Return and Risk: The Capital Asset Pricing Model (CAPM)

11

OPENING CASE

In early 2016, Lumber Liquidators, LinkedIn, and T. Rowe Price all made major announcements. Following such events, stock prices tend to change, and it was no different in these three cases.

Lumber Liquidators, the hardwood flooring retailer, announced that it had agreed to pay $2.5 million to California to settle charges related to the safety of laminate products that were sourced in China. The company's stock jumped about 16 percent on the day. For LinkedIn, the employment-related social media company, earnings came in above analysts' estimates and EPS was $.12, beating the expected breakeven EPS. Even so, the stock fell about 44 percent. T. Rowe Price, the well-known mutual fund company, announced that its earnings for the fourth quarter of 2015 were down 4 percent, but the company's stock rose 3.7 percent.

LinkedIn's announcement seemed like good news, but its stock price fell. The news from Lumber Liquidators and T. Rowe Price seemed bad, but the stock prices for these companies rose. So when is good news really good news? The answer is fundamental to understanding risk and return, and—the good news is—this chapter explores it in some detail.

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11.1 INDIVIDUAL SECURITIES

In the first part of Chapter 11, we will examine the characteristics of individual securities. In particular, we will discuss:

1. *Expected Return.* This is the return that an individual expects a stock to earn over the next period. Of course, because this is only an expectation, the actual return may be either higher or lower. An individual's expectation may be the average return per period a security has earned in the past.

Alternatively, it may be based on a detailed analysis of a firm's prospects, on some computer-based model, or on special (or inside) information.

2. *Variance and Standard Deviation.* There are many ways to assess the volatility of a security's return. One of the most common is variance, which is a measure of the squared deviations of a security's return from its expected return. Standard deviation is the square root of the variance.
3. *Covariance and Correlation.* Returns on individual securities are related to one another. Covariance is a statistic that measures the interrelationship between two securities. Alternatively, this relationship can be restated in terms of the correlation between the two securities. Covariance and correlation are building blocks to an understanding of the beta coefficient.

11.2 EXPECTED RETURN, VARIANCE, AND COVARIANCE

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Expected Return and Variance

Suppose financial analysts believe that there are four unequally likely states of the economy next year: depression, recession, normal, and boom times. The returns on the Supertech Company, R_A, are expected to follow the economy closely, while the returns on the Slowpoke Company, R_B, are not. The return predictions are as follows:

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STATE OF ECONOMY	PROBABILITY OF STATE OF ECONOMY	SUPERTECH RETURNS R_A	SLOWPOKE RETURNS R_B
Depression	.10	−30%	0%
Recession	.20	−10	5
Normal	.50	20	20
Boom	.20	50	−5

Variance can be calculated in four steps.[1] An additional step is needed to calculate standard deviation. (The calculations are presented in Table 11.1.) The steps are:

1. Calculate the expected returns, $E(R_A)$ and $E(R_B)$, by multiplying each possible return by the probability that it occurs, and then add them up:

   Supertech

   $$.10(-.30) + .20(-.10) + .50(.20) + .20(.50) = .15 = 15\% = E(R_A)$$

   Slowpoke

   $$.10(.00) + .20(.05) + .50(.20) + .20(-.05) = .10 = 10\% = E(R_B)$$

2. As shown in the fourth column of Table 11.1, we next calculate the deviation of each possible return from the expected returns for the two companies.
3. Next, take the deviations from the fourth column and square them as we have done in the fifth column.

4. Finally, multiply each squared deviation by its associated probability and add the products up. As shown in Table 11.1, we get a variance of .0585 for Supertech and .0110 for Slowpoke.
5. As always, to get the standard deviations, we just take the square roots of the variances:

Supertech

$$\sqrt{.0585} = .242 = 24.2\% = SD(R_A) = \sigma_A$$

Slowpoke

$$\sqrt{.0110} = .105 = 10.5\% = SD(R_B) = \sigma_B$$

TABLE 11.1 Calculating Variance and Standard Deviation

(1) STATE OF ECONOMY	(2) PROBABILITY OF STATE OF ECONOMY	(3) RATE OF RETURN	(4) DEVIATION FROM EXPECTED RETURN	(5) SQUARED VALUE OF DEVIATION	(6) PRODUCT (2) × (5)
		SUPERTECH (EXPECTED RETURN = .15)			
		R_A	$R_A - E(R_A)$	$(R_A - E(R_A))^2$	
Depression	.10	−.30	−.45	.2025	.02025
Recession	.20	−.10	−.25	.0625	.01250
Normal	.50	.20	.05	.0025	.00125
Boom	.20	.50	.35	.1225	.02450
				$\text{Var}(R_A) = \sigma_A^2 =$	.05850
		SLOWPOKE (EXPECTED RETURN = .10)			
		R_B	$R_B - E(R_B)$	$(R_B - E(R_B))^2$	
Depression	.10	.00	−.10	.0100	.00100
Recession	.20	.05	−.05	.0025	.00050
Normal	.50	.20	.10	.0100	.00500
Boom	.20	−.05	−.15	.0225	.00450
				$\text{Var}(R_B) = \sigma_B^2 =$	.01100

Covariance and Correlation

Variance and standard deviation measure the variability of individual stocks. We now wish to measure the relationship between the return on one stock and the return on another. Enter **covariance** and **correlation**.

Covariance and correlation measure how two random variables are related. We explain these terms by extending our Supertech and Slowpoke example presented earlier.

EXAMPLE 11.1

Calculating Covariance and Correlation

We have already determined the expected returns and standard deviations for both Supertech and Slowpoke. (The expected returns are .15 and .10 for Supertech and Slowpoke, respectively. The standard deviations are .242 and .105, respectively.) In addition, we calculated the deviation of each possible return from the expected return for each firm. Using these data, covariance can be calculated in two steps. An extra step is needed to calculate correlation.

1. For each state of the economy, multiply Supertech's deviation from its expected return and Slowpoke's deviation from its expected return together. For example, Supertech's rate of return in a depression is −.30, which is −.45 (= −.30 − .15) from its expected return. Slowpoke's rate of return in a depression is .00, which is −.10 (= .00 − .10) from its expected return. Multiplying the two deviations together yields .0450 [= (−.45) × (−.10)]. The actual calculations are given in the last column of Table 11.2. This procedure can be written algebraically as

   $$(R_A - E(R_A)) \times (R_B - E(R_B)) \qquad [11.1]$$

   where R_A and R_B are the returns on Supertech and Slowpoke. $E(R_A)$ and $E(R_B)$ are the expected returns on the two securities.

TABLE 11.2 Calculating Covariance and Correlation

(1)	(2)	(3)	(4)	(5)	(6)
STATE OF ECONOMY	PROBABILITY OF STATE OF ECONOMY	DEVIATIONS FROM EXPECTED RETURNS		PRODUCT OF DEVIATION	PRODUCT (2)× (5)
		SUPERTECH	SLOWPOKE		
Depression	.10	−.45	−.10	.0450	.00450
Recession	.20	−.25	−.05	.0125	.00250
Normal	.50	.05	.10	.0050	.00250
Boom	.20	.35	−.15	−.0525	−.01050
			$\text{Cov}(R_A,R_B) = \sigma_{A,B} =$		−.001

2. Once we have the products of the deviations, we multiply each one by its associated probability and sum to get the covariance.

Note that we represent the covariance between Supertech and Slowpoke as either $\text{Cov}(R_A,R_B)$ or $\sigma_{A,B}$. Equation 11.1 illustrates the intuition of covariance. Suppose Supertech's return is generally above its average when Slowpoke's return is above its average, and Supertech's return is generally below its average when Slowpoke's return is below its average. This is indicative of a positive dependency or a positive relationship between the two returns. Note that the term in Equation 11.1 will be *positive* in any state where both returns are *above* their averages. In addition, [11.1] will still be *positive* in any state where both terms are *below* their averages. Thus, a positive relationship between the two returns will give rise to a positive value for covariance.

Conversely, suppose Supertech's return is generally above its average when Slowpoke's return is below its average, and Supertech's return is generally below its average when Slowpoke's return is above its average. This is indicative of a negative dependency or a negative relationship between the two returns. Note that the term in Equation 11.1 will be *negative* in any state where one return is above its average and the other return is below its average. Thus, a negative relationship between the two returns will give rise to a negative value for covariance.

Finally, suppose there is no relation between the two returns. In this case, knowing whether the return on Supertech is above or below its expected return tells us nothing about the return on Slowpoke. In the covariance formula, then, there will be no tendency for the deviations to be positive or negative together. On average, they will tend to offset each other and cancel out, making the covariance zero.

Of course, even if the two returns are unrelated to each other, the covariance formula will not equal zero exactly in any actual history. This is due to sampling error; randomness alone will make the calculation positive or negative. But for a historical sample that is long enough, if the two returns are not related to each other, we should expect the covariance to come close to zero.

Our covariance calculation seems to capture what we are looking for. If the two returns are positively related to each other, they will have a positive covariance, and if they are negatively related to each other, the covariance will be negative. Last, and very important, if they are unrelated, the covariance should be zero.

The covariance we calculated is −.001. A negative number like this implies that the return on one stock is likely to be above its average when the return on the other stock is below its average, and vice versa. However, the size of the number is difficult to interpret. Like the variance figure, the covariance is in squared deviation units. Until we can put it in perspective, we don't know what to make of it.

We solve the problem by computing the correlation.

3. To calculate the correlation, divide the covariance by the product of the standard deviations of the two securities. For our example, we have

$$\rho_{A,B} = \text{Corr}(R_A, R_B) = \frac{\text{Cov}(R_A, R_B)}{\sigma_A \times \sigma_B} = \frac{-.001}{.242 \times .105} = -.039 \qquad [11.2]$$

Now consider two stocks, each with an expected return of 10 percent. The expected return on a portfolio composed of these two stocks must be 10 percent, regardless of the proportions of the two stocks held. This result may seem obvious at this point, but it will become important later. The result implies that you do not reduce or *dissipate* your expected return by investing in a number of securities. Rather, the expected return on your portfolio is a weighted average of the expected returns on the individual assets in the portfolio.

Variance and Standard Deviation of a Portfolio

THE VARIANCE The formula for the variance of a portfolio composed of two securities, *A* and *B*, is:

The Variance of the Portfolio

$$\text{Var (portfolio)} = X_A^2\sigma_A^2 + 2X_AX_B\sigma_{A,B} + X_B^2\sigma_B^2 \qquad [11.4]$$

Note that there are three terms on the right-hand side of the equation (in addition to X_A and X_B, the investment proportions). The first term involves the variance of A (σ_A^2), the second term involves the covariance between the two securities ($\sigma_{A,B}$), and the third term involves the variance of $B(\sigma_B^2)$. (As stated earlier in this chapter, $\sigma_{A,B} = \sigma_{B,A}$. That is, the ordering of the variables is not relevant when expressing the covariance between two securities.)

