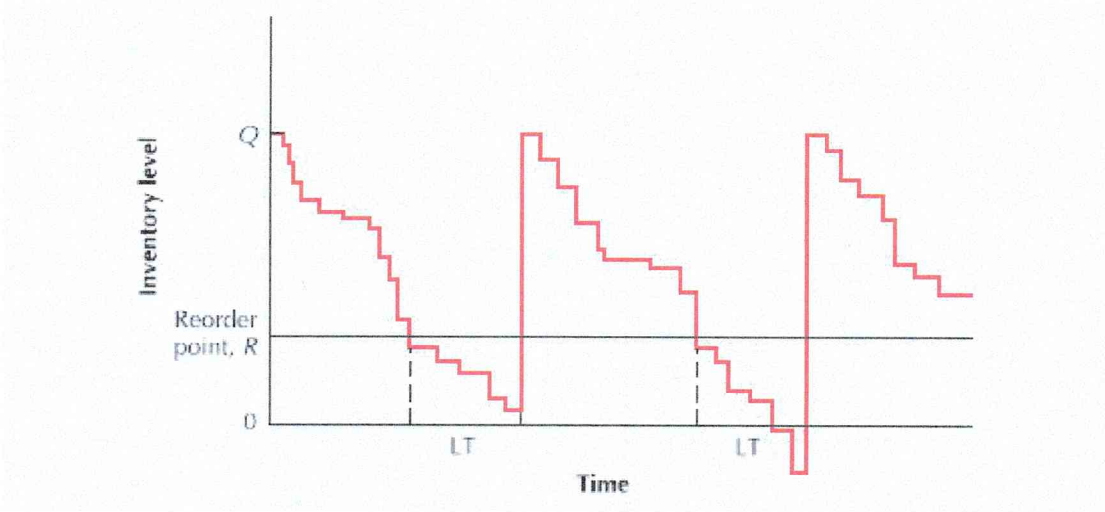
$$\begin{aligned} R &= dL \\ &= \left(\frac{10{,}000}{311}\right)(10) \\ &= 321.54 \text{ gallons} \end{aligned}$$

When the inventory level falls to approximately 321 gallons of paint, a new order is placed. Notice that the reorder point is not related to the optimal order quantity or any of the inventory costs.

Safety Stocks

In Example 13.5, an order is made when the inventory level reaches the reorder point. During the lead time, the remaining inventory in stock will be depleted at a constant demand rate, such that the new order quantity will arrive at exactly the same moment as the inventory level reaches zero. Realistically, demand—and, to a lesser extent lead time—are uncertain. The inventory level might be depleted at a faster rate during lead time. This is depicted in **Figure 13.6** for uncertain demand and a constant lead time.

