Chapter 11
Assessing and Teaching Mathematics*

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Learning Outcomes

11.1 Identify factors that influence math success.

11.2 Describe progress monitoring and assessment practices in mathematics.

11.3 Explain prenumber skills.

11.4 Describe the numeration concepts students need to progress in arithmetic.

11.5 List factors contributing to difficulties with problem solving and describe how teachers can assist students in learning problem-solving strategies.

11.6 Articulate how math interventions can be used to improve students' performance.

Sanjay, a third-grade student who has behavior disorders, spends most of his day in the general education third-grade classroom. He is in the top math group and is proud of this achievement. His special education teacher is pleased that he is fulfilling his behavior contract and has not had any serious disturbances since he has been included in the general education classroom.

Claudia, a seventh-grade student who spends part of her day in a classroom for students with learning disabilities, is not as successful in math. In fact, when asked what her favorite academic time is during the day, she says, "I love to write. In fact, I think I will be an author. I have already written several books for the classroom, and one was even selected for the library." When asked what she thinks of math, she looks away and says, "No way! Don't even mention it. I can't do math. We don't get along."

Claudia has had difficulty with mathematics since she was in the primary grades. Her first-grade teacher thought that she simply was not interested in math and suggested that her parents obtain special tutoring help during the following summer. Her parents found that the special help did little good, and when Claudia continued to have serious difficulty with math in second grade, the teacher referred her for assessment for possible learning disabilities. The assessment results suggested that she had difficulty with spatial relations and using memory to recall rote math facts. She has received special help in math for the past 4 years, and though she seems to have made progress, her math skills are still her weakest academic area.

Some students with learning and behavior problems have difficulty with language arts (reading, writing, and spelling), some have difficulty with mathematics, and some have difficulty with both. Students with math difficulty tend to have slower growth in math skills and fail to catch up to their typically developing peers without math difficulty (Nelson & Powell, 2018). Compared with students who have only math problems, students who have both math and reading problems are far more likely to continue to have math problems in later grades (Jordan & Hanich, 2003), and these students have lower academic functioning and grades than students with reading or math disabilities in isolation (Willcutt et al., 2013). With an increased need for students to understand problem solving for success in the workplace, the need for all students to demonstrate proficient math skills is high.

Factors Influencing Math Success

Math success can be influenced by several factors, including teacher preparation and knowledge of math instruction, school curriculum and materials, and students' characteristics that influence their understanding and retention of math. Let's consider each of these and what you might do to address them in your educational setting.

One consideration for effective math instruction is teachers who are knowledgeable about math concepts and recognize the prerequisite skills for constructs they are teaching as well as how the constructs they are teaching relate to subsequent knowledge and skills. For example, if students are having difficulty with division, it is often because they lack the prerequisite skills related to multiplication or because they have difficulty understanding the construct of dividing into groups. Teachers who have the mathematical facility to identify key components that students need to know to successfully complete mathematics problems are more effective. Thus, teachers' knowledge and understanding of mathematic influence their instruction.

School curriculum and materials also influence students' performance in mathematics. Schools that have a well-conceptualized math curriculum with the necessary materials and resources for teachers are more likely to develop successful math instruction.

The third factor, and perhaps the one that teachers can most influence, is the learning characteristics of students and how instruction can be modified to support their acquisition of mathematics knowledge and skills. What are some of the characteristics of students that influence math acquisition? These vary somewhat for students who have significant reading disabilities, but typically these students display the following deficits (Geary, 2010; Vukovic & Siegel, 2010):

1. *Cognitive factors,* such as distractibility and cognitive learning strategies.
2. *Education factors,* such as the quality and amount of instructional intervention across the range of areas of mathematics (e.g., computation, measurement, time, and problem solving).
3. *Self-regulations factors,* such as persistence, attitudes toward mathematics, and math anxiety.
4. *Neuropsychological patterns,* such as perception and visual spatial skills.

Considering these factors, it is not surprising that many students with learning and behavior problems have difficulty in math. Interestingly, although students with learning disabilities demonstrate significant difficulties with math, they do not report lower self-perceptions of their math skills than those of average-achieving students (Montague & van Garderen, 2003). Because much of their educational intervention has focused on computation, they often have limited exposure to other elements of math, including measurement, time, and practical problem solving. Many students with learning and behavior problems struggle with applying computation skills to everyday math problems. Persistence and motivation to succeed are associated with good math performance, and many students with learning and behavior problems lack these qualities. The fourth factor that was identified, unique neuropsychological patterns, characterizes many students with learning and behavior problems.

Language development also plays an important role in learning mathematics. Reduced vocabulary levels and difficulties with reasoning and conceptual abstractions interfere with learning. The language of teacher directions, curricular materials, and mathematics does not aid students with learning and behavior problems to understand and learn the concepts and skills they need to succeed in mathematics. This is especially applicable to students with lower cognitive abilities and those with difficulties in understanding English (i.e., English language learners; ELLs). Teachers can simplify the vocabulary they use within a lesson because students with lower cognitive abilities and who are ELLs may have difficulty understanding oral and written concepts (Tucker, Singleton, & Weaver, 2006). The teacher can use the more readily understood vocabulary and follow it with a less complex term that has the same meaning (e.g., "Show me the fraction that equals one half" becomes "Show me the fraction or part that equals or is the same as one half"). The teacher can also use an example that is visually easy to image (e.g., "parallel lines" or "like railroad tracks running side by side"). Teachers can make revisions to the directions provided in texts, thereby allowing better understanding. However, if a difficult word is essential, then a marginal note (a simpler word written in the margin and tied to the difficult word by a line) can be provided to increase understanding.

Moreover, students can benefit from developmental activities wherein they are given opportunities to develop mathematical experiences that will become the basis for generalizing mathematical concepts and skills. These activities help students to develop connections to what they already know and can more readily allow them "to see" how new learning is related to learning that is already understood (Tucker et al., 2006). Mental imagery is built by developmental activities allowing the student to have a better picture of what the teacher is saying (they can see that the real world is this way). As a teacher builds mental imagery, concrete thinking is facilitated. The use of developmental activities increases memory of skills learning and reduces the amount of practice required, allowing learning to occur (Tucker et al., 2006).

Barnes et al. (2006) reported that students with math disabilities are likely to have problems with both math facts and math procedures. Also, in a study by Badian and Ghublikian (1983), students with significantly lower math than reading skills (low math) were compared with two other groups: students with significantly higher math skills than reading skills and students who had similar math and reading skills. The results indicated that the low-skill math group demonstrated lower scores overall on the following abilities:

- Paying sustained attention
- Working in a careful and organized manner
- Accepting responsibility

These findings partially explain why students with learning and behavior problems have difficulty with mathematics. Students with learning disabilities often have difficulty applying learning strategies and are frequently characterized as having perceptual and neurological complications. Students with emotional disturbances may have greater difficulties with mathematics than with other subjects because it requires persistence and concentration. Students with math difficulties frequently display:

- Number deficits
- Problems with retrieving math facts
- Conceptual misunderstandings of math

Remember that students with math disabilities, besides having difficulties within the area of mathematics, often have other related problems that cause interference in learning mathematics. Students with math disabilities only are less at risk than are students with both math and reading disabilities. Yet, math disabilities, as with any disability, demonstrate wide diversity among students. This requires much attention to and development of instructional interventions to meet specific needs.

Teaching Considerations

Teachers need to consider a number of factors when developing math programs for students with special needs, regardless of the students' ages or the curricular program being used.

Comprehensive Programming Mr. Noppe was not happy with his math program. He taught students in special education at an elementary school, and 90% of his math program consisted of teaching math computation. In discussing his math program with a coteacher, he said, "I know I need to include more than just computation, but I'm not sure what else I should be teaching. I guess I should ask the students to apply some of their math computation. Next year, I want to concentrate on my math program and make it more comprehensive."

Students need to be taught and be involved in a full range of mathematics skills, including basic facts, operations, algebra, word problems, mathematical reasoning, time, measurement, fractions, and math application. Teachers should not focus their entire mathematics program on math facts and the four basic operations of addition, subtraction, multiplication, and division. The **Common Core Math Standards** provide guidelines for math instruction content and processes. These standards (listed in Apply the Concept 11.1) can be useful to teachers as they design curriculum and instruction for students with special needs. Having students work on the developmental and conceptual foundations of mathematics will aid them in understanding the big ideas underlying mathematics (e.g., the use

11.1 Apply the Concept

Common Core Math Standards

Instructional programs from prekindergarten through grade 12 in the following areas should enable all students to use the following concepts (**www.corestandards.org/math**):

1. Counting and Cardinality
2. Operations and Algebraic Thinking
3. Number and Operations in Base Ten
4. Number and Operations—Fractions
5. Measurement and Data
6. Geometry
7. Ratios and Proportional Relationship
8. The Number System
9. Expressions and Equations
10. Functions
11. Statistics and Probability

In addition to the above identified Content Areas, mathematic success is set to rest on the following processes and proficiencies:

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Model with mathematics.
- Use appropriate tools strategically.
- Attend to precision.
- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.

of like units in their computations and problem solving) that they will need in learning and practicing new skills. Building the foundational scaffold onto which learners can fuse new skills and concepts is a key element to success in mathematics learning.

Remember that a relevant feature of a comprehensive math program is to teach concepts and vocabulary—to make sure that students understand the language of mathematics. Students with disabilities are less likely to know the relevant concepts and specific vocabulary related to mathematics learning. Thus, when they participate in problem solving in real-life contexts, they may not understand key words such as *perimeter* or *diameter;* younger students may not understand words such as *minus, half,* or *percentage.* Pairing the difficult concept with lots of examples and nonexamples and the difficult vocabulary with simpler words aids in understanding.

Math Language and Vocabulary It is important for teachers to also consider the language demands of mathematics, as the vocabulary terms and concepts make understanding and communicating in math difficult for students with math difficulty (Riccomini, Smith, Hughes, & Fries, 2015). Math often requires the uses of specialized vocabulary words that have different meanings in general English than they do in mathematics (Harmon, Hedrick, & Wood, 2005). For example, the word negative is synonymous with bad in general English but has a specialized definition in math that means a real number less than 0. Thus, it is of utmost importance for teachers to understand the complexity of mathematics language and to teach vocabulary to help increase students' understanding and use of mathematical concepts (Riccomini et al., 2015). Because of the importance of language in mathematical learning, it is crucial that teachers use clear and concise mathematical language in instruction (Hughes, Powell, & Stevens, 2016). This helps ensure that students develop an accurate conceptual and procedural understanding of mathematics (Hughes et al., 2016). For example, when teaching counting, Hughes and colleagues (2016) suggest that teachers refrain from using the word "first," as this implies that no number can have a value less than the first number and to instead say "start."

Individualization Sanjay and Claudia, the two students described at the beginning of this chapter, have very different needs in math. Individualization in math programming refers not just to the task but also to the way in which the task is presented. Some students learn math facts through rote drill, whereas other students learn math facts by associating them with known facts. We often assume that an individualized program means that a student works alone; but individualization actually means that the program is designed to meet the individual needs of the student. It is often beneficial for students to work in small groups to explore and develop patterns and relationships, and hypothesize and test those hypotheses in the process of learning new skills and rehearsing and practicing problems. In addition, small groups that focus on solving the same problem can include students of different abilities, particularly when the teacher creates a cooperative environment for solving the problems and allowing the students to learn from each other.

Correction and Feedback Receiving immediate feedback about performance is particularly important in mathematics. If students are performing an operation incorrectly, they should be told which parts are correct and which parts are incorrect. Showing students patterns in their errors is an important source of feedback. Students also need to learn to check their own work and monitor their errors. Working in pairs can help in this process of checking and monitoring because students benefit from a peer's help when they may not from the teacher. Remember, feedback includes pointing out improvements as well as needed changes.

Students in Ms. Wong's math class were given a worksheet to practice their new skill of using dollar signs and decimal points in their subtraction problems. Ms. Wong told the students to do only the first problem. After they completed the problem, they were to check it and make any necessary changes. If they thought that their answer to the problem was correct, they were to write a small *c* next to their answer; if not, they were to write a small *i* for incorrect next to the answer. They were also to indicate with a check mark where they thought they had made a mistake. Ms. Wong moved quickly from student to student, checking their work. Students who had the first problem correct were given task-specific praise and directions for the rest of the problems: "Good for you. You got the first problem correct, and you had the confidence, after checking it, to call it correct. I see you remembered to use the dollar sign and decimal points where they were needed. After you finish the first row, including checking your problems, meet with another student to see how your answers compare. Do you know what to do if there is a discrepancy in your answers? That's right. You'll need to check each other's problem to locate the error." For the students who had solved the problem incorrectly, Ms. Wong stopped by each student's desk and said, "Tell aloud how you did this. Start from the beginning, and as you think of what you're doing, say it aloud so I can follow." Ms. Wong finds that students often notice their own errors, or she will identify some faulty thinking by the students that keeps them from correctly solving the problem. Once the error has been found and corrected, Ms. Wong asks the student to do the next problem, again saying what is being done aloud. If correct, Ms. Wong gives task-specific praise and directions for the rest of the problems.

Alternative Approaches to Instruction If a student is not succeeding with one instructional approach or program, the teacher should not hesitate to make a change. Most students learn best when they are provided prerequisite skills to support the math processes and opportunities to practice with feedback. Consider changing resources if students are having difficulty, including adjusting textbooks, workbooks, math stations, and manipulatives.

Applied Mathematics Concrete and representational materials and real-life applications of math problems make math relevant and increase the likelihood that students will transfer skills to applied settings such as home and work. Students can continue to make progress in mathematics throughout their school years when they have the underlying foundational scaffold from which to build their skills and problem solving. The emphasis needs to be on problem solving rather than on rote drill and practice activities.

The term *situated cognition* refers to the principle that students will learn complex ideas and concepts in the contexts in which they occur in day-to-day life (real-world application). Students need many opportunities to practice what they learn in the ways in which they will eventually use what they learn. This is a critical way to promote the generalization of mathematical skills. For example, when teaching measurement, a teacher can give students real-world application opportunities to use the mathematics they are learning, such as measuring rooms for carpet, determining the mileage to specific locations, and so on.

When Ms. Wong's students successfully used dollar signs and decimals in subtraction, she gave each of them a mock checkbook, which included checks and a ledger for keeping the balance. In each of their checkbooks, she wrote the amount of $100.00. During math class for the rest of the month, she gave students "money" for their checkbook when their assignments were completed and their behavior was appropriate. She asked them to write her checks when they wanted supplies (pencils, erasers, chalk) or privileges (going to the bathroom, free time, meeting briefly with a friend). Students were asked to maintain the balances in their checkbooks. Students were penalized $5.00 for each mistake the "bank" located in the checkbook ledgers at the end of the week, much like a charge a real bank would make for an overdrawn account.

Generalization Generalization, or transfer of learning, needs to be taught. As most experienced teachers know, students often can perform skills in the special education room but cannot perform them in a regular classroom. To facilitate the transfer of learning between settings, teachers must provide opportunities to practice skills by using a wide range of materials, such as textbooks, workbooks, manipulatives (e.g., blocks, rods, tokens, real money), and word problems. For example, the teacher could have students measure different objects with things (unsharpened pencils, sheets of construction paper, or newspaper pages) rather than rulers or yardsticks. Teachers also need to systematically reduce the amount of help they provide students in solving problems. When students are first learning a math concept or operation, teachers provide a lot of assistance in performing it correctly. As students become more skillful, they need less assistance. Teachers must remember that generalization or transfer of learning must be planned for rather than "teach and hope" that it will occur.

When Ms. Wong's students correctly applied subtraction with dollars and decimals in their checkbooks, she asked them to perform similar problems for homework. Ms. Wong realized that before she could be satisfied that the students had mastered the skill, they needed to perform it outside her classroom and without her assistance.

Participation in Goal Selection Allowing students to participate in setting their own goals for mathematics is likely to increase their commitment to achieving goals. Students who selected their own mathematics goals improved their performance on math tasks over time more than did those students whose mathematics goals were assigned to them by a teacher (L. S. Fuchs, Bahr, & Rieth, 1989). Even very young children can participate in selecting their overall mathematics goals and can keep progress charts on how well they are performing.

Instructional Approaches Students in the United States have scored well below others (Taipei, South Korea, Singapore, Hong Kong, Japan) in mathematics proficiency in grades 4 and 8 on the international assessment of mathematics, Trends in International Mathematics and Science Study (Provasnik et al., 2016). We need to examine how we teach. It may be advantageous for math teachers to consider focusing instruction on the development of fewer mathematical topics that are the more important ones so that students become truly proficient. This approach is used in many other countries that have demonstrated successful outcomes in mathematics (Ginsburg, Cooke, Leinwand, Noell, & Pollock, 2005).

The National Research Council (NRC) (National Research Council, 2001) indicates that "mathematical proficiency" is the essential goal of instruction. What is mathematical proficiency? It is what any student needs to acquire mathematical understanding. The NRC describes five interwoven strands that compose proficiency. Consider how you are integrating these strands into your instruction. Also consider how you might determine whether the students you teach are making progress along each of these strands.

1. *Conceptual understanding* refers to understanding mathematic concepts and operations.
2. *Procedural fluency* is the ability to accurately and efficiently conduct operations and mathematics practices.
3. *Strategic competence* is the ability to formulate and conduct mathematical problems.
4. *Adaptive reasoning* refers to thinking about, explaining, and justifying mathematical work.
5. *Productive disposition* is appreciating the useful and positive influences of understanding mathematics and how one's disposition toward mathematics influences success.

See Apply the Concept 11.2 for suggested instructional practices.

It is particularly important for teachers to design mathematics programs that enhance learning for all students, especially those with diverse cultural or linguistic backgrounds. See the next section for suggestions on how to do this.