The formula indicates an important point. The variance of a portfolio depends on both the variances of the individual securities and the covariance between the two securities. The variance of a security measures the variability of an individual security's return. Covariance measures the relationship between the two securities. For given variances of the individual securities, a positive relationship or covariance between the two securities increases the variance of the entire portfolio. A negative relationship or covariance between the two securities decreases the variance of the entire portfolio. This important result seems to square with common sense. If one of your securities tends to go up when the other goes down, or vice versa, your two securities are offsetting each other. You are achieving what we call a *hedge* in finance, and the risk of your entire portfolio will be low. However, if both your securities rise and fall together, you are not hedging at all. Hence, the risk of your entire portfolio will be higher.

The variance formula for our two securities, Super and Slow, is:

$$\text{Var (portfolio)} = X_{\text{Super}}^2\sigma_{\text{Super}}^2 + 2X_{\text{Super}}X_{\text{Slow}}\sigma_{\text{Super, Slow}} + X_{\text{Slow}}^2\sigma_{\text{Slow}}^2$$

Given our earlier assumption that an individual with \$100 invests \$60 in Supertech and \$40 in Slowpoke, $X_{\text{Super}} = .6$ and $X_{\text{Slow}} = .4$. Using this assumption and the relevant data from the previous box, the variance of the portfolio is:

$$.0223 = .36 \times .0585 + 2 \times [.6 \times .4 \times (-.001)] + .16 \times .0110$$

STANDARD DEVIATION OF A PORTFOLIO We can now determine the standard deviation of the portfolio's return. This is:

$$\sigma_P = \text{SD (portfolio)} = \sqrt{\text{Var (portfolio)}} = \sqrt{.0223}$$
$$= .1495, \text{ or } 14.95\% \qquad [11.5]$$

The interpretation of the standard deviation of the portfolio is the same as the interpretation of the standard deviation of an individual security. The expected return on our portfolio is 13 percent. A return of −1.95 percent (=13% − 14.95%) is one standard deviation below the mean, and a return of 27.95 percent (=13% + 14.95%) is one standard deviation above the mean. If the return on the portfolio is normally distributed, a return between −1.95 percent and +27.95 percent occurs about 68 percent of the time.[2]

THE DIVERSIFICATION EFFECT It is instructive to compare the standard deviation of the portfolio with the standard deviation of the individual securities. The weighted average of the standard deviations of the individual securities is:

$$\text{Weighted average of standard deviations} = X_{Super}\sigma_{Super} + X_{Slow}\sigma_{Slow} \qquad [11.6]$$
$$.187 = .6 \times .242 + .4 \times .105$$

One of the most important results in this chapter concerns the difference between Equations 11.5 and 11.6. In our example, the standard deviation of the portfolio is *less* than a weighted average of the standard deviations of the individual securities.

We pointed out earlier that the expected return on the portfolio is a weighted average of the expected returns on the individual securities. Thus, we get a different type of result for the standard deviation of a portfolio than we do for the expected return on a portfolio.

It is generally argued that our result for the standard deviation of a portfolio is due to diversification. For example, Supertech and Slowpoke are slightly negatively correlated ($\rho = -.039$). Supertech's return is likely to be a little below average if Slowpoke's return is above average. Similarly, Supertech's return is likely to be a little above average if Slowpoke's return is below average. Thus, the standard deviation of a portfolio composed of the two securities is less than a weighted average of the standard deviations of the two securities.

The above example has negative correlation. Clearly, there will be less benefit from diversification if the two securities exhibit positive correlation. How high must the positive correlation be before all diversification benefits vanish?

To answer this question, let us rewrite Equation 11.4 in terms of correlation rather than covariance. First, note that the covariance can be rewritten as:

$$\sigma_{Super,\,Slow} = \rho_{Super,\,Slow}\sigma_{Super}\sigma_{Slow} \qquad [11.7]$$

The formula states that the covariance between any two securities is the correlation between the two securities multiplied by the standard deviations of each. In other words, covariance incorporates both (1) the correlation between the two assets and (2) the variability of each of the two securities as measured by standard deviation.

From our calculations earlier in this chapter, we know that the correlation between the two securities is $-.039$. Thus, the variance of our portfolio can be expressed as:

$$\text{Variance of the portfolio's return}$$
$$= X_{Super}^2\sigma_{Super}^2 + 2X_{Super}X_{Slow}\rho_{Super,\,Slow}\sigma_{Super}\sigma_{Slow} + X_{Slow}^2\sigma_{Slow}^2 \qquad [11.8]$$
$$.0223 = .36 \times .0585 + 2 \times .6 \times .4 \times (-.039) \times .242 \times .105$$
$$+ .16 \times .0110$$

The middle term on the right-hand side is now written in terms of correlation, ρ, not covariance.

Suppose $\rho_{Super,\ Slow} = 1$,the highest possible value for correlation. Assume all the other parameters in the example are the same. The variance of the portfolio is:

$$\text{Variance of the portfolio's return} = .035 = .36 \times .0585 + 2 \times (.6 \times .4 \times 1 \times .242 \times .105) + .16 \times .0110$$

The standard deviation is:

$$\text{Standard deviation of portfolio's return} = \sqrt{.035} = .187 = 18.7\%$$

Note that Equations 11.9 and 11.6 are equal. That is, the standard deviation of a portfolio's return is equal to the weighted average of the standard deviations of the individual returns when $\rho = 1$. Inspection of Equation 11.8 indicates that the variance and hence the standard deviation of the portfolio must fall as the correlation drops below 1. This leads to the following result: page 324

As long as ρ < 1, the standard deviation of a portfolio of two securities is *less* than the weighted average of the standard deviations of the individual securities.

TABLE 11.3
Standard Deviations for Standard & Poor's 500 Index and for Selected Stocks in the Index, 2006–2015

ASSET	STANDARD DEVIATION
S&P 500 Index	15.09%
IBM	17.99
Walt Disney	22.58
Microsoft	24.59
BP	28.98
Google	30.49
Apple	33.02
Amazon.com	38.48
Bank of America	50.02
Ford	58.63

In other words, the diversification effect applies as long as there is less than perfect correlation (as long as $\rho < 1$). Thus, our Supertech-Slowpoke example is a case of overkill. We illustrated diversification by an example with negative correlation. We could have illustrated diversification by an example with positive correlation—as long as it was not *perfect* positive correlation.

AN EXTENSION TO MANY ASSETS The preceding insight can be extended to the case of many assets. That is, as long as correlations between pairs of securities are less than 1, the standard deviation of a portfolio of many assets is less than the weighted average of the standard deviations of the individual securities.

Now consider Table 11.3, which shows the standard deviation (based on annual returns) of the Standard & Poor's 500 Index and the standard deviations of some of the individual securities listed in the index over a recent 10-year period. Note that all of the individual securities in the table have higher standard deviations than that of the index. In general, the standard deviations of most of the individual securities in an index will be above the standard deviation of the index itself, though a few of the securities could have lower standard deviations than that of the index.

11.4 THE EFFICIENT SET

The Two-Asset Case

Our results on expected returns and standard deviations are graphed in Figure 11.2. In the figure, there is a dot labeled Slowpoke and a dot labeled Supertech. Each dot represents both the expected return and the standard deviation for an individual security. As can be seen, Supertech has both a higher expected return and a higher standard deviation.

The box or "□" in the graph represents a portfolio with 60 percent invested in Supertech and 40 percent invested in Slowpoke. You will recall that we have previously calculated both the expected return and the standard deviation for this portfolio.

The choice of 60 percent in Supertech and 40 percent in Slowpoke is just one of an infinite number of portfolios that can be created. The set of portfolios is sketched by the curved line in Figure 11.3.

Consider Portfolio *1*. This is a portfolio composed of 90 percent Slowpoke and 10 percent Supertech. Because it is weighted so heavily toward Slowpoke, it appears close to the

Slowpoke point on the graph. Portfolio *2* is higher on the curve because it is composed of 50 percent Slowpoke and 50 percent Supertech. Portfolio *3* is close to the Supertech point on the graph because it is composed of 90 percent Supertech and 10 percent Slowpoke.

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FIGURE 11.2
Expected Returns and Standard Deviations for Supertech, Slowpoke, and a Portfolio Composed of 60 Percent in Supertech and 40 Percent in Slowpoke

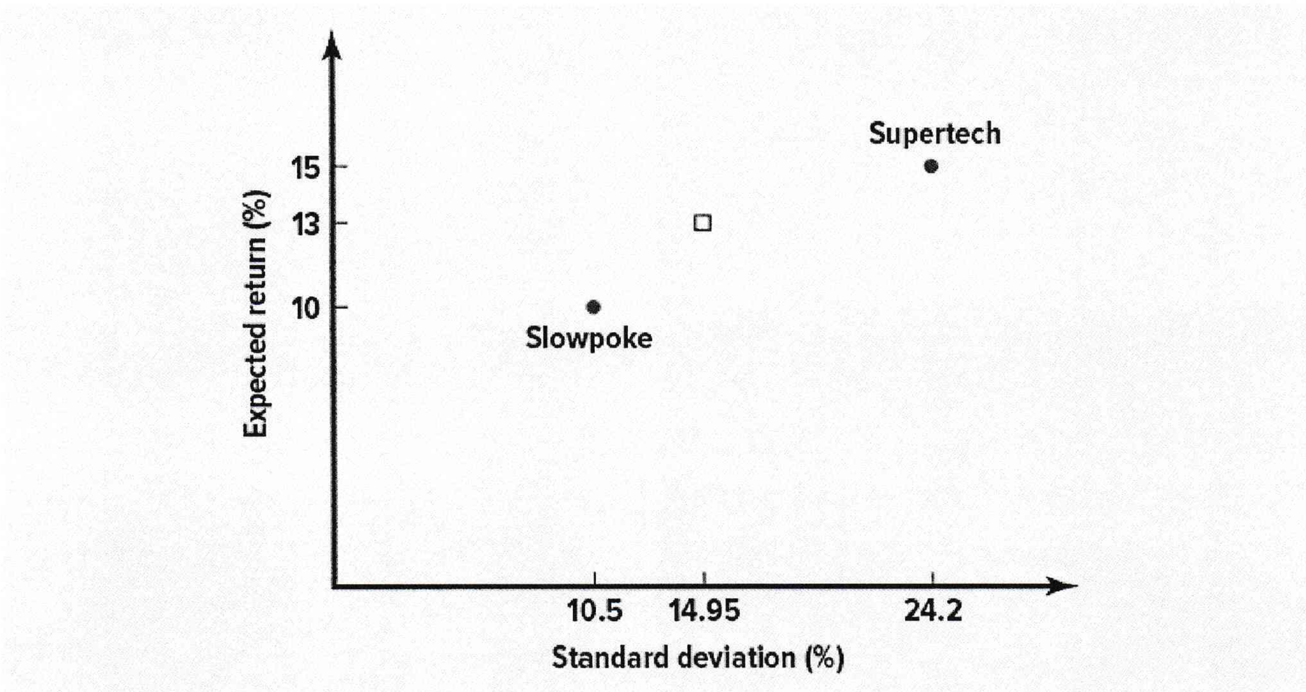
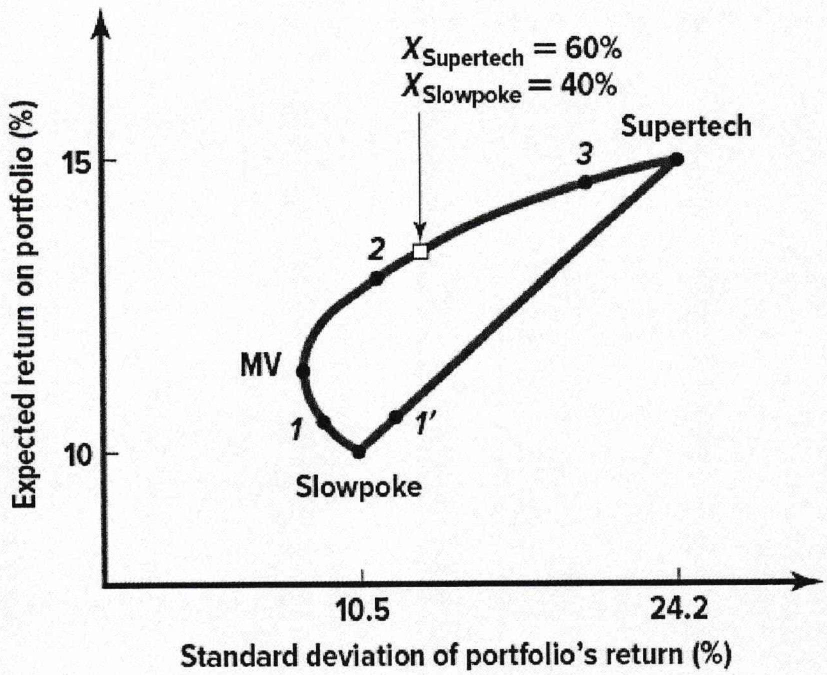