FIGURE 13.6 Variable Demand with a Reorder Point

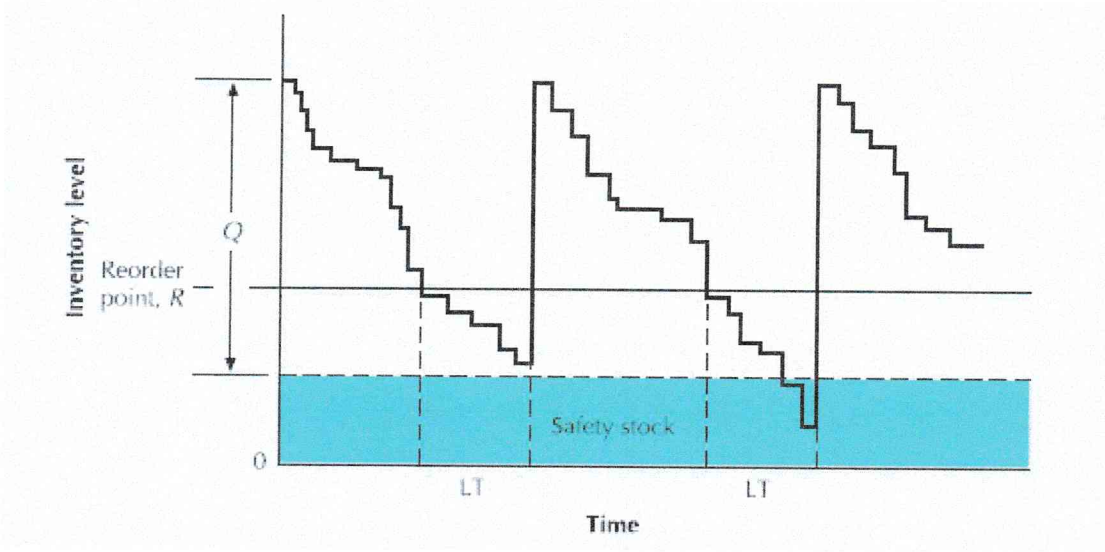


Stockout An inventory shortage.

Safety stock A buffer added to the inventory on hand during lead time.

Notice in the second order cycle that a **stockout** occurs when demand exceeds the available inventory in stock. As a hedge against stockouts when demand is uncertain, a **safety stock** of inventory is frequently added to the expected demand during lead time. The addition of a safety stock to the stockout occurrence shown in **Figure 13.6** is displayed in **Figure 13.7**.

FIGURE 13.7 **Reorder Point with a Safety Stock**



Service Level

Service level The probability that the inventory available during lead time will meet demand.

There are several ways to determine the amount of the safety stock. One popular method is to establish a safety stock that will meet a specified **service level**. The service level is the probability that the amount of inventory on hand during the lead time is sufficient to meet expected demand—that is, the probability that a stockout will not occur. The term *service* is used, since the higher the probability that inventory will be on hand, the more likely that customer demand will be met—that is, that the customer can be served. A service level of 90% means that there is a 0.90 probability that demand will be met during the lead time, and the probability that a stockout will occur is 10%. The service level is typically a policy decision based on a number of factors, including carrying costs for the extra safety stock and lost sales if customer demand cannot be met.

Reorder Point with Variable Demand

To compute the reorder point with a safety stock that will meet a specific service level, we will assume the demand during each day of lead time is uncertain, independent, and can be described by a normal distribution. The average demand for the lead time is the sum of the average daily demand for the days of the lead time, which is also the product of the average daily demands multiplied by the lead time. Similarly, the variance of the distribution is the sum of the daily variances for the number of days in the lead time. Using these parameters, we can compute the reorder point to meet a specific service level as

$$R = \bar{d}\,L + z\varsigma_d\sqrt{L}$$

where

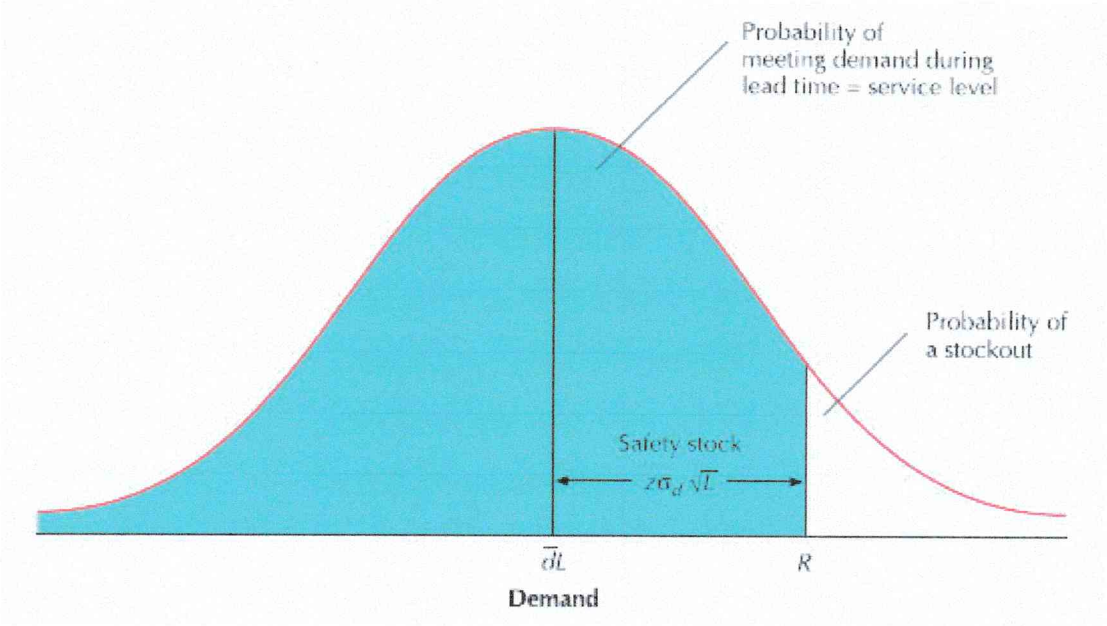
$$\begin{aligned}
\bar{d} &= \text{average daily demand} \\
L &= \text{lead time} \\
\varsigma_d &= \text{the standard deviation of daily deman} \\
z &= \text{number of standard deviations corresponding to the service level probability} \\
z\varsigma_d\sqrt{L} &= \text{safety stock}
\end{aligned}$$