Considerations for Students Who Are Culturally and Linguistically Diverse: Enhancing Skills in Mathematics

An essential part of successful mathematics learning is to provide instructional practices and assignments that facilitate the learning of mathematics for all students. An important caution when dealing with students with diverse backgrounds is that their prior experiences will also be diverse (Tucker, Singleton, & Weaver, 2006). To build on particular prior experiences, you must ensure that all students have had those same experiences. To take for granted that they have them is a common error of many teachers. To accomplish successful learning, you must consider the needs of students from diverse cultures and language backgrounds; suggestions for doing so include:

- Assign individuals from different backgrounds to work in pairs or small groups, providing opportunities in mathematical problem solving that will ensure similarity of prior learning experiences and social interaction, with infrequent failure.
- Use manipulatives to provide common firsthand experiences to concretely explore the meaning of mathematical symbols and problems. Manipulatives enhance learning, increase social interaction, and provide an easy means of crossing potential language barriers.
- Model an enthusiastic and positive attitude about appreciating and learning more about the cultures and languages of other groups.
- Consider ways to infuse aspects of the various cultures of the students in your classroom and of students not represented into the mathematics curriculum provided by your school. Students will appreciate the inclusiveness you develop, feel accepted, be more engaged, and learn a great deal about each other and other cultures when cultural diversity is a part of their daily learning routines.
- Infuse aspects of mathematics and story problems that reflect names and events from diverse cultures. Encourage students to design mathematics and story problems that reflect other cultures. Books and newspaper or magazine articles or stories about individuals, families, groups, and data from other cultures can be incorporated to design mathematically based story problems.
- Link students' accomplishments to their hard work and effort. Remind students that they performed a task well because they worked hard, persisted, reread, rethought, revisualized, modeled, and so forth.

11.2 Apply the Concept

Instructional Practices for Mathematics

On the basis of several syntheses of mathematics instruction for students with learning and behavior problems (Baker, Gersten, & Lee, 2002; Gersten et al., 2009), we recommend the following 10 instructional practices:

1. ***Use data to make decisions about instruction and progress.*** Teachers and/or students should use data to determine if instructional changes are needed. The links to changes in instruction are central to the use of data-based decision making.

2. ***Involve peers*** in working together to develop understandings and a foundational scaffold from which to engage the learning of mathematics skills and practice computation and word problems. In addition to developing a foundational scaffold, practicing computations, and problem solving, peers can provide support by teaching each other self-monitoring, correcting answers, and charting data for progress monitoring.

3. ***Inform parents*** about students' progress and success in mathematics *and involve parents in their student's learning* so that they can enhance interest and practice of mathematics at home. Provide parents with information about students' success in mathematics so that they can recognize those accomplishments at home. Have students demonstrate (teach) the mathematics that they are learning at school to their parents rather than having parents just help with homework. Many times, the parents' mathematics knowledge is based on rote memorization that is not helpful to their child's learning and understanding.

4. ***Use instructional routines*** that focus on developmental and cognitive behavioral techniques that benefit students with learning and behavior problems and engage them in the learning process.

5. ***Instructional design features*** are effective ways to achieve the following: teach students to develop a foundational scaffold to support additional learning, differentiate problem types, use a wide range of examples, separate confusing elements, and provide opportunities for students to reach performance levels before introducing more new principles.

6. ***Teach students the principles of mathematics to mastery*** and then move to more advanced principles. Many students with special needs are given the same instructional mathematics curriculum (e.g., subtraction with regrouping) from second through ninth grades. Upper-level students need to learn mathematics at levels beyond this, including prealgebra and algebra. Some may need to focus on functional mathematics.

7. ***Establish realistic goals*** for progress in mathematics with students by providing information to students about their present performance and what they need to learn and how to learn best (e.g., paired with peers, or use of manipulatives, number lines, or calculators).

8. ***Monitor progress*** on a weekly basis through graphing or visual display so that students can chart and see how they are performing. Make adjustments in teaching, materials, grouping, or other features of instruction if students are not making adequate progress.

9. ***Provide evidence*** that hard work and effort yield good outcomes and progress. Students can also learn to reinforce themselves for setting and meeting goals in mathematics.

10. ***Use computer-assisted instruction*** as an effective way to learn and practice arithmetic computation and mathematical problem solving.

- Use culturally relevant materials as a springboard for mathematics learning. The ways in which mathematics is practiced in various cultures, the mathematical games that are played, and the use of mathematics cross-culturally can be embedded into mathematics instruction.
- Use the students' languages within the instruction. Ask students to provide the mathematical word that means the same as "______" in their home language, and then use both words when referring to the term. Encourage all students in the room to learn the term and to use and apply the terms that represent the languages of the students in the class. Communicate to students that you value their home language and culture.
- Use technology to enhance learning and understanding of mathematical principles. Computers provide language that typically is available to most students. Encourage expertise on the computer, and provide multiple opportunities to practice skills. **MathPad Plus** is designed for students in kindergarten through grade 8 to enable learners to do arithmetic directly on the computer. This program is ideal for learners who need help organizing or navigating through math problems or who have difficulty using a pencil and paper with math.

Assessing and Progress Monitoring Mathematics Performance

How is mathematics performance assessed, and how is progress monitored? Mr. Sebeny is a first-year special education teacher. He is fortunate to work in a middle school with three other special education teachers who have been at the school for several years and are used to team teaching. They've asked Mr. Sebeny whether he would

MyLab Education
Video Example 11.1
In this video, a teacher monitors students' understanding of new concepts informally. What is the role of such assessment and progress monitoring in teaching math?

be comfortable teaching mathematics to all of the special education students. He quickly realized that his first task would be to determine the mathematics performance level of all of his students. Mr. Sebeny knows that he needs to select a measure that will tell him what students know and don't know and how they compare with other students in their grade.

There are many ways in which Mr. Sebeny could obtain the information required to develop instructional programs for his students. One of the first questions Mr. Sebeny needs to address is whether he has the time to use an individually administered assessment or whether he must use a group-administered measure. For students with special needs, individually administered measures yield the most information.

Second, Mr. Sebeny needs to determine whether the measure is designed for students in the age range of the students he is teaching. Figure 11.1 provides a list of mathematics measures, states whether they are administered to groups or individuals, and lists the age range for which they are appropriate. Figure 11.2 provides a list of progress-monitoring measures for math. The **National Center on Intensive Intervention website** also provides tool charts for academic screening and progress monitoring in mathematics.

How can teachers best make decisions about whether students are learning mathematics effectively? How can teachers monitor the progress of their students so that they can document the rate of progress they are making in

Figure 11.1 Measures to Assess Mathematics Performance Progress Monitoring

Test Name	How Administered	Age/Grade Appropriate	Other Information
Comprehensive Math Assessment	Group	Grades 2–8	Based largely on the NCTM'S critical elements in mathematics instruction
Diagnostic Achievement Battery	Individual	Most grade levels	Provides normative data on student performance but not specific information for designing strengths and weaknesses
Wide Range Achievement Test	Individual or group	Most grade levels	Provides normative data on student performance but difficult to identify students' needs for instruction
Woodcock Johnson III Tests of Achievement	Individual	Most grade levels	Provides normative data on student performance but may not provide adequate information for designing instruction
Test of Early Mathematics Ability	Individual	Ages 3–9	Provides information to assist with designing and monitoring instruction
BRIGANCE Diagnostic Comprehensive Inventory of Basic Skills—Revised	Individual	Prekindergarten–grade 9	Provides information to assist with designing and monitoring instruction
Comprehensive Mathematical Ability Test	Individual	Grades 1–12	Provides information to assist with designing instruction
Key Math—Revised	Individual	Grades 1–12	Provides information to assist with designing instruction
Test of Mathematical Abilities	Individual	Grades 3–12	Provides information to assist with designing instruction
Math—Level Indicator: A Quick Group Math Placement Test	Group	Grades 4–12	Takes approximately 30 minutes, and because it is group administered, it quickly determines the performance levels of a large group of students. The problems are based on the NCTM standards.

Figure 11.2 Progress-Monitoring Measures for Math

Test Name	Concepts Addressed	Grade	Website	Forms
Monitoring Basic Skills Progress	Math computation Math concepts	Grade 1 and above	http://www.proedinc.com	30 forms per grade
PASeries Math	Numbers Operations Geometry Algebra Data analysis Measurement	Grades 3–12	http://www.paseries.com	6 forms per grade
Star Math	Computation Application Concepts	Grades 1–12	http://www.renlearn.com	Unlimited forms
AIMSweb Systems	Oral counting Number identification Quantity discrimination Missing number Basic skill areas	Grades K–8	http://www.aimsweb.com	33–50 forms for each construct
AIMSweb Plus	Math facts Concepts and applications	Grades K–8	http://www.aimswebplus.com	24 forms
FAST Early Math	Matching Number sequence Numeral identification	Grades K–1	http://www.fastbridge.org	20 forms
I-Ready Diagnostic for Mathematics	Many dimensions of math	Grades 1–8	www.curriculumassociates.com	Multiple forms
mClass	Many dimensions of math	Grades K–3	www.amplify.com	20 forms

mathematics? Considerable and growing evidence indicates that when teachers use curriculum-based measurement (CBM) to monitor their students' progress and to adjust their instruction accordingly, students make gains at much more rapid rates than when CBM is not used (Shapiro, Keller, Lutz, Santoro & Hintze, 2006).

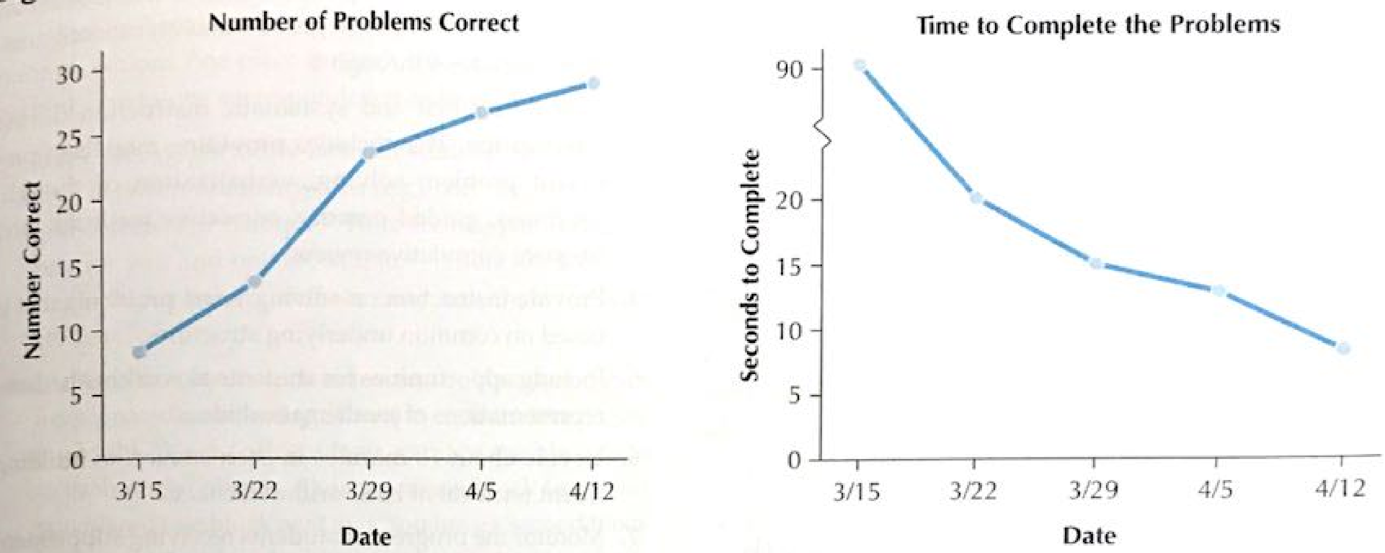
What is CBM for math? Simply stated, it is a way of documenting the extent to which the student is learning the critical elements in the curriculum that you have targeted. To illustrate, let's consider the case of Ricky, a fifth-grade boy with learning and attention problems, who has been struggling with math. His goals for the next 10 weeks are to know all subtraction facts up to 100 automatically and quickly, to be able to do addition with regrouping word problems, and to be able to appropriately use basic measurement terms, such as *inches*, *feet*, and *yards*. Here is how Ricky and his teacher use CBM.

Ricky's teacher pretested on all 100 subtraction facts in random order, timing him while he completed the worksheet. She then showed Ricky how to graph his performance in two ways: by graphing how long it took him to complete the worksheet and by graphing the number of correct problems. Together, they agreed that Ricky would take a version of this test once every week to determine whether he could decrease the amount of time he needed to complete the test and increase the number of problems he got correct. Together, they established a schedule of work assignments and practice sessions. Figure 11.3 shows the graph that Ricky kept.

Ricky's teacher followed a similar procedure with measurement and problem solving to determine what Ricky knew and what he needed to know. Then the teacher established a simple graph that Ricky could complete to monitor his progress. Ricky and his teacher frequently discussed Ricky's progress and modified assignments and instruction to facilitate his learning.

Computerized applications of CBM procedures are available for mathematics as well as spelling and reading (L. S. Fuchs, Hamlett, & Fuchs, 1990; Gersten et al., 2009). One Web resource for math activities that is loaded with explanations, interactive practice, games, and teacher resources is **AAA Math**.

Figure 11.3 Ricky's Progress-Monitoring Chart



Assessing Number Sense

Assessing number sense can be an effective way to monitor the progress of young children in mathematics and to determine who has mathematics difficulties. *Number sense* refers to whether a student's understanding of a number and of its use and meaning is flexible and fully developed. One definition of number sense is "a child's fluidity and flexibility with numbers, the sense of what numbers mean, and an ability to perform mental mathematics and to look at the world and make comparisons" (Gersten & Chard, 1999, p. 19). In terms of assessment, number sense is particularly important because it assists teachers in determining which students currently have mathematical difficulty and even serves as a predictor for students who may have learning difficulties in the future (Geary, 2010).

Measures that can be used to quickly determine students' understanding of number sense are available (L.S. Fuchs et al., 2007; Gersten et al., 2012). Each of these measures can easily be constructed and changed by the teacher to determine what a student knows. These include the following five:

1. *Number identification.* In this task, students must orally identify numbers between 0 and 20 when these are presented randomly on a piece of paper.
2. *Number writing.* Students are asked to write the number when given a number orally between 1 and 20.
3. *Quantity discrimination.* This task requires students to name which of two numbers is the larger (or smaller).
4. *Missing number.* Students are provided with a string of numbers and are asked to identify which number is missing.
5. *Computation.* Students are asked to complete computations that are representative of their grade level. Students have 2 minutes to complete as many problems as possible.

In addition, several counting measures can be used as effective screening tools for students with mathematical difficulties or to monitor students' progress in this area (Clarke & Shinn, 2004):

1. *Count to 20.* This is a beginning-level skill requiring students to count to 20 while the teacher records the correct and incorrect numbers in the sequence.
2. *Count by 3 and 6.* This skill requires students to count from a predetermined number, such as from 5, in increments of 3 or 6. The teacher records the accuracy and speed with which the students perform this task.
3. *Count by 2, 5, or 10.* This skill requires student to count by the designated number—2, 5, or 10—in increments up to a specified number, such as 20 for 2s, 50 for 5s, or 100 for 10s. The teacher records the accuracy with which students perform this task.

How Effective Are Test Accommodations in Mathematics for Students with Disabilities?

Assessment is an important part of the instructional routine. Teachers use assessment to assist them in determining what students know and can do and what they need to know and do. Appropriate assessments allow teachers to monitor students' progress and to make effective instructional decisions that will improve the students' performances. Having the results of daily, weekly, and monthly progress assessments along with more long-term assessments (e.g., state-mandated tests) makes planning effective instruction easier. Assessments can also tell teachers how students compare to others at their same age or grade level.

The idea behind test accommodations is that they are more responsive to the individual needs of students with disabilities. Elbaum (2007) reports that when mathematics tests are read aloud to students with disabilities and their performance on these tests is compared with that of students without disabilities, the read-aloud condition is more helpful to elementary students with disabilities than elementary students without disabilities. However, the reverse is true for secondary students with disabilities; their improved performance with accommodations is overall lower than that for students without disabilities. Keep in mind that a teacher is bound by the student's individualized education program (IEP) to make the accommodations and modifications that are stated in the document. Many times the accommodation consists of extended time (1.5 or 2 times the normal time allotted), having the mathematics assessment read to the student, and spreading the assessment over several days.

Response to Intervention and Math

Response to intervention (RTI) is a way to more quickly identify students who need additional instruction and to provide the necessary instructional intervention within the classroom to help ensure mastery. RTI has been applied most frequently to the academic area of early reading. However, some schools and districts are currently using RTI in mathematics at both the elementary and the middle school levels. How can RTI be used in math? You can learn more about RTI and mathematics by exploring this **IRIS module**. Initially, many of the same principles that applied to the use of RTI in reading also can be applied to math. These include:

- *Screening.* Students can be screened to determine if they have math problems in numeracy, math calculations, and/or problem solving.
- *Evidence-based math.* Schools and districts can ensure that math instruction for all students is based on the best research available.
- *Interventions.* When students have difficulties that are not adequately addressed through the evidence-based math program in the classroom, additional instruction through short-term interventions (10 to 20 weeks) can be implemented.
- *Progress monitoring.* Students' progress in the classroom and in interventions can be documented to ensure that they are staying on track and meeting curriculum benchmarks.

Gersten et al. (2009) provided a set of recommendations to aid teachers, principals, and others in their use of RTI for early identification of students who need help in mathematics and for focused mathematics intervention in elementary and middle schools. These recommendations include:

1. Screen all students to identify those at risk for potential mathematics difficulties and to provide interventions to students identified as at risk.
2. Focus instructional materials for students receiving interventions on in-depth treatment of whole numbers in kindergarten through grade 5 and on rational numbers in grades 4 through 8.
3. Provide explicit and systematic instruction during intervention. This includes providing models of proficient problem solving, verbalization of thought processes, guided practice, corrective feedback, and frequent cumulative review.
4. Provide instruction on solving word problems that is based on common underlying structures.
5. Include opportunities for students to work with visual representations of mathematical ideas.
6. Devote about 10 minutes in each session to building fluent retrieval of basic arithmetic facts.
7. Monitor the progress of students receiving supplemental instruction and other students who are at risk.
8. Include motivational strategies in Tier II and Tier III interventions.

