Chapter 13 Investment Project Evaluation and Risk Management

Entrepreneurial Finance: Fundamentals of Financial Planning and Management for Small Business, First Edition. M. J. Alhabeeb.

ALL FIRMS, large and small, would at some point have to make some sort of long-term investment. In addition to the commonly known investment in financial securities, firms are often engaged in a more important investment that is crucial to their operations and survival. That is the investment in both current and fixed assets, which takes a variety of forms and sizes and shapes. As the effectiveness of current assets is tied to the operating cycle within a year, current expenditures is defined as the outlay which would reap in benefits during a year, such as the account receivable, prepaid expenses, and inventory. Fixed assets, on the other hand, are those assets that would serve the operations over a prolonged period of time, extending beyond a year and for many operating cycles. Therefore, the outlay of funds that is expected to continue to reap services for the long run is called **capital expenditures**. Since firms are usually involved in investment allocation of both assets, a systematic process is needed to enable the firm to evaluate and select the right investment at the right time and according to the firm's priorities.

Investment project evaluation is the firm's internal process to prioritize, assess, select, and allocate funds for the long-term investment in its assets to achieve its goals. Capital budgeting is a similar concept about the process to justify and use the capital expenditures to generate higher gains where the present value of the cash inflow exceeds the current value of the cash outflow. More specifically, **capital budgeting** would be the process to analyze the firm's potential investment projects in order to enable managers in their decisions to enlist different projects in the capital budget.

There are many examples of capital outlay for a firm's investment projects such as

- Funds to acquire another company or to finance the purchase of land, building, and equipment.
- Funds to expand business operations, enhance the working capital system, better manage inventories, and control account receivable.

Funds to upgrade research and development or to pay for promotion and advertisement.

Each and every one of these investment projects can be a good reason for the firm's capital spending. The following are the most common purposes for the firm's investment projects which would satisfy various needs any firm might face at some point of its lifetime.

13.1 Categories of Investment Purposes

Replacement Projects

This category requires capital expenditures on the replacement of obsolete, outdated, worn out, and broken machines and equipment. Most often the managerial decision to spend on replacement is assessed against the cost and benefit of the alternative way of repairing and reconditioning.

Renewal Projects

This is another alternative to replacement or partial repair of machines and equipment. It involves a complete rebuilding, overhauling, or retrofitting an entire plant or section of machinery to improve the efficiency of operation.

Expansive Projects

These projects require capital spending on introducing a new product or developing another line of an existing product. It also includes venturing into a new market and extending the service outlets and expanding the distribution and storage facilities.

Cost-Reduction Projects

This category includes all projects aiming at reducing the cost of production through lower costs of labor, raw material, and energy, as well as through a better technology. Lower costs of production could also be achieved by moving production to less-expensive locations and establishing a better personnel training.

Conforming Projects

This category includes all the projects that require capital expenditures to comply with federal and state regulations and honoring production and services according to the required codes and standards, especially in the fields of health, safety, and environmental reservation.

Other projects

These are the projects that may result in increasing sales and boosting revenues such as advertisement projects, or those that may revolutionize operations such as research and development, or those that may increase efficiency such as managerial consulting and staff training.

13.2 Steps for Project Selection

After recognizing a certain need and identifying the category of projects which would satisfy that need, firm's usually solicit ideas and proposals either internally or externally. The internal solicitation could be directed to all employees or limited to specialized committees. Project proposals submitted by outside investors would be highly competitive and often selected for their capacity to reduce cost and deliver high quality results. Regardless of who would carry out the projects, firms usually follow the standard procedures of decision-making for consistency and efficiency. The following nine steps would briefly describe the process of project selection:

1. Exploring a pool of proposed investment projects, and generating a list of the most qualified proposals to form the alternative projects under consideration. By focusing on the initially strong and promising projects, this step would also be a screening step to eliminate all unfeasible or unworthy proposals.
2. Estimating the project cash flow which is a stream of returns that would occur in a future time. It should, therefore, be considered with the appropriate level of uncertainty, risk, and bias. A major bias could be the natural tendency to be over-optimistic, which may lead to underestimation of costs and overestimation of benefits. It is essential to rely on professionals and unbiased experts who use objective measures to minimize any over- or underestimation. This step usually emphasizes three important criteria:

    a. Cash flow measurement should be done on an incremental basis. It means that a project's cash flow takes a marginal sense as the difference between the firm's cash flow before and after the initiation of that project.
    b. Cash flow estimation should be considered on after tax basis using the firm's marginal tax rate and should include the effects of depreciation and all other noncash expenses that would be considered for income tax purposes.
    c. Cash flow estimation should include all indirect effects of the project throughout its lifetime. For example, the possible interference and overlap with other products, services, or functions of the firm.

3. Determining the firm's cost of capital that would serve as the discount rate for converting the value of cash flow from the future to the present time. It is equivalent to the required rate of return by the firm's investors.
4. Assessing the various potential risks that may be associated with certain projects.

5. Evaluating the alternative investment projects in order to choose and accept the best alternative project that would yield the maximum return for the firm. The fundamental criterion for establishing the measure of desirability and preference for a specific project is the comparison of the present value of the expected cash inflows with the initial cash outflow. The project that wins the allocation of capital has to show that the present value of its expected returns is higher than the initial capital outlay.

6. Preparing and writing a summary report that would highlight the top alternative projects and justify their fit to the firm and its need. Often this report comes with the final recommendations that would be submitted to the ultimate decision maker in the firm.
7. After the decision has been made in selecting of a project, the chosen proposal would be submitted to the appropriation department to secure the fund and signal for the implementation stage.
8. Following up the implementation process and monitoring its phases in order to assure a match between what is expected and what is obtained.
9. Devising and taking some corrective action should the actual results deviate from the expected ones. If any corrective action is not feasible, terminating the project would be the last choice to cut the costs and minimize the damages.

13.3 Types of Projects

The investment projects that may be submitted for approval can be classified according to their relationship to each other. In other words, projects would be judged according to their independence, and divided into independent projects and mutually exclusive projects.

Independent Projects

An **independent project** is a project that does not compete with other submitted projects for the firm's approval. In other words, it is a project the acceptance of which by the firm would not necessarily eliminate or reduce the opportunity for other projects to get approved. In this case, the firm's capacity to fund projects would not be an issue of consideration in the capital allocations. Such a project would be evaluated and judged only by its merit to meet the required criteria.

Mutually Exclusive Projects

A **mutually exclusive project** is a project that competes with other submitted projects to gain the firm's approval for funding. The firm's capacity to fund such projects would play a major role to determine the outcome of project competition. For example, if a firm receives five proposals, but its allocated capital is enough only to fund at most two projects, then approving two projects out of five would eliminate the chance of the other three to get approved regardless of the project's merit to meet the firm's criteria. It is just the limitation of capital in this example that imposed this status of being mutually exclusive. The firm can also be limited by its need. For example, if only one piece of equipment is needed, then the firm would choose only one project that would satisfy this need in the best way. That specific project would become mutually exclusive of all other projects that would be able to satisfy the firm's need for that piece of equipment.

Firms generally have two approaches for their decision-making in project evaluation.

- The first is the Yes/No approach in which the firm either grants its approval to a specific project or delivers its rejection. This means that there is no in-between decision such as a partial funding or conditional approval.
- The second approach is the flexible approach in which the firm would rank projects based on certain criteria, where either some or all projects get approved but according to a certain ranking that is ranging from top to bottom. For example, a number of projects could be ranked according to the size of return, or speed of completion, or least of costs or any other criteria. The highly ranked proposal would get the highest funding and the lowly ranked would get the least funding.

Firms are different in terms of the size of capital available for funding their investment projects. Firms with plenty of capital would be able to accept all the independent projects to satisfy their needs. Firms with limited funds would have to be extra careful in choosing the projects to be funded. They would most likely follow a policy of capital rationing to allocate their investment capital for the necessary projects.

13.4 Patterns of Cash Flow

The major premise for project evaluation and capital budgeting is the comparison between the capital initially allocated for investment and the future returns of that investment. It is basically comparing two cash flows, the outflow and the inflow, and needless to say that any firm would like to see the inflow larger than the outflow as an essential measure of investment success. The standard principle of time value of money requires unifying the values of the two flows back into the present time. In other words, for a plausible comparison of the two flows, all the future inflows or returns have to be discounted to their current values at the firm's cost of capital. Figure 13.1 illustrates this concept. An investment project yielding five annual future returns, all of which are transformed from their future values to their current values so that the present value of their sum is compared to the current value of the initial investment (cash outflow). The hope here is that the present value of all inflows would turn out to be larger or at least equal to the current value of the initially invested capital.

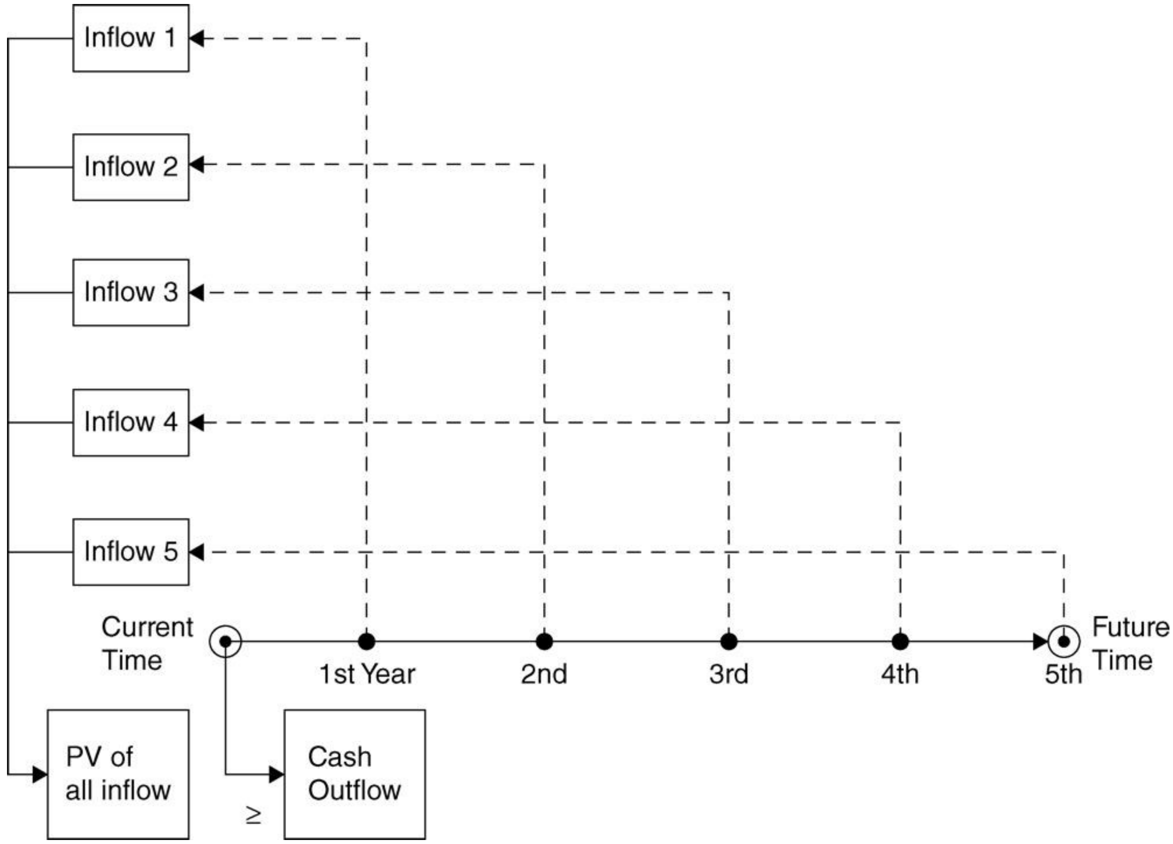

Figure 13.1 Cash Outflow and the Present Value of Future Cash Inflows

Cash flow pattern can be different from one project to another. The following are the two major and most common patterns:

The Conventional Pattern

Cash flow is structured in this pattern as on initial cash outflow and a series of future cash inflows. These cash inflows or returns can be either equal in equal intervals to form an annuity pattern or unequal in terms of amount and intervals. The following couple of timeline diagrams (Figures 13.2 and 13.3) show two types of the conventional pattern. In the first annuity type, an initial capital of $35,000 is spent, and five equal annual returns of $10,000 are expected to be received for the

next five years. In the second, non-annuity pattern, an initial investment of $14,000 is made and four unequal returns of $2000, $8000, $3000, and $5000 are expected to be received in the first, third, fourth, and sixth year, respectively.