The term $\varsigma_d\sqrt{L}$ in this formula for the reorder point is the square root of the sum of the daily variances during lead time:

$$\begin{aligned}
\text{Variance} &= (\text{daily variance}) \times (\text{number of days of lead time}) \\
&= \varsigma_d^2 L \\
\text{Standard deviation} &= \sqrt{\varsigma_d^2 L} \\
&= \varsigma_d\sqrt{L}
\end{aligned}$$

The reorder point relative to the service level is shown in **Figure 13.8**. The service level is the shaded area, or probability, to the left of the reorder point, R.

FIGURE 13.8 Reorder Point for a Service Level

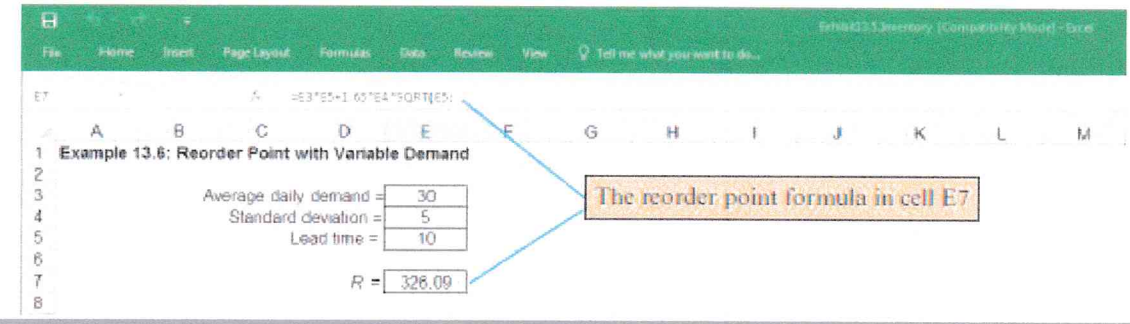


Determining the Reorder Point with Excel

Excel can be used to determine the reorder point for variable demand. Exhibit 13.5 shows the Excel screen for Example 13.6. Notice that the reorder point is computed using the formula in cell E7, which is shown on the formula bar at the top of the screen.

EXHIBIT 13.5

Excel File



EXAMPLE 13.6 | Reorder Point for Variable Demand

For the ePaint Internet Store in Example 13.2, we will assume that daily demand for Ironcoat paint is normally distributed with an average daily demand of 30 gallons and a standard deviation of 5 gallons of paint per day. The lead time for receiving a new order of paint is 10 days. Determine the reorder point and safety stock if the store wants a service level of 95%—that is, the probability of a stockout is 5%.