Key principles for secondary intervention include:

- Instructional explicitness
- Instructional design that minimizes the learning challenge
- Strong conceptual basis for procedures taught
- Emphasis on drill and practice
- Cumulative review as part of drill and practice
- Motivators to help students regulate their attention and behavior and to work hard

Web Resources

To provide additional practice to aid students' proficiency in mathematics, visit a website located at **http://superkids.com/**. From the left navigation bar, you can access math worksheets for additional practice in addition, subtraction, multiplication, division, fractions, telling time, and many other topics.

Prenumber Skills

What are the prenumber skills students need to progress in arithmetic? Many students come to school with few experiences that allow them to develop important prenumber skills, such as one-to-one correspondence, classification, and seriation. These skills are essential to success in learning other mathematics concepts and skills.

One-to-One Correspondence

Matching one object with another is a core skill in any mathematics curriculum. It eventually leads the student to a better understanding of numeration and representation.

One-to-one correspondence is used in life when we set a table, one place setting for each person; go to the theater, one ticket and seat for each person; and distribute paper in the classroom, one piece for each student. Activities for teaching one-to-one correspondence include the following:

- Use every opportunity to teach students the relationship between number words (e.g., *one, two, three, four*) and objects. For example, "Here are two paintbrushes: one for you and one for Madju." "There are five students in our group, and we need one chair for each student."
- Use familiar objects such as toy cars or blocks, and give a designated number (e.g., three) to each student. Pointing to the objects, ask students to place one block next to each of the objects. "You have one block here, and you placed one block next to it. You have a second block here, and you placed a block next to it. And you have a third block here, and you placed a block next to it."
- Give the student a set of cards with numbers that the student recognizes. Ask the student to put the correct number of blocks on top of each number card. Reverse the task by giving objects to students and asking them to put the correct number card next to the objects.

Classification

Classification is the ability to group or sort objects on the basis of one or more common properties. For example, classification can be done by size, color, shape, texture, or design. Classification is an important prenumber skill because it focuses on common properties of objects and requires students to reduce large numbers of objects to smaller groups. Most students are naturally interested in sorting, ordering, and classifying. Activities for teaching classification include the following:

- Ask students to sort different-colored, shaped, and sized objects into groups. Ask them which rule they used for sorting their objects.
- Give each student an empty egg carton and a box of small objects. Ask the students to sort the objects according to one property (e.g., color). Now ask them to think of another way to sort the objects (e.g., size, texture).
- Using an assortment of objects, ask a student to classify several of the objects into one group. Other students then try to guess the property or properties that qualify the objects for the group.
- Students can use pictures for sorting tasks. Animals, foods, plants, toys, and people can all be sorted by different properties.
- Board games and bingo games can be played by sorting or classifying shapes, colors, or pictures.

Seriation

Seriation is similar to classification in its dependence on the recognition of common attributes or properties of objects. With seriation, ordering depends on the degree to which the object possesses the attribute. For example, seriation can occur by length, height, color, or weight. Activities for teaching seriation include the following:

- Give students some objects of varied length, like sharpened pencils or used crayons, and ask the students to put the objects in order from shortest to longest.
- Ask students to stack their books from largest, on the bottom, to smallest, on the top.
- Using a peg and some rings of varied sizes, ask students to put the rings on the peg, from largest to smallest.
- Fill jars of the same size with varied amounts of sand, rice, water, or marbles, and ask students to put them in order.

Algebraic Principles

Most teachers think that algebra is a subject taught in secondary schools. Yet the Common Core Standards indicate that all students should understand the foundations of algebra. Students should understand mathematical patterns, relationships, and functions. Students should also be able to view mathematical structures and situations and represent and analyze them using algebraic symbols. Moreover, students should be able to represent and understand quantitative relationships through the use of mathematical models in their problem solving. The foundations of algebra and its basic principles can be learned at many different ages and within many math topics.

You might not point out to young students that they are solving algebraic problems when they are asked to fill in the symbol that completes a pattern (e.g., XX OO XX O_) or provide a number that creates an equality (e.g., 2 + [_] = 3). However, just thinking that they can do algebra could be that great motivator that jump starts their learning of more mathematics.

Math Concepts and Computation

Numeration and Place Value

What are several numeration concepts? Teachers and parents often assume that children understand numbers because they can count or name them. Understanding numbers is an essential basic concept; many children who have trouble with computation and word problems are missing numeral concepts. For example, Nadia's beginning experiences with math were positive. She had learned to read and write numbers and even to perform basic addition and subtraction facts. However, when Nadia was asked to perform problems that involved addition with regrouping, she demonstrated that she had very little knowledge of numbers and their meaning (her errors are shown here) and thus quickly fell behind her peers in math.

$$\begin{array}{r} 48 \\ +26 \\ \hline 614 \end{array} \quad \begin{array}{r} 37 \\ +55 \\ \hline 812 \end{array} \quad \begin{array}{r} 68 \\ +17 \\ \hline 715 \end{array}$$

Her computations are correct, but she lacks the understanding of the concepts *numeration* and *place value* and what to do when the result exceeds the place value (tens and ones). The use of manipulatives would aid her understanding. She could use bundled and loose sticks to represent the problem; she could take 10 of the 14 (8 + 6) loose sticks and make a new bundle of ten to be placed with the other bundled tens (4 and 2). This would aid her in understanding numeration and place value.

Understanding numeration and place value is necessary in the following areas of mathematics:

- *Progress in computation.* Like Nadia, many students fail to make adequate progress in math because they lack understanding of numbers and place value.
- *Estimation* (i.e., "number sense"). Many students with learning difficulties in math do not have a sense of how much $1.00 is, what it means to have 35 eggs, or "about" how much 24 plus 35 equals. They cannot check their answers by looking at problems and determining which answers could not be correct because the answer doesn't make "sense."
- *Reducing conceptual errors.* Students who understand the meaning of the numbers 43 and 25 would be less likely to make the following error:

$$\begin{array}{r} 48 \\ -25 \\ \hline 22 \end{array}$$

- *Understanding place value.* Students who know the meaning of the number 28 are going to have far less difficulty understanding the value of the 2 and the 8. Students need to understand that the 2 in 28 represents two 10s and the 8 represents eight 1s.
- *Understanding regrouping.* Regrouping errors, such as those following, are less likely to occur if a student understands numeration.

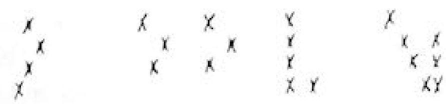
$$\begin{array}{r} 39 \\ +27 \\ \hline 516 \end{array} \quad \begin{array}{r} 56 \\ -18 \\ \hline 42 \end{array} \quad \begin{array}{r} 41 \\ -24 \\ \hline 23 \end{array}$$

- *Application of math computation to everyday problems.* Students who do not understand the real meaning of numbers have difficulty applying computation to everyday problems.
- *Understanding zero.* Students need to understand that 0 (zero) has more meaning than just "nothing." For example, in the number 40, they need to understand the meaning of the 0 as a placeholder (zero ones).

Readiness for Numeration: Eighteen Concepts Engelhardt, Ashlock, and Wiebe (1984) identified 18 numeration readiness concepts that can be assessed through paper-and-pencil assessment and interview. A list of the behaviors that correspond to each concept, along with examples of how the concept can be assessed, follows each concept.

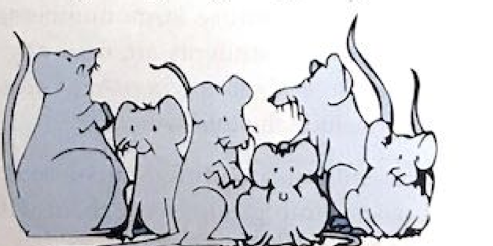
Concept 1: Cardinality The face value of each of the 10 digits (0 through 9) tells how many. It is a good idea to get students to learn that zero is a number and is the empty set.

1. Identify sets with like numerousness (1 through 9). "Circle the groups with the same number of *x*s."

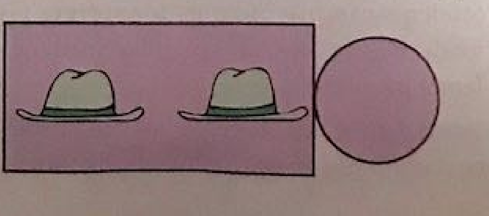


2. Represent and name the numerousness of the empty set (zero). "The box has 2 hats in it. Make 0 hats in the circle." Zero can be illustrated as [ ] or the empty set.
3. Identify, write, and name the numeral that corresponds to the numerousness of a set (1 through 9). "Circle the numeral showing how many mice there are. How many mice are there?"

3 2 6 4 9



4. Construct sets with a given numerousness (1 through 9). "Draw five dots." Remember to include the zero set in these examples.
5. Recognize sets of one to five without counting. Place from one to five objects behind a book. Say to the students, "As soon as I move this book, I want you to tell me how many objects there are." Without counting, the students should tell you how many objects are in the group. Remember to include the zero set in these examples.



Concept 2: Grouping Pattern When representing quantity, objects are grouped into sets of a specified size (base) and sets of sets.

1. Form sets of 10 from a random set of objects or marks. "Circle the *x*s to make as many groups of 10 as possible. Now circle the *o*s, *A*s, and the *s to make as many groups of 10 as possible."

xxxxxxxxxxxx xxxxx xxxxxx
ooooooooooooo oooooo ooooooooo
AAAAAAAAAA AAAAA AAAAAA
************* ****** *********

2. Construct appropriate groups to show how many. Give students about 125 Popsicle® sticks and rubber bands. Say, "Bundle these sticks so that it will be easy to tell how many there are." Ask the students what was the basis (the number in each set) they used for their bundling the sticks together.

Concept 3: Place Value The position of a digit in a multidigit number determines its value (places are assigned values).

1. Given two multidigit numbers with the same digits but in different orders, identify the position of the digits as distinguishing the two numbers. "How are 145 and 154 alike? How are they different?"
2. Explain that the value of a digit in a multidigit number is dependent on its position. Using the numeral 5, place it in each column, and ask, "How does the number change? How much is it worth?" Repeat with several other numerals (e.g., 3, 8) and then with other multidigit numbers (e.g., 125, 372).

100s	10s	1s

Concept 4: Place Value (Base 10) A power of 10 is assigned to each position or place (the place values). It is a good idea to provide the position or place names: thousands, hundreds, tens, and ones. Furthermore, it is a good idea to build from single-digit numbers to double-, then triple-, and four-digit numbers. With middle school students who have mastery of the base 10 concept, other base numbers (e.g., base 5 or 2) can be introduced.

1. Identify, name, and show the values for each place in a multidigit number. Show the number 1,829, and say, "What is the name of the place the numeral 8 is in?"
2. Select the place having a given value. Show the number 6,243, and say, "Circle or point to the numeral in the thousands place."

Concept 5: One Digit per Place Only one digit is written in a position or place.

1. Identify and name which numerals (digits) can be assigned to a place. Say to the students, "Tell me the numerals that can be written in the tens place."
2. State that no more than one digit should be written in a place or position. Then ask, "What's wrong with these problems?"

$$\begin{array}{r}85\\+39\\\hline1114\end{array}\qquad\begin{array}{r}27\\+35\\\hline512\end{array}\qquad\begin{array}{r}13\\+48\\\hline511\end{array}$$

3. Rewrite or restate a nonstandard multidigit numeral (or its representation) as a numeral with only one digit in each place or position. Say to the students, "Write the numeral for this:"

10s	1s
5	4
7	8

Bundled and loose straws or 10 blocks and unit blocks could also be used to help gain this understanding of one digit per place.

Concept 6: Places—Linear/Ordered The places (and their values) in a multidigit (whole-number) number are linearly arranged and ordered from right to left.

1. Identify the smaller-to-larger ordering of place values in a multidigit number. Show the number 6,666 to the students, and say, "Underline the 6 that is worth the most, and circle the 6 that is worth the least."
2. Describe how the place values are ordered. Show the number 8,888 to the students, and say, "Point to the 8 that is 8 hundreds; point to the 8 that is 8 tens." Then ask, "Which of these 8s is worth more?"
3. State or demonstrate that the places in a multidigit number are linearly arranged. Say to the students, "Rewrite this problem correctly."

```
      7       1
          2       4
      6   3       8
  +           0
  ________________
```

Concept 7: Decimal Point The decimal point in a decimal fraction indicates the location of the units (ones) and tenths and hundredths places. Use the table to illustrate where this occurs in order to make a link to prior knowledge. Have the students place the numerals in the correct place. Start with the number 13, then 13.5, and several more numbers with tenths; once the tenths are understood, use numbers with hundredths, like 26.34, 57.92, and 10.01. Ask the students to tell you the value of different numbers.

1. Given a decimal fraction, identify the digit in the units (ones) place. Show the number 29.64 to the students, and say, "Circle the numeral in the ones place." Repeat this with several other numbers like 45.72, 26.89, and 19.03.

10s	1s	tenths	hundredths

2. Given juxtaposed digits and a digit's value, identify and place the decimal point to show the appropriate multidigit number. Show the number 284 to the students, and say, "Place the decimal in the correct place to show 2 tens, 8 ones, and 4 tenths." After tenths, you can use numbers that include hundredths. As students become more proficient, you can show a number like 3201 and say, "Place the decimal in the correct place to show 3 tens, 2 ones, and 1 hundredth."
3. State the meaning (function) of the decimal point.

Concept 8: Place Relation/Regrouping Each place in a multidigit number has a value 10 times greater than that of the place to its right and one-tenth the value of the place to its left (place relationships and regrouping).

1. Describe the relationships between the values of two adjacent places in a multidigit number. Show the number 222 to the students, and say, "How does the first 2 in the number compare with the second 2?"
2. Express the value of a multidigit number in several ways. Give the following problem to the students: 1 hundred, 8 tens, and 6 ones can also be expressed as ____ tens and ____ ones.

Concept 9: Implied Zeros All numbers have an infinite number of juxtaposed places, each occupied by an expressed or implied digit. In places to the left of nonzero digits in whole numbers written as numerals, zeros are understood; in places to the right of nonzero digits and the decimal point in decimal fractions, zeros are understood.

1. Name the digit in any given place for any multidigit number. Tell the students to rewrite each number and show a digit in each place.

1000s	100s	10s	1s	tenths	hundredths
683					
27					
79					
4.3					
351.84					

2. Rewrite a given number with as few digits as needed. Tell the students to cross out the zeros that are not needed.

0301	004	01.30
1010	105	0246.080

3. State a rule for writing zeros in a multidigit number. Ask the students, "When do we need to write zeros in a number?"

Concept 10: Face Times Place The value of any digit in a multidigit number is determined by the product of its face and place values (implied multiplication).

1. Show, name, and identify the value of a specified digit within a multidigit number. Show the number 1,468 to the students, and ask, "How much is the 6 worth: 0, 6, 10, 60, or 16?" Ask, "What is the 4 worth: 4, 40, 400, or 4,000?" Then use more examples. Also, include a decimal number, like 25.79, and ask, "How much is the 7 worth: 70, 7, 7 tenths, or 7 hundredths?
2. Name and identify the operation that is used to determine the value of a digit in a multidigit number. Ask the students, "In 1,468 how do we find that 6 is worth 60? Do we add, subtract, multiply, or divide?"
3. State a rule for finding the value of a specified digit in a multidigit number. Ask the students, "How do you know that 6 is worth 60 in the number 1,468?"

Concept 11: Implied Addition The value of a multidigit number is determined by the sum of the values of each digit (implied addition).

1. Express any multidigit number as the sum of the values of each digit.
   294 = ____ ones + ____ tens + ____ hundreds
   Also use a decimal number, like
   38.4. 38.4 = ____ tenths + ____ ones + ____ tens.
2. Express the sum of digit values as a multidigit number.
   4 ones + 3 tens + 6 hundreds = ____
   Also use a decimal number, 2 tenths, 4 ones, 5 tens, __ (remind students who may forget the decimal point).
3. Identify the operation that is used to determine the value of a multidigit number. Ask the students, "To know the value of 287, do we add, subtract, multiply, or divide the value of each numeral?"

Concept 12: Order Multidigit numbers are ordered.

1. Order multidigit numbers. Say to the students, "Put these numbers in the correct order from smallest to largest: 1689, 1001, 421, 1421." Remember to use multidigit decimal numbers, too (e.g., 32.4, 101.45, 7.1, 78.23, 45.34, 45.23, 45.31, 45.07).

2. Describe a procedure for determining which of two unequal multidigit numbers is larger. Show the numbers 984 and 849 to the students, and ask, "How do you know which is larger?" Also use unequal multidigit decimal numbers such as 3.45 and 3.06, and ask, "How do you know which is larger?"

Concept 13: Verbal Names (0 through 9) In English, the verbal names for the numbers 0 through 9 are unique.

1. Identify the oral/written names of the 10 digits. Ask the students to write the name next to each digit:

0___	4___	7___
1___	5___	8___
2___	6___	9___
3___		

2. State the names for the 10 digits.

Concept 14: Verbal Names with Places In English, the verbal names for multidigit numbers (except 10 through 12) are closely associated with the written numbers (i.e., combining face and place names).

1. Give a multidigit number. Identify the verbal name for one of the digits that includes both a face and place name. Show the number 2,847, and ask the students, "How is the 8 read?"
   a. eight
   b. eight hundred
   c. eighty
   d. eighteen

2. Identify the digit in a multidigit number that is stated first in giving the verbal name. Say to the students, "Write the numeral that is said first in reading the number."

44___	6,186___
284___	37___

3. Select two-digit numbers whose naming pattern is different from most. Ask the students to circle the numbers that when read aloud are different from the others: 17, 43, 126, 11, 281.

Concept 15: Periods and Names Beginning with the ones place, clusters of three (whole numbers) adjacent places are called *periods* and are named by the place value of the rightmost member of the number triad (e.g., ones, thousands, millions).

1. Given a multidigit number, insert commas to form periods. Ask the students to put commas in the correct places: 28146, 682, 7810, 192642.