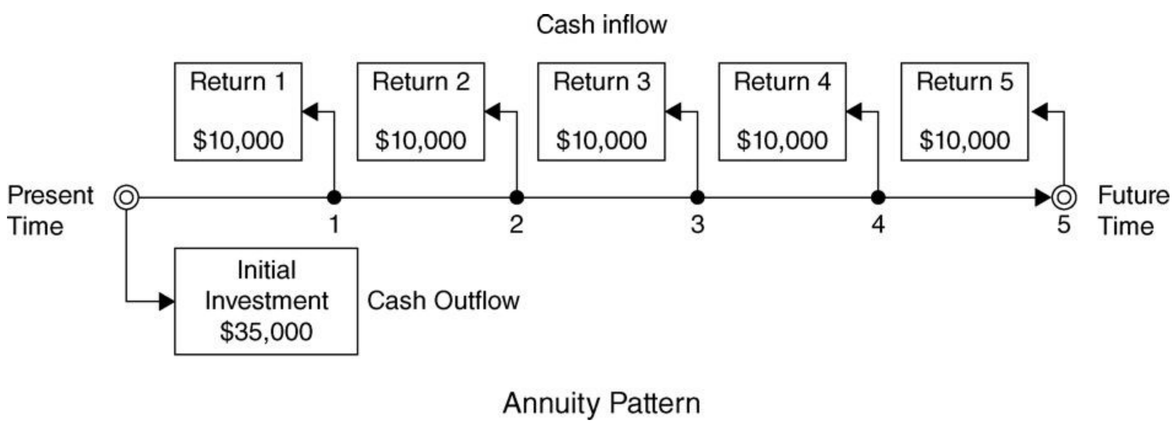

Figure 13.2 The Annuity Pattern of the Conventional Cash Flow

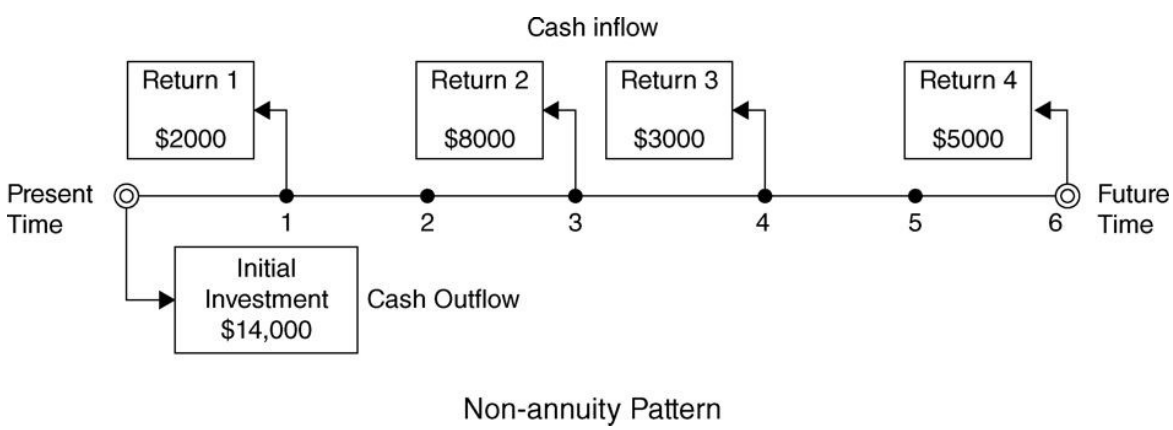

Figure 13.3 The Non-Annuity Pattern of the Conventional Cash Flow

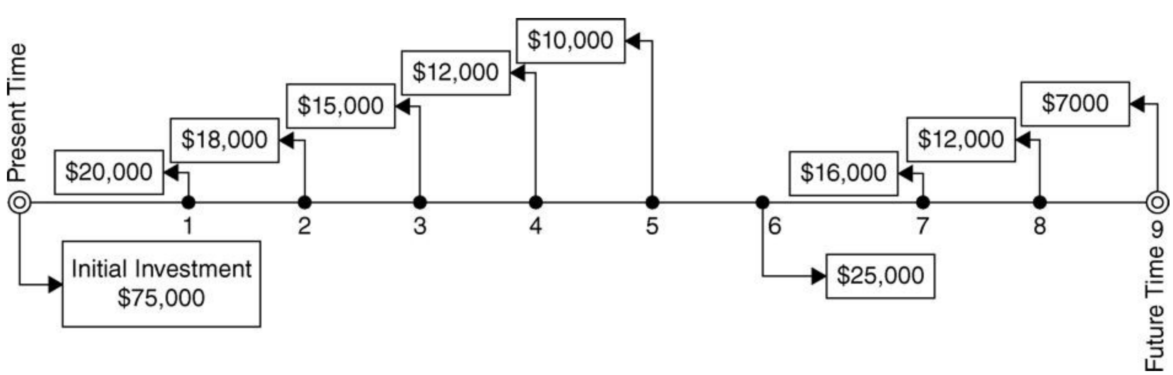

Figure 13.4 The Non-Conventional Pattern of Cash Flow

The Non-conventional Pattern

In this pattern, cash flow is not necessarily structured as an initial outflow followed by a series of inflows. Sometimes an investment requires more than one outflow, each of which may be followed by different series of inflows. The typical example of this pattern is the investment in a long-life productive machine that at some point in the future may require some capital expenditures to renew its capacity, such as in case of rebuilding or overhauling it. The series of returns out of this machine after each capital spending are most likely to be different from each other. **Figure 13.4** shows this type of non-conventional cash flow pattern. A piece of machinery is purchased for $75,000. It would generate $20,000, $18,000, $15,000, $12,000, and $10,000 during the next five years, respectively. In the sixth year, the machine is overhauled for a cost of $25,000, and it continues to generate $16,000, $12,000, and $7000 for the next three years, respectively.

13.5 Project Evaluation Techniques

There are two groups of evaluation techniques. The first group does not consider any time value of money adjustment, and the second group does. That is why the second group is called the **value-adjusted techniques** or the **sophisticated methods**. Most businesses, especially small firms, are using the first group which is represented by two techniques: The Average Rate of Return (ARR) and the Payback (PB) techniques. The value-adjusted techniques are more technical and require knowing the firm cost of capital, which makes it harder to apply. They are represented by the net present value (NPV), the Internal Rate of Return (IRR), and the Profitability Index (PI). Let us first discuss the first group which is represented by the ARR and PB before we turn to detailing the second group as represented by NPV, IRR, and PI.

Average Rate of Return

In this technique, a firm compares a calculated average rate of return for the proposed project with its own predetermined minimum acceptance rate of return. A project would be approved if its calculated rate of return equals or exceeds the firm's minimum rate of return, and it would be rejected if the calculated rate of return turns out to be less than the firm's acceptance rate. The average rate of return of the proposed project is calculated by dividing the average of expected earnings after depreciation and taxes (**AEADT**) by the average capital that would be initially invested in that project (AIInv).

$$\boxed{\text{ARR} = \frac{\text{AEADT}}{\text{AIInv}}}$$

Using the data given in **Table 13.1**, we can use this technique to determine which project, I or II, would be approved for funding by the firm which has put its acceptance rate of return at 26%.

The average earning after depreciation and taxes is obtained by the sum of all annual earnings after depreciation and taxes ($\sum EADT$) divided by the number of years (n = 5).

$$\text{Project I:} \qquad \text{AEADT} = \frac{\sum EADT}{n} = \frac{11{,}625 + 7050 + 7800 + 7650 + 7500}{5}$$
$$= \frac{41{,}625}{5} = 8325$$
$$\text{Project II:} \qquad \text{AEADT} = \frac{\sum EADT}{n} = \frac{32{,}000 + 4000 + 1000 + 950 + 900}{5}$$
$$= \frac{38{,}850}{5} = 7770$$

The average initial investment is obtained by dividing the initial investment for each project by 2:

$$\text{Project I:} \qquad \text{AIInv} = \frac{\text{InInv}}{2} = \frac{60{,}000}{2} = 30{,}000$$
$$\text{Project II:} \qquad \text{AIInv} = \frac{\text{InInv}}{2} = \frac{62{,}000}{2} = 31{,}000$$

Table 13.1 Average Rate of Return (ARR) for Two Projects

	Project I	Project II
Year	AEADT	AEADT
1	11,625	32,000
2	7050	4000
3	7800	1000
4	7650	950
5	7500	900
∑AEADT	**41,625**	**38,850**
AEADT	8325	7770
InInv (Initial investment)	60,000	62,000
AIInv	60,000/2 = 30,000	62,000/2 = 31,000
ARR	27.75%	25.06%

The average rate of return for each project is obtained by dividing the average earnings after depreciation and taxes (AEADT) by the average initial investment (AIInv).

$$\text{Project I:} \qquad \text{ARR} = \frac{\text{AEADT}}{\text{AIInv}} = \frac{8325}{30{,}000} = 27.75\%$$

$$\text{Project II:} \qquad \text{ARR} = \frac{\text{AEADT}}{\text{AIInv}} = \frac{7{,}770}{31{,}000} = 25.06\%$$

Project I would be approved and project II would be rejected according to the firm's criteria for the minimum acceptable rate of return of 26%.

Payback Period

Instead of having the project rate of return as a measure of merit for investment, this technique relies on the length of time in years required for fully recovering all investment. Payback is recognized for its simplicity and straightforwardness. **PB period** refers to the expected number of operational years during which the initial investment can be recovered. In particular, as a criterion for project selection, the shorter the PB the better the project. The short recovery time would be a crude measure of liquidity of the project. It can also be an indication of a less potential risk. The fewer the number of years in which the initial capital can be fully recovered, the more the project can cut off the potential risk that may lie ahead.

The PB is obtained by the following two methods:

1. For projects yielding an equal amount of cash inflow during the project's lifetime, PB is obtained by dividing the initial investment or the proposed capital (*I*) by the annual cash inflow (CIF):

$$\text{PB} = \frac{\text{I}}{\text{CIF}}$$

2. For projects yielding variable cash inflows throughout the years of the project's operation, the PB can be obtained by

$$\text{PB} = (f-1) + \left[\frac{I - \sum_{t=1}^{f-1} \text{CIF}_t}{\text{CIF}_f}\right]$$

   where f is the year in which the initial investment can be fully recovered, f–1 the year before the year of full recovery of the initial investment, I the initial investment or the capital proposed for allocation, CIF_t the cash inflow during period t up to the year before the year of full recovery of initial investment, and CIF_f the cash inflow in the year of full recovery of the initial investment.

Sunshine Company is considering the following two projects for capital allocation, Project X is asking for $64,000 and Project Y is asking for $68,000. Let us calculate the PB for both projects, as the detailed information is given in **Table 13.2**.

Table 13.2

	Project X ($64,000)		Project Y ($68,000)	
Year	Expected Profits (After taxes) ($)	Cash inflows ($)	Expected profits (after taxes) ($)	Cash inflows ($)
1	9500	16,000	21,500	40,800
2	10,200	16,000	9000	12,200
3	10,200	16,000	5500	10,000
4	8000	16,000	4500	10,000
5	7100	16,000	4000	8000
	9000	16,000	8900	16,000

For project X:

$$\begin{aligned} PB &= \frac{I}{CIF} \\ &= \frac{64{,}000}{16{,}000} \\ &= 4 \text{ years} \end{aligned}$$

For project Y:

$$\begin{aligned} PB &= (f-1) + \left[\frac{I - \sum_{t=1}^{f-1} CIF_t}{CIF_f}\right] \\ &= 3 + \left[\frac{68{,}000 - (40{,}800 + 12{,}200 + 10{,}000)}{10{,}000}\right] \\ &= 3.5 \text{ years} \end{aligned}$$

where f is the fourth year since the $68,000 would be within the accumulated cash inflows by the end of the fourth year. Therefore, $\sum_1^3 CIF_t = (\$40{,}800 + \$12{,}200 + \$10{,}000 + \$10{,}000 = \$73{,}000)$. $f - 1$ is the third year (the year before the fourth) and CIF_f the annual cash inflow ($10,000) in the fourth year as the year in which the initial investment would be fully recovered.

In addition to the major shortcoming of the payback method, wherein the time value of money is ignored, it has also been criticized for ignoring the dynamics of the cash inflows obtained during the years after the year of full recovery.

Net Present Value

An investment project's **net present value** is the present value of all future returns or cash inflows minus the initial capital invested in the project. The stream of future cash inflows must be discounted back to the current time using the firm's cost of capital as the discount rate. This rate is determined by the firm based on its assessment of the risk involved in each investment project. High risk projects are assigned higher discount rate and low risk projects are assigned lower discount rates. NPV is probably the most common technique used to assess how worthy a project is and whether it would be accepted or not. If the present value of all future returns (the cash inflows) is larger than the initial cost of the project (the cash outflows), the NPV would be positive and the project is acceptable. Otherwise, if the present value of the cash inflows is smaller than the cash outflows, the NPV would be negative and the project cannot be accepted.

Let us recall that the current or discounted value (CV) of any future return (FV) can be obtained by

$$\text{CV} = \frac{\text{FV}}{(1-r)^n}$$

where *r* is the interest rate used for discounting or bringing the value of return from future back to the present and *n* is the number of terms such as in years and quarters alike. If we refer to an annual cash inflow of a project by CF, and to the project cash outflow by *I*, then the NPV would be the difference between the discounted streams of both flows throughout the lifetime of the project (*t*).

$$\text{NPV} = \sum_{i=1}^{t}\left[\frac{\text{CF}_i}{(1+r)^n}\right] - \sum_{i=1}^{t}\left[\frac{I_i}{(1+r)^n}\right]$$

If the project takes into account only the initial capital outlay, then it would already be in its current value, I_0, and does not need to be discounted.

$$\text{NPV} = \sum_{i=1}^{t}\left[\frac{\text{CF}_i}{(1+r)^n}\right] - I_0$$

The criteria for project acceptability are for the NPV to be non-negative:

$$\text{NPV} \geq 0$$

Example

A proposal to expand a fast food restaurant calls for the investment of an initial capital of $42,000 and promises to deliver a return of at least $14,000 per year for the next 5 years. Would the franchise company approve such an expansion project if its cost of capital is 11.5%?