Solution:

$$\begin{aligned} \bar{d} &= 30 \text{ gallons per day} \\ L &= 10 \text{ days} \\ \varsigma_d &= 5 \text{ gallons per day} \end{aligned}$$

For a 95% service level, the value of z (from the Normal table in Appendix A) is 1.65. The safety stock is:

$$\begin{aligned} \text{Safety stock} &= z\varsigma_d\sqrt{L} \\ &= (1.65)(5)\left(\sqrt{10}\right) \\ &= 26.1 \text{ gallons} \end{aligned}$$

The reorder point is computed as follows:

$$\begin{aligned} R &= \bar{d}\,L + z\varsigma_d\sqrt{L} \\ &= 30(10) + (1.65)(5)\left(\sqrt{10}\right) \\ &= 300 + 26.1 \\ &= 326.1 \text{ gallons} \end{aligned}$$

Along the Supply Chain

Multi-Echelon Inventory Optimization at Procter & Gamble

The various EOQ models presented in this chapter are known as single-stage models; they only attempt to optimize inventory of a single item at a single location. The items are considered in isolation, and not how this single item's inventory level might affect the entire supply chain. This is a somewhat simplistic but necessary view of inventory optimization for many companies, because of the complexity involved in considering the relationship among many different inventory items at many locations for an end-to-end supply chain. However, many *Fortune 500* and other companies across a wide range of industries are now approaching inventory optimization from a total supply chain perspective. This relatively new approach, called *multi-echelon* inventory optimization, uses various software packages designed for the purpose to scientifically determine the minimum inventory levels for multiple materials, parts, subassemblies, and finished goods across

the entire supply chain. One company that has had great success with the multi-echelon inventory optimization approach is Procter & Gamble.

Procter & Gamble, headquartered in Cincinnati, Ohio, is one of the world's leading and best-known consumer products companies with sales over $80 billion in 180 countries for such products as Crest, Tide, Pantene, Pampers, Charmin, Cascade, Duracell, Cover Girl, and Gillette, among many others. Each of its three global business units—beauty and grooming, household care, and health and well-being—individually is large enough to be on the *Fortune 200* list. P&G's supply chain network consists of 500 supply chains that include 145 P&G-owned manufacturing facilities and 300 contract manufacturers. Inventory management across a supply chain network of this size and magnitude is a complex process. P&G's logistics management workforce that plans material supply, capacity, inventory, and logistics across its supply chain network has over 5000 individuals. P&G's inventory management process is implemented in a two-step approach. First, spreadsheet–based inventory models (employing assorted mathematical inventory optimization tools) are applied locally to optimize each stage in the supply chain. These models are used in about 70% of P&G's business units. Next, multi-echelon inventory optimization software is implemented in about 30% of P&G's business units across more complex supply chains.

The North American supply chain for cosmetics liquid makeup in P&G's beauty products division includes 500 stages with 8 raw materials, 10 uncolored work-in-process materials, 24 colored work-in-process materials, 150 packaging materials, 18 intermediate partially assembled products, and 75 finished goods that move from packaging to U.S. and Canadian distribution centers and then to retail customers. Material lead times ranged from 7 days to 8 weeks, production times from 1 to 2 days, review periods from 7 to 28 days, transportation times from 1 to 7 days, and quality assurance from 1 to 5 days. Demand forecasts were based on the immediate past 13 weeks of shipments and a forecast for the future 13 weeks. Among a number of improvements, P&G's application of a multi-echelon inventory approach changed the location of safety stocks within the supply chain and reduced safety stock investment for the supply chain by 17% and total inventory by 5%, while maintaining its 99.5% service level. The multi-echelon approach at Procter & Gamble reduced inventory in its beauty division by $100 million, and in one year alone saved the company $1.5 billion in inventory costs across all of its supply chains.

Discuss some of the problems a company such as Procter & Gamble might face in managing inventory across such a large, complex global supply chain.

Source: Based on I. Farsyn, S. Humair, J.L. Kahn, J.J. Neale, O. Rosen, J. Ruark, W. Tarlton, W. Van de Velde, G. Wegryn, and S. Willems, "Inventory Optimization at Procter & Gamble: Achieving Real Benefits Through User Adoption of Inventory Tools," Interfaces 41 (1) (January–February 2011), pp. 66–78; and Sean R. Williams,"How Inventory Optimization Opens Pathways to Profitability," Supply Chain Management Review, 5 (March/April 2011), pp. 30–36.