2. Name the periods of a given multidigit number. Ask the students, "Which number represents the periods?"

   284,000,163___

   (ones, hundreds, thousands, millions)

Concept 16: Naming in the Ones Period Numerals in the ones period are named by stating, from left to right, each digit's name (except zero) followed by its place name (ones being omitted; special rules exist for naming tens).

1. Name three-digit numbers (tens digit not being a 1 or 0). Tell the students to write the name for 683.
2. Name three-digit numbers (tens digit being 1). Tell the students to write the name for 718.
3. Name three-digit numbers (tens digit being 0). Tell the students to write the name for 502.

Concept 17: Naming Multidigit Numbers In naming a multidigit number, the digits in each period are read as if they were in the ones period, followed by the period name (ones period name being omitted).

1. Name multidigit numbers up to six digits. Tell the students to write the name for 284,163.
2. Name multidigit numbers over six digits. Ask the students to read the following numbers:

1,846,283	27,219,143
103,600,101	3,078,420

Concept 18: Decimal Places and Their Verbal Names Beginning with the decimal, the first place to the right is the tenths place. The adjacent places (to the right) are hundredths, thousandths, ten thousandths, hundred thousandths. In reading the decimal numbers, they are named by the place value of the rightmost member of the decimal number (e.g., 25.6 is twenty-five and six tenths; 65.39 is sixty-five and thirty-nine hundredths).

1. Give a decimal number, identify the verbal name for the decimal digits place. Show the number 43.7, and ask the students, "How is the 7 read?"
   a. seven hundredths
   b. seven tenths
   c. seventy
   d. seventeen

2. Identify the decimal digit name in a multidigit decimal number that is stated by giving the verbal name. Say to the students, "Write the decimal verbal name that is said in reading the number."

4.63___	6.18___
28.41___	1.373___

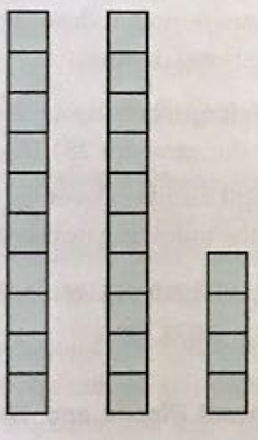
Teaching Place Value Place value is directly related to the students' understanding of numeration. Students need to be able to do the following:

- *Group by ones and tens.* Using manipulatives, pictures, and then numerals, students need practice and instruction in grouping by ones and tens. Students can sort manipulatives such as buttons or sticks in groups of 10. Students can also use a table grid to record their answers.

Tens	Ones	Answer
2	3	23
6	2	62
4	7	47

Source: J. M. Engelhardt, R. B. Ashlock, & J. H. G. Wiebe, *Helping Children Understand and Use Numerals* (Boston: Allyn & Bacon, 1984), pp. 89–149. Copyright 1984 by Allyn & Bacon. Adapted by permission.

Use "tens blocks" and "single blocks" to represent numbers. For example, 24 can be represented as follows:



Flannel boards can also be used to group tens and ones.

- *Naming tens.* Teach students to identify numbers by the number of tens. For example, 6 tens is 60, 4 tens is 40, 8 tens is 80, and so on. Give students opportunities to count by tens and then name the number. For example, "Count by tens 3 times." "Ten, twenty, thirty." "Count by tens 7 times." "Ten, twenty, thirty, forty, fifty, sixty, seventy." Also give students opportunities to draw picture diagrams that represent the place values of tens and ones and to identify the number from diagrams.
- *Place value beyond two digits.* Once students can accurately group and identify numbers at the two-digit level, introduce them to three- and four-digit numbers. It is a good idea to be certain students have mastered the concept of *two-digit place value* before introducing numbers and place values greater than two digits. Many of the principles that students have learned in terms of two-digit place value will generalize to three digits and beyond. Give students plenty of opportunity to group, orally name, and sequence three- and four-digit place values.
- *Place value with older students.* Because place value is a concept that is taught during the primary grades, students who have not adequately learned the skill will likely have problems with computation and word problems. Students need opportunities to learn place value. Many of the games and activities that have been designed to teach place value focus on young children and are less appropriate for older students. Following are five sources of numbers that may be useful for teaching place value to older students:

1. An odometer
2. Numbers from students' science or social studies texts
3. Numbers from the population of your school (e.g., number of freshmen, sophomores, juniors, seniors)
4. Population data from your town, county, state, or country
5. The financial data page from a newspaper

Addition, Subtraction, Multiplication, and Division

Most of students' time in math instruction is spent on computation: memorizing facts and practicing addition, subtraction, multiplication, and division problems; and completing math sheets, workbook pages, and problems copied from books that require the continued practice and application of math computation principles. It is probably for this reason that many students find math boring and miss its applicability to everyday life. Providing appropriate instruction and instructional activities is important to turning the learning of computation into a more desirable process for students.

Teachers can help to make computation exercises and fact learning more engaging by using computer-assisted instruction, which is equally effective for teaching basic arithmetic facts as the conventional drill and practice (Okolo, 1992). Students are likely to be more persistent in solving math problems and have a better attitude toward math when they participate in computer-assisted instruction (Cawley, Foley, & Doan, 2003).

Teachers can also improve students' understanding of computation as they improve students' conceptual knowledge and understanding of mathematics. One intervention for third-grade students with math disabilities addressed conceptual knowledge (Kaufmann, Handl, & Thony, 2003). Students were taught counting principles, the use of arithmetic symbols, memorization of numerals that equaled 10, strategies for memorizing facts, complex multistep calculations, and procedural language for using memorized facts. Students who participated in the intervention made significant gains.

> Web Resources
>
> You can access a website that may be useful in locating sources of information and lesson plans for teaching a wide variety of mathematics. Go to **http://www.awesomelibrary.org/**, then click on *Mathematics*.

Computing math problems is much easier for students if they understand numeration and place value and if they are given frequent practical application of the math problems. When students are having difficulty performing math computation, it may be for the following three reasons:

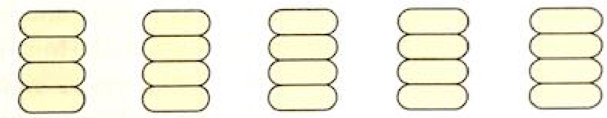
1. They do not have an understanding of numeration and/or place value.
2. They do not understand the operation they are performing.
3. They do not know basic math facts and their application to more complicated computation. Students with attention-deficit disorder often demonstrate difficulty in computation because they fail to automatize computational skills at an appropriate age (Ackerman, Anhalt, & Dykman, 1986). By *automatize*, we mean learning computational facts so that they are automatic, quickly done in one's head. Students with attention-deficit disorder require more time for computation.

> Web Resources
>
> For a website that may be beneficial in planning instruction to help with computation, go to **http://www.aaamath.com/**. The site has lesson plans by grade level that may be useful. Another site claims to be the best website for homework practice. It can be found at **http://www.ixl.com/**. Maybe you should check it out, as well.

Understanding the Operation Students should be able to demonstrate their understanding of an operation by drawing picture diagrams and illustrating it with manipulatives. For example, Sara was able to write the correct answer to the multiplication fact $3 \times 2 = 6$. However, when asked to draw a picture to represent the problem, she drew three flowers and two flowers. She seemed totally undisturbed that the number of flowers she drew was different from the answer she wrote. When asked why she had drawn the number of flowers she did, Sara said, "I drew three flowers for the 3 and two flowers for the 2." When the teacher questioned her further, she discovered that Sara had no understanding of multiplication. By rote, she had memorized the answers to some of the elementary multiplication facts. The teacher used manipulatives such as chips and interlocking cubes to illustrate multiplication.

The following activities can be helpful in teaching students to understand mathematical operations:

- The following drawing illustrates how chips in rows can be used to illustrate multiplication. For example, ask, "How many 4s make 20? Fours are placed on the board __ times."



4 x __ = 20

- Have students talk aloud about what is involved in solving a problem. Do not let them merely *read* the problem; ask them to *explain* what it means. For example, "63 – 27 means that someone had 63 jelly beans and gave 27 of them to a friend."
- Have students explain the process to another student by using block manipulatives. For example, "24 + 31 is the same as adding 4 one-block pieces to 1 one-block piece and 2 ten-block pieces to 3 ten-block pieces."
- Have students close their eyes and use noises to illustrate operations. For example, to illustrate multiplication, the teacher can tap in groups of six and ask, "How many times did I tap a group of 6?"

  tap-tap-tap-tap-tap-tap

  tap-tap-tap-tap-tap-tap

  tap-tap-tap-tap-tap-tap

  "Yes, I tapped a group of 6 three times. Now I am going to tap a group of 6 three times again, and I want you to tell me how many taps there are altogether. Yes, when you tap a group of 6 three times there are 18 total taps." This same process can be used for addition and subtraction (Bley & Thornton, 1981).

In addition, for a student who is more visual in learning style or has a hearing impairment, you could show the 3 rows of 6 stars and ask similar questions. "How many rows of 6 stars are there? Now count the rows by sixes. Six, twelve, eighteen; that is correct." For the more kinesthetic learners, the teacher could have the students stand in three groups of six students each, and ask the questions and count off.

Knowing Basic Math Facts Two of the reasons students may have difficulty with computation have been discussed: They do not understand numeration and/or place value, and they do not understand the computation process. A third reason students may have difficulty with computation is they do not know basic math facts. A common instructional misconception is that if students learn basic arithmetic facts they will no longer have difficulties with other arithmetic operations and problems. Arithmetic facts do not help students in analyzing or understanding the application of arithmetic operations; however, they do aid in the acquisition and speed of performing arithmetic operations. Students who do not know basic math facts are going to be considerably slower and less accurate in math computation and less likely to solve problems effectively. It is difficult for students to understand the math process because so much of their attention is focused on computing one small segment of problems. This is not unlike the student who, when reading, spends so much time decoding an unknown word that comprehension suffers.

Using thinking strategies assists in the acquisition and retention of basic math facts (Thornton & Toohey, 1985). Without direct instruction, students with learning disabilities often do not discover and use these strategies and relationships for learning and retaining math facts (Thornton, 1978). Some thinking strategies that are used by students who are successful at solving basic math facts (Thornton & Toohey, 1985; Thornton, Tucker, Dossey, & Bazik, 1983) can be taught to students who are having difficulties:

- *Using doubles.* Students can learn to use doubles to solve basic math facts. If a student knows 6 + 6 = 12, then the student can easily compute 6 + 7.
- *Counting-on.* Students do not need to resort to counting from one to solve math facts. They can learn to count on from the largest number in an addition fact. For example:

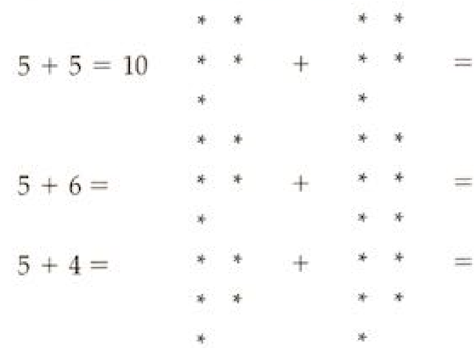
  7 + 2 = ____

  The student counts on two more from 7: "seven, eight, nine." Students can use this same principle when subtracting, only they count backward. For example:

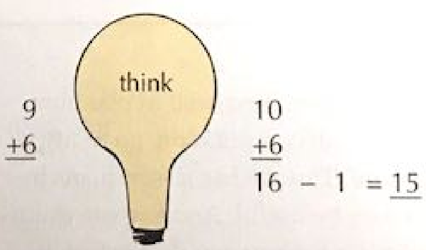
  7 − 2 = ____

  The student counts backward two from 7: "seven, six, five." Students can be taught counting-on before operations, and then they will only need to learn to apply the principle.
- *Using the commutative idea.* The commutative property means that adding or multiplying any two numbers always yields the same answer regardless of their order. Students can be taught that with addition and multiplication, if they know it one way, they know it the other. For example:

  3 + 5 = 8
  5 + 3 = 8
  2 × 9 = 18
  9 × 2 = 18
- *Thinking one more or less than a known fact.* Rasheed knew most of the easy math facts and several of the basic math facts but had trouble with the more difficult ones. When his teacher taught him how to use the math facts he knew to solve the more difficult ones, his math performance improved. For example, Rasheed knew 5 + 5 = 10, but when he was presented with 5 + 6, he began counting on his fingers. His teacher taught him to think of 5 + 6 as one more than 5 + 5, and 5 + 4 as one less than 5 + 5. Pictures such as the following can help to illustrate the principle:



- *Using tens.* Students can learn that 10 + any single-digit number merely changes the 0 in the 10 to the number they are adding to it.
- *Using nines.* Students can apply two strategies to addition facts that involve nines. First, they can think of the 9 as a 10 and then subtract 1 from the answer. As illustrated here, the student is taught to think of the 9 as a 10:



- *Counting by twos, threes, and fours.* Being able to count by multiples helps in addition, multiplication, and division. Multiplication facts can be taught by interpreting 3 × 4 as counting by threes 4 times. Division facts, such as 8 / 2, can be interpreted as, "How many times do you count by twos before you reach 8?"
- *Relationships between addition and subtraction, between multiplication and division.* After students learn addition facts, you can show them the relationship between the addition fact and subtraction. For example, if students know 7 + 6 = 13, they can learn the relationships between the known addition fact and the subtraction fact, 13 − 7 = ____. Whenever possible, reinforce this principle as students are working, "You know 8 + 4 = 12, so 12 − 4 = must be ____." Give students known addition facts, and ask them to form subtraction problems. These sample relationships can be used to teach multiplication and division facts (see Figure 11.4).

Figure 11.4 Relationships Between Addition and Subtraction; Multiplication and Division

Known Addition Facts	*Made-Up Subtraction Facts*
5 + 5 = 10	10 − 5 = 5
3 + 2 = 5	______
8 + 8 = 16	______
6 + 4 = 10	______
Known Multiplication Facts	*Made-Up Division Facts*
7 × 4 = 28	28 ÷ 4 = 7
8 × 8 = 64	______
5 × 9 = 45	______
5 × 10 = 50	______

Van Luit and Naglieri (1999) have been successful in teaching students to compute multiplication and division by ensuring that they learn the following six concepts:

1. Multiplication is repeated addition.
2. Reversibility means that 4 × 7 is the same as 7 × 4.
3. The need to memorize the basic multiplication facts below 100.
4. Division is repeated subtraction.
5. The need to memorize the basic division facts below 100.
6. Ways to apply multiplication and division in real-life problems.

If you think these strategies for assisting students in learning math facts seem logical and automatic, you are right. For most students, they are. However, students with learning difficulties in math do not automatically use these strategies, and this prevents them from acquiring the math facts they need for accurate and speedy computation.

When these strategies are taught directly, students' math performance improves. When Ms. Pappas taught strategies to students who were having difficulty in addition, she used the strategies summarized in Apply the Concept 11.3, and she enlisted the help of other students who were performing the math skill accurately. She interviewed students who knew how to perform the skill and asked them to talk aloud while they solved the problems so that she could learn what strategies they used. She then taught these strategies to students who were having problems.

Peer-Assisted Instructional Practices Perhaps one of the most effective procedures for teaching math facts to students with learning and behavior problems is the use of cross-age tutors. Cross-age tutors are older students, often those who do not have learning or behavior problems, who serve as tutors for younger students with learning difficulties. Cross-age tutors are particularly effective in teaching math facts because the skills they need to be effective can be acquired quickly, in as little as two 45-minute periods. In a study that successfully used cross-age tutors to teach addition facts to students with learning disabilities (Beirne-Smith, 1991), tutors were trained to do the following:

- Use contingent reinforcement.
- Use task and error correction procedures.
- Use procedures for counting-on.
- Use procedures for rote memorization of facts.
- Repeat skills and instruction until mastery.

Cooperative learning groups, usually small groups of students (three to five per group), can be used to have students work together to solve problems. Maheady, Harper, and Sacca (1988) conducted a cooperative learning math instruction program for ninth- and tenth-grade students with mild disabilities. Students who participated in the cooperative teams performed better in mathematics and received higher grades than those who did not.

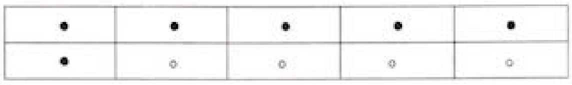
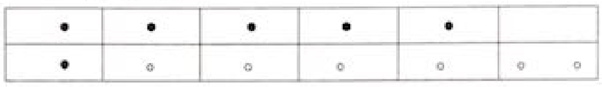
Constant Time-Delay Procedure Constant time delay is a procedure for teaching math facts that provides for systematic assistance from the teacher through near errorless control of the prompt to ensure the successful performance of the student (Gast, Ault, Wolery, Doyle, & Belanger, 1988; Schuster, Stevens, & Doak, 1990; Stevens & Schuster, 1988; Wolery, Cybriwsky, Gast, & Boyle-Gast, 1991). Students are presented with a math problem and are allowed a specific amount of time to give the correct answer. If students do not respond within the allotted time, a controlling prompt (typically, the teacher modeling the correct response) is provided. Students then repeat the model. Correct responses before and after the prompt are reinforced; however, only correct responses provided before the prompt are counted toward criterion.