Solution:

$$\begin{aligned}
\text{NPV} &= \sum_{i=1}^{t}\left[\frac{\text{CF}_i}{(1+r)^n}\right] - I_0 \\
&= \frac{\text{CF}_1}{(1+r)^1} + \frac{\text{CF}_2}{(1+r)^2} + \frac{\text{CF}_3}{(1+r)^3} + \frac{\text{CF}_4}{(1+r)^4} - I_0 \\
&= \frac{14{,}000}{(1+0.115)^1} + \frac{14{,}000}{(1+0.115)^2} + \frac{14{,}000}{(1+0.115)^3} + \frac{14{,}000}{(1+0.115)^4} + \frac{14{,}000}{(1+0.115)^5} \\
&\quad - 42{,}000 \\
&= [12{,}556.05 + 11{,}261.03 + 10{,}099.58 + 9{,}057.95 + 8{,}123.69] - 42{,}000 \\
&= 51{,}098.27 - 42{,}000 \\
&= 9{,}098.27
\end{aligned}$$

Since the NPV is positive, the expansion project would be accepted.

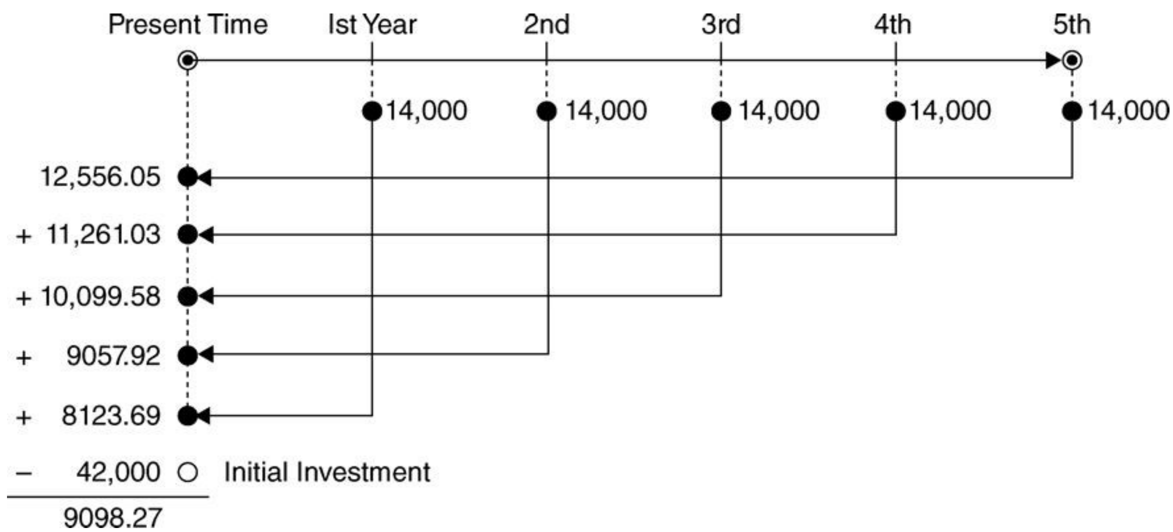

Figure 13.5

Example

The development committee in a construction company is studying two investment proposals whose cash inflows for the next 4 years are as projected in Table 13.3. Both projects require a capital allocation of $200,000, given that the cost of capital for the first project is 8%, and for the second is 7.5%. Which of the two proposals would get an approval?

Project I:

$$
\begin{aligned}
\text{NPV} &= \sum_{i=1}^{t}\left[\frac{\text{CF}_i}{(1+r)^n}\right] - I_0\\
&= \frac{35{,}000}{(1+0.08)^1} + \frac{40{,}000}{(1+0.08)^2} + \frac{50{,}000}{(1+0.08)^3} + \frac{120{,}000}{(1+0.08)^4} - 200{,}000\\
&= 32{,}407.40 + 34{,}293.55 + 39{,}691.61 + 88{,}203.58 - 42{,}000\\
&= 194.596 - 200{,}000\\
&= -5404.26
\end{aligned}
$$

Project II:

$$
\begin{aligned}
\text{NPV} &= \frac{40{,}000}{(1+0.075)^1} + \frac{40{,}000}{(1+0.075)^2} + \frac{95{,}000}{(1+0.075)^3} + \frac{100{,}000}{(1+0.075)^4} - 200{,}000\\
&= 37{,}209.30 + 34{,}613.30 + 76{,}471.25 + 74{,}880.05 - 200{,}000\\
&= 223{,}173.90 - 200{,}000\\
&= 23{,}173.90
\end{aligned}
$$

Table 13.3

	Project I		Project II	
Year	Cash Inflows ($) $r = 8\%$	Cash Outflows ($)	Cash Inflows ($) $r = 7.5\%$	Cash Outflows ($)
1	35,000	200,000	40,000	200,000
2	40,000		40,000	
3	50,000		95,000	
4	120,000		100,000	

Project I has a negative NPV and project II has a positive NPV. The committee would accept the second project and reject the first.

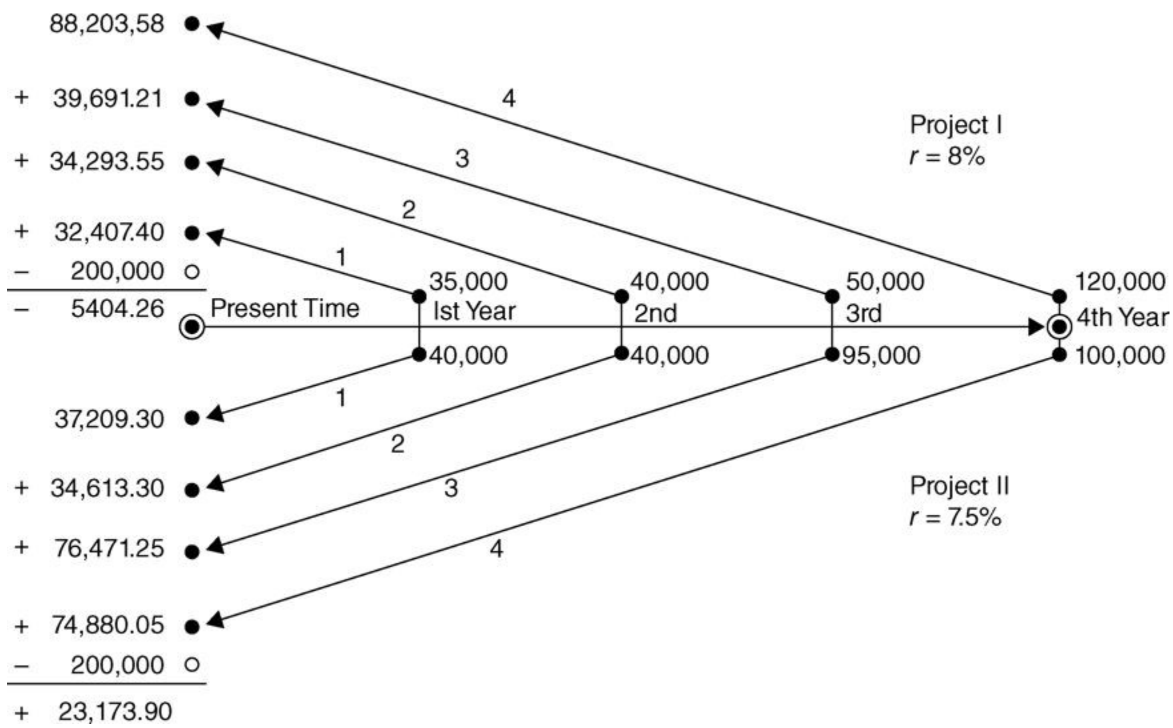

Figure 13.6

The cash inflow in the NPV formula could be replaced by the firm's profit for any period (π_i) and adjusted for depreciation and taxes.

Since $\pi_i = TR_i - TC_i$

$$NPV = \sum_{i=1}^{t} \left\{ \frac{[TR_i - TC_i](1-t) + D_i}{(1+r)^n} \right\} - I_0$$

where TR_i and TC_i are the firm's total revenue and total cost for the ith period, respectively; t is the firm's marginal tax rate; D_i is the firm's capital depreciation; and I_0 is the initial investment capital allocated for the project.

Example

A firm received a proposal for an investment project showing estimated projections of the gross profit based on the total cost, total revenue, and depreciation allowances for the first 5 years as shown in **Table 13.4**. How would the firm decide

 275

on accepting or rejecting the request for an initial capital of $650,000, given that the cost of capital is 8.25% and the firm's marginal income tax is 32%? 276

$$
\begin{aligned}
\text{NPV} &= \frac{124{,}600}{(1+0.0825)^1} + \frac{157{,}300}{(1+0.0825)^2} + \frac{172{,}400}{(1+0.0825)^3} + \frac{183{,}700}{(1+0.0825)^4} \\
&\quad + \frac{205{,}000}{(1+0.0825)^5} - 650{,}000 \\
&= [115{,}103.93 + 134{,}237.21 + 135{,}910.67 + 133{,}781.95 + 137{,}915.90] \\
&\quad - 650{,}000 \\
&= 656{,}949.66 - 650{,}000 \\
&= 6949.66
\end{aligned}
$$

The proposal would be approved for yielding a non-negative NPV of $6,949.66, as shown in Figure 13.7.

Table 13.4

Year	TR_i (\$)	TC_i (\$)	π_i (3-2)	$(1-t) =$ $1-0.32$	D_i (\$)	$\pi_i(1-t) + D_i$ $(4 \times 5) + 6$ (\$)
1	600,000	460,000	140,000	0.68	29,400	124,600
2	660,000	475,000	185,000	0.68	31,500	157,300
3	625,000	420,000	205,000	0.68	33,000	172,400
4	620,000	400,000	220,000	0.68	34,100	183,700
5	668,000	418,000	250,000	0.68	35,000	205,000

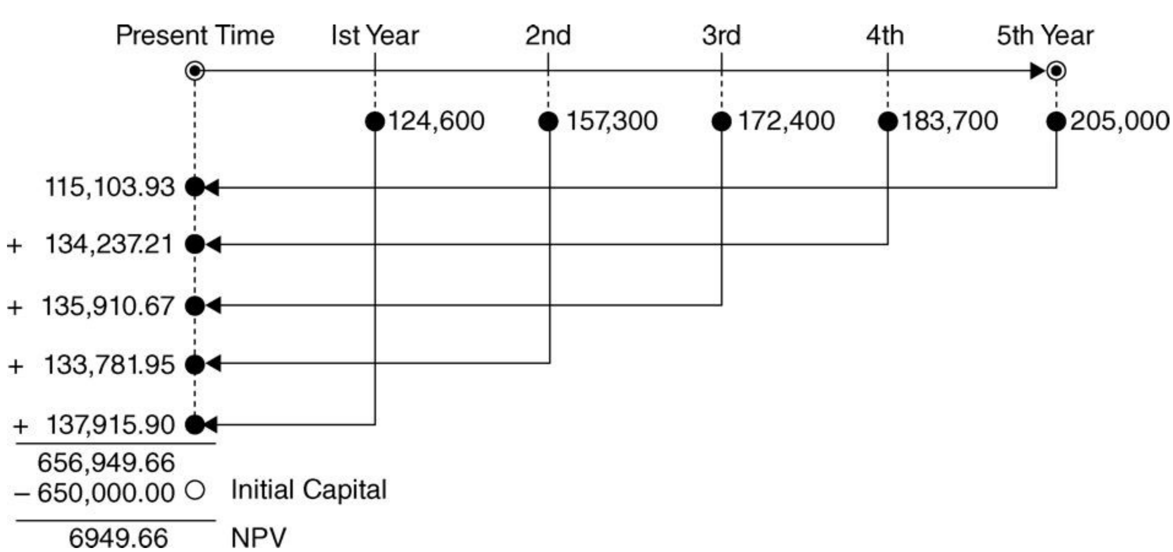

Figure 13.7

Internal Rate of Return

Another method used to determine the acceptability of a proposed investment is to compare the internal rate of return with the firm's cost of capital. The criterion for the project to be accepted is that it must yield an internal rate of return larger than or at least equal to the firm's cost of capital.

$$\text{IRR} > r$$

The **Internal Rate of Return** is sometimes called "**the profit rate**" or "**the marginal efficiency of investment**" is defined as the rate that equates the present value of cash inflows and the cash outflows. In the NPV formula, such a rate that makes the two cash flows equal must make the NPV equal to zero. This would mean that the project is not capable of delivering an earning rate higher than the cost of capital.

Using the NPV formula, we can now replace the firm's cost of capital (r) with the IRR and set the NPV to zero.

$$\text{NPV} = \sum_{i=1}^{n} \left[\frac{\text{CF}_i}{(1 + \text{IRR})^n} \right] - I_0 = 0$$

To find the right IRR that makes the NPV equal to zero, we have to solve the above equation. IRR can be found by many ways such as using the table values, trial and error, interpolation, and so on. Computers and sophisticated business calculators can easily find IRR. However, an equation is developed to get the estimated IRR. Once we get such an estimate, we can keep iterating until we get the exact rate that makes the value of NPV zero.

$$\text{IRR} = \left[\frac{\sum_{i=1}^{n} \text{CF}_i - I_0}{\sum i\text{CF}_i} \right]$$

where CF is cash inflow for the period I, and I is the number of any year of the discounted term. i = 1, 2, 3,..., n and I_0 is the initial investment.