FIGURE 11.3
Set of Portfolios Composed of Holdings in Supertech and Slowpoke (correlation between the two securities is −.039)



Portfolio *1* is composed of 90 percent Slowpoke and 10 percent Supertech ($\rho = -.039$).
Portfolio *2* is composed of 50 percent Slowpoke and 50 percent Supertech ($\rho = -.039$).
Portfolio *3* is composed of 10 percent Slowpoke and 90 percent Supertech ($\rho = -.039$).
Portfolio *1'* is composed of 90 percent Slowpoke and 10 percent Supertech ($\rho = 1$).
Point MV denotes the minimum variance portfolio. This is the portfolio with the lowest possible variance. By definition, the same portfolio must also have the lowest possible standard deviation.

There are a few important points concerning this graph:

1. We argued that the diversification effect occurs whenever the correlation between the two securities is below 1. The correlation between Supertech and Slowpoke is −.039. The diversification effect can be illustrated by comparison with the straight line between the Supertech point and the Slowpoke point. The straight line represents points that would have been generated had the correlation coefficient between the two securities been 1. The diversification effect is

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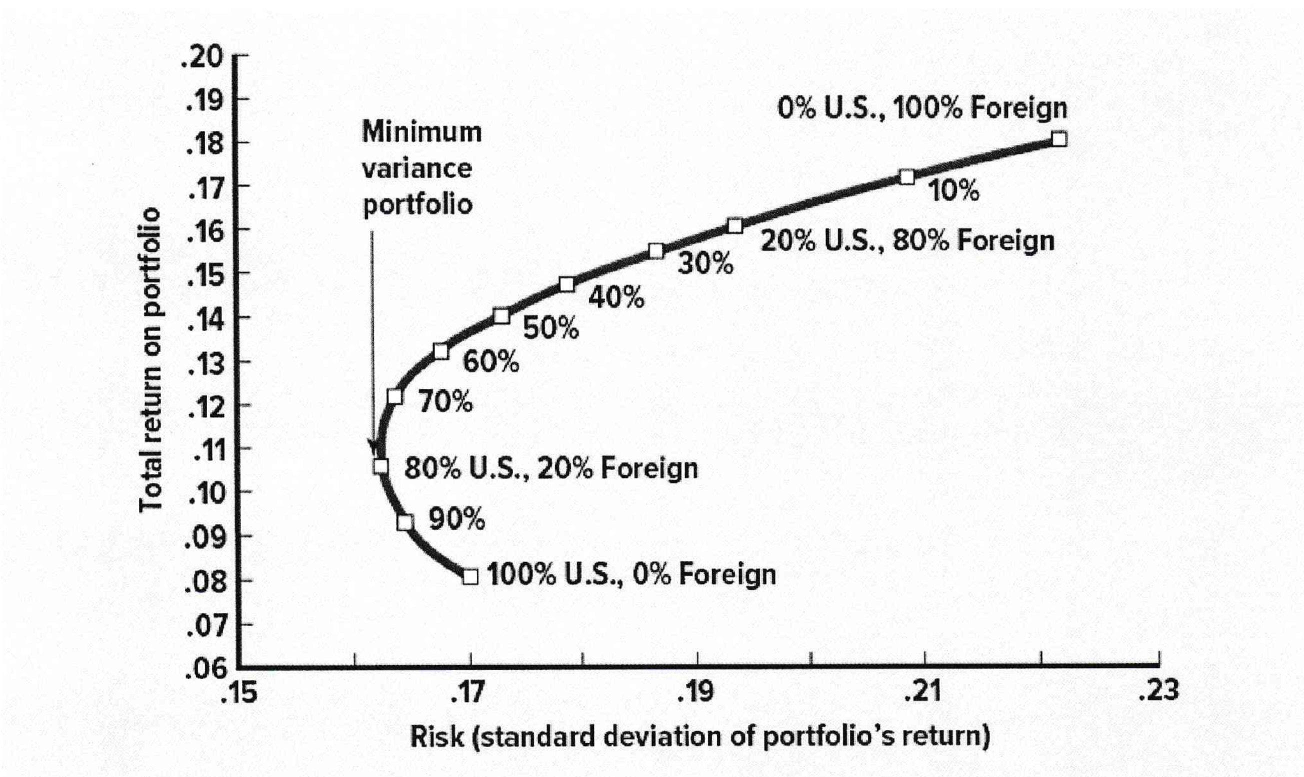
combining a small percentage of the foreign stock portfolio with the U.S. portfolio actually reduces risk, as can be seen by the backward-bending nature of the curve. In other words, the diversification benefits from combining two different portfolios more than offset the introduction of a riskier set of stocks into one's holdings. The minimum variance portfolio occurs with about 80 percent of one's funds in American stocks and about 20 percent in foreign stocks. Addition of foreign securities beyond this point increases the risk of one's entire portfolio.

FIGURE 11.5
Return/Risk Trade-off for World Stocks: Portfolio of U.S. and Foreign Stocks



The backward-bending curve in Figure 11.5 is important information that has not bypassed American money managers. In recent years, pension fund and mutual fund managers in the United States have sought out investment opportunities overseas. Another point worth pondering concerns the potential pitfalls of using only past data to estimate future returns. The stock markets of many foreign countries have had phenomenal growth in the past 40 years. Thus, a graph like Figure 11.5 makes a large investment in these foreign markets seem attractive. However, because abnormally high returns cannot be sustained forever, some subjectivity must be used when forecasting future expected returns.

The Efficient Set for Many Securities

The previous discussion concerned two securities. We found that a simple curve sketched out all the possible portfolios. Because investors generally hold more than two securities, we should look at the same graph when more than two securities are held. The shaded area in Figure 11.6 represents the opportunity set or feasible set when many securities are considered. The shaded area represents all the possible combinations of expected return and standard deviation for a portfolio. For example, in a universe of 100 securities, Point 1 might represent a portfolio of, say 40 securities. Point 2 might represent a portfolio of 80 securities. Point 3 might represent a different set of 80 securities, or the same 80 securities held in different proportions, or something else. Obviously, the combinations are virtually endless. However, note that all possible combinations fit into a confined region. No security or combination of securities can fall outside of the shaded region. That is, no one can choose a portfolio with an expected return above that given by the shaded region. Furthermore, no one can choose a portfolio with a standard deviation below that given in the shaded area. Perhaps more surprisingly, no one can choose an expected return below that given in the curve. In other words, the capital markets actually prevent a self-destructive person from taking on a guaranteed loss.[3]

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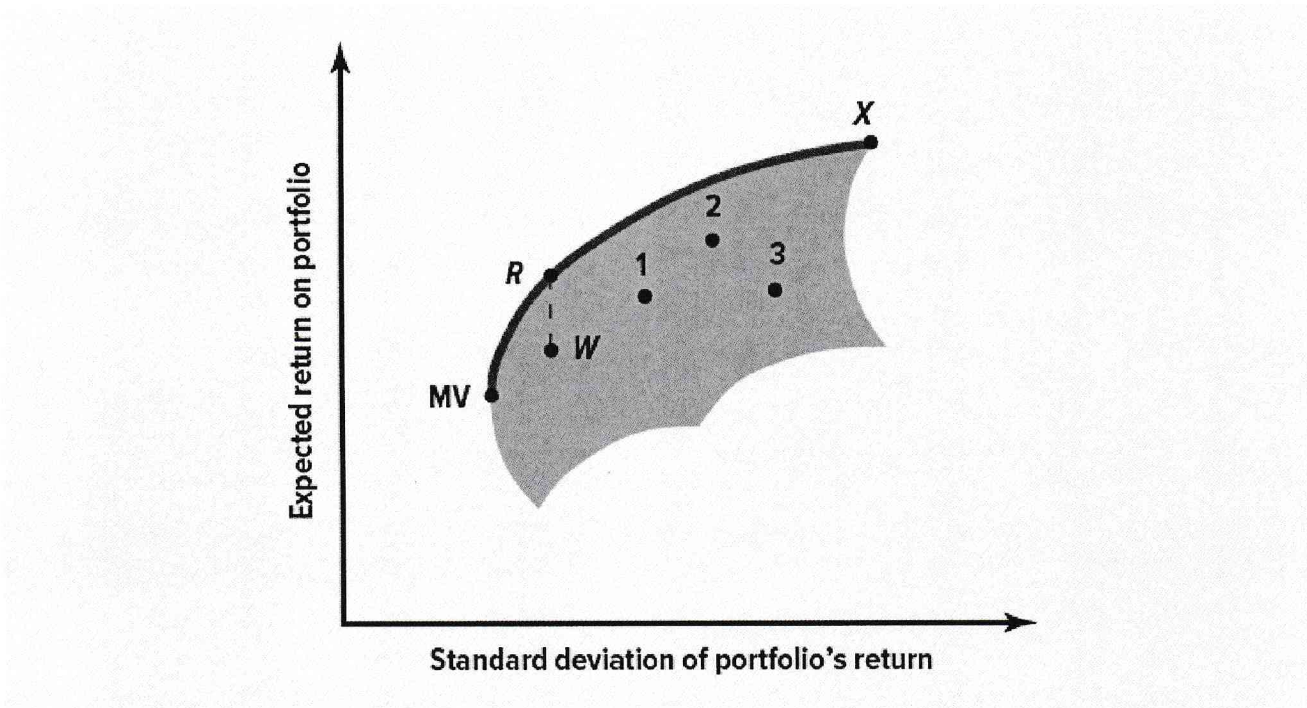
FIGURE 11.6
The Feasible Set of Portfolios Constructed from Many Securities



So far, Figure 11.6 is different from the earlier graphs. When only two securities are involved, all the combinations lie on a single curve. Conversely, with many securities the combinations cover an entire area. However, notice that an individual will want to be somewhere on the upper edge between MV and *X*. The upper edge, which we indicate in Figure 11.6 by a thick curve, is called the *efficient set*. Any point below the efficient set would receive less expected return and the same standard deviation as a point on the efficient set. For example, consider *R* on the efficient set and *W* directly below it. If *W* contains the risk you desire, you should choose *R* instead in order to receive a higher expected return.