Order Quantity for a Periodic Inventory System

We defined a continuous, or fixed-order-quantity, inventory system as one in which the order quantity was constant and the time between orders varied. So far this type of inventory system has been the focus of our discussion. The less common *periodic*, or *fixed-time-period, inventory system* is one in which the time between orders is constant and the order size varies. Small

retailers often use this sytem. Drugstores are one example of a business that sometimes uses a fixed-period inventory system. Drugstores stock a number of personal hygiene- and health-related products such as shampoo, toothpaste, soap, bandages, cough medicine, and aspirin.

Normally, the vendors who provide these items to the store will make periodic visits—every few weeks or every month—and count the stock of inventory on hand for their product. If the inventory is exhausted or at some predetermined reorder point, a new order will be placed for an amount that will bring the inventory level back up to the desired level. The drugstore managers will generally not monitor the inventory level between vendor visits but instead will rely on the vendor to take inventory.

Under this system, the vendor would bundle many small, low-cost items into a single order and delivery, thereby saving costs. Since the items are generally of low value, larger safety stocks will not pose a significant cost. Also, if the items are noncritical, even if there is a stockout, it is not a big deal. However, inventory might be exhausted early in the time period between visits, resulting in a stockout that will not be remedied until the next scheduled order. As a result, a larger safety stock for more critical items is sometimes required for the fixed-interval system.

Order Quantity with Variable Demand

If the demand rate and lead time are constant, then the fixed-period model will have a fixed-order quantity that will be made at specified time intervals, which is the same as the fixed-quantity (EOQ) model under similar conditions. However, as we have already explained, the fixed-period model reacts differently from the fixed-order model when demand is a variable.

The order size for a fixed-period model given variable daily demand that is normally distributed is determined by

$$Q = \bar{d}\left(t_b + L\right) + z\varsigma_d\sqrt{t_b + L} - I$$

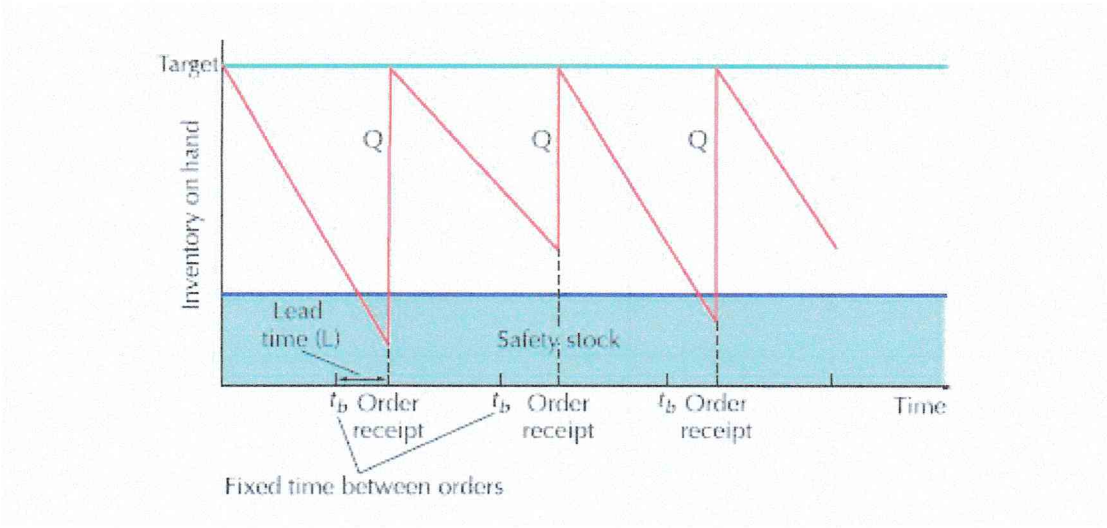
where

$$\begin{aligned} \bar{d} &= \text{average demand rate} \\ t_b &= \text{the fixed time between orders} \\ L &= \text{lead time} \\ \varsigma_d &= \text{standard deviation of demand} \\ z\varsigma_d\sqrt{t_b + L} &= \text{safety stock} \\ I &= \text{inventory in stock} \end{aligned}$$