Example

A firm is studying an investment proposal that requires $15,000 as initial capital. The projections for cash inflows during the first 5 years are shown in **Table 13.5**.

$$\text{IRR} = \left[\frac{\sum_{i=1}^{5} \text{CF}_i - I_0}{\sum i\text{CF}_i} \right]$$

$$\text{IRR} = \left[\frac{27{,}100 - 15{,}000}{91{,}200} \right]$$

$$\text{IRR} = 13.3\%$$

Table 13.5

Cash Inflows		
i	CF_i	$i\,CF_i\ (1 \times 2)$
1	3600	3600
2	4200	8400
3	5500	16,500
4	6300	25,200
5	7500	37,500
$\sum_{i=1}^{5} CF_i$	27100	91,200

But this is only a rough estimate for the IRR. However, we can reach the exact rate after some trial and error attempts by calculating NPV until it reaches zero. That would be possible when the present value of all cash inflows is exactly equal to the initial investment, in this case $15,000.

The first step is to calculate the present value of cash inflows at the discount rate of 13.3% to see how close it would bring the present value to the initial investment of $15,000.

$$\begin{aligned} PV &= \frac{3{,}600}{(1+0.133)^1} + \frac{4{,}200}{(1+0.133)^2} + \frac{5{,}500}{(1+0.133)^3} + \frac{6{,}300}{(1+0.133)^4} + \frac{7{,}500}{(1+0.133)^5} \\ &= 3{,}177 + 3{,}272 + 3{,}782 + 3{,}823 + 4{,}017 \\ &= 18{,}071 \end{aligned}$$

Therefore, a rate of 13.3 produces a present value of the cash inflows larger than the initial investment of $15,000. Since the rate of discount has an inverse relationship with the present value, we have to increase the rate in the next try to reduce the present value hoping to let it reach $15,000.

At a rate of 18%, the present value of cash inflows would be

$$\begin{aligned} PV &= \frac{3{,}600}{(1+0.18)^1} + \frac{4{,}200}{(1+0.18)^2} + \frac{5{,}500}{(1+0.18)^3} + \frac{6{,}300}{(1+0.18)^4} + \frac{7{,}500}{(1+0.18)^5} \\ &= 3{,}051 + 3{,}016 + 3{,}347 + 3{,}249 + 3{,}278 \\ &= 15{,}942 \end{aligned}$$

Therefore, we are getting much closer to $15,000. We need to raise the rate a little more to bring the present value to exactly $15,000. We can also use the $PVIF_{r,\,n}$ table value (v^n) to ease the tedious calculations. **Table A2** in the appendix shows the discount factor of a dollar for many combinations of r and n.

$$PVIF = v^n = \frac{1}{(1+r)^n}$$

For example, to get the discounted cash inflow in the third year, we can either divide 5500 by $(1 + 0.18)^3$ or get the discount factor from **Table A2** for $r = 18\%$ and $n = 3$, and multiply it by 5500.

$$\begin{aligned} PV &= FV[PVIF_{r,n}] \\ &= 5{,}500[PVIF_{0.18,3}] \\ &= 5{,}500[0.6086] \\ &= 3{,}347 \end{aligned}$$

Other few tries to get the exact rate revealed that at 20%, PV = 15,151 and at exactly 20.4%, PV = 15,000.

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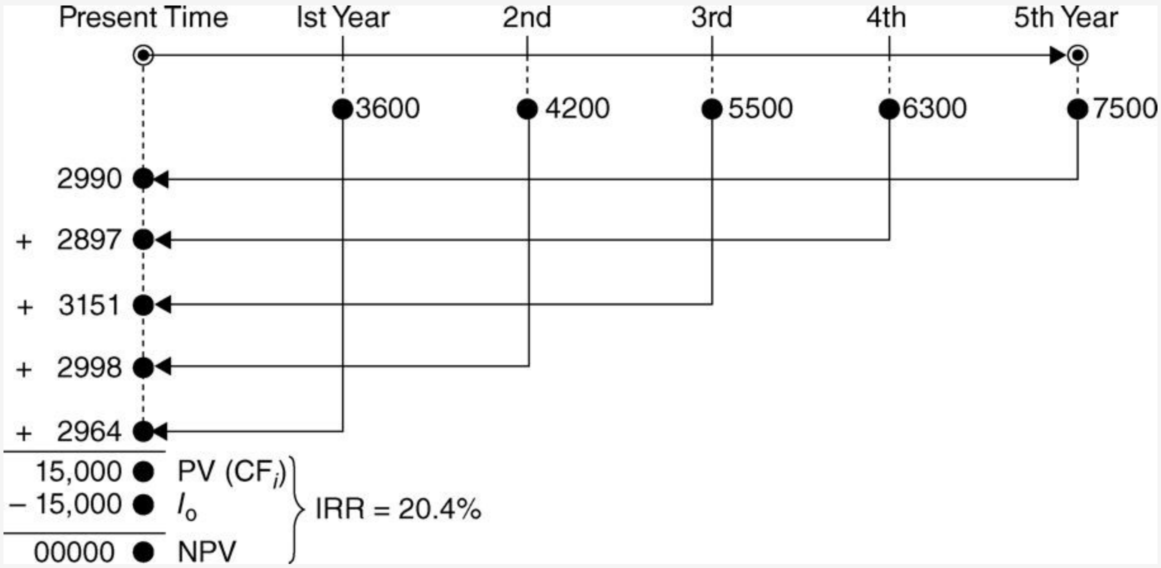

Figure 13.8

Here, 15,000 is the IRR that brings about the equality between the present value of cash inflows and the initial investment, making the NPV equal to zero.

$$\begin{aligned}\text{NPV} &= \left[\frac{3600}{(1+0.204)^1} + \frac{4200}{(1+0.204)^2} + \frac{5500}{(1+0.204)^3} + \frac{6300}{(1+0.204)^4} + \frac{7500}{(1+0.204)^5}\right] - 15{,}000 \\ &= [2990 + 2897 + 3151 + 2998 + 2964] - 15{,}000 \\ &= 15{,}000 - 15{,}000 = 0\end{aligned}$$

NPV versus IRR for Mutually Exclusive Projects

The criterion for accepting or rejecting an investment project can be based on either the highest NPV or the highest IRR. It would make no difference to the firm, whichever the measure followed, simply because they reflect each other consistently. Having a positive value for the NPV means having an IRR that exceeds the firm's cost of capital and having a negative value of the NPV refers to having an IRR lower than the firm's cost of capital.

$$\text{If IRR} > \text{MCC} \rightarrow \text{NPV} > 0$$
$$\text{IRR} < \text{MCC} \rightarrow \text{NPV} < 0$$

Therefore, either measure would be fine if followed, but that is especially true if the firm is assessing only a single independent project. However, if the firm wants to assess two or more projects that are mutually exclusive, then the measures of NPV and IRR may not mean the same. **Mutually exclusive projects** are those projects that compete to earn the decision of approval. In other words, the firm can only choose one project among the alternatives. Let us calculate both the NPV and IRR for the two projects whose 5-year cash inflows are shown in **Table 13.6**, given that both are vying with each other for the $250,000 initial capital that is to be invested at least at the firm's 9.5% cost of capital.

Table 13.6

Year	Project I	Project II	
0	250,000	250,000	Initial Capital ($)
1	−25,000	87,500	Cash Inflows ($)
2	0	87,500	
3	125,000	87,500	
4	125,000	87,500	
5	350,000	87,500	Discount Rate 9.5%
NPV ($)	131,653	85,973	
IRR	20.2%	22.1%	

$$\text{NPCV} = \sum_{i=1}^{t}\left[\frac{\text{CF}_i}{(1+r)^n}\right] - I_0$$

$$\text{NPV}_\text{I} = \left[\frac{-25{,}000}{(1+0.095)^1} + \frac{0}{(1+0.095)^2} + \frac{125{,}000}{(1+0.095)^3} + \frac{125{,}000}{(1+0.095)^4} + \frac{350{,}000}{(1+0.095)^5}\right] - 250{,}000$$

$$\text{NPV}_\text{I} = [-22{,}831 + 0 + 95{,}207 + 86{,}947 + 222{,}330] - 250{,}000$$

$$\text{NPV}_\text{I} = 131{,}653$$

$$\text{IRR}_\text{I} = 20.2\%\ \text{(obtained using a calculator)}$$

$$\text{NPV}_\text{II} = \left[\frac{87{,}500}{(1+0.095)^1} + \frac{87{,}500}{(1+0.095)^2} + \frac{87{,}500}{(1+0.095)^3} + \frac{87{,}500}{(1+0.095)^4} + \frac{87{,}500}{(1+0.095)^5}\right] - 250{,}000$$

$$\text{NPV}_\text{II} = [79{,}908 + 72{,}976 + 66{,}645 + 60{,}862 + 55{,}582] = -250{,}000$$

$$\text{NPV}_\text{II} = 85{,}973$$

$$\text{NPV}_\text{II} = 22.1\%\ \text{(obtained using a calculator)}$$

The calculation shows that the NPV and the IRR are inconsistent across the two projects. While project I has a higher NPV (NPV_I = 131,653 compared with NPV_II = 85,973), project II has a higher IRR (IRR_II = 22.1 compared with IRR_I = 20.2).

Therefore, the firm has to decide which would be the better project. Theoretically, it would be better for the firm to decide project acceptability on the basis of the higher NPV rather than the higher IRR. One of the justifications for that is the assumption that the earned cash inflows are to be reinvested at a reasonable cost of capital rate and there would be no guarantee of reinvestment of the cash inflows earned by the other project at a higher rate of return. However, practically most financial managers in the business market tend to favor decisions based on the higher IRR.

One interpretation for such a tendency is the common reliance on relative change than on absolute change, which makes the "rates" more preferable than the actual dollar as in the case of the NPV.

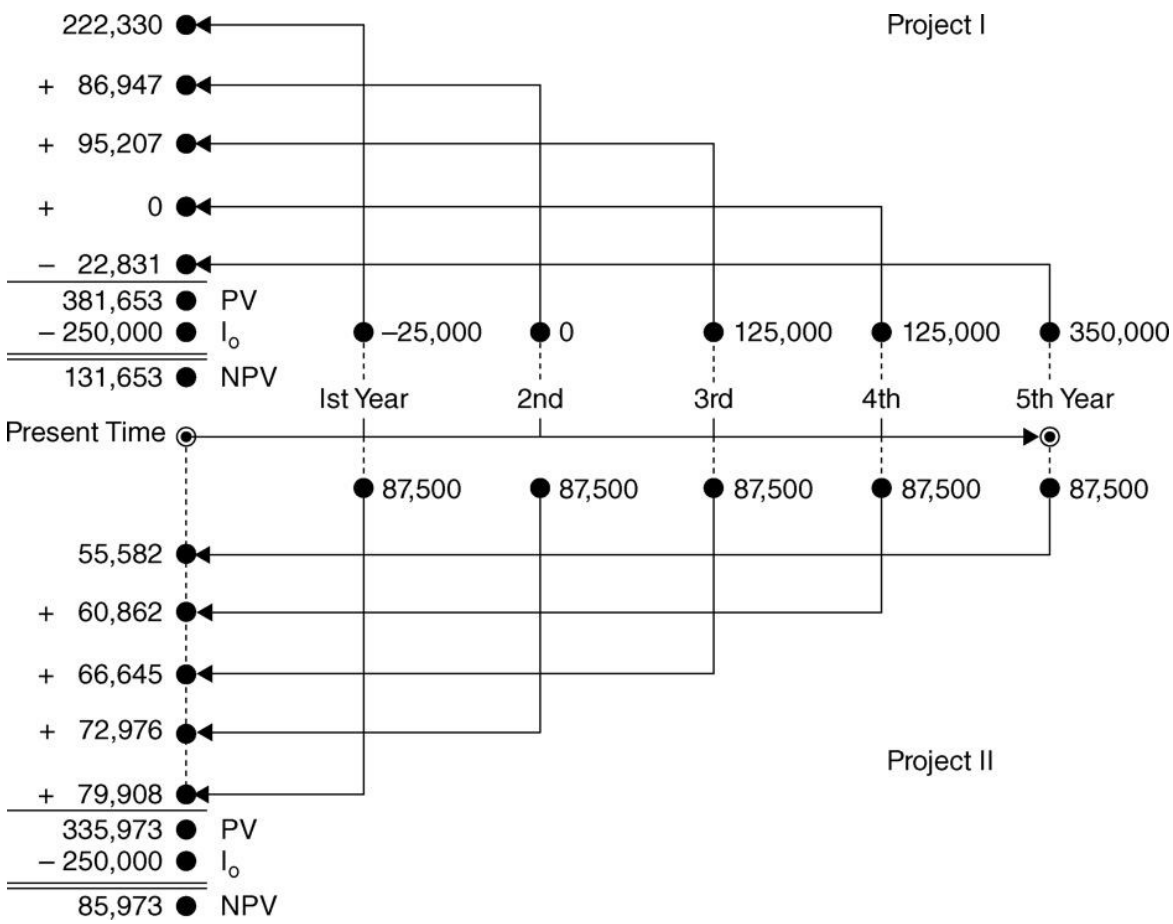

Figure 13.9

The relative measures can still remain reliable for comparison, especially when the firm faces many investment proposals but it cannot afford to fund all proposals except the most profitable one due to budgetary constraints. Let us consider a firm's six investment project proposals requiring different capitals and promising different NPVs as shown in Table 13.7. Suppose that a firm can only allocate a maximum of $800,000 and one of the proposals is asking for an initial capital of $800,000 and

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the rest of the proposals are asking for funds ranging from $150,000 to $400,000. In this case, it is better not to consider the absolute amounts of the NPV but to find the **yield** or the NPV per dollar by dividing the NPV by the amount invested or the initial capital allocated.