In the final analysis, Figure 11.6 is quite similar to Figure 11.3. The efficient set in Figure 11.3 runs from MV to Supertech. It contains various combinations of the securities Supertech and Slowpoke. The efficient set in Figure 11.6 runs from MV to *X*. It contains various combinations of many securities. The fact that a whole shaded area appears in Figure 11.6 but not in Figure 11.3 is just not an important difference; no investor would choose any point below the efficient set in Figure 11.6 anyway.

We mentioned before that an efficient set for two securities can be traced out easily in the real world. The task becomes more difficult when additional securities are included because the number of calculations quickly becomes huge. As a result, hand calculations are impractical for more than just a few securities. A number of software packages allow the calculation of an efficient set for portfolios of moderate size. By all accounts, these packages sell quite briskly, so our discussion above would appear to be important in practice.

11.5 RISKLESS BORROWING AND LENDING

Figure 11.6 assumes that all the securities on the efficient set are risky. Alternatively, an investor could combine a risky investment with an investment in a riskless or *risk-free* security, such as an investment in United States Treasury bills. This is illustrated in the following example:

EXAMPLE 11.3

Riskless Lending and Portfolio Risk

Ms. Bagwell is considering investing in the common stock of Merville Enterprises. In addition, Ms. Bagwell will either borrow or lend at the risk-free rate. The relevant parameters are:

	COMMON STOCK OF MERVILLE	RISK-FREE ASSET
Expected return	14%	10%
Standard deviation	.20	0

Suppose Ms. Bagwell chooses to invest a total of $1,000, $350 of which is to be invested in Merville Enterprises and $650 placed in the risk-free asset. The expected return on her total investment is a weighted average of the two returns:

$$\text{Expected return on portfolio composed of one riskless and one risky asset} = .114 = (.35 \times .14) + (.65 \times .10)$$

Because the expected return on the portfolio is a weighted average of the expected return on the risky asset (Merville Enterprises) and the risk-free return, the calculation is analogous to the way we treated two risky assets. In other words, Equation 11.3 applies here.

Using Equation 11.4, the formula for the variance of the portfolio can be written as:

$$X^2_{\text{Merville}}\sigma^2_{\text{Merville}} + 2X_{\text{Merville}}X_{\text{Risk-free}}\sigma_{\text{Merville, Risk-free}} + X^2_{\text{Risk-free}}\sigma^2_{\text{Risk-free}}$$

However, by definition, the risk-free asset has no variability. Thus, both $\sigma_{\text{Merville, Risk-free}}$ and $\sigma^2_{\text{Risk-free}}$ are equal to zero, reducing the above expression to:

$$\text{Variance of portfolio composed of one riskless and one risky asset} = X^2_{\text{Merville}}\sigma^2_{\text{Merville}} \quad [11.9]$$

$$= (.35)^2 \times (.20)^2$$

$$= .0049$$

The standard deviation of the portfolio is:

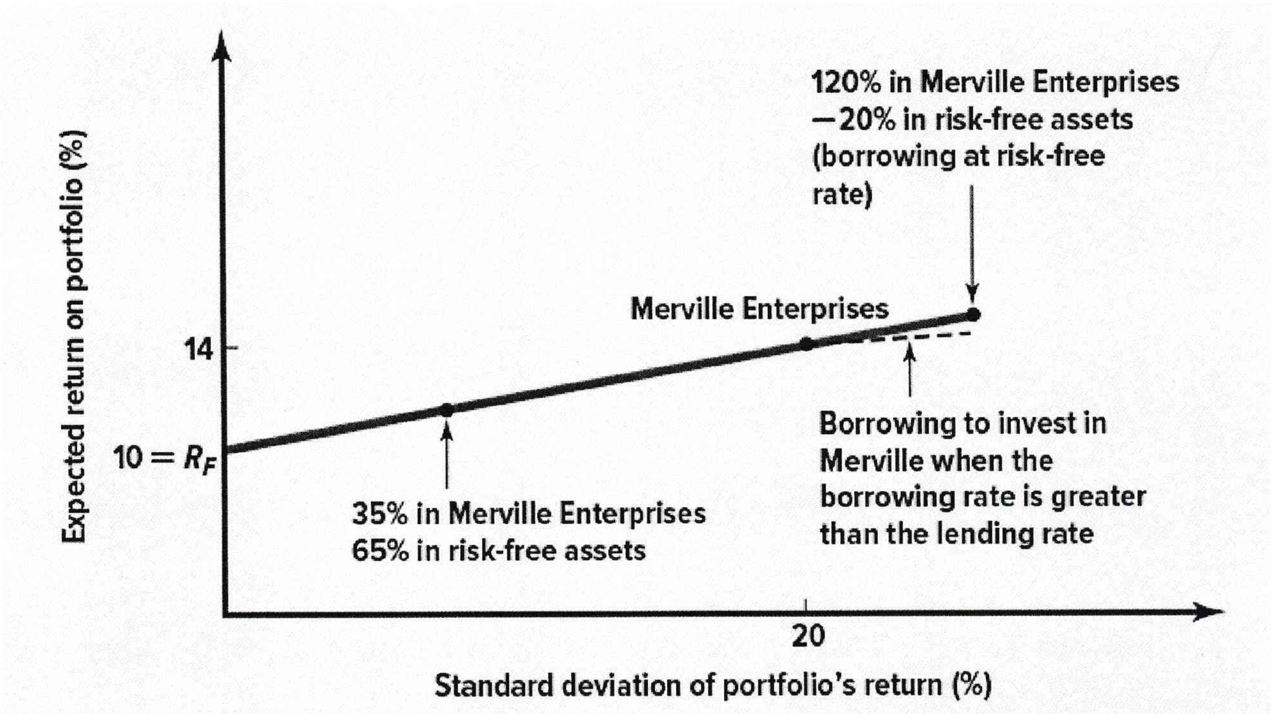
$$\text{Standard deviation of portfolio composed of one riskless and one risky asset} = X_{\text{Merville}}\sigma_{\text{Merville}} \quad [11.10]$$

$$= .35 \times .20$$

$$= .07$$

The relationship between risk and expected return for one risky and one riskless asset can be seen in Figure 11.7. Ms. Bagwell's split of 35-65 percent between the two assets is represented on a *straight* line between the risk-free rate and a pure investment in Merville Enterprises. Note that, unlike the case of two risky assets, the opportunity set is straight, not curved.

FIGURE 11.7
Relationship between Expected Return and Risk for a Portfolio of One Risky Asset and One Riskless Asset



Suppose that, alternatively, Ms. Bagwell borrows $200 at the risk-free rate. Combining this with her original sum of $1,000, she invests a total of $1,200 in Merville. Her expected return would be:

$$\text{Expected return on portfolio formed by borrowing to invest in risky asset} = .148 = 1.20 \times .14 + (-.2 \times .10)$$

Here, she invests 120 percent of her original investment of $1,000 by borrowing 20 percent of her original investment. Note that the return of 14.8 percent is greater than the 14 percent expected return on Merville Enterprises. This occurs because she is borrowing at 10 percent to invest in a security with an expected return greater than 10 percent.

The standard deviation is:

$$\text{Standard deviation of portfolio formed by borrowing to invest in risky asset} = .24 = 1.20 \times .2$$

The standard deviation of .24 is greater than .20, the standard deviation of the Merville investment, because borrowing increases the variability of the investment. This investment also appears in Figure 11.7.

So far, we have assumed that Ms. Bagwell is able to borrow at the same rate at which she can lend.[4] Now let us consider the case where the borrowing rate is above the lending rate. The dotted line in Figure 11.7 illustrates the opportunity set for borrowing opportunities in this case. The dotted line is below the solid line because a higher borrowing rate lowers the expected return on the investment.

The Optimal Portfolio

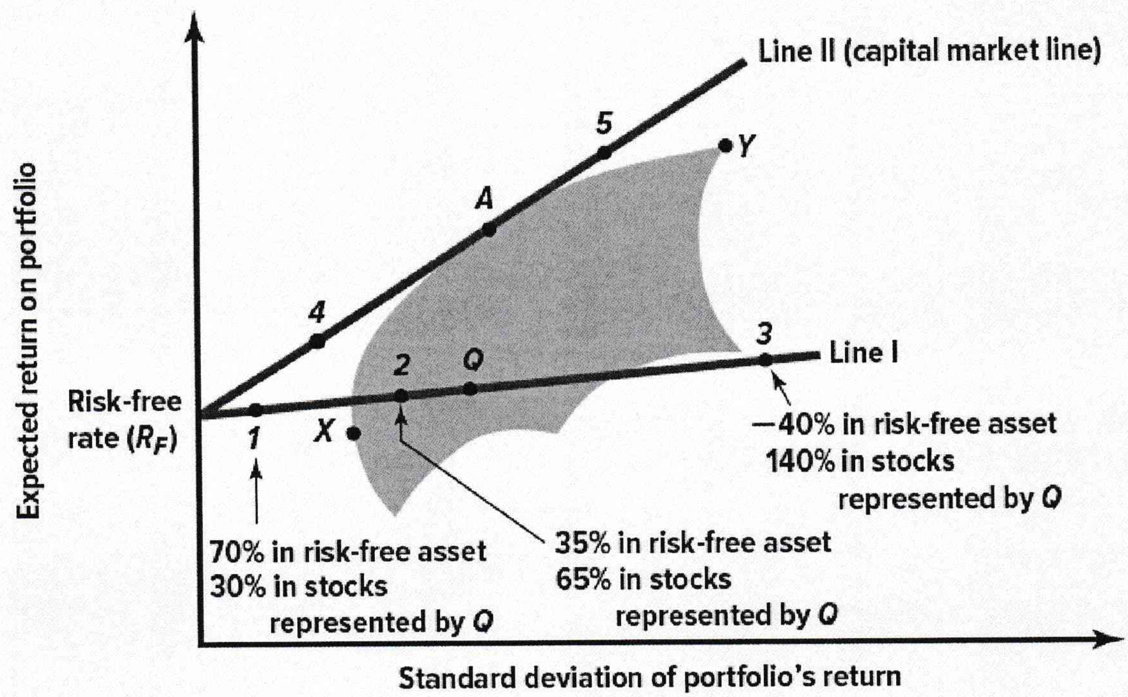
The previous section concerned a portfolio formed between one riskless asset and one risky asset. In reality, an investor is likely to combine an investment in the riskless asset with a *portfolio* of risky assets. This is illustrated in Figure 11.8.