The first term in this formula, $\bar{d}\left(t_b + L\right)$, is the average demand during the order cycle time plus the lead time. It reflects the amount of inventory that will be needed to protect against the entire time from this order to the next and the lead time until the order is received. The second term, $z\varsigma_d\sqrt{t_b + L}$, is the safety stock for a specific service level, determined in much the same way as previously described for a reorder point. These first two terms combined are a "target" level of inventory to maintain. The final term, I, is the amount of inventory on hand when the inventory level is checked and an order is made.

Figure 13.9 shows a periodic inventory system in which variable order sizes (Q) are placed at fixed time intervals (t_b), and Example 13.7 illustrates this system.

FIGURE 13.9 Periodic Inventory System

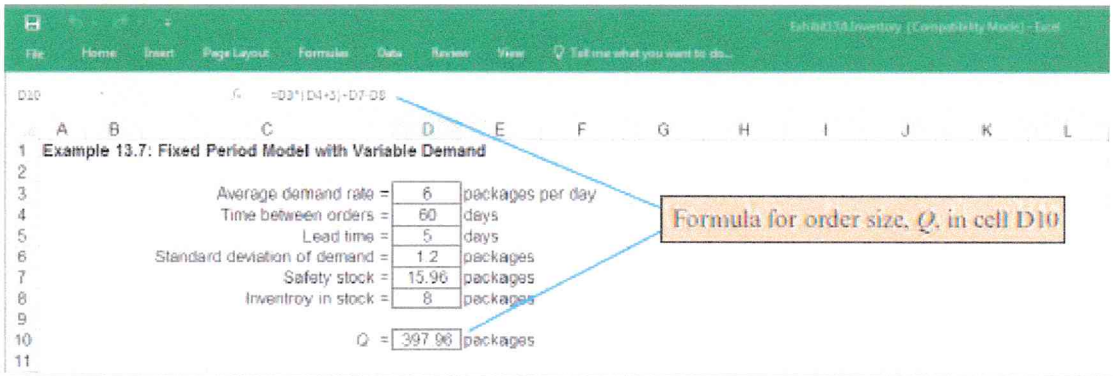


Determining the Order Quantity for the Fixed-Period Model with Excel

The order quantity for the fixed-period model with variable demand can be determined using Excel. The Excel screen for Example 13.7 is shown in Exhibit 13.6. Notice that the order quantity in cell D10 is computed with the formula shown on the formula bar at the top of the screen.

EXHIBIT 13.6

Excel File



EXAMPLE 13.7 | Order Size for Fixed-Period Model with Variable Demand

The KVS Pharmacy stocks a popular brand of over-the-counter flu and cold medicine. The average demand for the medicine is six packages per day, with a standard deviation of 1.2 packages. A vendor for the pharmaceutical company checks KVS's stock every 60 days. During one visit the store had eight packages in stock. The lead time to receive an order is five days. Determine the order size for this order period that will enable KVS to maintain a 95% service level.

Solution:

$$
\begin{aligned}
\bar{d} &= 6 \text{ packages per day} \\
\varsigma_d &= 1.2 \text{ packages} \\
t_b &= 60 \text{ days} \\
L &= 5 \text{ days} \\
I &= 8 \text{ packages} \\
z &= 1.65 (\text{for a } 95\% \text{ service level}) \\
Q &= \bar{d}\left(t_b + L\right) + z\varsigma_d\sqrt{t_b + L} - I \\
&= (6)(60 + 5) + (1.65)(1.2)\sqrt{60 + 5} - 8 \\
&= 397.96 \text{ packages}
\end{aligned}
$$

This will be rounded to 398 packages, or perhaps 400 if it is shipped in boxes of 100 packages.