Table 13.7

Projects	I_0 ($)	NPV ($)	NPV/I_0 (%)	
I	800,000	2,560,000	3.2	
II	290,000	725,000	2.5	
III	250,000	850,000	3.4	←
IV	350,000	770,000	2.2	3.6125
V	400,000	1,440,000	3.6	←
VI	150,000	600,000	4.0	←

Firm's total funding capacity: $800,000.

Table 13.8

	Net Present Value ($)	
Cost of Capital (%)	Project I	Project II
0	20,000	25,000
2.5	15,972	20,656
4	12,270	15,662
6	10,527	12,498
7.5	9057	10,775
8	8765	8765
10	5695	4307
11.5	2772	240
13	60	–3480
153	–2462	–6887
IRR	13.3%	11.7%

The firm can invest in project I only by investing all it has and getting a return of $3.2 per dollar invested, but a combination of more than one project not only reduces the risk that arises by concentration but also increases the return. Therefore, the available capital of $800,000 can be shared by projects III, V, and VI generating a total of $2,890,000 (850,000 + 1,440,000 + 600,000) NPV, which would be translated into a 3.6125% earning per dollar invested $\left(\frac{2{,}890{,}000}{800{,}000}\right)$. It is higher than 3.2% from the first project that required the entire investment budget. Projects I, II, and IV would be dismissed.

NPV Profile, Crossover Rate, and the Ranking Reversal

A **net present value profile** is the relationship between a project's NPV and several alternative costs of capital rates. It is expressed in Table 13.8 and Figure 13.10 for projects I and II. The table and figure show how the NPV changes in response to changes in the firm's cost of capital rate.

Looking at Table 13.8 and Figure 13.10, we can make the following observations:

1. Since the relationship between the NPV and the discount rate is negative, both curves, NPV_I and NPV_{II}, are sloping downwards from their initial values of $20,000 and $25,000

at zero rate, to negative values at the highest rate. It means that, as the firm's cost of capital increases, its NPV decreases.

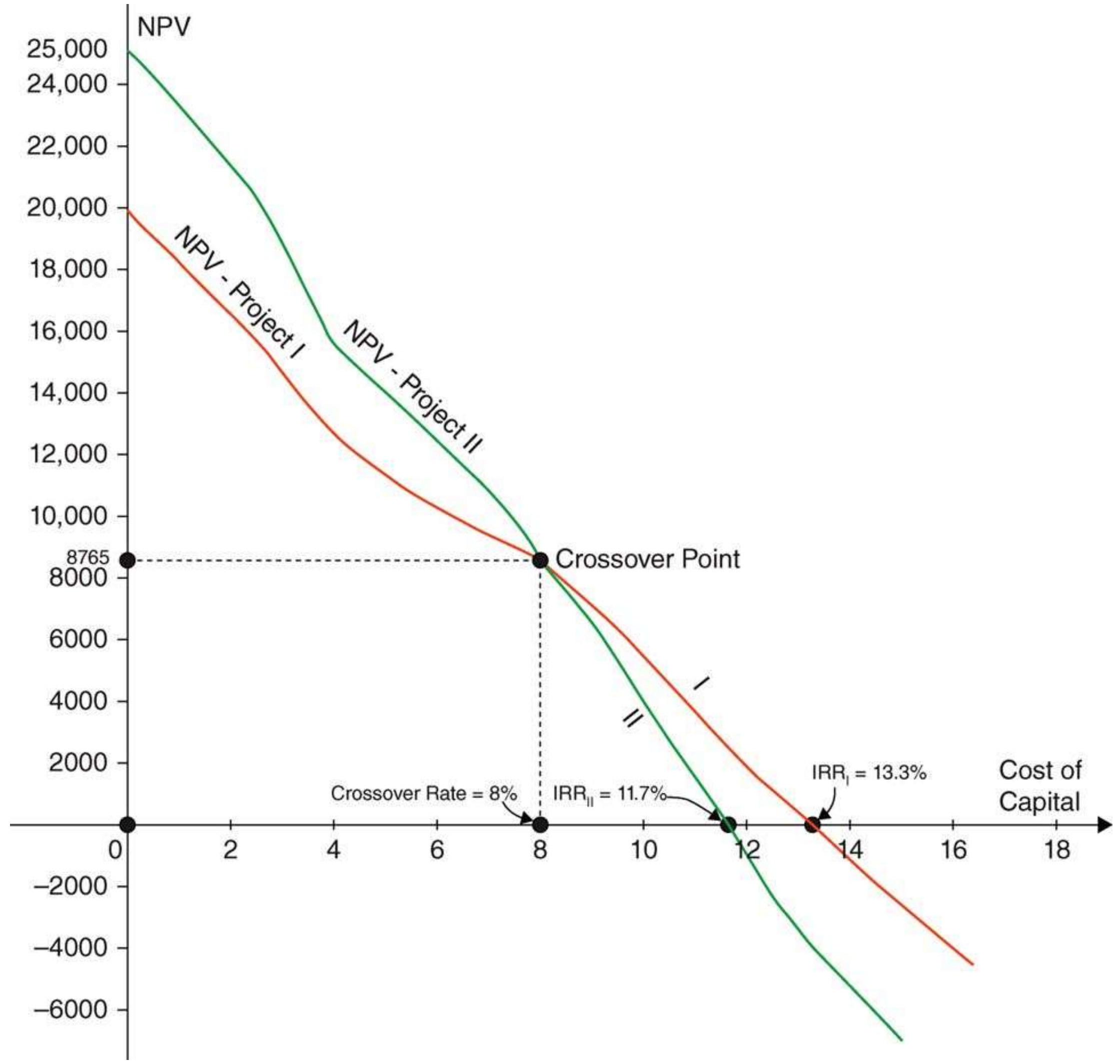

Figure 13.10 The NPV Profile and Crossover Point

2. The curve for project II is steeper than the curve for project I. This reflects that project II is more sensitive to the change in cost of capital rate than project I:

$$\frac{\partial \text{NPV}_{\text{II}}}{\partial r} > \frac{\partial \text{NPV}_{\text{I}}}{\partial r}$$

We can observe, for example, that as the cost of capital rate goes up by 92% (from 6% to 11.5%), the NPV for project II goes down by 98% while the NPV for project I declines by 74%.

$$\frac{11.5 - 6}{6} = 92\%, \frac{10{,}527 - 2{,}772}{10{,}527} = 74\%, \frac{12{,}498 - 240}{12{,}498} = 98\%$$

The difference in the project sensitivity to changes in the cost of capital is due to the differences in the magnitude as well as the timing of the cash inflows. Because the discounting process is just the reverse of the compounding process, the present value of cash inflows in its later years would decrease at a higher rate than those inflows in its earlier years. This is the reason for the curves to intersect at the crossover point.

1. When the NPVs of the two projects are equal, their curves intersect at the **crossover point**. The cost of capital rate corresponding to that point is called the crossover rate. Therefore, the **crossover rate** is defined as the rate at which the NPVs of two projects become equal to each other and where their curves intersect.
2. The crossover rate serves as the turning point where the reversal of project ranking occurs. For any cost of capital below the crossover rate, project II would be preferred for its higher NPV. For any cost of capital above the crossover rate, project I would be preferred for its higher NPV.
3. The NPVs of both projects would decline to zero when they cross the *x*-axis at their firm's IRR. The graph shows that the IRR for project I is 13.3%, and for project II the IRR is 11.7%, and those where the two curves cross the *x*-axis, respectively.

Profitability Index and Capital Rationing

We have briefly discussed the relative measure of the net present value in assessing the acceptability of investment projects. We also discussed the benefits of the net present value per dollar invested (the yield), where the NPV is divided by the initial investment to shed another light on the assessment process. Here we discuss another ratio that may seem similar to the yield. It is the PI, which is a ratio of the present value for cash inflows and outflows.

$$\mathrm{PI} = \frac{\sum_{i=1}^{t}\left[\frac{\mathrm{CF}_i}{(1+r)^n}\right]}{\sum_{i=1}^{t}\left[\frac{I_i}{(1+r)^n}\right]}$$

This is to say that it is a ratio between the present value of cash inflows (not the net present value) and the initial investment:

$$\mathrm{PI} = \frac{\mathrm{PV}(\mathrm{CF}_i)}{I_0}$$

$$\mathrm{PI} \geq 1$$

The criterion for project acceptability is the value of PI, whether the value is equal to or larger than 1, since a value equal to 1 would indicate the equality between the cost and benefit.

In two of the projects listed in **Table 13.7**, the NPV for one was $770,000, and the other was $600,000, but because the investment capital required for both were $350,000 and $150,000 respectively, their values for the firm would differ dramatically. Calculating the PI for both would reveal the difference. However, we need to restore the present value for cash flows by combining the NPV and the initial investment for each project:

$$\begin{aligned}
\text{PV} &= \text{NPV} + I_0 \\
\text{PV}_1 &= 770{,}000 + 350{,}000 = 1{,}120{,}000 \\
\text{PV}_2 &= 600{,}000 + 150{,}000 = 750{,}000 \\
\text{PI}_1 &= \frac{\text{PV}_1(\text{CF}_i)}{I_0} \\
&= \frac{1{,}120{,}000}{350{,}000} \\
&= 3.2 \text{ for project IV} \\
\text{PI}_2 &= \frac{750{,}000}{150{,}000} \\
&= 5 \text{ For project VI}
\end{aligned}$$

The conclusion is that while the NPV of the first project ($700,000) is 28% more than the NPV of the second project ($600,000), the PI of the second project (5) is 56% more than the PI for the first project (3.2). This would illustrate the benefits of the PI as a tool, especially when judgment by the NPV alone would not be conclusive. The relative measure of the NPV may rise again when the firm has some constraints on its capacity of investment. Usually, there is a certain limit to a firm's capacity for financing all its available feasible projects. The senior management of firms may fix a limit and determine the maximum capacity of capital investment, and decide whether project financing should be done by borrowing from banks and financial institutions or from the public in terms of corporate bonds and equity shares. Declaring and recognizing the limit on investment may mean recognizing some sort of capital scarcity which should most likely lead to seeking allocation efficiency and that is what is called capital rationing. **Capital rationing** is defined as the process of allocating scarce financial capital as efficiently as the firm's conditions and circumstances permit. It is obvious that capital rationing would be exercised when the total funds requested to finance all eligible projects exceeds the firm's affordability as set by the maximum level of funding. Capital rationing involves contemplating every possible combination of projects that can be funded, and choosing the best combination of projects, where

1. their total required capital would not exceed the firm's maximum level of funding;
2. their total NPV per dollar is the largest among the alternative combinations.

Tables 13.9 and 13.10 show how capital rationing is done. Table 13.9 shows five competing projects all of which were considered worthy, but the firm cannot fund all because the required total capital is $300,000 and the firm's maximum level of funding is $200,000. Table

13.10 lists the possible combinations of projects with their combined capital and combined NPV. It also lists the remaining funds out of the available $200,000 (Column 4) and their compounded future values (Column 5). Column 6 shows the sum of NPV and the accumulated future value, and finally

Column 7 shows the adjusted NPV per dollar of funded capital. This is the column that shows which combination of projects is the best based on the highest adjusted NPV per dollar. The combination containing projects (1, 4, and 5) shows the highest yield value (79%).

Table 13.9

Project	Initial Capital ($)	NPV ($)
1	100,000	60,000
2	75,000	40,000
3	50,000	35,000
4	45,000	30,000
5	30,000	15,000
	300,000	180,000

Investment Projects and Business Risk

Every business firm understands that approving any project might naturally carry a certain level of risk with it. This simple fact requires that the approving firm must consider such a possibility and be prepared to deal with it. Small firms usually face a greatest possibility of risk when compared with larger and more established firms. Many structural and functional reasons are behind the higher risk potentials for small businesses such as the naturally smaller size of their equity, and being less financially diversified, as well as having a lower credit status and less market power. The following are the small firms' biggest risks:

Having an inadequate cash flow that may render them unable to pay for their obligations.
Being in an insolvency state.
Facing lower rate of return on equity and losing the market.

As far as investment projects are concerned, the basic risk is always there in terms of the possibility of an approved project being unable to deliver on its promises. **Risk management** is concerned with gearing the firm's managerial and financial resources towards anticipating and dealing with the potential risk. Risk can be avoided, transferred, reduced, and absorbed. Each and every one of these treatments to risk can be applied to optimize the solution to any specific kind of risk in certain circumstances. Before we go over a variety of risk management methods, let us define risk and explain where it comes from.

Table 13.10

1	2	3	4	5	6	7
Project Combination	Combined Capital	Combined NPV	Remaining Fund (RM) 200 – (2)	Value of Invested Remaining Funds RM $(1 + r)^n$	Final NPV (3 + 5)	NPV Per $ Invested (6 ÷ 2)
1, 2	175	100	25	33.45	133	0.75
1, 3, 5	195	125	5	6.70	132	0.68
1, 3, 5	180	110	20	26.76	137	0.76
1, 4, 5	175	105	25	33.45	138	0.79
2, 3, 4, 5	200	120	0	0	120	0.60

13.6 Risk and its Sources

Risk refers to the condition in which there are multiple possible outcomes and the probability of each alternative outcome is either *known or can be estimated*. **Uncertainty** shares the first element with risk – the existence of multiple possible outcomes – but differs in the second element, so that the probability of each outcome is either *unknown or cannot be estimated*.

Managerial decisions often follow a certain scheme when it comes to facing conditions of risk or uncertainty. The scheme includes the following steps.

- Predicting a set of possible future conditions that would affect any possible managerial decision. These are often called the **states of nature**.
- Estimating the **expected payoff** for each possible decision and each possible state of nature.