Consider Point *Q*, representing a portfolio of securities. Point *Q* is in the interior of the feasible set of risky securities. Let us assume the point represents a portfolio of 30 percent AT&T, 45 percent General Electric (GE), and 25 percent IBM stock. Individuals combining investments in *Q* with investments in the riskless asset would achieve points along the straight line from R_F to *Q*. We refer to this as Line *I*. For example, point *1* on the line represents a portfolio of 70 percent in the riskless asset and 30 percent in stocks represented by *Q*. An investor with $100 who chooses Point *1* as his portfolio would put $70 in the riskless asset and $30 in *Q*. This can be restated as $70 in the riskless asset, $9 (= .3 × $30) in AT&T, $13.50 (= .45 × $30) in GE, and $7.50 (= .25 × $30) in IBM. Point 2 also represents a portfolio of the risk-free asset and *Q*, with more (65 percent) being invested in *Q*.

Point *3* is obtained by borrowing to invest in *Q*. For example, an investor with $100 of his own would borrow $40 from the bank or broker in order to invest $140 in *Q*.

This can be stated as borrowing \$40 and contributing \$100 of one's own money in order to invest \$42 (= .3 × \$140) in AT&T, \$63 (= .45 × \$140) in GE, and \$35 (= .25 × \$140) in IBM. page 332

FIGURE 11.8
Relationship between Expected Return and Standard Deviation for an Investment in a Combination of Risky Securities and the Riskless Asset



Portfolio Q is composed of 30 percent AT&T, 45 percent GE, and 25 percent IBM.

The above investments can be summarized as:

	POINT *Q*	POINT *1* (LENDING \$70)	POINT *3* (BORROWING \$40)
AT&T	\$ 30	\$ 9	\$ 42
GE	45	13.50	63
IBM	25	7.50	35
Risk-free	0	70	- 40
Total investment	\$100	\$100	\$100

Though any investor can obtain any point on Line *I*, no point on the line is optimal. To see this, consider Line *II*, a line running from R_F through *A*. Point *A* represents a portfolio of risky securities.

Line *II* represents portfolios formed by combinations of the risk-free asset and the securities in *A*. Points between R_F and *A* are portfolios in which some money is invested in the riskless asset and the rest is placed in *A*. Points past *A* are achieved by borrowing at the riskless rate to buy more of *A* than one could with one's original funds alone.

As drawn, Line *II* is tangent to the efficient set of risky securities. Whatever point an individual can obtain on Line *I*, he can obtain a point with the same standard deviation and a higher expected return on Line *II*. In fact, because Line *II* is tangent to the efficient set of risky assets, it provides the investor with the best possible opportunities. In other words, Line *II* can be viewed as the efficient set of *all* assets, both risky and riskless. An investor with a fair degree of risk aversion might choose a point between R_F and *A*, perhaps Point 4. An individual with less risk aversion might choose a point closer to *A* or even beyond *A*. For example, Point *5* corresponds to an individual borrowing money to increase his investment in *A*.

The graph illustrates an important point. With riskless borrowing and lending, the portfolio of *risky* assets held by any investor would always be Point *A*. Regardless of the investor's tolerance for risk, he would never choose any other point on the efficient set of risky assets (represented by Curve *XAY*) nor any point in the interior of the feasible region. Rather, he would combine the securities of *A* with the riskless assets if he had high aversion to risk. He would borrow the riskless asset to invest more funds in *A* if he had low aversion to risk.

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This result establishes what financial economists call the **separation principle**. That is, the investor's investment decision consists of two separate steps:

1. After estimating (*a*) the expected returns and variances of individual securities, and (*b*) the covariances between pairs of securities, the investor calculates the efficient set of risky assets, represented by Curve *XAY* in Figure 11.8. He then determines Point *A*, the tangency between the risk-free rate and the efficient set of risky assets (Curve *XAY*). Point *A* represents the portfolio of risky assets that the investor will hold. This point is determined solely from his estimates of returns, variances, and covariances. No personal characteristics, such as degree of risk aversion, are needed in this step.
2. The investor must now determine how he will combine Point *A*, his portfolio of risky assets, with the riskless asset. He might invest some of his funds in the riskless asset and some in Portfolio *A*. He would end up at a point on the line between R_F and *A* in this case. Alternatively, he might borrow at the riskfree rate and contribute some of his own funds as well, investing the sum in Portfolio *A*. In this case, he would end up at a point on Line *II* beyond *A*. His position in the riskless asset, that is, his choice of where on the line he wants to be, is determined by his internal characteristics, such as his ability to tolerate risk.

11.6 ANNOUNCEMENTS, SURPRISES, AND EXPECTED RETURNS

Now that we know how to construct portfolios and evaluate their returns, we begin to describe more carefully the risks and returns associated with individual securities. Thus far, we have measured volatility by looking at the difference between the actual return on an asset or portfolio, *R*, and the expected return, E(*R*). We now look at why those deviations exist.

Expected and Unexpected Returns

To begin, for concreteness, we consider the return on the stock of a company called Flyers. What will determine this stock's return in, say, the coming year?

The return on any stock traded in a financial market is composed of two parts. First, the normal, or expected, return from the stock is the part of the return that shareholders in the market predict or expect. This return depends on the information shareholders have that bears on the stock, and it is based on the market's understanding today of the important factors that will influence the stock in the coming year.

The second part of the return on the stock is the uncertain, or risky, part. This is the portion that comes from unexpected information revealed within the year. A list of all possible sources of such information would be endless, but here are a few examples:

News about Flyers's research.

Government figures released on gross domestic product (GDP).

The results from the latest arms control talks.

The news that Flyers's sales figures are higher than expected.

A sudden, unexpected drop in interest rates.

Based on this discussion, one way to express the return on Flyers's stock in the coming year would be:

$$\text{Total return} = \text{Expected return} + \text{Unexpected return}$$
$$R = \text{E}(R) + U \quad [11.11]$$

where R stands for the actual total return in the year, E(R) stands for the expected part of the return, and U stands for the unexpected part of the return. What this says is that the actual return, R, differs from the expected return, E(R), because of surprises that occur during the year. In any given year, the unexpected return will be positive or negative, but, through time, the average value of U will be zero. This means that on average, the actual return equals the expected return.

Announcements and News

We need to be careful when we talk about the effect of news items on the return. For example, suppose Flyers's business is such that the company prospers when GDP grows at a relatively high rate and suffers when GDP is relatively stagnant. In this case, in deciding what return to expect this year from owning stock in Flyers, shareholders either implicitly or explicitly must think about what GDP is likely to be for the year.

When the government actually announces GDP figures for the year, what will happen to the value of Flyers's stock? Obviously, the answer depends on what figure is released. More to the point, however, the impact depends on how much of that figure is *new* information.

At the beginning of the year, market participants will have some idea or forecast of what the yearly GDP will be. To the extent that shareholders have predicted GDP, that prediction will already be factored into the expected part of the return on the stock, E(R). On the other hand, if the announced GDP is a surprise, then the effect will be part of U, the unanticipated portion of the return. As an example, suppose shareholders in the market had forecast that the GDP increase this year would be .5 percent. If the actual announcement this year is exactly .5 percent, the same as the forecast, then the shareholders don't really learn anything, and the announcement isn't news. There will be no impact on the stock price as a result. This is like receiving confirmation of something that you suspected all along; it doesn't reveal anything new.

A common way of saying that an announcement isn't news is to say that the market has already "discounted" the announcement. The use of the word *discount* here is different from the use of the term in computing present values, but the spirit is the same. When we discount a dollar in the future, we say it is worth less to us because of the time value of money. When we discount an announcement or a news item, we say that it has less of an impact on the market because the market already knew much of it.

Going back to Flyers, suppose the government announces that the actual GDP increase during the year has been 1.5 percent. Now shareholders have learned something, namely, that the increase is one percentage point higher than they had forecast. This difference between the actual result and the forecast, one percentage point in this example, is sometimes called the *innovation* or the *surprise.*

This distinction explains why what seems to be good news can actually be bad news (and vice versa). For example, to open the chapter we compared Lumber Liquidators, LinkedIn, and T. Rowe

Price. For Lumber Liquidators, the company had been embroiled in negative publicity since it was discovered that laminated wood products from Chinese suppliers had levels of formaldehyde that exceeded California standards. The $2.5 million settlement was somewhat lower than expected. Additionally, although the settlement included provisions that Lumber Liquidators monitor its products more in the future, the company admitted no wrongdoing in the settlement. For LinkedIn, although the company had beat both income and earnings projections, it announced income projections for the next quarter which were lower than the previous quarter. In T. Rowe Price's

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case, the lower earnings were the result of increased costs. However, the company had increased its assets under management by 2 percent, which indicated potentially higher future earnings.

To summarize, an announcement can be broken into two parts, the anticipated, or expected, part and the surprise, or innovation:

$$\text{Announcement} = \text{Expected part} + \text{Surprise} \qquad [11.12]$$

The expected part of any announcement is the part of the information that the market uses to form the expectation, E(R), of the return on the stock. The surprise is the news that influences the unanticipated return on the stock, U. Henceforth, when we speak of news, we will mean the surprise part of an announcement and not the portion that the market has expected and therefore already discounted.

11.7 RISK: SYSTEMATIC AND UNSYSTEMATIC

The unanticipated part of the return, that portion resulting from surprises, is the true risk of any investment. After all, if we always receive exactly what we expect, then the investment is perfectly predictable and, by definition, risk-free. In other words, the risk of owning an asset comes from surprises—unanticipated events.

There are important differences, though, among various sources of risk. Look back at our previous list of news stories. Some of these stories are directed specifically at Flyers, and some are more general. Which of the news items are of specific importance to Flyers?

Announcements about interest rates or GDP are clearly important for nearly all companies, whereas the news about Flyers's research or sales is of specific interest to Flyers. We will distinguish between these two types of events, because, as we shall see, they have very different implications.

Systematic and Unsystematic Risk

The first type of surprise, the one that affects a large number of assets, we will label **systematic risk**. A systematic risk is one that influences a large number of assets, each to a greater or lesser extent. Because systematic risks have marketwide effects, they are sometimes called *market risks* (or correlated risks).

The second type of surprise we will call **unsystematic risk**. An unsystematic risk is one that affects a single asset or a small group of assets. Because these risks are unique to individual companies or assets, they are sometimes called *unique* or *asset-specific risks* (or uncorrelated risks). We will use these terms interchangeably.

As we have seen, uncertainties about general economic conditions, such as GDP, interest rates, or inflation, are examples of systematic risks. These conditions affect nearly all companies to some degree. An unanticipated increase, or surprise, in inflation, for example, affects wages and the costs of the supplies that companies buy; it affects the value of the assets that companies own; and it affects the prices at which companies sell their products. Forces such as these, to which all companies are susceptible, are the essence of systematic risk.

In contrast, the announcement of an oil strike by a company will primarily affect that company and, perhaps, a few others (such as primary competitors and suppliers). It is unlikely to have much of an effect on the world oil market, however, or on the affairs of companies not in the oil business, so this is an unsystematic event.

Systematic and Unsystematic Components of Return

The distinction between a systematic risk and an unsystematic risk is never really as exact as we make it out to be. Even the most narrow and peculiar bit of news about a company ripples through the economy. This is true because every enterprise, no matter how tiny, is a part of the economy. It's like the tale of a kingdom that was lost because one horse lost a shoe. This is mostly hairsplitting, however. Some risks are clearly much more general than others. We'll see some evidence on this point in just a moment.