Math Computation Errors How could students make the errors in Figure 11.5? It appears as though all the students did was guess. Yet the students who computed the problems can tell you what they did to get the answer. Most errors that students make are rule governed. Although the rule they are applying is not always obvious, the students are using some rule to tell them how to compute a problem. In problem A in Figure 11.5, Erika said, "I took 1 away from 7 to get 6 and took 0 away from 5 to get 5." In problem B, Jeff added across, adding the 3 and 1 to get 4 and adding the 2 and 3 to get 5. In problem C, Yolanda said, "I knew this wasn't right, but it was the best I knew how to do. I multiplied 3 × 4 to get 12, and then 6 × 4 to get 24." In problem D, Shawn knew that 7 plus 7 plus 7 was 21, but he was operating under the faulty rule that you always carry the smaller number, so he wrote the 2 in the ones column and carried the 1. In problem E, Jae said, "I added 4 plus 1 because it was easier than adding 4 plus 3." When given several similar problems, she had no concerns about placing the number in the ones or tens column, depending on where

11.3 Apply the Concept

Strategies for Teaching Addition Facts

Addition Facts Groups by Strategy for Recall

	Fact Group	Examples	Most Popular Strategy for Working Out Unknown Answers
	Count-Ons	(+1, +2, + 3, facts)	"Feel" the count
	Zero Facts	(6 + 0, 0 + 4)	Show it
No fingers needed!	Doubles	(4 + 4, 7 + 7)	Use pictures (e.g., 7 + 7 is the 2-week fact; 7 + 7 = 14)
	10 Sums	(especially 6 + 4)	Use 10-frame 
	9s	(4 + 9, 9 + 6)	Use pattern
	Near Doubles	(4 + 5, 7 + 8)	Relate to doubles (via pictures)
	Four Last Facts	(7 + 5, 8 + 4, 8 + 5, 8 + 6)	Make 10, add extra 

Note: Turnarounds (commutatives of facts within each group) would be learned before moving to a different group of facts.

Verbal Prompts Used in the Addition Program

Fact Group	Sample	Facts	Sentence Patterns (Verbal Prompts)
Count-Ons	8 +2	3 +7	Start BIG and count on.
Zeros	6 +0	0 +3	Plus zero stays the same.
Doubles	5 +5	7 +7	Think of the picture.
Near Doubles	5 +6	7 +8	Think doubles to help.
9s	4 +9	9 +7	What's the pattern?
Near 10s	7	6	Use 10 to help.

Source: C. A. Thornton & M. A. Toohey (1985), Basic math facts: Guidelines for teaching and learning, *Learning Disabilities Focus, 1*(1), pp. 50, 51. Reprinted with permission of the Division for Learning Disabilities.

Figure 11.5 Math Computational Errors

(A) 15 − 7 = 65	(B) 15 + 23 = 45	(C) 63 × 23 = 2412	
(D) 37 + 27 + 17 = 72	(E) 13 + 4 = 53		

it was easier for her to add. All of these students applied faulty rules as they performed math computations. Once the teacher discovered the faulty rules they were applying, she was able to teach them the underlying concepts and the correct rule for completing computations.

Teachers can learn a great deal about students' thinking in mathematics through an oral diagnostic interview (Lankford, 1974). Such an interview will provide information about what each student is doing and why the student is doing it that way. For the diagnostic interview to yield accurate, helpful information, the teacher must ask the student questions about math computation in a nonthreatening way. For example, "I am interested in learning what you say to yourself while you do this problem. Say aloud what you are thinking." It is often most effective to use a problem that is different from the one the student has performed incorrectly. The assumption behind this interview is that there is an underlying reason behind the mistakes, and understanding why a student is making errors provides valuable diagnostic information that leads directly to instruction. Roberts (1968) identified four common failure strategies in computation, which are summarized in Apply the Concept 11.4.

11.4 Apply the Concept

Errors in Computation

1. ***Wrong operation.*** The student attempts to solve the problem by using the wrong process. In this example, the student subtracted instead of adding.

$$\begin{array}{r} 24 \\ +\ 11 \\ \hline 13 \end{array}$$

2. ***Computational error.*** The student uses the correct operation but makes an error recalling a basic number fact.

$$\begin{array}{r} 24 \\ +\ 11 \\ \hline 58 \end{array}$$

3. ***Defective algorithm.*** The student attempts to use the correct operation but uses a wrong procedure for solving the problem. The error is not due to computation.

$$\begin{array}{r} 24 \\ -\ 17 \\ \hline 13 \end{array}$$

4. ***Random response.*** The student has little or no idea how to solve the problem, and writes numbers randomly.

$$\begin{array}{r} 304 \\ -\ 196 \\ \hline 396 \end{array}$$

Source: Based on G. H. Roberts (1968), The failure strategies of third-grade arithmetic pupils, *The Arithmetic Teacher, 15*, pp. 442–446.

Students with learning problems are slower but not necessarily less accurate when it comes to doing computation and learning math facts. When teaching mathematics in your classroom, teachers should consider that students with learning problems may need additional practice to learn math facts and more time to perform mathematics computations because they often lack the skill in automatization to perform math computation effectively and efficiently.

Language of Math Computation "What do you mean by 'find the difference'?" a student might ask. "Am I supposed to add or subtract? Why don't you just say it in plain English?" Many students with learning and behavior problems have difficulty with the language of computation. However, understanding the vocabulary is important for success in the regular classroom, application to math story problems, and communication with others. Understanding the terminology of the four basic operations as well as the symbols associated with the processes is important. Students also need to understand the vocabulary that is associated with the answer derived from each of these processes. Table 11.1 illustrates the relationship between the process, symbol, answer, and problem.

Table 11.1 Relationship of Process, Symbol, Answer, and Problem

Process	Symbol	Answer	Problem
Addition	+	Sum	6 + 4 =
Subtraction	−	Difference	5 − 3 =
Multiplication	×	Product	8 × 5 =
Division	÷	Quotient	12 ÷ 6 =

After teaching the information on the chart, the teacher can use the following three activities:

1. Cover one column (e.g., the symbols), and ask the student to write the answer.
2. Place each of the symbols, answers, and problems on a separate index card, and ask the student to sort them by process.
3. Play concentration with two columns. Two columns of index cards (e.g., the symbol cards and answer cards) are laid answer down, and the students take turns searching for matching pairs by selecting two cards. When a student picks up a corresponding pair, the student keeps the pair and takes another turn.

Use of Calculators Many students with learning and behavior problems let computation interfere with their ability to learn problem solving. They spend so much time learning to compute the problem accurately that they miss the more important aspects of mathematics, such as concept development and practical application.

Many teachers do not use calculators because they believe that the use of calculators threatens the acquisition of basic skills. Mr. Coffland, a third-grade teacher, put it this way: "If I let my students use calculators to solve problems, they will not have adequate practice in basic skills. They will become too dependent on using the calculator." Research suggests that Mr. Coffland has little to fear. The results of a summary of 79 studies (Hembree, 1986) on the use of calculators suggest the following:

- The use of calculators does not interfere with basic mathematics skill acquisition. In fact, calculator use can improve skill acquisition.

- Only in fourth grade does sustained calculator use interfere with skill development.
- The use of calculators in testing situations results in much higher achievement scores, particularly when students are low in problem-solving ability.
- The use of calculators improves students' attitudes toward mathematics.
- Calculators can be introduced at the same time that paper-and-pencil practice exercises are introduced.
- Students can use calculators to solve complex problems that they construct. This also provides support for improved self-concept with math skills.

In summary, as long as students get basic skills instruction, the use of calculators is a positive aid to mathematics instruction. Students who are having difficulty with mathematics instruction can use calculators in several ways:

1. *To develop a positive attitude.* Using a calculator removes the drudgery associated with solving computations and makes problem solving fun.
2. *To improve self-concept.* Being able to compute extremely complex problems on a calculator gives students confidence in their mathematics abilities.
3. *To improve practice in problem solving.* Students are willing to tackle difficult problem-solving tasks when they have a calculator to help solve the problem. Students still have to decide what numbers are used, what operation is involved, and whether additional operations are necessary. Using a calculator can free students from the burden of computation and allow more focus on thinking about the problem.
4. *To develop their own problems.* Using a calculator lets students develop their own problems. They can then exchange their problems with each other and use their calculators to solve them.

Fractions

The National Mathematics Advisory Panel (2008) states that a conceptual understanding of fractions and decimals, as with learning whole numbers, and the operational procedures for using them are mutually reinforcing; however, the concept of fractions is one of the most difficult math concepts for children and adults (Hecht, Vagi, & Torgeson, 2007). Moreover, the difficulties with fractions (including decimals and percents) is a great obstacle to further progress in mathematics (National Mathematics Advisory Panel, 2008). Teachers, through the use of their instructional materials, ensure the learning of conceptual and procedural knowledge of fractions and of proportional reasoning. Even though many teachers think the concept of fractions is difficult to teach, the concept of a fraction can be introduced before the actual fractions are even discussed. For example, Figure 11.6 shows the relationship between common fractional terminology and represented units. Moreover, through the availability of calculators, teachers can now place less emphasis on being able to compute fractions and more emphasis on understanding the meaning and use of fractions.

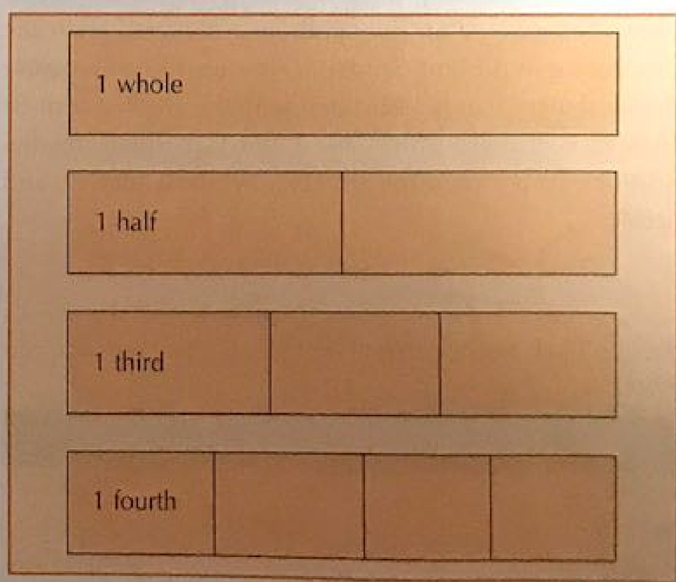
Children as young as 3, 4, and 5 are introduced to the concept of fractions as they receive halves or quarters of apples as part of lunch. They also help a parent in food preparation and cooking. "We use one cup of milk and one-half cup of flour." "You'll need to share the cookie with your brother. You each may have half of the cookie." When children enter school, teachers often use cooking activities to enhance students' understanding of fractions. Many manipulative aids can be used to teach fractions: colored rods, cardboard strips and squares, blocks, fractional circle wheels, cooking utensils such as measuring cups, and any unit dividers such as egg cartons and muffin pans.

Teaching fractions, like teaching most concepts, proceeds from concrete to abstract. Apply the Concept 11.5 demonstrates the teaching sequence.

However, the use of intuitive procedures for the acquisition of knowledge in fractions is unlikely to be successful with low achievers (Kelly, Gersten, & Carnine, 1990). Success

MyLab Education
Video Example 11.2
In this video, students use manipulatives to learning about equivalent fractions. What strategies does the teacher use to support learning? What additional strategies might be used?

Figure 11.6 Unit Representation of Fractions



11.5 Apply the Concept

Sequence for Teaching Fractional Concepts

The student

1. Manipulates concrete models (e.g., manipulating fractional blocks and pegs).
2. Matches fractional models (e.g., matching halves, thirds, fourths).
3. Points to the fractional model when the name is stated by another (e.g., the teacher says "half," and the student selects a model of "half" from several distractors).
4. Names fractional units when selected by another (e.g., the teacher points to a fractional unit such as a "fourth," and the student names it).
5. Draws diagrams or uses manipulatives to represent fractional units (e.g., the teacher says or writes fractional units such as "whole," "half," and "third," and the student uses manipulatives or drawings to represent these units).
6. Writes fraction names when given fractional drawings (e.g., next to the drawing, the student writes "half").
7. Uses fractions to solve problems (e.g., place 1½ cups of sugar in a bowl).
8. Uses the concept of like units to help solve computations with fractions.

in understanding fractions is likely to occur when the following three variables are presented (Kelly et al., 1990):

1. *Systematic practice in discriminating among different problem types.* Students with learning and behavior problems often confuse algorithms when computing fractions. For example, they learn to compute denominators and then use this procedure when adding, subtracting, multiplying, and dividing.
2. *Separation of confusing elements and terminology.* Much of the language of learning fractions is unfamiliar and confusing to youngsters. If the language is well explained and the concepts are well illustrated, students are more likely to be successful in learning fractions.
3. *Use a wide range of examples to illustrate each concept.* Students have a difficult time generalizing beyond the number of examples provided by the teacher; a wide range and large number of examples facilitate understanding.

The Institute for Education Sciences issued a practice guide on research-based practices for teaching fractions (Siegler et al., 2010). They provide the following guidance for instruction in fractions.

Recommendation 1 Build on students' informal understanding of sharing and proportionality to develop initial fraction concepts.

- Use equal-sharing activities to introduce the concept of fractions. Use sharing activities that involve dividing sets of objects as well as single whole objects.
- Extend equal-sharing activities to develop students' understanding of ordering and equivalence of fractions.
- Build on students' informal understanding to develop more advanced understanding of proportional reasoning concepts. Begin with activities that involve similar proportions, and progress to activities that involve ordering different proportions.

Recommendation 2 Help students recognize that fractions are numbers and that they expand the number system beyond whole numbers. Use number lines as a central representational tool in teaching this and other fraction concepts from the early grades onward.

- Use measurement activities and number lines to help students understand that fractions are numbers, with all the properties that numbers share.
- Provide opportunities for students to locate and compare fractions on number lines.
- Use number lines to improve students' understanding of fraction equivalence, fraction density (the concept that there are an infinite number of fractions between any two fractions), and negative fractions.
- Help students understand that fractions can be represented as common fractions, decimals, and percentages, and develop students' ability to translate among these forms.

Recommendation 3 Help students understand why procedures for computations with fractions make sense.

- Use area models, number lines, and other visual representations to improve students' understanding of formal computational procedures.
- Provide opportunities for students to use estimation to predict or judge the reasonableness of answers to problems involving computation with fractions.
- Address common misconceptions regarding computational procedures with fractions.

- Present real-world contexts with plausible numbers for problems that involve computing with fractions.

Recommendation 4 Develop students' conceptual understanding of strategies for solving ratio, rate, and proportion problems before exposing them to cross-multiplication as a procedure to use to solve such problems.

- Develop students' understanding of proportional relations before teaching computational procedures that are conceptually difficult to understand (e.g., cross-multiplication). Build on students' developing strategies for solving ratio, rate, and proportion problems.
- Encourage students to use visual representations to solve ratio, rate, and proportion problems.
- Provide opportunities for students to use and discuss alternative strategies for solving ratio, rate, and proportion problems.

Recommendation 5 Prioritize improving teachers' understanding of fractions and of how to teach them.

- Build teachers' depth of understanding of fractions and computational procedures involving fractions.
- Prepare teachers to use varied pictorial and concrete representations of fractions and fraction operations.
- Develop teachers' ability to assess students' understandings and misunderstandings of fractions.

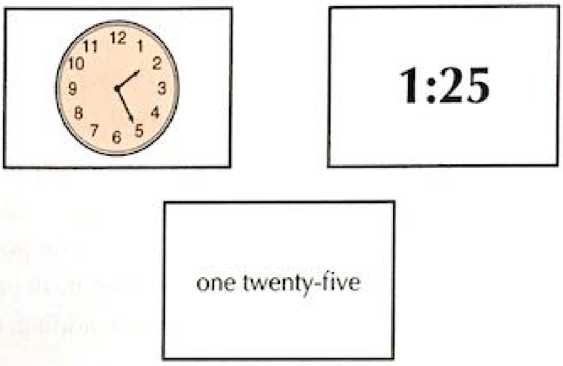
Measurement

Measurement includes weight, distance, quantity, length, money, and time. Measurement can be taught almost entirely with applied problems. For example, students learn time by using the clock in the classroom or by manipulating a toy clock; they learn money by making purchases with real or toy money; and they learn measures such as pint, liter, and teaspoon through following recipes. With each measurement unit that is taught (e.g., weight, distance, money), students need to learn the vocabulary and concepts for that unit. Only after students understand the terminology and concepts and have had experience applying the concepts in real measurement problems should they be exposed to measurement instruction through the use of less applied procedures such as textbooks and worksheets.

In addition, an instructional activity has been provided to help illustrate the use of a real-world problem in teaching the concept of fractions (see "Cake for Four—No, Make That Six!" in the Instructional Activities section later in the chapter). Another instructional activity shows how lots of examples help in teaching the concept of using like units with fractions (see "Finding Like Units" in the Instructional Activities section later in the chapter).

Time Even before coming to school, most children can tell time by the hour or know when the clock says that it is time to go to bed or time for dinner. The following teaching sequence assists students in understanding time:

1. *Teach students to sequence events.* Younger students can sequence the normal routine of the school day—for instance, "First we have a group story, then reading, then we go to recess." Additional practice in sequencing events can occur with story cards, events that occur at home, field trips, and so on.
2. *Ask students to identify which events take longer.* Name two events (e.g., math time and lining up for recess), and ask students to identify which event takes longer. Name several events, and ask students to put them in order from the event that takes the longest to the one that is quickest to complete.



A scope-and-sequence list of skills for teaching time is presented in Apply the Concept 11.6.

Money Students with learning disabilities often have difficulty applying money concepts because they have not mastered many of the earlier concepts, such as the value of coins, how coins compare (e.g., a quarter is more than two times as much as a dime), and how the value of the coins relates to what can be purchased. One parent reported that her child was frequently taken advantage of because he would trade coins of higher value for coins of lesser value. Students with learning difficulties often do not know the price of common goods. Although they may not need to know the exact price of a loaf of bread or a television set, they should be able to estimate what these items cost.

When initially teaching students to identify money, start with real coins. After they learn to recognize real coins, switch to play money and then to representations of money on workbook pages. The following sequence is useful in teaching money identification:

1. *Teach students to match the same coins.* Give students several different coins, and ask them to place all of the same coins in the same group.
2. *Ask students to point to the coin when you name it.* Depending on the students' skill level, you may want to start with two coins (e.g., a penny and a nickel) and then

11.6 Apply the Concept

Time: Scope and Sequence of Skills

The student

- Sequences events—first, and then next.
- Identifies duration of events—what takes longer and what is quicker to do.
- Tells time to the hour.
- Tells time to the half hour.
- Knows the days of the week.
- Knows the names of the months.
- Tells time to the quarter hour.
- Knows the number of days in a week.
- Knows the number of months in a year.
- Can use a calendar to answer questions about the date, the day, and the month.
- Writes time to the hour.
- Writes time to the half hour.
- Writes time using 5-minute increments.
- Writes time accurately.
- Can solve story problems using time.

progress to three and four. At this point, students do not need to be able to tell the name of the coin; they merely need to be able to locate it when it is named.