Organizing these elements for each managerial decision would form a **payoff matrix**.

Suppose that a manager has to choose among three possible decisions: opening a new branch for the company, expanding and updating the existing branch, or just keeping the current branch with no change. Also suppose that the company sees the possibility of the economy descending into a recession during the next period. If the company can estimate the alternative outcomes in terms of the amount of total profits that can be earned in each of the possibilities, a payoff matrix can be constructed like **Table 13.11**.

Table 13.11

Decision/State of Nature	Recession	No Recession
Opening a new branch	$290,000	$350,000
Expanding the current branch	$480,000	$400,000
Keeping the current branch as it is	$300,000	$320,000

Comparing the option of no change with that of building a new branch reveals that doing nothing would do slightly better in a recession and slightly worse in no recession. However, the picture is significantly different when considering an expansion and renovation project. It would do much better than keeping the current state of the company in both economic conditions. However, comparing the expansion and renovation to establishing a new branch seems much better in recession and slightly better in no recession. This concludes and confirms that going with the expansion and renovation project is the winning decision no matter where the economy is heading.

Where Would Risk Come From?

Many possible sources can introduce certain potentials of risk to the managerial decision-making process. Most of these sources are external to the firm. We can group the most common sources into four categories.

Economic Sources

This category includes a variety of sources that are related to the economic environment of a country. The fluctuations in the financial market pose a credible risk to the value of assets in the current and future periods. Such a risk is known as "**market risk**." Major economic factors such as inflation and interest rate pose yet other significant impacts on prices and the value of lending and borrowing and their impact on earnings. Changes in the credit obligation and in the state of liquidity can also introduce credit risk and liquidity risk, in addition to the currency risk that can stem from changes in the exchange rate between the domestic and foreign currencies. In addition, the state of competition in the same industry or region poses another type of economic risk.

Political Sources

This type of risk is related to the government policies, domestically and internationally. Some policies may introduce certain risks to an industry, in particular, or to the economy as a whole. One specific risk is the change in tax policies. Another possible risk is the "**expropriation risk**," where a government abroad may seize the property, restrict the rights, or remove the privileges of the hosted firm. Terrorism nowadays constitutes a significant political risk on business activities of all firms, domestically and globally.

Social Sources

This category includes the cultural, religious, and environmental sources such as certain social norms or trends that affect consumer preferences and demand. For example, certain food or clothing items or even weather-related products may not have any chance to be marketed in certain areas or countries, which can be a big reason to determine the orientation of businesses dealing with such products.

Consumer taste and preferences are usually subject to change, domestically and globally, and any business that cannot respond and keep up with those changes would face the risk of losing its market share.

Internal Sources

A variety of risks can arise from within a certain firm or from within the industry for commercially competitive reasons, or for power conflicts and political struggles among the leadership personnel. They can also be related to the firm's inability to cope with the rising cost of production, declining demand of its products, or its failure to keep up with the technological advances in its own industry.

13.7 Methods of Risk Management

Here, we discuss three common methods of risk management. They are the risk-adjusted rate, the risk-adjusted returns (RARs), and the statistical adjustment method.

Risk-Adjusted Discount Rate

This method is based on adjusting the discount rate used in calculating the net present values of the proposed projects. The adjustment here means raising up the rate for the projects that would somehow be considered risky. Riskiness of the projects would be measured, in one way, by the **coefficient of variation of the expected returns (*V*)**. The coefficient of variation would be calculated as

$$V = \frac{\sigma_k}{\bar{k}}$$

where σ_k is the standard deviation of returns, and $\bar{k}$ is the expected rate of return. This coefficient is a measure of relative dispersion of the expected returns, where the higher the coefficient of variation, the riskier is the project. This means that the use of the risk-adjusted discount rate (RADR) method requires the firm to have estimated the level of potential risk involved in each proposed project by calculating the coefficient of variation in the project's expected returns. Applying this method would be raising the discount rate to calculate the net present value for each project by a certain percentage above the risk-free rate used by the firm. This percentage is called the **risk premium**, which would be different from one project to another depending on how risky the project is perceived. A higher premium is assigned to the riskier project so that the total discount rate becomes larger which would lower the calculated net present value. This is a way to see if the risky projects can still produce a positive net present value under the use of higher discount rate. **Figure 13.11** shows the possibilities of the risk premium that a firm can choose above the risk-free rate on the vertical axis. Assuming that the risk-free rate of return is 5%, meaning that this exceptional rate would be considered only for projects that have zero risk, as risk is measured on the horizontal axis by the coefficient of variation (*V*). As the positively sloped market risk–return curve shows, the higher the level of perceived risk, the higher the given risk premium (R_p) and therefore the higher the adjusted rate of return (k).

$$k = r + R_p$$

where r is the risk-free rate and R_p is the assigned risk premium that can be defined as the added percentage by which the required discount rate for a risky project exceeds the risk-free rate. For example, there are two projects on the market risk-return curve, A and B. Since A has a coefficient of variation equal to 0.7, its assigned risk premium is 5% above the risk-free rate of 5%, which makes the required discount rate for this project equal to 10%.

$$\begin{aligned} k_A &= r + R_p^A \\ &= 0.05 + 0.05 = 0.10 \end{aligned}$$

Project B is riskier because its coefficient of variation is 1.0. Therefore, the risk premium assigned to it has to be greater than that premium assigned to project A. It is shown that its risk premium is 8%, which would make the required discount rate 13%.

$$\begin{aligned} k_B &= r + R_p^B \\ &= 0.05 + 0.08 = 0.13 \end{aligned}$$

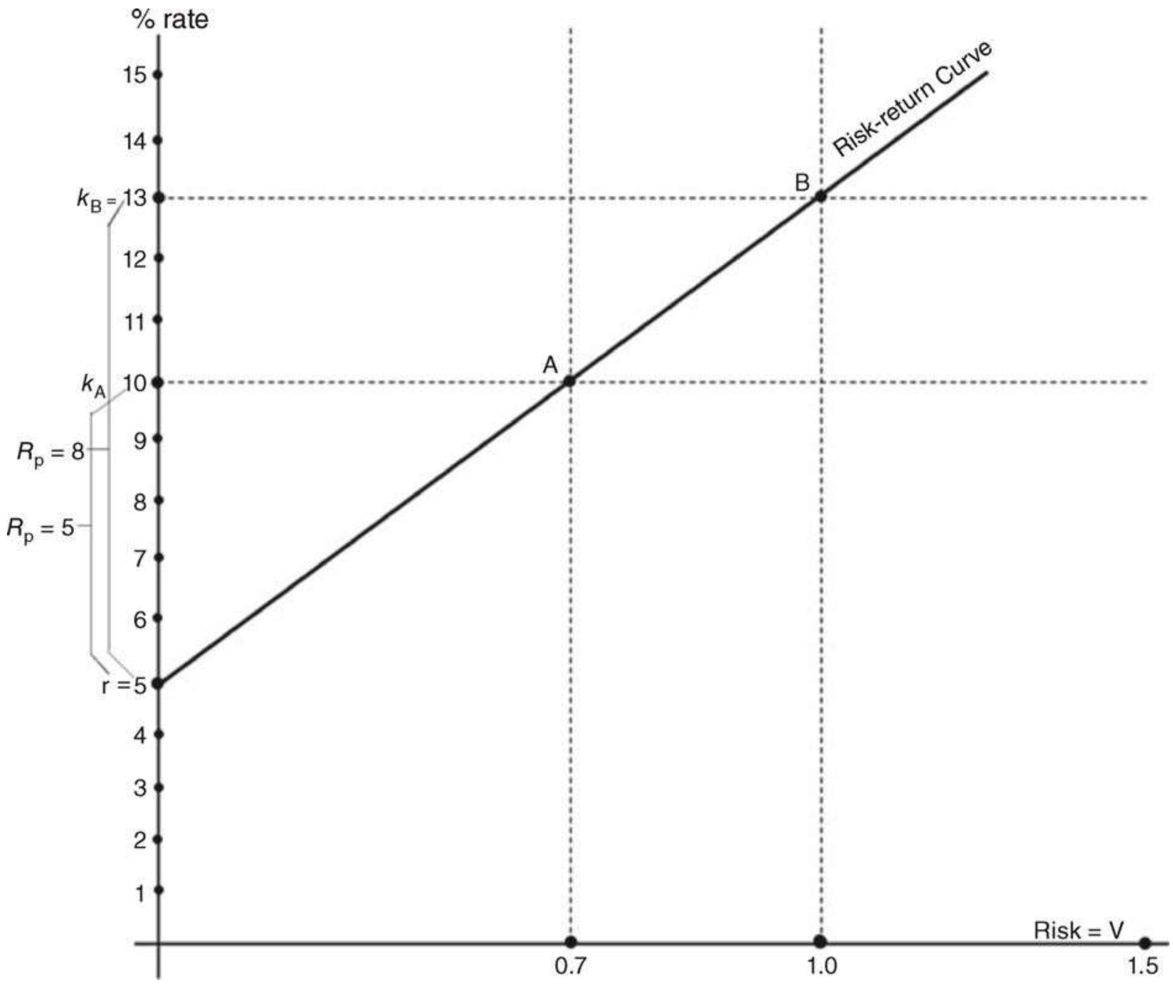

Figure 13.11 Risk Premium and the Risk-Free Rate of Return

Example

Table 13.12 shows two proposed investment projects, each of which will yield profits for the next 5 years. They require initial investments of $420,000 and $500,000, respectively. Since further investigation revealed that certain risk elements are associated with both projects, the management team found it necessary to adjust for risk by assigning risk premiums of 2.5% and 3.5% to both projects, respectively. Which project would win the funding approval if the risk-free rate is 6%?

Table 13.12

Time	Project A Cash Inflows	Project B Cash Inflows
Year 1	126,000	280,000
2	126,000	108,000
3	112,000	90,000
4	98,000	80,000
5	84,000	70,000
Initial Investment	420,000	500,000
R	6%	6%
R_p	2.5%	3.5%

First, we calculate the NPV of the cash inflows for both projects for the time of their yields using the firm's risk-free rate (r = 6%). Then, we calculate the same NPVs using the risk-adjusted rate (k) for both projects.

$$k = r + R_p$$
$$k_A = 0.06 + 0.025 = 0.085 \text{ for project A}$$
$$k_B = 0.06 + 0.035 = 0.095 \text{ for project B}$$

$$\begin{aligned}
NPV_A &= \sum_{t=1}^{5} \frac{\pi_i}{(1+r)^t} - C_0 \\
&= \frac{\pi_1}{(1+r)^1} + \frac{\pi_2}{(1+r)^2} + \frac{\pi_3}{(1+r)^3} + \frac{\pi_4}{(1+r)^4} + \frac{\pi_5}{(1+r)^5} - C_0 \\
&= \frac{126{,}000}{(1+0.06)^1} + \frac{126{,}000}{(1+0.06)^2} + \frac{112{,}000}{(1+0.06)^3} + \frac{98{,}000}{(1+0.06)^4} \\
&\quad + \frac{84{,}000}{(1+0.06)^5} - (420{,}000) \\
&= 118{,}868 + 112{,}140 + 94{,}037 + 77{,}625 + 62{,}767 - 420{,}000 \\
&= 465{,}440 - 420{,}000 \\
&= 45{,}440
\end{aligned}$$

$$\begin{aligned}
NPV_B &= \frac{280{,}000}{(1+0.06)^1} + \frac{108{,}000}{(1+0.06)^2} + \frac{90{,}000}{(1+0.06)^3} + \frac{80{,}000}{(1+0.06)^4} \\
&\quad + \frac{70{,}000}{(1+0.06)^5} - (500{,}000) \\
&= 264{,}151 + 96{,}120 + 75{,}565 + 63{,}367 + 52{,}308 - 500{,}000 \\
&= 551{,}511 - 500{,}000 \\
&= 51{,}511
\end{aligned}$$

$$\begin{aligned}
NPV^{adj} &= \sum_{t=1}^{5} \frac{\pi_i}{(1+k)^t} - C_0 \\
NPV_A^{adj} &= \frac{126{,}000}{(1+0.085)^1} + \frac{126{,}000}{(1+0.085)^2} + \frac{112{,}000}{(1+0.085)^3} + \frac{98{,}000}{(1+0.085)^4} \\
&\quad + \frac{84{,}000}{(1+0.085)^5} - (420{,}000) \\
&= 116{,}129 + 107{,}031 + 87{,}685 + 70{,}714 + 55{,}864 - 420{,}000 \\
&= 17{,}423
\end{aligned}$$

$$\begin{aligned}
NPV_B^{adj} &= \frac{280{,}000}{(1+0.095)^1} + \frac{108{,}000}{(1+0.095)^2} + \frac{90{,}000}{(1+0.095)^3} + \frac{80{,}000}{(1+0.095)^4} \\
&\quad + \frac{70{,}000}{(1+0.095)^5} - (500{,}000) \\
&= 255{,}707 + 90{,}073 + 68{,}549 + 55{,}646 + 44{,}466 - (500{,}000) \\
&= 514{,}440 - 500{,}000 \\
&= 14{,}440
\end{aligned}$$

At the risk-free rate of 6%, project B would win the approval of the managerial team since its NPV ($51,511) is larger than that of project A ($45,440). However, after considering the expected risk involved in both projects, the team would give its approval to project A because of its larger risk-adjusted NPV ($17,423) than that of project B ($14,440).