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The distinction between the types of risk allows us to break down the surprise portion, *U*, of the return on the Flyers stock into two parts. Earlier, we had the actual return broken down into its expected and surprise components:

$$R = E(R) + U$$

We now recognize that the total surprise component for Flyers, *U*, has a systematic and an unsystematic component, so:

$$R = E(R) + \text{Systematic portion} + \text{Unsystematic portion} \quad [11.13]$$

To see why the distinction between systematic and unsystematic risks is important, we need to return to the subject of portfolio risk.

11.8 DIVERSIFICATION AND PORTFOLIO RISK

We've seen earlier that portfolio risks can, in principle, be quite different from the risks of the assets that make up the portfolio. We now look more closely at the riskiness of an individual asset versus the risk of a portfolio of many different assets. We will once again examine some market history to get an idea of what happens with actual investments in U.S. capital markets.

For more on risk and diversification, visit www.investopedia.com/university.

The Effect of Diversification: Another Lesson from Market History

In our previous chapter, we saw that the standard deviation of the annual return on a portfolio of 500 large common stocks has historically been about 20 percent per year. Does this mean that the standard deviation of the annual return on a typical stock in that group of 500 is about 20 percent? As you might suspect by now, the answer is no. This is an extremely important observation.

To allow examination of the relationship between portfolio size and portfolio risk, Table 11.4 illustrates typical average annual standard deviations for equally weighted portfolios that contain different numbers of randomly selected NYSE securities.

In Column 2 of Table 11.4, we see that the standard deviation for a "portfolio" of one security is about 49 percent. What this means is that if you randomly selected a single NYSE stock and put all your money into it, your standard deviation of return would typically be a substantial 49 percent per year. If you were to randomly select two stocks and invest half your money in each, your standard deviation would be about 37 percent on average, and so on.

The important thing to notice in Table 11.4 is that the standard deviation declines as the number of securities is increased. By the time we have 100 randomly chosen stocks, the portfolio's standard deviation has declined by about 60 percent, from 49 percent to about 20 percent. With 500 securities, the standard deviation is 19.27 percent, similar to the 20 percent we saw in our previous chapter for the large common stock portfolio. The small difference exists because the portfolio securities and time periods examined are not identical.

The Principle of Diversification

Figure 11.9 illustrates the point we've been discussing. What we have plotted is the standard deviation of return versus the number of stocks in the portfolio. Notice in Figure 11.9 that the benefit in terms of risk reduction from adding securities drops off as we add more

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and more. By the time we have 10 securities, most of the effect is already realized, and by the time we get to 30 or so, there is very little remaining benefit.

TABLE 11.4
Standard Deviations of Annual Portfolio Returns

(1) NUMBER OF STOCKS IN PORTFOLIO	(2) AVERAGE STANDARD DEVIATION OF ANNUAL PORTFOLIO RETURNS	(3) RATIO OF PORTFOLIO STANDARD DEVIATION TO STANDARD DEVIATION OF A SINGLE STOCK
1	49.24%	1.00
2	37.36	.76
4	29.69	.60
6	26.64	.54
8	24.98	.51
10	23.93	.49
20	21.68	.44
30	20.87	.42
40	20.46	.42
50	20.20	.41
100	19.69	.40
200	19.42	.39
300	19.34	.39
400	19.29	.39
500	19.27	.39
1,000	19.21	.39

Sources: M. Statman, "How Many Stocks Make a Diversified Portfolio?" *Journal of Financial and Quantitative Analysis* 22 (September 1987), pp. 353–63. Derived from E. J. Elton and M. J. Gruber, "Risk Reduction and Portfolio Size: An Analytic Solution," *Journal of Business* 50 (October 1977), pp. 415–37.

FIGURE 11.9
Portfolio Diversification

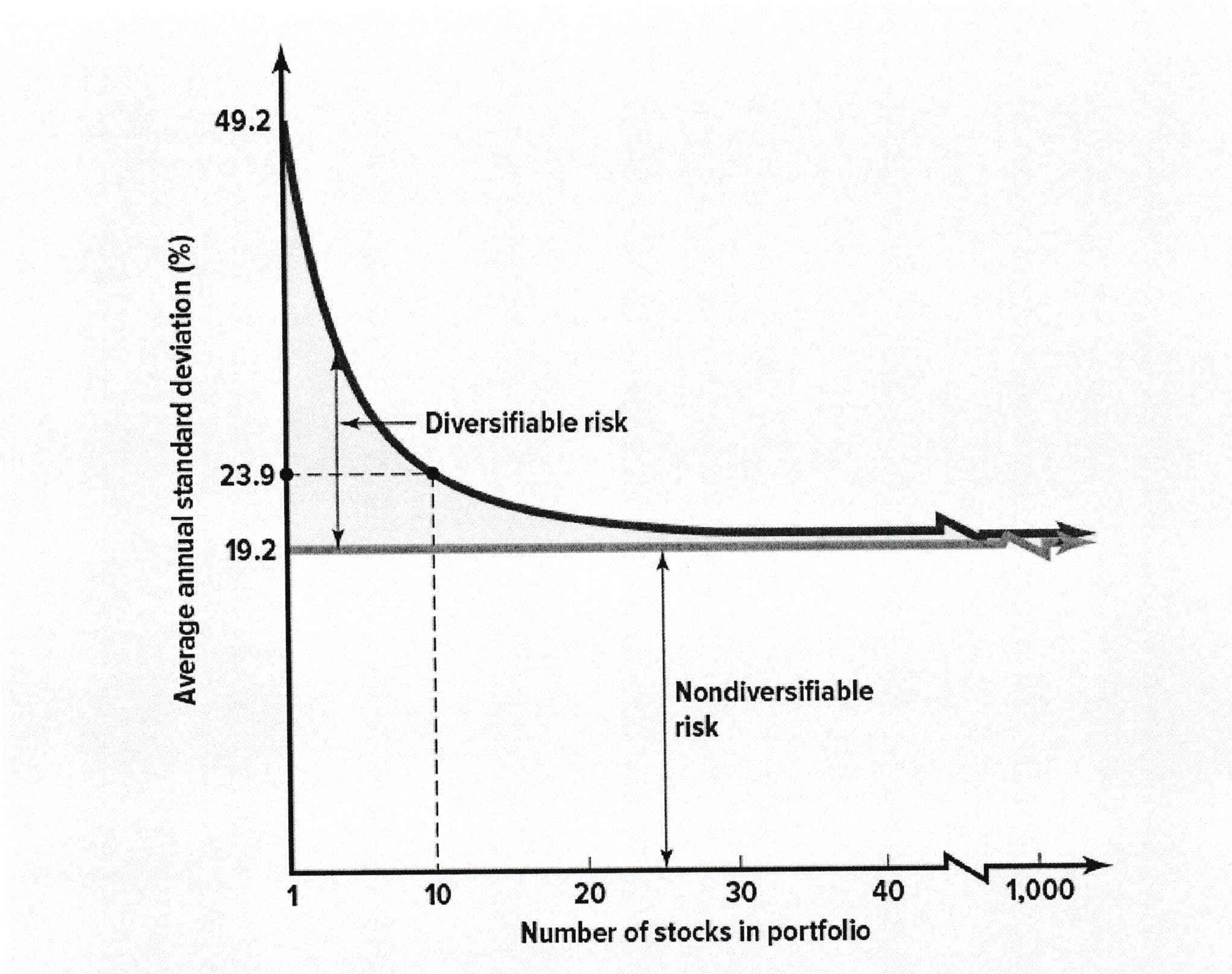

Figure 11.9 illustrates two key points. First, some of the riskiness associated with individual assets can be eliminated by forming portfolios. The process of spreading an

investment across assets (and thereby forming a portfolio) is called *diversification.* The **principle of diversification** tells us that spreading an investment across many assets will eliminate some of the risk. The purple shaded area in Figure 11.9, labeled "diversifiable risk," is the part that can be eliminated by diversification.

The second point is equally important. There is a minimum level of risk that cannot be eliminated by diversifying. This minimum level is labeled "nondiversifiable risk" in Figure 11.9. Taken together, these two points are another important lesson from capital market history: Diversification reduces risk, but only up to a point. Put another way, some risk is diversifiable and some is not.

To give a recent example of the impact of diversification, the Dow Jones Industrial Average (DJIA), which is a widely followed stock market index of 30 large, well-known U.S. stocks, was down about 2 percent in 2015. As we saw in our previous chapter, this loss represents a poor year for a portfolio of large-cap stocks. The biggest individual winners for the year were McDonald's (up 33 percent) and Nike (up 32 percent). But not all 30 stocks were up: The losers included Walmart (down 26 percent) and American Express (down 24 percent). Again, the lesson is clear: Diversification reduces exposure to extreme outcomes, both good and bad.

Diversification and Unsystematic Risk

From our discussion of portfolio risk, we know that some of the risk associated with individual assets can be diversified away and some cannot. We are left with an obvious question: Why is this so? It turns out that the answer hinges on the distinction we made earlier between systematic and unsystematic risk.

Here is the important observation: If we only held a single stock, then the value of our investment would fluctuate because of company-specific events. If we hold a large portfolio, on the other hand, some of the stocks in the portfolio will go up in value because of positive company-specific events and some will go down in value because of negative events. The net effect on the overall value of the portfolio will be relatively small, however, because these effects will tend to cancel each other out.

Now we see why some of the variability associated with individual assets is eliminated by diversification. When we combine assets into portfolios, the unique, or unsystematic, events—both positive and negative—tend to "wash out" once we have more than just a few assets.

This is an important point that bears repeating:

Unsystematic risk is essentially eliminated by diversification, so a portfolio with many assets has almost no unsystematic risk.

In fact, the terms *diversifiable risk* and *unsystematic risk* are often used interchangeably.

Diversification and Systematic Risk

We've seen that unsystematic risk can be eliminated by diversifying. What about systematic risk? Can it also be eliminated by diversification? The answer is no because, by definition, a systematic risk affects almost all assets to some degree. As a result, no matter how many assets we put into a portfolio, the systematic risk doesn't go away. Thus, for obvious reasons, the terms *systematic risk* and *nondiversifiable risk* are used interchangeably.

Because we have introduced so many different terms, it is useful to summarize our discussion before moving on. What we have seen is that the total risk of an investment, as measured by the standard deviation of its return, can be written as:

Total risk = Systematic risk + Unsystematic risk **[11.14]**

Systematic risk is also called *nondiversifiable risk* or *market risk.* Unsystematic risk is also called *diversifiable risk,unique risk*, or *asset-specific risk.* For a well-diversified portfolio, the unsystematic risk is negligible. For such a portfolio, essentially all of the risk is systematic.

11.9 MARKET EQUILIBRIUM

Definition of the Market Equilibrium Portfolio

Financial economists often imagine a world where all investors possess the *same* estimates on expected returns, variances, and covariances. Though this can never be literally true, it can be thought of as a useful simplifying assumption in a world where investors have access to similar sources of information. This assumption is called **homogeneous expectations.**[5]

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If all investors had homogeneous expectations, Figure 11.8 would be the same for all individuals. That is, all investors would sketch out the same efficient set of risky assets because they would be working with the same inputs. This efficient set of risky assets is represented by the Curve *XAY.* Because the same risk-free rate would apply to everyone, all investors would view Point *A* as the portfolio of risky assets to be held.