SUMMARY

The two types of systems for managing inventory are continuous and periodic, and we presented several models for determining how much to order and when to order for each system. However, we focused our attention primarily on the more commonly used continuous, fixed-order-quantity systems with EOQ models for determining order size and reorder points for determining when to order.

The objective of these order quantity models is to determine the optimal tradeoff between inventory carrying costs and ordering costs that would minimize total inventory cost. However, a drawback of approaching inventory management in this manner is that it can delude management into thinking that if it determines the minimum cost order quantity, it has achieved all it can in reducing inventory costs, which is not the case. Management should continually strive both to accurately assess and to reduce individual inventory costs. If management has accurately determined carrying and order costs, then it can seek ways to lower them that will reduce overall inventory costs regardless of the order size and reorder point.

Key Terms

ABC system A method for classifying inventory items according to their dollar value to the firm based on the principle that only a few items account for the greatest dollar value of total inventory.

carrying costs The cost of holding an item in inventory, including lost opportunity costs, storage, rent, cooling, lighting, interest on loans, and so on.

continuous inventory system A system in which the inventory level is continually monitored; when it decreases to a certain level, the reorder point, a fixed amount is ordered.

dependent demand Typically, component parts or materials used in the process to produce a final product.

economic order quantity (EOQ) A fixed-order quantity that minimizes total inventory costs.

independent demand Final or finished products whose demand is not a function of, or dependent on, internal production activity.

inventory A stock of items kept by an organization to meet internal or external customer demand.

order cycle The time between the receipt of orders in an inventory system.

ordering costs The cost of replenishing the stock of inventory including requisition cost, transportation and shipping, receiving, inspection, handling, and so forth.

periodic inventory system A system in which the inventory level is checked after a specific time period and a variable amount is ordered, depending on the inventory in stock.

production quantity model Also known as the production lot-size model; an inventory system in which an order is received gradually and the inventory level is depleted at the same time it is being replenished.

quantity discount A pricing schedule in which lower prices are provided for specific (higher) order quantities.

reorder point A level of inventory in stock at which a new order is placed.

safety stock An amount added to the expected amount demanded during the lead time (the reorder point level) as a hedge against a stockout.

service level The probability that the amount of inventory on hand during the lead time is sufficient to meet expected demand.

shortage costs Temporary or permanent loss of sales that will result when customer demand cannot be met.

stockout An inventory shortage occurring when demand exceeds the inventory in stock.

Key Formulas

Basic EOQ Model

$$\begin{aligned} \text{TC} &= \frac{C_o D}{Q} + \frac{C_c Q}{2} \\ Q_{\text{opt}} &= \sqrt{\frac{2C_o D}{C_c}} \end{aligned}$$

EOQ Model with Noninstantaneous Receipt

$$\begin{aligned} \text{TC} &= \frac{C_o D}{Q} + \frac{C_c Q}{2}\left(1 - \frac{d}{p}\right) \\ Q_{\text{opt}} &= \sqrt{\frac{2C_o D}{C_c\left(1-\frac{d}{p}\right)}} \end{aligned}$$

Inventory Cost for Quantity Discounts

$$\text{TC} = \frac{C_o D}{Q} + \frac{C_c Q}{2} + PD$$

Reorder Point with Constant Demand and Lead Time

$$R = dL$$

Reorder Point with Variable Demand

$$R = \bar{d}\,L + z\varsigma_d\sqrt{L}$$

Fixed-Time-Period Order Quantity with Variable Demand

$$Q = \bar{d}\left(t_b + L\right) + z\varsigma_d\sqrt{t_b + L} - I$$

Solved Problems

1 Basic EOQ Model

Electronic Village stocks and sells a particular brand of personal computer. It costs the store $450 each time it places an order with the manufacturer for the personal computers. The annual cost of carrying the PCs in inventory is $170. The store manager estimates that annual demand for the PCs will be 1200 units. Determine the optimal order quantity and the total minimum inventory cost.