Risk-Adjusted Returns

Instead of adjusting the discount rate for potential levels of risk as we have seen in the RADR method, the adjustment would be here on the stream of cash flow or returns as they are treated by certain levels of certainty. The major instrument for this adjustment is the **certainty-equivalent coefficient** (α). In a game context, certainty-equivalent is defined as the compensation that renders a player indifferent to a risky gamble, or it is the sure sum which is equal to the expected value of a risky outcome. In the business risk context, certainty-equivalent means the portion of estimated return which project managers would prefer to obtain "as certain" over any other return that is "just probable." It indicates a certain preference of certainty over uncertainty, and in this sense it is related to the concepts of risk aversion and risk taking.

Let us assume that a proposed project calls for an initial investment of $30,000, and let us further assume that an evaluation team estimates the probability of its success and failure at 50/50 between earning $100,000 and zero, respectively. We can calculate the project's expected value as

$$E(v) = (0.50)100{,}000 + (0.50)0 = 50{,}000$$

If this project wins an approval for funding, it would mean that the approving company is knowingly and willingly prefer to trade off the certainty of spending $30,000 for a risky expected return of $50,000. In other words, the decision maker would be here indifferent between the two prospects.

The certainty-equivalent coefficient (α) can be obtained by dividing the **certainty-equivalent (CE) amount** by the expected risky return $E(v)$.

$$\alpha = \frac{\text{CE}}{E(v)}$$

For our example, the certainty-equivalent coefficient would be

$$\alpha = \frac{30{,}000}{50{,}000} = 0.6$$

This coefficient would refer to the decision-maker's attitude toward risk between risk aversion and risk taking. To illustrate this concept, let us assume that the requested fund for the project is $40,000 instead of $30,000. It would be plausible then to think that in comparison to $\alpha = 0.6$, an alpha of 0.8 (40,000/50,000) would refer to a higher value of the spent dollar when compared with the expectedly earned dollar. Decision makers who consider this high value may not agree to fund the project and may be identified as risk averters. Otherwise, having a lower alpha such as 0.4 (whether due to requesting less funding (20,000/50,000 or due to having a higher expected return (30,000/75,000)) would make it easier to take the risk in the project. 293

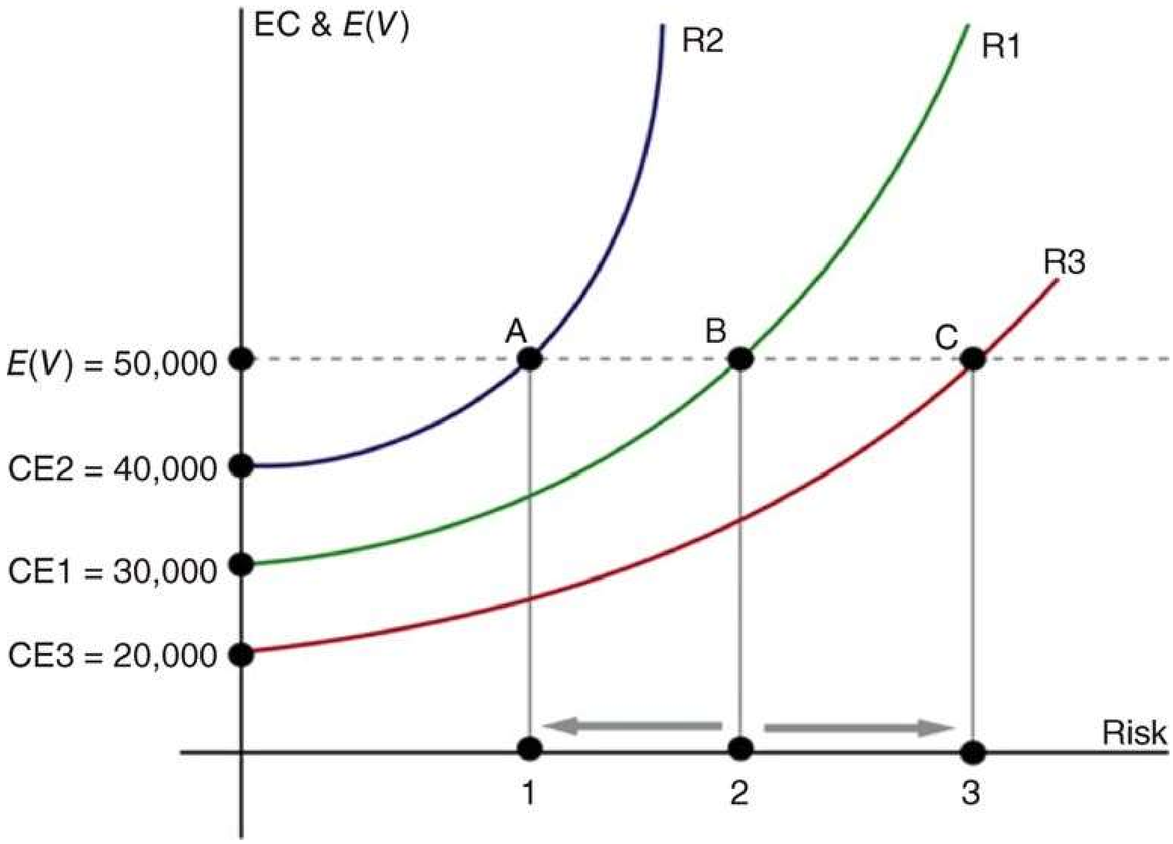

Figure 13.12 The Certainty Equivalence and Attitudes Towards Risk

Figure 13.12 shows the above three levels of the certainty-equivalent coefficients as they are associated with three attitudes toward risk.

The risk-averse decision maker on R_2 and alpha of 0.8 would try to protect his spent money (investment amount) more because an alpha of 0.8 means that for every dollar of the expected return, he would spend 80¢. On the other hand, a risk taker on R_3 with an alpha of 0.4 would view his investment as worth only 40¢ per dollar of earnings. This is the reason why he dares to take a higher risk. In other words, the risk-taker would value the return on investment more than the risk-averter. That is $2.50 in return per dollar of investment for the risk taker (50,000/20,000) versus $1.25 in return per dollar of investment for the risk averter (50,000/40,000).

Generally speaking, we can state the rules for the certainty-equivalent coefficient (α) as

- When α = 0, it is an indication that the probability of getting the expected return does not exist, and therefore, the project is too risky to be pursued.

- When $\alpha = 1$, it refers to the equality between the certainty equivalent and the expected value of return of the risky project. When the manager or investor gets his return equal to what he assigns as a certainty equivalent, the project is considered risk free.
- When $0 < \alpha < 1$, it is an indication that there is some level of risk. The project is riskier as α is closer to zero, and less risky if it is closer to 1. It would depend on how smaller the certainty equivalent is compared to the expected value of the risky return ($E(v)$).

Table 13.13

Time	A			B		
Year	Expected Return	Certainty Equivalent	α	Expected Return	Certainty Equivalent	α
1	126,000	123,000	0.98	280,000	240,000	0.86
2	126,000	123,000	0.98	108,000	100,000	0.93
3	112,000	106,000	0.95	90,000	86,000	0.95
4	98,000	95,000	0.97	80,000	76,000	0.95
5	84,000	82,000	0.98	70,000	68,000	0.97
Initial Investment		420,000			500,000	
r		6%			6%	

Technically, the adjustment for risk would occur by introducing α to the numerator of the net present value formula as a multiplier to the expected return, or profit, or cash flow, while the denominator of the formula would keep the risk-free rate (*r*).

$$\text{NPV} = \sum_{t=1}^{n} \frac{\alpha \pi_i}{(1+r)^t} - C_0$$

Example

Looking at the table above, let us assume that a manager assigned certainty-equivalent sums to each and every annual return of 5 years in both projects, A and B. As shown in Table 13.13, alpha values are calculated by dividing the assigned certainty equivalent by the corresponding expected return.

Applying those calculated alphas, we get

$$
\begin{aligned}
\text{NPV}_\text{A} &= \sum_{t=1}^{n} \frac{\alpha \pi_i}{(1-r)^t} - C_0 \\
&= \frac{(0.98)(126{,}000)}{1+0.06^1} + \frac{(0.98)(126{,}000)}{1+0.06^2} + \frac{(0.95)(112{,}000)}{1+0.06^3} + \frac{(0.97)(98{,}000)}{1+0.06^4} \\
&\quad + \frac{(0.98)(84{,}000)}{1+0.06^5} - 420{,}000 \\
&= (116{,}490 + 109{,}897 + 89{,}335 + 75{,}296 + 61{,}514) - 420{,}000 \\
&= 452{,}532 - 420{,}000 \\
&= 32{,}532
\end{aligned}
$$

$$
\begin{aligned}
\text{NPV}_\text{B} &= \frac{(0.86)(280{,}000)}{(1+0.06)^1} + \frac{(0.93)(108{,}000)}{(1+0.06)^2} + \frac{(0.95)(90{,}000)}{(1+0.06)^3} + \frac{(0.95)(80{,}000)}{(1+0.06)^4} \\
&\quad + \frac{(0.97)(70{,}000)}{(1+0.06)^5} - 500{,}000 \\
&= (227{,}170 + 98{,}391 + 71{,}787 + 60{,}199 + 50{,}739) - 500{,}000 \\
&= 499{,}286 - 500{,}000 \\
&= -714
\end{aligned}
$$

Considering the expected risk for both projects in terms of estimating the certainty equivalent and calculated α for each return reveals that project A is more worthwhile for yielding a positive value of $32,532 when compared with project B for its negative net value of –714.

The Statistical Adjustment

Business risk can be accounted for by statistical tools such as standard deviation and coefficient of variation which can be instrumental in measuring the variability of returns to investment in relation to a certain expected return. The **standard deviation (**σ_i**)** measures the dispersion around the expected value of a project's stream of cash inflows, while the **coefficient of variation (*V*)** measures the relative dispersion of the cash inflows, as it is calculated by dividing the standard deviation by the expected value of returns. Knowing these instruments accurately requires a lot of detailed information, especially on the probability of each potential return. Since most business firms, and particularly small firms, cannot afford collecting and processing such elaborate data, it became plausible to assume some subjective probabilities that can be reasonable representative of a normal distribution of three possibilities.

- The optimistic estimation, which is how probable is to get the best returns.
- The pessimistic estimation, which is how probable is to get the worse returns.
- The moderate estimation, which is how probable is to get the ordinary returns that would be in-between and the most likely occurrence.

The original idea for this kind of subjective probabilities follows the three possibilities of the state of economy, the boom, recession, and the normal times.

Tables 13.14, **13.15**, and **13.16** present the cash inflow or returns for a proposed project. Each annual return for the first three years of the project production is listed in three estimates corresponding to three probabilities 0.25, 0.50, and 0.25 as they stand for the good times, normal, and bad times, respectively. The expected value of each return is calculated as

$$E(R_i) = R_{i1}\,P_{r1} + R_{i2}P_{r2} + R_{i3}P_{r3}$$

Table 13.14 First-Year Return in Three Possible States

State of Estimation	Probability of Estimation Pr_i	Return 1 R_1	$R_1 - ER_1$	$(R_1 - ER_1)^2$	$(R_1 - ER_1)^2\ Pr_i$
Optimistic	0.25	53,000	4,000	16,000,000	4,000,000
Normal	0.50	49,000	0	0	0
Pessimistic	0.25	45,000	−4000	16,000,000	4,000,000
	Expected value of return $ER_1 = 49,000$				$\sum(R_1 - ER_1)^2\ Pr = \sigma_1^2 = 8,000,000$

$$\sigma_1 = \sqrt{\sigma_1^2} = \sqrt{8,000,000} = 2,828 \quad V_1 = \frac{\sigma_1}{ER_l} = \frac{2,828}{49,000} = 0.057$$

Table 13.15 Second-Year Return

State of Estimation	Pr_i	R_2	$R_2 - ER_2$	$(R_2 - ER_2)^2$	$(R_2 - ER_2)^2\ Pr_i$
Optimistic	0.25	55,000	4500	20,250,000	5,062,500
Normal	0.50	50,000	−500	250,000	125,000
Pessimistic	0.25	47,000	−3,500	12,250,000	3,062,500
Expected value of return $R_2 = 50{,}500$					$\sigma_2^2 = 8,250,000$
		$\sigma_2 = 2{,}872$	$V_2 = .057$		

As an example, the expected value of the first year returns is

$$E(R_1) = 53{,}000(0.25) + 49{,}000(0.50) + 45{,}000(0.25) = 49{,}000$$

The standard deviation (σ_i) for the possibilities of each return is calculated by

$$\sigma_i = \sqrt{\sum_{i=1}^{3} \left[R_i - E(R_i)\right]^2 \mathrm{Pr}_i}$$

and the coefficient of variation (V_i) is calculated by

$$V_i = \frac{\sigma_i}{E(R_i)}$$

The larger the standard deviation, the riskier the project. This is because a standard deviation means that the projected returns would be more dispersed around the expected value of the return. By the same token, the smaller the standard deviation, the less risky the project because smaller reading reflects that the projected returns are staying much closer to the expected value. Looking at the calculated standard deviation, we can conclude that the projected returns of the first year are more reliable or less risky (σ = 2,828) than the returns of the other two years where the standard deviations are 2872 and 4242, respectively. The coefficient of variation measures the risk relative to the expected value and it would be particularly useful when the projects differ in their sizes. Since the higher the coefficient of variation, the higher the risk, the third year projected returns with a V_3 = 0.082 would be the riskier when compared with the first and second years, which came with equal Vs of 0.057.

The standard deviation and coefficient of variation can also be applied to the projected net present values of competing projects to see which project is riskier.