This Point *A* takes on great importance because all investors would purchase the risky securities that it represents. Those investors with a high degree of risk aversion might combine *A* with an investment in the riskless asset, achieving Point *4*, for example. Others with low aversion to risk might borrow to achieve, say, Point *5.* Because this is a very important conclusion, we restate it:

In a world with homogeneous expectations, all investors would hold the portfolio of risky assets represented by Point *A*.

TABLE 11.5 Estimates of Beta for Selected Individual Stocks

STOCK	BETA
American Electric Power	.22
General Mills	.63
Procter & Gamble	.66
Facebook	.73
Pfizer	1.02
Allstate	1.08

Citigroup	1.64
Amazon.com	1.65

The beta is defined as Cov(R_j, R_M)/Var(R_M), where Cov(R_j, R_M) is the covariance of the return on an individual stock, R_j and the return on the market, R_M. Var(R_M) is the variance of the return on the market, R_M.

Source: finance.yahoo.com

If all investors choose the same portfolio of risky assets, it is possible to determine what that portfolio is. Common sense tells us that it is a market value weighted portfolio of all existing securities. It is the **market portfolio**. In terms of our discussion of diversification, the market portfolio is perfectly diversified by having eliminated all unsystematic risk.

In practice, financial economists use a broad-based index such as the Standard & Poor's (S&P) 500 as a proxy for the market portfolio. Of course, all investors do not hold the same portfolio. However, we know that a large number of investors hold diversified portfolios, particularly when mutual funds or pension funds are included. A broad-based index is a good proxy for the highly diversified portfolios of many investors.

Definition of Risk When Investors Hold the Market Portfolio

The previous section states that many investors hold diversified portfolios similar to broad-based indexes. This result allows us to be more precise about the risk of a security in the context of a diversified portfolio.

Researchers have shown that the best measure of the risk of a security in a large portfolio is the *beta* of the security. We illustrate beta by an example:

EXAMPLE 11.4

Beta

Consider the following possible returns on both the stock of Jelco, Inc., and on the market:

STATE	TYPE OF ECONOMY	RETURN ON MARKET (PERCENT)	RETURN ON JELCO, INC. (PERCENT)
I	Bull	15	25
II	Bull	15	15
III	Bear	−5	−5
IV	Bear	−5	−15

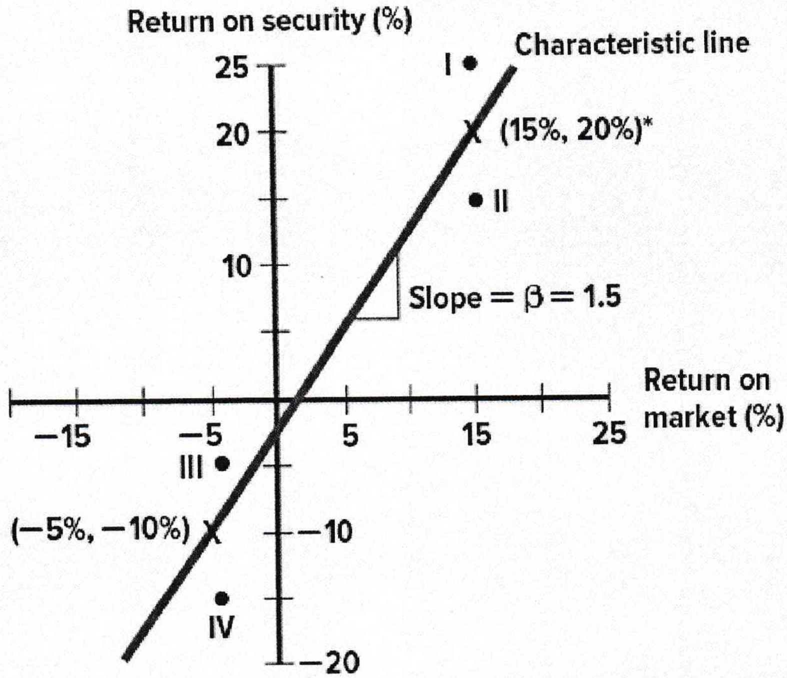
Though the return on the market has only two possible outcomes (15% and −5%), the return on Jelco has four possible outcomes. It is helpful to consider the expected return on a security for a given return on the market. Assuming each state is equally likely, we have:

TYPE OF ECONOMY	RETURN ON MARKET (PERCENT)	EXPECTED RETURN ON JELCO, INC. (PERCENT)
Bull	15	20% = 25% ×.50 + 15% × .50
Bear	−5	−10% = −5% ×.50 + (−15%) × .50

Jelco, Inc., responds to market movements because its expected return is greater in bullish states than in bearish states. We now calculate exactly how responsive the security is to market movements. The market's return in a bullish economy is 20 percent [= 15% − (−5%)] greater than the market's return in a bearish economy. However, the expected return on Jelco in a bullish economy is 30 percent [= 20% − (−10%)] greater than its expected return in a bearish state. Thus, Jelco, Inc., has a responsiveness coefficient of 1.5 (= 30%/20%).

This relationship appears in Figure 11.10. The returns for both Jelco and the market in each state are plotted as four points. In addition, we plot the expected return on the security for each of the two possible returns on the market. These two points, each of which we designate by an X, are joined by a line called the characteristic line of the security. The slope of the line is 1.5, the number calculated in the previous paragraph. This responsiveness coefficient of 1.5 is the beta of Jelco.

FIGURE 11.10
Performance of Jelco, Inc., and the Market Portfolio



The two points marked X represent the expected return on Jelco for each possible outcome of the market portfolio. The expected return on Jelco is positively related to the return on the market. Because the slope is 1.5, we say that Jelco's beta is 1.5. Beta measures the responsiveness of the security's return to movements in the market.

*(15%, 20%) refers to the point where the return on the market is 15 percent and the return on the security is 20 percent.

The interpretation of beta from Figure 11.10 is intuitive. The graph tells us that the returns of Jelco are magnified 1.5 times over those of the market. When the market does well, Jelco's stock is expected to do even better. When the market does poorly, Jelco's stock is expected to do even worse. Now imagine an individual with a portfolio near that of the market who is considering the addition of Jelco to his portfolio. Because of Jelco's *magnification factor* of 1.5, he will view this stock as contributing much to the risk of the portfolio. (We will show shortly that the beta of the average security in the market is 1.) Jelco contributes more to the risk of a large, diversified portfolio than does an average security because Jelco is more responsive to movements in the market.

Further insight can be gleaned by examining securities with negative betas. One should view these securities as either hedges or insurance policies. The security is expected to do well when the market does poorly and vice versa. Because of this, adding a negative beta security to a large, diversified portfolio actually reduces the risk of the portfolio.[6]

Table 11.5 presents empirical estimates of betas for individual securities. As can be seen, some securities are more responsive to the market than others. For example, Allstate has a beta of 1.08. This means that, for every 1 percent movement in the market, Allstate is expected to move 1.08 percent in the same direction. Conversely, Procter & Gamble has a beta of only .66. This means that, for every 1 percent movement in the market, Procter & Gamble is expected to move .66 percent in the same direction.

We can summarize our discussion of beta by saying:

Beta measures the responsiveness of a security to movements in the market portfolio.

Betas are easy to find on the Web. Try finance.yahoo.com and money.cnn.com.

You can find beta estimates at many sites on the web. One of the best is finance.yahoo.com. We went there and entered the ticker symbol FNMA for Fannie Mae, and followed the "Key Statistics" link. Here is part of what we found:

STOCK PRICE HISTORY	
Beta	4.94
52-week change	−41.18%

INCOME STATEMENT

Revenue (ttm):	23.18B
Revenue per share (ttm):	4.02
Qtrly revenue growth (yoy):	49.30%
Gross profit (ttm):	N/A
EBITDA (ttm):	N/A
Net income avl to common (ttm):	−262.00M
Diluted EPS (ttm):	−.05
Qtrly earnings growth (yoy):	88.00%

BALANCE SHEET

Total cash (mrq):	82.83B
Total cash per share (mrq):	14.43
Total debt (mrq):	3.20T
Total debt/euity (mrq):	78,790.22
Current ratio (mrq):	23.28
Book value per share (mrq):	−23.05

CASH FLOW STATEMENT

Operating cash flow (ttm):	−6.67B
Levered free cash flow (ttm):	N/A

The reported beta for Fannie Mae Corporation is 4.94, which means Fannie Mae has about 494 percent more systematic risk than the average stock. Perhaps you would expect that a mortgage company such as Fannie Mae would be very risky because of its dependence on mortgages. Looking at the numbers, we agree. Fannie Mae's operating cash flow is negative, the company has $3.2 *trillion* in debt, and a debt-equity ratio of 78,790! It has negative book value of equity, stemming from an accumulation of losses. Fannie Mae's success going forward will depend on the health of the real estate market and the company's ability to manage its massive debt burden. In all, Fannie Mae seems to be a good candidate for a high beta. For more about "real-world" betas, see the nearby *Finance Matters* box.

The Formula for Beta

Our discussion so far has stressed the intuition behind beta. The actual definition of beta is

$$\beta_i = \frac{\mathrm{Cov}(R_i, R_M)}{\sigma^2(R_M)} \qquad [11.15]$$

For more on beta, visit money.msn.com.

where $\mathrm{Cov}(R_i, R_M)$ is the covariance between the return on Asset i and the return on the market portfolio and $\sigma^2(R_M)$ is the variance of the market. This formula reinforces the notion that the risk of an individual security does not depend on its own variance (or standard deviation), only on its covariance with the market.

One useful property is that the average beta across all securities, when weighted by the proportion of each security's market value to that of the market portfolio, is 1. That is:

$$\sum_{i=1}^{N} X_i \beta_i = 1 \qquad [11.16]$$

where X_i is the proportion of Security i's market value to that of the entire market and N is the number of securities in the market.

For more on beta, visit money.msn.com. Equation 11.16 is intuitive, once you think about it. If you weight all securities by their market values, the resulting portfolio is the market. By definition, the beta of the market

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portfolio is 1. That is, for every 1 percent movement in the market, the market must move 1 percent—*by definition.*

FINANCE MATTERS

BETA, BETA, WHO'S GOT THE BETA?

Based on what we've studied so far, you can see that beta is a pretty important topic. You might wonder then, are all published betas created equal? Read on for a partial answer to this question.

We did some checking on betas and found some interesting results. The Value Line *Investment Survey* is one of the best-known sources for information on publicly traded companies. However, with the explosion of online investing, there has been a corresponding increase in the amount of investment information available online. We decided to compare the betas presented by Value Line to those reported by Yahoo! Finance (finance.yahoo.com), Google (finance.google.com), and CNN Money (money.cnn.com). What we found leads to an important note of caution.