3. *Students name the coin.* At this level, the students tell the name of the coin.

When students can accurately identify the name of the coins, the value of the coins is discussed. Coins and dollars are discussed in terms of both their purchase power and how they relate to each other. Activities and problems that require students to use money and make change assure students that they can apply what they have learned about money. For example, students can learn to keep and balance a checkbook and to give change when role-playing a clerk in a store.

A scope-and-sequence list for teaching money is presented in Apply the Concept 11.7.

Problem Solving

What factors contribute to difficulties with problem solving, and how can teachers assist students in learning problem-solving strategies? Many students with learning problems have trouble with traditional story problems in mathematics because their difficulty in reading makes understanding the math problem almost impossible. In addition, students with learning problems often have difficulty with logical reasoning, which is the basis of many story problems. It is also common that their mathematics education has focused primarily on operations and not on understanding the reasons for operations or even a thorough understanding of the numbers involved in operations. Because of their difficulties with reading and logical reasoning and perhaps because of insufficient instruction in mathematics, students with learning problems often find problem solving the most difficult aspect of mathematics.

Despite its difficulties, problem solving may be the most important skill we teach students who have learning and behavior problems. Whereas most other students can apply the operations they learn to real-life problems with little direct instruction, students with learning problems will be less able to apply these skills without instruction, rehearsal, and practice. Students with learning disabilities lack metacognitive knowledge about strategies for math problem solving. Poor math performance is not solely a function of math computation difficulties (Montague et al., 2011;

11.7 Apply the Concept

Money: Scope and Sequence of Skills

The student

- Correctly identifies penny, nickel, dime, and quarter.
- Knows how many cents are in a penny, nickel, dime, and quarter.
- Can add to the correct amount when shown combinations of pennies, nickels, dimes, and quarters.
- Can describe items that can be purchased with combinations of pennies, nickels, dimes, and quarters.
- Can solve simple word problems involving pennies, nickels, dimes, and quarters.
- Can identify a dollar bill, a $5 bill, a $10 bill, and a $20 bill.
- Can identify the value of combinations of coins and various dollar bills.
- Can solve verbal math problems involving combinations of coins and various dollar bills.

Vukovic & Siegel, 2010). Students who are taught strategies for problem solving are more likely to be successful than are students who are taught the sequence for solving problems (Montague et al., 2011; Powell et al., 2009).

Students need to know when and how to add, subtract, multiply, and divide. Knowing *when* involves understanding the operation and applying it in the appropriate situation. Knowing *how* is the accurate performance of the operation. Most students are better at *how* than at *when;* problem solving gives students practice at these skills.

Factors Affecting Successful Verbal Problem Solving

Teachers need to consider the factors that affect successful story problem solving when writing and selecting story problems and instructing students. Use the following strategies:

MyLab Education
Video Example 11.3

The student in this video demonstrates her ability to use verbal and visual problem solving to manipulate fractions and solve word problems. How does the teacher help her recognize her error and guide her to a correct solution?

- *Teach big ideas.* When students understand the big idea or principle, all of the subordinate concepts around that big idea make more sense and are easier to learn and remember (Carnine, 1997). An example of a big idea is volume. You can teach students the principle of volume and then provide examples of real-life problems that students can solve by applying the big-idea principles they learn about volume.
- *Sameness analysis.* Carnine and colleagues determined the importance of sameness in mathematical problem solving through a series of research investigations (Engelmann, Carnine, & Steely, 1991). The idea is to connect math concepts so that students see the ways in which aspects of mathematical problem solving are the same. Identify types of word problems, and then explicitly teach students the ways in which these word problems are alike.
- *Cue words.* The presence or absence of cue words can significantly affect students' abilities to solve verbal word problems. The cue word *altogether* is illustrated in the following example: "Maria has 4 erasers. Joe has 7 erasers. How many erasers do they have *altogether?*" The cue word *left* is illustrated in the following example: "Jasmine has 9 pieces of candy. She gave 3 pieces to Lin. How many does she have *left?*" Students need to be taught to look for cue words that will guide them in solving problems.
- *Reasoning.* Ask students to think about the idea behind the story problem. Does it appear that the person in the problem will get more or less? Why? What operation will help to solve this? What numbers in the story do we have to use? Are there numbers that we do not have and need to compute? Ask students to explain the way in which they set up and calculated the problem so that they can justify what they've done and why they've done it that way.
- *Syntactic complexity.* The sentence structure within the story problem needs to be kept simple. The sentence length and vocabulary can also affect verbal problem solving.
- *Extraneous information.* Extraneous information in word problems causes difficulties because the majority of students attempt to use all of the information in solving the problem. For example: "Mary's mother baked 10 cookies. Mary's sister baked 8 cookies. Mary's brother baked 3 cupcakes. How many *cookies* were baked?" The information regarding Mary's brother baking three cupcakes is extraneous, yet many students will use the information in attempting to solve the problem. Extraneous information in story problems is associated with decreases in accuracy and computation speed with students. Students will construct the problem using the numbers available in the story problem, disregarding the question and the content available in the story problem. When students can complete story problems successfully without extraneous information, teach them to complete story problems with extraneous information.
- *Content load.* The *content load* refers to the number of ideas contained within a story problem. The story problem should not be overloaded with concepts. Students need to be taught to discriminate between relevant and irrelevant concepts.
- *Suitable content.* Story problems should contain content that is interesting and appealing to students and relevant to the types of real problems that students have or are likely to encounter.
- *Monitor progress.* Use weekly tests of word problem solving to monitor study progress on each type of word problem students have mastered and/or are learning. Reteach when necessary.
- *Provide guided practice.* Use diagrams to demonstrate how to solve the problem, and guide students through the development and use of these diagrams. As students demonstrate increasing proficiency with independent use of diagrams and strategies for effective problem solving, reduce the amount of support provided.

- *Use computer-assisted instruction.* Computer-assisted instruction gives students opportunities to practice computation and problem solving independently and provides correction and feedback. Many students prefer to do mathematics with the computer. *My Math* (Cawley, 2002) is an example of a computer software program that incorporates three mathematical components: computation problems, arithmetic word problems, and arithmetic story problems.

Web Resources

Students who want to get online and work on mathematics can find a website that provides games, flashcards, homework help, and worksheets, along with options to purchase software for use at home. This website is located at **http://www.aplusmath.com/**. You may find it worth a look and possibly send your students to it during class time or for use at home.

Methods of Teaching Story Problem Solving

Students also need to learn specific strategies that will assist them in using a successful process for mastering story problems in class and applying those principles to the mathematics of everyday life. The Institute for Education Sciences provides a guide for how to teach problem solving in mathematics based on current research (Woodward et al., 2012), and the recommendations from this brief follow.

Recommendation 1 Prepare problems and use them in whole-class instruction.

1. Include both routine and nonroutine problems in problem-solving activities.
2. Ensure that students will understand the problem by addressing issues students might encounter with the problem's context or language.
3. Consider students' knowledge of mathematical content when planning lessons.

Recommendation 2 Assist students in monitoring and reflecting on the problem-solving process.

1. Provide students with a list of prompts to help them monitor and reflect during the problem-solving process.
2. Model how to monitor and reflect on the problem-solving process.
3. Use student thinking about a problem to develop students' ability to monitor and reflect.

Recommendation 3 Teach students how to use visual representations.

1. Select visual representations that are appropriate for students and the problems they are solving.
2. Use think alouds and discussions to teach students how to represent problems visually.
3. Show students how to convert the visually represented information into mathematical notation.

Recommendation 4 Expose students to multiple problem-solving strategies.

1. Provide instruction in multiple strategies.
2. Provide opportunities for students to compare multiple strategies in worked examples.
3. Ask students to generate and share multiple strategies for solving a problem.

Recommendation 5 Help students recognize and articulate mathematical concepts and notation.

1. Describe relevant mathematical concepts and notation, and relate them to the problem-solving activity.
2. Ask students to explain each step used to solve a problem in a worked example.
3. Help students make sense of algebraic notation.

A step-by-step strategy for teaching sixth-grade students to solve story problems is illustrated in Apply the Concept 11.8 (E. M. Smith & Alley, 1981). Students first need to learn the strategies, then practice them with support from a teacher, and finally practice them independently until they can apply the principles with success. After continued success, students make adaptations in or condense the steps they use.

Montague and Bos (1986a, 1986b) demonstrated the efficacy of the learning strategy approach described in Apply the Concept 11.9, for secondary adolescents with learning disabilities.

Powell and Fuchs (2018) identify two practices that can be used to teach word problem solving: attack strategies and schema instruction. Attack strategies provide simple steps for students to use when they approach a new problem. For example, one common attack strategy is called "RUN," which stands for (1) **R**ead the problem, (2) **U**nderline the question, and (3) **N**ame the problem type (Fuchs et al., 2014). Because many students with math difficulty lack systematic approaches to problem solving, attack strategies provide a method for understanding how to solve problems. Schema instruction involves teaching students to recognize different types of problems (e.g., additive schemas or multiplicative schemas) to help them solve word problems

11.8 Apply the Concept

Steps for Teaching Students to Solve Story Problems

Story Problem: Mark had $1.47 to spend. He spent $0.34 on gum. How much money does he have left?

- **I.** Read the problem.
  - **A.** Find unknown words.
  - **B.** Find cue words (e.g., left).
- **II.** Reread the problem.
  - **A.** Identify what is given.
    1. Is renaming needed?
    2. Are there unit changes?
  - **B.** Decide what is asked for.
    1. What process is needed?
    2. What unit or category is asked for? (e.g., seconds, pounds, money)
- **III.** Use objects to show the problem.
  - **A.** Decide what operation to use.
- **IV.** Write the problem.
- **V.** Work the problem.

11.9 Apply the Concept

Teaching Adolescents to Solve Story Problems

The eight steps in the verbal math problem-solving strategy are directed to the student and are described below:

1. ***Read the problem aloud.*** Ask the teacher to pronounce or define any word you do not know. (The teacher pronounces and provides meanings for any words if the student asks.)

   Example: In a high school there are 2,878 male and 1,943 female students enrolled. By how many students must the enrollment increase to make the enrollment 5,000?

2. ***Paraphrase the problem aloud.*** State important information, giving close attention to the numbers in the problem. Repeat the question part aloud. A self-questioning technique such as "What is asked?" or "What am I looking for?" can help you provide focus on developing the solution.

   Example: Altogether there are a certain number of kids in high school. There are 2,878 boys and 1,943 girls. The question is by how many students must the enrollment increase to make the total enrollment 5,000. What is asked? How many more students are needed to total 5,000 in the school?

3. ***Visualize.*** Graphically display the information. Draw a representation of the problem.

4. ***State the problem.*** Complete the following statements aloud: "I have . . ." "I want to find . . ." Underline the important information in the problem.

   Example: I have the number of boys and the number of girls who go to the school now. I want to find how many more kids are needed to total 5,000.

5. ***Hypothesize.*** Complete the following statements aloud: "If I . . ." "Then . . ." "How many steps will I use to find the answer?" Write the operation signs.

   Example: If I add 2,878 boys and 1,943 girls, I'll get the number of kids now. Then I must subtract that number from 5,000 to find out how many more must enroll. First add, then subtract. + −. This is a two-step problem.

6. ***Estimate.*** Write the estimate. My answer should be around . . . or about . . . Underline the estimate.

   Example: 2,800 and 2,000 are 4,800. 4,800 from 5,000 is 200. My answer should be around 200.

7. ***Calculate.*** Show the calculation and label the answer. Circle the answer. Use a self-questioning technique such as, "Is this answer in the correct form?" Correct labels for the problems should be reinforced.

   Example:

   $$\begin{array}{r} 2{,}878 \\ +1{,}943 \\ \hline 4{,}821 \end{array} \qquad \begin{array}{r} 5{,}000 \\ -4{,}821 \\ \hline 179 \text{ students} \end{array}$$

8. ***Self-check.*** Refer to the problem, and check every step to determine accuracy of operation(s) selected and correctness of response and solution. Check computation for accuracy. Use the self-questioning technique by asking whether the answer makes sense.

Source: M. Montague & C. S. Bos (1986), The effect of cognitive strategy training on verbal math problem-solving performance of learning disabled adolescents, *Journal of Learning Disabilities, 19*, pp. 26–33.

(Powell & Fuchs, 2018). By combining both attack strategies and schema instruction, teachers can provide more effective problem-solving instruction.

In summary, when teaching story problems to students with learning and behavior problems, teachers should keep the following guidelines in mind:

- Be certain the students can perform the arithmetic computation before introducing the computation in story problems.
- Teach students to use common attack strategies for solving problems.
- Develop a range of story problems that contain the type of problem or schema you want students to learn to solve.
- Instruct with one type of problem until mastery is attained.
- Teach the students to read through a word problem and visualize the situation. Ask them to read the story aloud and tell what it is about.
- Ask the students to reread the story—this time to get the facts.
- Identify the key question. In the beginning stages of problem solving, the students should write the key question so that they can refer to it when the computation is complete.
- Identify extraneous information.
- Reread the story problem, and attempt to state the situation in a mathematical sentence. The teacher plays an important role in this step by asking the students questions and guiding them in formulating the arithmetic problem.
- Tell the students to write the arithmetic problem and compute the answer. (Students can compute some problems in their heads without completing this step.)
- Tell the students to reread the key question and be sure that they have completed the problem correctly.
- Ask the students whether their answer is likely, based on their estimate.

Teaching math story problems does not have to be limited to the content area of math and can be integrated into reading comprehension instruction. For example, a story about a mother duck and her babies was part of a student's reading lesson. During mathematics, the teacher made minor changes in the story and used it for instruction in story problems in mathematics (see Figure 11.7).

This same procedure can be used with junior high and high school students' content area textbooks. Math story problems can be taken from social studies and science tests; Cawley and Miller (1986) refer to these as knowledge-based problems. Usually, these problems require specific knowledge in the content area. Cawley (1984) identifies the integration of math into other content areas as an important means of promoting generalization of math concepts.

Figure 11.7 Example of Teacher-Altered Story for Use in Story Problem Instruction

The mother duck went to the pond with her *eight* babies. They looked for their new friend. *Two* more baby ducks joined them. How many baby ducks were there?

Pictures can be used to facilitate processing information in solving mathematical word problems. For example, using Figure 11.8, a teacher could say, "The small monkeys have four bananas, and the large monkeys have six bananas. How many bananas would they have if they put them all together?"

Instructional manipulatives can also be used to assist students with learning problems in solving mathematical word problems. Cuisenaire rods, base 10 blocks, fraction strips, and multilink cubes can be used to represent the numerical values in the problem and to assist students in better understanding and solving mathematical word problems (Van de Walle & Lovin, 2007).

Improving Math Performance

How can math interventions be used to improve math performance? One way to conceptualize developing and implementing effective interventions for students with math disabilities is to consider approaches that incorporate cognitive, behavioral, and alternative instructional delivery systems, which include cooperative learning, computer-assisted instruction, and interactive video games.

Figure 11.8 Pictures Help to Solve Math Story Problems

Cognitive Approaches

Cognitive behavior modification can be used with instructional procedures in mathematics. CBM often takes the form of self-instruction, which relies on using internalized language to facilitate the problem-solving process. When self-talk is said aloud, these verbalizations are beneficial to the arithmetic process. For example, teachers can model math procedures by thinking aloud and describing their steps in solving the problem.

$$8 + 17 = \_\_\_\_$$

"The first thing I notice is that the sign is a plus sign, which means that I'm going to add the two numbers together. I know that my answer is going to be larger than 17 because that's the bigger of the two numbers. I'm going to start with 17 and count up, making a slash for each number until I have 8 slashes. That will be my answer."

Using these types of think alouds with students and then demonstrating how to proceed will help them to develop procedures for solving problems.

Leon and Pepe (1983) taught a five-step self-instructional sequence to special education teachers. Students receiving arithmetic instruction from these teachers who were trained in the sequence improved greatly both in arithmetic computation and in generalizing the skills they acquired. Second-grade students with learning disabilities became more proficient at learning addition through strategy instruction than through drill and practice. This study demonstrated that even very young children with learning disabilities can benefit from cognitive approaches to math instruction (Tournaki, 2003). The following sequence for using self-instruction in mathematics is a modification of the approach used by Leon and Pepe (1983):

1. *Modeling.* The teacher demonstrates how to compute a problem by using overt self-instruction. This overt self-instruction, or talking aloud about the process, assists students who have learning problems in knowing what they should say to themselves and what questions they should ask to keep themselves focused on the process.
2. *Coparticipation.* The teacher and students compute the problem together by using overt self-instruction. This step helps the students to put the procedure in their own words, yet supplies the support of the teacher while the students are still learning the process.
3. *Student demonstration.* The students compute the problem alone by using overt self-instruction, and the teacher monitors the students' performance. The students are more independent in this step; however, the teacher is still available to give correction and feedback.
4. *Fading overt self-instruction.* The students continue to demonstrate the computation of the problem with internal self-instruction. Often students have a check sheet of symbols or key words to cue them to the key points.
5. *Feedback.* The students complete the problem independently by using covert self-instruction and providing self-reinforcement for a job well done.

Behavioral Approaches

Cues Behavioral techniques are also available to improve students' math performance. As you know, stimulus cues precede responses and often control or provide information to control responses. In arithmetic instruction, teachers need to identify relevant cues and determine whether the students are aware of these cues and are using them appropriately. In Figure 11.9, three different problems are presented; a student must understand and attend to many different cues before accurately performing these problems. For example, in problem 1 of Figure 11.9, the student must know what "+" means, what the numbers represent, and what procedure to follow to perform the problem. In problem 2, the student must know what the picture represents, the difference between the short and long hands of the clock, and what each of the numbers represents. Problem 3 requires the student to understand the cue *long* and to know what type of tool is needed to address the problem. Math provides many stimulus cues, and teachers need to be certain that students recognize and understand the cues and attend to them.