Tables 13.17 and 13.18 present a comparison of two projects, I and II, as it lists three estimates of their net present values, following the same subjective probabilities that we have seen in the last example. The results of the standard deviation shows that project I with a higher standard deviation (16,135) is riskier than project II with much lower standard deviation (4033). This conclusion is also confirmed by the results of the coefficients of variation. Project I with a higher coefficient of variation (2.06) is more risky than project II with a much lower coefficient of variation (0.516).

Table 13.16 Third-Year Return

State of Estimation	Pr_i	R_3	$R_3 - ER_3$	$(R_3 - ER_3)^2$	$(R_3 - ER_3)^2\ Pr_i$
Optimistic	0.25	58,000	6000	36,000,000	9,000,000
Normal	0.50	52,000	0	0	0
Pessimistic	0.25	46,000	−6000	36,000,000	9,000,000
Expected value of return $ER_3 = 52,000$					$\sigma_3^2 = 18,000,000$
		$\sigma_3 = 4,242$	$V_3 = .082$		

Table 13.17

State of Estimation	Probability of Estimation Pr_i	Project I			
		NPV_1	$NPV_1 - E(NPV_1)$	$[NPV_1 - E(NPV_1)]^2$	$[NPV_1 - E(NPV_1)]^2 Pr_i$
Optimistic	0.25	30,636	22,828	520,661,124	130,165,281
Normal	0.50	7818	0	0	0
Pessimistic	0.25	−15,000	−22,818	520,661,124	130,165,281
		$E(NPV_1) = 7818$			$\sigma_1^2 = \sum[NPV_1 - E(NPV_1)]^2 Pr_i$
					$\sigma_1^2 = 260,330,562$

$$\sigma_1 = \sqrt{\sigma_1^2} = \sqrt{\sum[NPV_1 - E(NPV_1)]^2\ Pr_i} = \sqrt{260,330,562} = 16,135$$

$$V_1 = \frac{\sigma_1}{E(NPV_1)} = \frac{16,135}{7818} = 2.06$$

13.8 Sensitivity Analysis, Scenario Analysis, and Simulation

Sensitivity analysis is a technique to get a general sense of risk in potential business projects using the variability among the outcomes such as the project's future returns and estimated net present values. One of the most simple examples in the application of sensitivity analysis is to use the simple range of estimates to make a general comparison. Projects yielding a wide range in their projected returns or net present values would be considered riskier than those projects exhibiting a narrow range. This idea is close to the idea of using the statistical measures such as the standard deviation and coefficient of variation in that the tighter the estimates among themselves the more reliable and safe they would be in comparison to the estimates which are spread out. Table 13.19 compares project X to project Y by obtaining the ranges of both the return estimates and the net present values. The range value is obtained by subtracting the worst estimate of the pessimistic probability from the best estimate of the optimistic probability. The results are clearly pointing out to Project X as riskier than project Y since both ranges are much larger in Project X than in project Y. That is 90,000 in returns and 28,902 in net present value, as compared to 30,000 in returns and 7014 in net present value for project Y.

Table 13.18

		Project II			
State of Estimation	Probability of Estimation Pr_i	NPV_2	NPV_2- $E(NPV_2)$	$[NPV_2$- $E(NPV_2)]^2$	$[NPV_2$- $E(NPV_2)]^2 Pr_i$
Optimistic	0.25	13,522	5704	32,535,616	8,133,904
Normal	0.50	7818	0	0	0
Pessimistic	0.25	2113	−5705	32,535,616	8,133,904
	$E(NPV_2) = 7{,}818$				$\sigma_2^2 = \sum[NPV_2 - E(NPV_2)]^2 Pr_i$ $\sigma_2^2 = 16{,}267{,}808$

$$\sigma_2 = \sqrt{\sigma_2^2} = \sqrt{[NPV_3 - E(NPV_3)]^2\, Pr_i} = \sqrt{16{,}267{,}808} = 4033$$

$$V_2 = \frac{\sigma_2}{E(NPV_2)} = \frac{4033}{7818} = 0.516$$

Table 13.19

State of Estimation	Project X	Project Y
	Returns	
Optimistic	100,000	65,000
Normal	45,000	45,000
Pessimistic	10,000	35,000
Range	90,000	30,000
	Net Present Value	
Optimistic	19,402	8564
Normal	4950	5000
Pessimistic	−9500	1550
Range	28,902	7014

Sensitivity analysis can also focus on randomizing the value of one key variable in a financial model in order to test the impact of such a single change on the rest of the related variables in the model. In this sense, it would be able to establish how sensitive the model is in its response to that random change. For example, a sensitive analysis can be performed on the next present value model through changing the unit price of the product frequently and tracking down the impact of such changes on the returns that would be an essential variable in the NPV calculation. Another similar technique is called **Scenario Analysis**, which differs from the sensitivity analysis only by extending the randomization of change to more than one variable and tracking down the multiple impacts simultaneously. Scenario analysis is considered as a short version of financial simulation. Both sensitivity and scenario analyses are simpler and more practical and affordable than the more comprehensive simulation.

Simulation is a thorough mathematical technique that utilizes predetermined probability distributions and random values to assess the state of probable future events where projects' rates of return and risk indices can be estimated.

Similar to the iconic models in the physical world of design and engineering, which are made to reduce the potential risks, explore flaws, and enhance the positive features, as well as to estimate costs, mathematical simulation models are designed to mimic the realities of the business world and deal with their changes. They present yet another technique that assists the decision maker in exploring all the possibilities surrounding the problem at hand when it comes to dealing with risky and uncertain conditions. The essential features of the real world can be translated into a multivariable model complete with estimations of the probability distributions of the key variables. The model can be tested repeatedly with random variables given to the variables in each test until a probability distribution and risk for the general model are estimated so that they can be used to calculate the expected outcome for any given variables.

Table 13.20

	Project A			Project B	
Probability Pr_i	PV_A	$PV_A\ Pr_i$	Pr_i	PV_B	$PV_B\ Pr_i$
0.25	−37,500	−9375	0.15	−100,000	−15,000
0.35	350,000	122,500	0.35	280,000	98,000
0.40	250,000	100,000	0.50	125,000	62,500
Expected Present Value A		213,125	Weighted Present Value B		145,500
Initial Capital		−190,000	Initial Capital		−120,000
Expected NPV_A		23,125	Expected NPV_B		25,500

Decision Tree

The decision tree technique allows the decision maker to map out the decision inputs and their probabilities of occurrence in a branched-out fashion that resembles the tree branches. In other words, **decision tree** is a graphical representation of the possible paths of alternative decisions and subsequent decisions according to their potential probabilities and values and under possible conditions. Table 13.20 presents two competing projects with three probable estimates for the present value of their returns. The decision to choose the best project is made by calculating the expected net present value for each, which would require subtracting the initial capital investment for each from the expected present value of their cash inflows. The information on Table 13.20 is mapped out in a decision tree as it is in Figure 13.13, which shows an ordered layout of both alternatives from the start on the left-hand side to the end of the right-hand side. The end is narrowed down to a comparison of the two calculated figures representing the expected net present value of the two projects. The larger figure of project B (25,500) would be the final choice over project A (23,125).

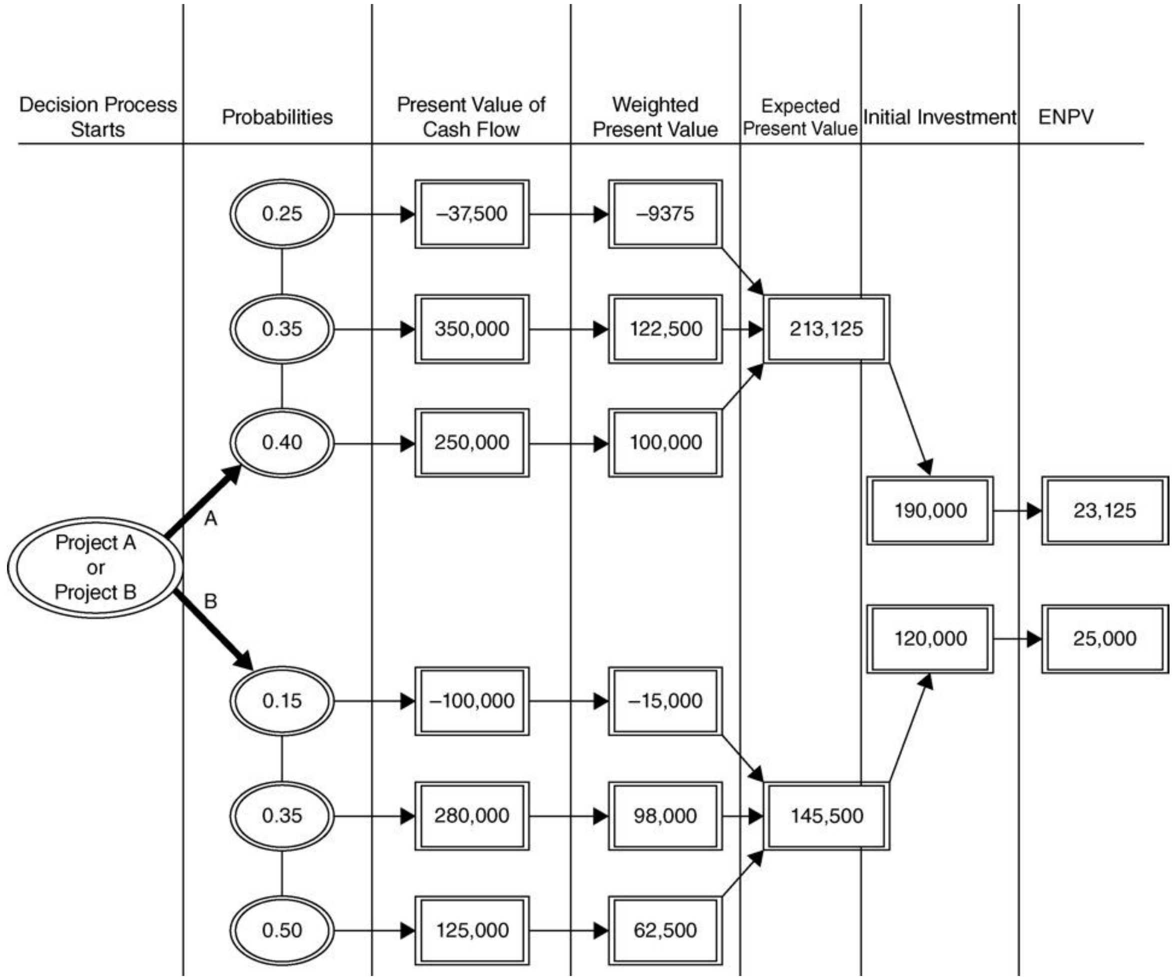

Figure 13.13 The Decision Tree

13.9 Summary

In this chapter, we discussed the investment project evaluation and risk management. We started off by defining the project evaluation and its connection to capital budgeting and explained the purpose of the investment projects as they usually come up for any business firm. Those purposes were categorized into six categories: the replacement, renewal, expansion, cost-reduction, conforming, and all other projects. We then described the formal process of project selection in nine systematic steps and classified the projects into independent and mutually exclusive ones. The next topic was to explain the cash flow criteria of assessment for both, the conventional and non-conventional patterns of order in cash returns. Project evaluation techniques came next with the most common methods in two groups: In the group that does not consider the time value of money, two methods were discussed, the Average Rate of Return (ARR) and the Payback period. In the group that takes into account the time value of money, three methods were discussed: The net present value (NPV), the Internal Rate of Return (IRR), and the Profitability Index (PI). A comparison was made between the net present value and Internal Rate of Return for the mutually exclusive projects, and other related topics were discussed. Among these topics were the net present value profile, the crossover rate, reversal of ranking, and capital rationing. The second part of the chapter was dedicated to business risk. What is risk and what are its sources was the first topic, followed by methods of risk management. These methods included the RADR, RAR, and the statistical method that uses the standard deviation and the coefficient of variation. Also discussed in this chapter were the sensitivity analysis, scenario analysis, and simulation, while the last subject was a brief description of the decision tree with an illustrative diagram.

Key Concepts

Project evaluation   Capital budgeting   Replacement projects
Renewal projects   Expansive project   Cost-reduction projects
Conforming projects   Independent projects   Mutually exclusive projects
Conventional pattern   Non-conventional pattern
Value-adjusted techniques   Sophisticated methods
Average Rate of Return (ARR)   AEADT   Payback
Net Present Value (NPV)   Internal Rate of Return (IRR)
Marginal Efficiency of Investment   Crossover rate
Reversal of ranking   Crossover point   Profitability index
Capital rationing   Risk   Uncertainty   States of nature
Payoff matrix   Risk-Adjusted Discount Rate (RADR)
Risk-Adjusted Returns (RARs)   Statistical method
Coefficient of variation   Sensitivity analysis   Scenario analysis
Simulation   Decision tree

Discussion Questions

1. What is project evaluation? And how important is it for the firm?
2. What are the most common purposes for the firm's investment funds?
3. Is there a systematic process for the project evaluation and selection? And what are the steps to be taken into this process towards the final project selection?
4. What is the major difference between the independent project and the mutually exclusive project? And would the evaluation process be different for each?
5. List the methods of project evaluation that are considered value-adjusted or sophisticated, and explain their major difference from those methods that are not value-adjusted.
6. Describe the payback technique and give a simple example to illustrate how this technique works.
7. Briefly describe the Net Present Value, and the Internal Rate of Return methods to evaluate investment project, and explain the major difference between them.

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