Teachers can also provide cues to assist students in learning new skills. For example, the following list illustrates cues a teacher provided when students were first learning long division:

÷ (divide) $6\overline{)478}$ $8\overline{)521}$
× (multiply)
− (subtract)
↓ (bring down)

Figure 11.9 Math Problems Enlisting Various Cues

(1)
$$\begin{array}{r} 37 \\ 24 \\ +\ 89 \\ \hline \end{array}$$

(2)


What time is it?__________

(3) How long is this line?__________

Corrective Feedback Providing corrective feedback reinforces student performance. Corrective feedback involves telling the students what they are doing well, including procedures, accuracy of responses, and work style. It also involves identifying areas in which a student needs further assistance. Corrective feedback should be given frequently. Effective teachers do not wait until students have completed tasks but give feedback while they are working on the task. Effective teachers provide precise feedback. Rather than saying, "You are doing a good job," the teacher says, "You remembered to carry. All of the answers in the first row are correct. Good job."

Task Analysis Task analysis is a process of specifying the behaviors needed for a particular task that can help to shape student responses. Students are taught behaviors from the simple to the more complex until they can perform the target behavior. For example, a teacher's goal may be for a student to complete two-place addition with carrying by solving a verbal math problem. The student's present level of performance is knowledge of math facts when adding numbers between 0 and 9. Through task analysis, the teacher identifies the prerequisite concepts that need to be known and the many problem-solving skills that need to be shaped through instruction and practice before the student is performing the target behavior, in this case a verbal math problem with two-place addition:

- Number concepts for 0–9
- Number concepts for 10–100
- Place value
- Simple oral word problems, requiring addition knowledge for 0–9
- Simple written word problems, requiring addition knowledge for 0–9
- Two-place addition problems
- Oral-addition word problems requiring knowledge of two-place addition
- Written-addition word problems requiring knowledge of two-place addition

The teacher decided that it would take approximately 3 months to reach the goal. He knew that his mathematics program would focus on other skills during that period (e.g., time, measurements, and graphs). Apply the Concept 11.10 lists several things teachers can do to improve their students' math performance.

Focus on Real-World Mathematics

Many students with learning and behavior problems manage to graduate despite having only a minimal understanding of mathematics skills. Many, relieved to escape formal education in mathematics, have the unfortunate misconception that they are finished with mathematics. Soon they find that functioning as an adult requires managing money, interest on loans, and credit cards, as well as filing taxes, completing employment forms for deductions, and using basic math skills in their jobs.

Many of the skills that are most important for students with learning disabilities are not part of general mathematics curricula because they do not need to be taught through direct instruction to students who do not have learning disabilities. What are some of the real-world issues that need to be part of the math curriculum for most students with learning and behavior problems? Start with topics that are of high importance to them. For example, ask students to identify three items they would like to buy. Ask them to determine how much these items cost at several different stores. Be sure to consider extras like whether tax and shipping apply.

11.10 Apply the Concept

Improving Math Performance

Baker, Gersten, and Lee (2002) conducted a synthesis of all of the empirical research on teaching mathematics to students with math difficulties. They reported several themes from these studies that teachers should consider in their instructional routines:

- Use ongoing progress-monitoring data in mathematics. These data allow teachers to determine how students are progressing, adjust instruction, and give feedback to students on their performance.
- Use peer-assisted learning to provide support for mathematical learning. When peers work together on organized practices of computing and problem solving, both peers benefit.
- Use explicit and systematic instruction in the elements of mathematics, which is associated with improved outcomes in math for students. This type of instruction guides students through problems and calculations rather than relying on students to figure it out independently.
- Provide families with information on how their students are performing, and engage families as the supporters and motivators for their children's progress in mathematics.

As students move through the grades, providing them with many opportunities to practice real-world mathematics is essential. Start with asking them to estimate how much their "take-home" pay would be based on an hourly rate and 40 hours per week of employment. Ask them to identify the monthly prices of apartments in an area where they would like to live. Now compute how much they would have left if they rented an apartment in that area. These types of real-world issues are prevalent and prepare students for thinking about everyday life.

Math instruction for students with disabilities requires teachers to consider the functional math skills that students need. Apply the Concept 11.11 presents an outline of content for teaching functional math.

11.11 Apply the Concept

Content for Teaching Functional Math

Consumer Skills

- Using computer to enhance life skills and access to banking and other personal decision making
- Making change
- Determining cost of sale items using percentages (e.g., "25% off")
- Determining tax amounts
- Doing cost comparisons
- Buying "on time"
- Balancing a checkbook
- Determining total cost of purchases

Homemaking Skills

- Measuring ingredients
- Budgeting for household expenses
- Calculating length of cooking and baking time when there are options (e.g., for a cake using two 9" round pans vs. two 8" round pans)
- Measuring material for clothing construction
- Doing cost comparisons

Health Care

- Weighing oneself and others
- Calculating caloric intake
- Determining when to take medication

Auto Care

- Calculating cost of auto parts
- Measuring spark plug gaps
- Determining if tire pressure is correct
- Figuring gas mileage

Home Care

- Determining amount of supplies (paint, rug shampoo) to buy
- Determining time needed to do projects
- Measuring rods and drapes
- Finding cost of supplies
- Finding cost of repairs

Vocational Needs

- Calculating payroll deductions
- Determining money owed
- Knowing when to be at work
- Doing actual math for various jobs

Leisure Activities

- Comparing travel expenses
- Magazine and newspaper costs
- Membership fees
- Entertainment: movies, video rentals, sporting and artistic events

Home Management

- Determining where to live
- Moving expenses
- Move-in expenses
- Utilities
- Insurance
- Furniture
- Additional expense

Transportation

- Public or automobile
- Maintenance
- Insurance

Sources: Adapted from J. R. Patton, M. E. Cronin, D. S. Bassett, & A. E. Koppel (1997), A life skills approach to mathematics instruction: Preparing students with learning disabilities for the real-life demands of adulthood, *Journal of Learning Disabilities, 30*(2), pp. 178–187; and S. E. Schwartz & D. Budd (1981), Mathematics for handicapped learners: A functional approach for adolescents, *Focus on Exceptional Children, 13*(7), pp. 7–8. Reproduced by permission of Love Publishing Company.

Curricula and Materials

Traditional math curricula have presented problems for students with learning disabilities. What are some of the typical difficulties with math curricula and how might you address these issues?

- The text is challenging to read and understand. You can interpret the text with students and provide concrete examples.
- The sequencing of math procedures and the approaches are poor. Teach the math procedures in a sequence that allows for maximizing learning and retention.
- The reading vocabulary is difficult, and the reading level is too high.
- The sequencing of material presented is poor, multiple concepts are introduced, and the focus skips from one concept to another.
- An insufficient number of problems covers each concept.
- Insufficient opportunities and problems focus on application.
- The page formatting is too varied.
- Students often do not have the prerequisite skills that the text assumes they possess.

Teachers who attempt to use traditional curricula with students who have learning difficulties will need to control for these factors in their teaching. This means that the teacher may have to carefully and thoughtfully select the concepts, skills, and problems as well as the instructional explanations from the traditional math curricula materials or choose alternative math materials.

A number of curricula have been developed that focus on teaching math skills to students with learning difficulties. Some of these curricula are described in Apply the Concept 11.12.

11.12 Apply the Concept

Sources of Curricula and Materials

- *Connecting Math Concepts: Comprehensive Edition 2012* (Engelmann, Engelmann, Carnine, & Kelly, 2012) is designed to provide explicit instruction and explanations of basic math concepts and the relationships between concepts for students in grades kindergarten through 8. Mastered concepts are then used to build problem-solving skills. The lessons proceed in small, incremental steps with continuous review. Materials include teacher's guides, student textbooks and workbooks, and fact and independent worksheets for additional practice. All lessons are scripted for teachers and provide systematic instruction in story problems.
- *Math Exploration and Applications,* developed by Bereiter, Hilton, Rubinstein, and Willoughby (1998), provides instruction, games, and manipulatives for building fluency in math skills. It is also available in Spanish.
- *The Corrective Mathematics Program,* by Engelmann, Carnine, and Steely (2005), provides remedial basic math for students in grades 3 through 12, addressing seven areas: addition, subtraction, multiplication, division, basic fractions, decimals and percentages, and rational numbers and equations.
- *Structural Arithmetic,* by Stern, Stern, and Gould (1998), involves students in prekindergarten through third grade in making, discovering, and learning math concepts and facts. Colorful blocks are used to assist students in discovering math concepts.
- *Cuisenaire rods,* developed by M. Georges Cuisenaire, help impart conceptual knowledge of the basic structure of mathematics.
- *Saxon Math* for kindergarten through secondary grades was developed to provide math instruction that continuously builds on previous instruction while increasing the complexity to learn math concepts in depth. Math concepts, problem solving, and applications are sequential.

Instructional Activities

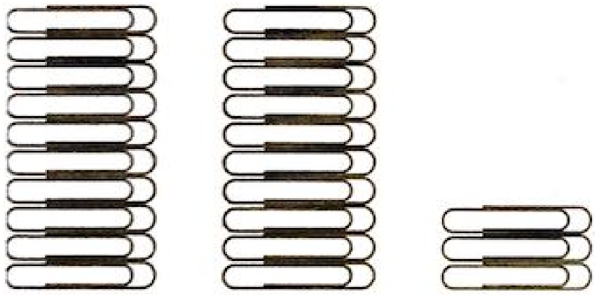
This section provides instructional activities related to mathematics. Some of the activities teach new skills; others are best suited for practice and reinforcement of already acquired skills. For each activity, the objective, materials, and teaching procedures are described.

Two-Digit Numbers: Focus on Reversals

Objective: To help students understand and use two-digit numbers successfully (for use with students who write 23 for 32, 41 for 14, etc.)

Grades: Primary

Materials: Objects that can be grouped by tens (e.g., pencils, paper, chips, sticks)



Teaching Procedures: Four steps are recommended: First, tell the students to group objects such as Popsicle sticks or chips in tens, and then ask them to say the number of tens and the number of ones left over. Next, the students count orally by tens and use objects to show the count (e.g., 2 tens is 20, 6 tens is 60, and so on). When multiples of ten are established, extra ones are included (e.g., 2 tens and 3 is 23). Because of naming irregularities, teens are dealt with last.

The students then group objects by tens and write to describe the grouping. The tens–ones labels, used in early stages, are gradually eliminated. On separate sheets of paper, the students write the number that corresponds to the grouping. The students use objects (tens and ones) to help compare and sequence numbers.

Two Up

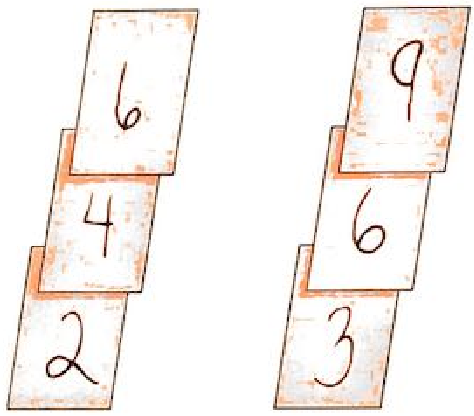
Objective: To practice multiplication facts by rehearsing counting by twos, threes, fours, and so on

Grades: Primary through intermediate

Prerequisite Behaviors: Counting by twos, threes, and so on

Materials: A set of 48 cards made by printing the multiples of 2 from 1 to 12, using a different color of pen for each set (e.g., 2, 4, . . . 12 in red, blue, green, and brown)

Teaching Procedures: Directions for playing the game are as follows: The cards are shuffled and dealt, giving an equal number of cards to each player. The player who has the red 2 starts the game by placing the red 2 in the middle of the table. The next player must place a red 4 on top of the red 2 or pass. Next a red 6 is needed, and so on. Each of the players plays in a similar manner. A player can play only one card each turn.



The object of the game is to play the cards from 2 on up. The first player to play all of his or her cards is the winner.

Adaptations: This game can be played with decks of threes, fours, sixes, and so on, called "Three Up," "Four Up," and so on. A different deck of cards must be made for each multiple.

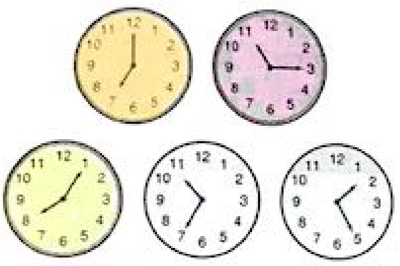
Clock-Reading Bingo

Objective: To give students practice in associating the time on a clock face with its written and spoken form

Grades: Primary

Materials: Cards that show times on a standard clock, large game boards with 16 squares and with times written at the bottom of each square, 16 "clock" chips (made by placing gummed labels on cardboard chips and drawing a clock on the face of the label), markers

Teaching Procedures: A caller holds up a clock face. The players must decide whether the time shown by that clock is on their game board. If it is, the player places a marker in the square that contains the written form. The winner is the first person who correctly completes a row in any direction and reads the time in each winning square.



three o'clock	seven o'clock	fifteen minutes after four	twenty minutes after twelve
fifteen minutes after six	twenty minutes after twelve	twenty minutes to eleven	twenty minutes after eleven
five minutes after five	fifteen minutes after ten	twenty-five minutes after one	twenty-five minutes to eleven
nine thirty	five minutes to seven	twenty-five minutes after twelve	ten o'clock

Coin Concentration

Objective: To practice reading money amounts in four different notations and to reinforce coin recognition

Grades: Primary, intermediate to high school (see "Adaptations" for older students)

Prerequisite Behaviors: Coin value, value placement, coin recognition of dollars and cents

Materials: Money picture card, money word card, money decimal card, money cents card

Teaching Procedures: The game of Coin Concentration can be played at several levels of difficulty, with varying skill emphasis, depending on specific classroom needs. At the simplest level, use only one kind of money card. (Make two copies of the card, and cut it apart on the solid lines so students play with a total of 20 cards.) Decide on the number and type of cards to be used, and place them face down on the table.

The first player turns over two cards, one at a time, trying to match values. If the cards match, the player keeps them. If not, the player turns them back over on the table in their original location. Then the next player tries to make a match by turning over two more cards and so on, until all the cards are matched with their pair.

The winner is the player with the most matched cards. For variety, ask the students to add the total value of their cards, and the player with the highest value wins. To add variety and increase difficulty, put different type cards down, and players can match 4¢ to $.04 or to *four cents.*

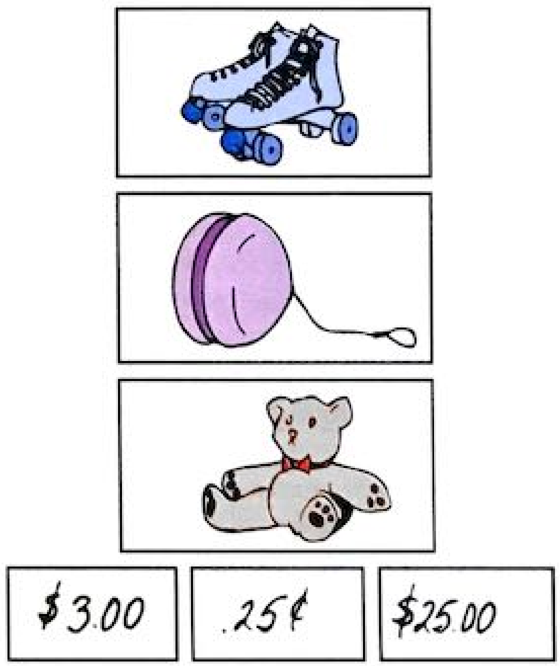
Adaptations: This activity can be used with older students by increasing the difficulty of the coin values represented and by adding a fifth card. On the fifth card is the name of an object that costs the corresponding amount. For example, if the money value is $329.00, the fifth card may have an MP3 written on it.

Shopping Spree

Objective: To give students practice using money and understanding the concept of addition and subtraction with money

Grades: Elementary-age students who are having difficulty with the money concepts (see "Adaptations" for use with older students)

Materials: Coins (and dollar bills when teaching the more advanced concepts), pictures of items



Teaching Procedures: Cut out magazine pictures of things children would like to buy, and put a price on each picture. Start with easy amounts, such as 5¢, 10¢, 25¢; then in future lessons, increase the complexity of the amounts to such things as 63¢ and 51¢. For higher grades, use dollar amounts. Have the class divide into two groups, with half the students serving as store clerks and the other half as shoppers. The shoppers buy picture items from the clerks. The shoppers are responsible for giving the correct amount of money. The clerks are responsible for giving the correct change. Then have students trade roles.

At a later date, distribute specific amounts of money, and ask students to select several items without going over their designated amount. Or, ask students to show two or three different items and tell which item they can afford with their amount of money.

Adaptations: For older students, you can distribute pretend checkbooks. Students receive a specified amount in their individual checkbooks and must make appropriate deductions as they make purchases.

99

Objective: To generalize and practice adding numbers in one's head or on paper

Grades: Intermediate to high school

Materials: Playing cards, paper, and pencils

Teaching Procedures: Explain that the objective of this game is to add cards up to a score of 99. Establish the following rules:

Jacks and queens = 10

Kings = 99

Nines = free-turn pass—to be used anytime

Fours = pass

Aces = 1

Other cards = face value

Each player is dealt three cards. The rest of the cards go face down on a draw pile. The players take turns discarding one card from their hand face up on a discard pile and drawing one card from the draw pile to put back into their hand. As a player places the card in the discard pile, the player must add the number from the card to any previous score acquired up to that point in the game and say the new score out loud. Note the exceptions: If a player plays a nine, the player receives a free-turn pass. If a player plays a four, the player has to pass a turn with no score. The first player to score higher than 99 loses the game.

Shopping

Objective: To provide practice in addition, subtraction, and comparing prices (problem solving)

Grades: Junior high

Materials: Supermarket sale ads that include the price per item (optionally mounted individually on cardboard and covered with clear plastic), made-up shopping lists to hand out to the class, pencil, and paper

Teaching Procedures: Divide the class into small groups. Tell the students that their shopping list contains the items that they will need this week. Assign each group a designated amount for groceries (e.g., $30.00). The object is to buy everything on the list while spending the least amount of money. Place on each desk the supermarket sale ads, each with the name of its store. After students buy an item, they record its price and the store where they bought it. (It's easier if one student in each group buys the meats, one buys the dairy products, and so on.) When the students have bought all the items on the list, tell them to total their bills and be ready to present the results.

Cake for Four—No, Make That Six!

By Sandra Stroud

Objective: Developing students' concept of fractions by having them partition an object into equal parts

Grades: Second through fourth grades (possibly higher)

Materials: For each student, a 6-inch paper circle; five strips of construction paper, 1 inch wide by 8 inches long, in a color contrasting to that of the paper circle; eight small cookies, placed in a small sandwich bag

Teaching Procedures: Students move desks together so that each student has a partner with whom to compare work. Materials are distributed. The teacher introduces the lesson by telling the students that they are going to take part in a "Let's Pretend" activity that will help them to learn that when they eat a piece of cake that has been divided into equal parts, they are actually eating a fraction of that cake.

The students are asked to imagine that they have just helped to bake a cake. It is their favorite kind of cake, and because there are four people in their pretend family, they are planning to divide it into four equal pieces. They are asked to think of the paper circle on their desk as the top of the cake and to show the teacher—and their partners—how they would use the strips of paper to divide the cake into four equal parts. When each child has successfully demonstrated this first partitioning task, they are asked what fraction of the whole cake each piece is and how that fraction is written.

Next, they are asked to imagine that their grandmother and grandfather have arrived unexpectedly and that the grandparents have accepted the family's invitation to stay for supper. The family certainly wants to share the cake with their grandparents, so into how many pieces will they now divide their cake? The teacher makes sure that each student shows six equal portions and that they understand that each piece of cake is now 1/6 of the whole—just enough for the six people at the dinner table. However, before that cake is served, Uncle Bob and Aunt Doris arrive! Now the cake will be divided into how many equal pieces? Finally, the time comes to decorate the cake with the cookies and to cut and serve the cake. (As a reward for all their good thinking, the students now get to eat the decorations—1/8 at a time!)

Students enjoy the story associated with this activity, and they enjoy comparing their partitioned cakes with those of their peers. This is a good example of cooperative learning. Students especially enjoy eating their cookies at the end of this activity.

The Values of Coins

By Ae-hwa Kim

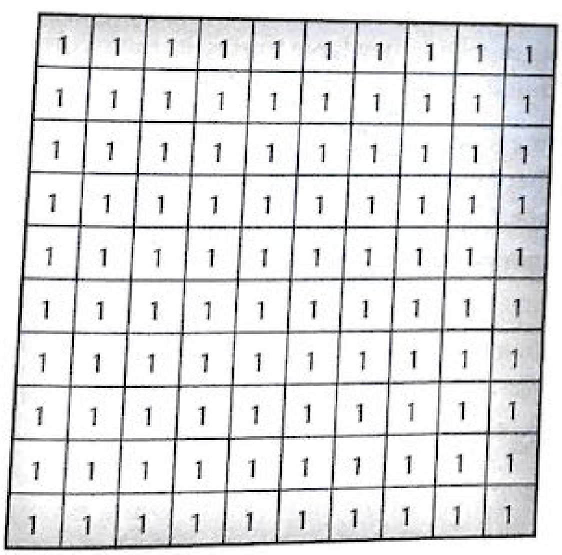
Objective: To help students learn the relative values of coins (e.g., students will determine that a quarter is worth 25 times as much as one cent)

Grades: Primary

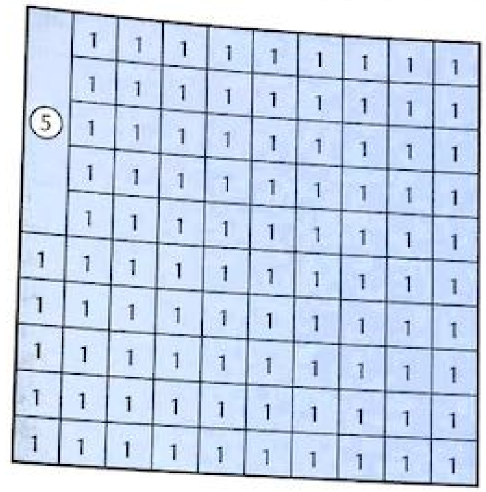
Materials: Models with real coins for each step

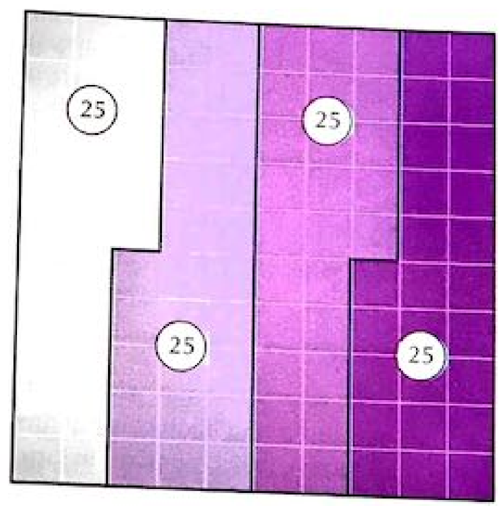
Teaching Procedures: For the initial instruction, the teacher shows all models and coins and addresses the objective of the lesson. During the lesson, the teacher models and verbally explains each step, provides students with guided and independent practice, and gives them feedback.

1. Show students proportionate models to represent the values of coins.



2. Teach the values of coins and their relative values with models, which visually represent the values of coins and their relative worth. For example, one nickel is worth five pennies and so takes up the space of five pennies.





3. Teach the value of a set of coins with models, which visually represent the value of a set of coins.
4. Teach the students to compare the values of sets of coins with models.
5. Teach the students to use models to create a set of coins with a given value. Allow the students to use different combinations of coins to make the given value.
6. Teach the students to create a set of coins with a given value by using the fewest coins with models.

Source: Based on an activity by R. L. Drum & W. G. Petty (1997), Teaching the value of coins, *Teaching Children Mathematics*, 5(5), pp. 264–268.

Learning Addition

By Ae-hwa Kim

Objective: To help students understand how to do addition with three-digit numbers and to provide practice in addition through activities

Grades: Primary

Materials: Pictures of three different types of animals; three lengths of bricks (short, medium, and long); scratch paper

Teaching Procedures:

1. Seat three volunteer "animals" (e.g., zebra, giraffe, and deer) on chairs side by side in front of a chalkboard. Hang a sign with a picture of each animal around the neck of the student who acts as that animal. The sign on the right (deer) also has the word *ones* or *1s*, signifying the units place of the number. The middle sign (giraffe) shows the word *tens* or *10s*, signifying the tens place of the number. The sign on the left (zebra) has the word *hundreds* or *100s*, signifying the hundreds place of the number. Throughout the activity, the place-value words are visible.
2. Give each animal a supply of bricks. Long bricks signify hundreds; medium bricks signify tens; and short bricks signify single units. Give two long bricks to the hundreds zebra, five medium bricks to the tens giraffe, and eight short bricks to the ones deer. (First number = 258.)
3. Ask each animal to tell what he or she has been given as the teacher writes the combined number on the chalkboard.
4. Teach students the rules to this activity:
   - All business exchanges begin with the ones deer, then the tens giraffe, and finally the hundreds zebra. The animals receive their shipments in turn and take inventory of their bricks as they are received.
   - The inventory process ensures that the ones deer never has 10 ones bricks (short), the tens giraffe never has 10 tens bricks (medium), and the hundreds zebra never has 10 hundreds bricks (i.e., each animal's total is 9 or less). If any animal has more than 10 bricks, that animal must trade 10 bricks for 1 brick of the next greater value.
5. The brick suppliers will arrive to deliver more bricks to each animal. For example, the supplier may bring 4 hundreds bricks, 7 tens bricks, and 3 ones bricks. (Second number = 473.)
6. When the ones deer inventories 11 bricks, a teacher reminds the deer of the rules and establishes that the ones deer must deliver a stack of 10 short bricks back to the supplier in exchange for 1 medium brick, which must then be given to the tens giraffe. The single short brick that remains is recorded on the chalkboard. When the 7 newly delivered medium bricks are added to the original 5 medium bricks and the medium brick is passed from the ones deer, the tens giraffe then has 13 medium bricks. Therefore, the giraffe delivers one set of 10 bricks to the supplier in exchange for 1 long brick, which is then given to the hundreds zebra. The tens giraffe then reports an inventory of 3 medium bricks remaining. Our hundreds zebra then reports a total inventory of 7 long bricks, or hundreds. (Final answer: 258 + 473 = 731.)

Source: Based on an activity by M. M. Bartek (1997), Hands-on addition and subtraction with the Three Pigs, (2), pp. 68–71.

Addition and America

By Ae-hwa Kim

Objective: To motivate students to solve addition problems as well as to increase their accuracy of solving problems

Grades: Second through fourth grades (possibly higher)

Materials: Map of the United States, tickets made of cards on which math addition facts are printed

Teaching Procedures:

1. Show students the map of the United States to get their attention.
2. Explain the rules of game:
   - The students try to move from their home state to another state across the nation.
   - Students have to have their ticket to travel from state to state.
   - Students must solve the addition problem printed on the ticket and read the name of the state. (The name of the state will be printed on the map, so students just need to read the word.)
   - Only when students get the right answer are they allowed to move to the next state. If students miss the problem, they have to stay in their current state until their next trial time.
3. Let the students play a game. During the game, the teacher assists them and also records the speed and accuracy of their answers.

Note: This game can be extended to subtraction, multiplication, and division.

Source: Based on an activity by D. E. Miller (1997), Math across America, *Teaching Exceptional Children, 24*(2), pp. 47–49.

The Value of Numbers

By Ae-hwa Kim

Objective: To help students understand the value of numbers (ones value, tens value, and hundreds value)

Grades: Primary

Materials: Popsicle sticks; rubber bands to group the Popsicle sticks; a sign; number cards; three boxes to hold ones, tens, and hundreds of Popsicle sticks

Teaching Procedures:

Practice

1. Count the number of Popsicle sticks.
2. Model putting a rubber band around a group of 10 sticks; then ask students to put a rubber band around each new group of 10 sticks.
3. Model putting 1s in the ones box, 10s in the tens box, and so on. Ask students to put 1s in the ones box, 10s in the tens box, and so on.

Activity

1. Show the students a sign that says "Thank you for the ___ Popsicle sticks" (e.g., 157).
2. Model putting that number of Popsicle sticks in the boxes; then ask students to put the number of Popsicle sticks in the boxes.
3. Change the numbers on the sign repeatedly, and allow students to practice grouping Popsicle sticks according to the sign.

Source: Based on an activity by C. Paddock (1997), Ice cream stick math. *Teaching Exceptional Children, 24*(2), pp. 50–51.

Finding Like Units

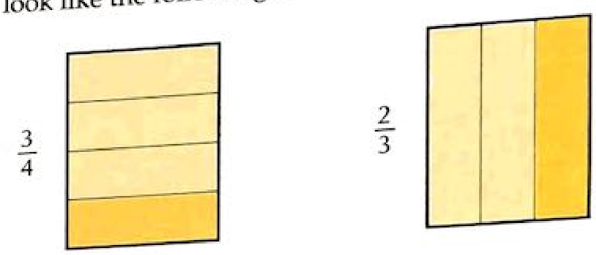
Objective: To help students understand the need to use like units in working with fractions (common denominator for addition, subtraction, multiplication, and division of fractions)

Grades: Middle and upper grades

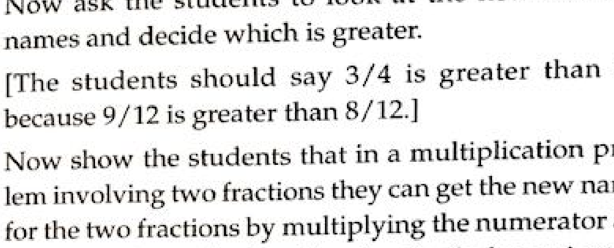
Materials: Whiteboard (chalkboard), dry-erase markers (chalk), overhead projector (document camera/projector), and transparencies

Teaching Procedures:

1. Write on the whiteboard the terms "8 feet" and "98 inches." Then below those write "12 yards" and "24 yards." Point to the top two lengths, and ask, "Which is longer?" Then point to the bottom two lengths, and repeat the question, "Which is longer?" Ask, "Why was the second group easier to compare?" Students should indicate that the same units were used.
2. Now write "48 eggs" and "60 eggs" on the whiteboard. Next, below that write "51 eggs" and "5 dozen eggs," and then point to the top two amounts, and ask, "Which is more?" Now point to the bottom amounts, and ask, "Which is more?" Ask, "Why are the first two egg amounts easier to compare?" Students should indicate that the same units were used. Next, explain that when comparing quantities, it is always easier when like units are compared.
3. Write two fractions on the whiteboard (e.g., 4/7 and 2/7). Below these write two other fractions with different denominators (e.g., 1/3 and 3/8). Point to the top pair of fractions, and ask, "Which is greater?" Then point to the bottom pair of fractions, and ask, "Which is greater?" Ask why the first pair of fractions is easier to compare. Students should indicate that the same units were used. Repeat to the students, "It is always easier to compare quantities if the units are the same."
4. Next, write "13 inches" and "1 foot" on the whiteboard. Ask the students, "How many inches are there in a foot?" Next mark out the "1 foot," and write "12 inches" above it. Tell the students how much easier it is to compare two lengths when we rewrite them using the same units.
5. Write the fractions "2/3" and "3/4" on the whiteboard. Tell the students that you are going to show them how to rewrite these two fractions so that they will have the same units. Next, get two transparencies. One should be lined horizontally with four equal segments, and the other should be lined with three equal segments that look like the following illustration:



6. Next, tell the students that you want them to cut the pieces so that you will have the same size pieces in both of the fractions. Show them that you can cut the first fraction vertically into three equal parts, and then you can cut the second fraction horizontally into four equal parts. Do this with each transparency (see the following illustration). After doing this, write the new names for each of the fractions.



Now ask the students to look at the new fractions' names and decide which is greater.

[The students should say 3/4 is greater than 2/3 because 9/12 is greater than 8/12.]

7. Now show the students that in a multiplication problem involving two fractions they can get the new names for the two fractions by multiplying the numerator and the denominator by the other fraction's denominator.

$$\frac{3 \times 3}{4 \times 3} = \frac{9}{12} \qquad \frac{2 \times 4}{3 \times 4} = \frac{8}{12}$$

Activity

1. Show the students two new fractions, and ask them which is greater. Then give two more new ones, and ask which is the lesser one.
2. Pair the students, and have them create two fractions for each other, and have them ask which one is greater or lesser.
3. Have the students generate multiplication of two fractions for each to solve using multiplication of the numerator and the denominator by the other fraction's denominator.

Source: Based on an activity by B. F. Tucker, A. H. Singleton, & T. L. Weaver (2006), *Teaching Mathematics to All Children: Designing and Adapting Instruction to Meet the Needs of Diverse Learners* (Upper Saddle River, NJ: Pearson Education, Inc.).

MyLab Education Self-Check 11.1
MyLab Education Self-Check 11.2
MyLab Education Self-Check 11.3
MyLab Education Self-Check 11.4
MyLab Education Self-Check 11.5
MyLab Education Self-Check 11.6
MyLab Education Application Exercise 11.1: Enhancing Skills in Mathematics

MyLab Education Application Exercise 11.2: Problem Solving

MyLab Education Application Exercise 11.3: Math Interventions

Summary

- Factors influencing math success include cognitive, educational, self-regulation, and/or neurological factors that may manifest in difficulties understanding math concepts and vocabulary; reasoning, or dealing with abstract concepts that prevail in math; poor memory, making it difficult to remember new concepts; and symbolism difficulties, interfering with learning what symbols refer to. Many students with learning disabilities and behavior problems also have difficulty in sustaining attention, working carefully, and accepting responsibility. Furthermore, because of their difficulty with math, many of these students have received an overabundance of instruction in basic skills but have not been exposed to essential math concepts and problem-solving strategies. In planning curricula for these students, teachers should consider such factors as comprehensive yet individualized programming, providing correction and feedback, generalizing examples to real-life situations, allowing students to participate in goal selection, and using discovery instead of didactic instruction.
- Assessment helps teachers determine what students know and need to know as well as how students compare to others of the same age or grade level. In addition, appropriate assessments allow teachers to monitor students' progress and make effective instructional decisions based on the information they have gathered. When curriculum-based measurement is used to monitor students' progress and adjust instruction accordingly, students make gains at much more rapid rates. Assessments that measure number sense include counting measures, number identification measures, and number writing.
- Prenumber skills that facilitate students' growth in math include one-to-one correspondence, classification, and seriation.

- Math concepts and computation include the numeration concepts of cardinality; grouping patterns; place value; one digit per place; linear order; decimal point; place relation; implied zeros; implied addition; order; name of numbers; periods and names; and understanding "zero."
- Mathematical problem solving can be affected by reading problems, poorly developed logical reasoning skills, and instruction that focuses primarily on computation. Teachers can increase students' problem-solving abilities by teaching attack strategies and schema, teaching big ideas, using sameness analysis, teaching cue words, teaching reasoning strategies, simplifying the sentence structure of word problems, eliminating extraneous information, and monitoring the number of concepts presented as well as the interest level. Computer-assisted instruction is a motivational way to provide practice in problem solving and feedback on performance.
- Teachers can best improve students' math performance by combining a variety of techniques that are appropriate for individual students and the skills they need to develop. Cognitive approaches to math instruction rely on verbalizing or making explicit steps or strategies in solving math problems. Behavioral approaches use the idea of stimulus–response learning to focus on the cues that students need to know in order to succeed in math. Alternative ways to deliver math instruction include cooperative learning, computer-assisted instruction, and interactive video. New curricula have been developed specifically for teaching math skills to students with learning and behavior problems.