Consider Guess, Inc., the clothing company. Value Line reported the company's beta at 1.15, a fairly ordinary number. But the beta for the company reported online was .50, a much lower value. Guess, Inc., wasn't the only stock that showed a divergence in betas. In fact, for most of the technology companies we looked at, Value Line reported betas that were significantly lower than their online cousins. For example, the online beta for Autodesk was 2.07, while Value Line reported a beta of 1.30. Similarly, the online beta for Advanced Micro Devices was 1.91 versus a Value Line beta of 1.45. Interested in something less high tech? The online beta for Principal Financial Group was 1.85, compared to Value Line's beta of 1.30.

We also found some unusual, and even hard-to-believe, estimates for beta. Southern Company, an electric utility, had a very low online beta of .03 (Value Line reported .60). The online beta for Ventas, Inc., a nursing and hospital real estate company, was −.12, compared to Value Line's .80. Perhaps the most ridiculous numbers were the ones reported for Cardia Bioplastics and Vensen Pharma. The estimated betas for those companies were 9,767 and −3,249 (notice the minus sign!), respectively. Value Line did not report a beta for either company. How do you suppose we should interpret a beta of −3,249?

There are a few lessons to be learned from all of this. First, not all betas are created equal. Some are computed using weekly returns and some using daily returns. Some are computed using 60 months of stock returns; some consider more or fewer returns. Some betas are computed by comparing the stock to the S&P 500 Index, while others use alternative indexes. Finally, some reporting firms (including Value Line) make adjustments to raw betas to reflect information other than just the fluctuation in stock prices.

The second lesson is perhaps more subtle and comes from the betas of Cardia Bioplastics and Vensen Pharma. We are interested in knowing what the beta of the stock will be in the future, but betas have to be estimated using historical data. Anytime we use the past to predict the future, there is the danger of a poor estimate. In our case, it is very unlikely that Cardia Bioplastics has a beta anything like 9,767 or that Vensen Pharma has a beta of −3,249. Instead, the estimates are almost certainly bad. The moral of the story is that, as with any financial tool, beta is not a black box that should be taken without question.

A Test

We have put these questions on past corporate finance examinations:

1. What sort of investor rationally views the variance (or standard deviation) of an individual security's return as the security's proper measure of risk?
2. What sort of investor rationally views the beta of a security as the security's proper measure of risk?

A good answer might be something like the following:

A rational, risk-averse investor views the variance (or standard deviation) of her portfolio's return as the proper measure of the risk of her portfolio. If for some reason or another the investor can hold only one security, the variance of that security's return becomes the variance of the portfolio's return. Hence, the variance of the security's return is the security's proper measure of risk.

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If an individual holds a diversified portfolio, she still views the variance (or standard deviation) of her portfolio's return as the proper measure of the risk of her portfolio. However, she is no longer interested in the variance of each individual security's return. Rather, she is interested in the contribution of an individual security to the variance of the portfolio.

Under the assumption of homogeneous expectations, all individuals hold the market portfolio. Thus, we measure risk as the contribution of an individual security to the variance of the market portfolio. This contribution, when standardized properly, is the beta of the security. While very few investors hold the market portfolio exactly, many hold reasonably diversified portfolios. These portfolios are close enough to the market portfolio so that the beta of a security is likely to be a reasonable measure of its risk.

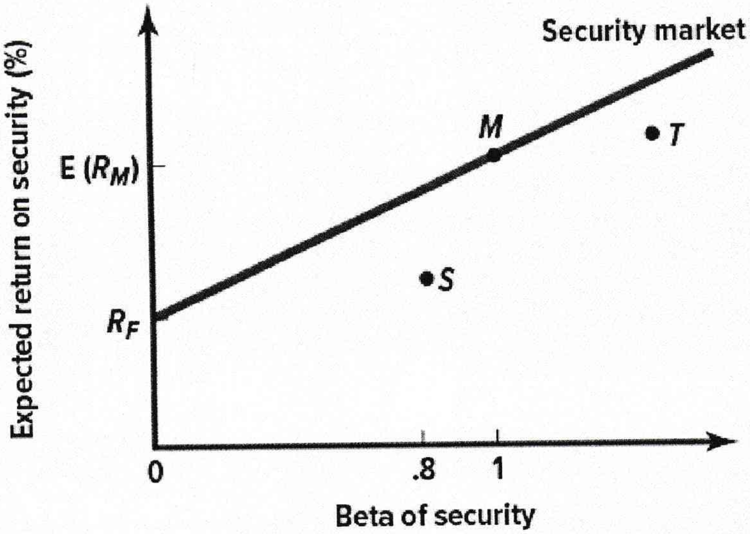
11.10 RELATIONSHIP BETWEEN RISK AND EXPECTED RETURN (CAPM)

It is commonplace to argue that the expected return on an asset should be positively related to its risk. That is, individuals will hold a risky asset only if its expected return compensates for its risk. In this section, we estimate expected returns on individual securities.

Expected Return on Individual Security

What is the expected return on an individual security? We start with the intuition that the expected return on an individual security can be represented as $E(R_i) = R_F +$ Risk premium. In words, the expected return on any individual security (or portfolio) is the sum of the risk-free rate plus some compensation for systematic risk. We have argued that the beta of a security is the appropriate measure of risk in a large, diversified portfolio. Since most investors are diversified, the expected return on a security should be positively related to its beta. This is illustrated in Figure 11.11.

FIGURE 11.11
Relationship between Expected Return on an Individual Security and Beta of the Security



The security market line (SML) is the graphical depiction of the capital asset pricing model (CAPM).
The expected return on a stock with a beta of 0 is equal to the risk-free rate.
The expected return on a stock with a beta of 1 is equal to the expected return on the market.

Actually, financial economists can be more precise about the relationship between expected return and beta. They posit that, under plausible conditions, the relationship between expected return and beta can be represented by the following equation:

> For more on CAPM and the equity risk premium, visit www.investopedia.com/terms/c/capm.asp.

Capital Asset Pricing Model:

$E(R)$	=	R_F	+	β	×	$E(R_M) - R_F$	
Expected return on a security	=	Risk-free rate	+	Beta of the security	×	Difference between expected return on market and risk-free rate, or the market risk premium	[11.17]

This formula, which is called the **capital asset pricing model** (or CAPM for short), implies that the expected return on a security is linearly related to its beta. Since the average return on the market has been higher than the average risk-free rate over long periods of time, the market risk premium $E(R_M) - R_F$ is presumably positive. Thus, the formula implies that the expected return on a security is *positively* related to its beta. The formula can be illustrated by assuming a few special cases:

- *Assume that* $\beta = 0$. Here $E(R) = R_F$, that is, the expected return on the security is equal to the risk-free rate. Because a security with zero beta has no relevant risk, its expected return should equal the risk-free rate.
- *Assume that* $\beta = 1$. Equation 11.17 reduces to $E(R) = E(R_M)$. That is, the expected return on the security is equal to the expected return on the market. This makes sense since the beta of the market portfolio is also 1.

Equation 11.17 can be represented graphically by the upward-sloping line in Figure 11.11. Note that the line begins at R_F and rises to $E(R_M)$ when beta is 1. This line is frequently called the **security market line (SML)**.

As with any line, the SML has both a slope and an intercept. R_F, the risk-free rate, is the intercept. Because the beta of a security is the horizontal axis, $E(R_M) - R_F$ is the slope. The line will be upward sloping as long as the expected return on the market is greater than the risk-free rate. Because the market portfolio is a risky asset, theory suggests that its expected return is above the risk-free rate. As mentioned, the empirical evidence of the previous chapter showed that the average return per year on the market portfolio (e.g., U.S. large-company stocks) from 1900–2010 was 7.2 percent above the risk-free rate.

EXAMPLE 11.5

CAPM

The stock of Aardvark Enterprises has a beta of 1.5, and that of Zebra Enterprises has a beta of .7. The risk-free rate is assumed to be 3 percent, and the difference between the expected return on the market and the risk-free rate is assumed to be 8 percent. The expected returns on the two securities are:

Expected Return for Aardvark

$15.0\% = 3\% + 1.5 \times 8\%$

Expected Return for Zebra

$8.6\% = 3\% + .7 \times 8\%$

Three additional points concerning the CAPM should be mentioned:

1. *Linearity.* The intuition behind an upwardly sloping curve is clear. Because beta is the appropriate measure of risk, high-beta securities should have an expected return above that of

low-beta securities. However, both Figure 11.11 and Equation 11.17 show something more than an upwardly sloping curve; the relationship between expected return and beta corresponds to a *straight* line.

It is easy to show that the line of Figure 11.11 is straight. To see this, consider Security *S* with, say, a beta of .8. This security is represented by a point below the security market line in the figure. Any investor could duplicate the beta of Security *S* by buying a portfolio with 20 percent in the risk-free asset and 80 percent in a security with a beta of 1. However, the homemade portfolio would itself lie on the SML. In other words, the portfolio dominates Security *S* because the portfolio has a higher expected return and the same beta.

Now consider Security *T* with, say, a beta greater than 1. This security is also below the SML in Figure 11.11. Any investor could duplicate the beta of Security *T* by borrowing to invest in a security with a beta of 1. This portfolio must also lie on the SML, thereby dominating Security *T*.

Because no one would hold either *S* or *T*, their stock prices would drop. This price adjustment would raise the expected returns on the two securities. The price adjustment would continue until the two securities lay on the security market line. The preceding example considered two overpriced stocks and a straight SML. Securities lying above the SML are *underpriced.* Their prices must rise until their expected returns lie on the line. If the SML is itself curved, many stocks would be mispriced. In equilibrium, all securities would be held only when prices changed so that the SML became straight. In other words, linearity would be achieved.

- *Portfolios as well as securities.* Our discussion of the CAPM considered individual securities. Does the relationship in Figure 11.11 and Equation 11.17 hold for portfolios as well?

Yes. To see this, consider a portfolio formed by investing equally in our two securities, Aardvark and Zebra. The expected return on the portfolio is:

Expected Return on Portfolio

$$11.8\% = .5 \times 15.0\% + .5 \times 8.6\%$$

The beta of the portfolio is a weighted average of the betas of the two securities. Thus, we have:

Beta of Portfolio

$$1.1 = .5 \times 1.5 + .5 \times .7$$

Under the CAPM, the expected return on the portfolio is:

$$11.8\% = 3\% + 1.1 \times 8\%$$

Because the expected return in Equation 11.17 is the same as the expected return in the above equation, the example shows that the CAPM holds for portfolios as well as for individual securities.

- *A potential confusion.* Students often confuse the SML in Figure 11.11 with Line *II* in Figure 11.8. Actually, the lines are quite different. Line *II* traces the efficient set of portfolios formed from both risky assets and the riskless asset. Each point on the line represents an entire portfolio. Point *A* is a portfolio composed entirely of risky assets. Every other point on the line represents a portfolio of the securities in *A* combined with the riskless asset. The axes on Figure 11.8 are the expected return on a *portfolio* and the standard deviation of a *portfolio.* Individual securities do not lie along Line *II.*

The SML in Figure 11.11 relates expected return to beta. Figure 11.11 differs from Figure 11.8 in at least two ways. First, beta appears in the horizontal axis of Figure 11.11, but standard deviation appears in the horizontal axis of Figure 11.8. Second, the SML in Figure 11.11 holds both for all individual securities and for all possible portfolios, whereas Line *II* in Figure 11.8 holds only for efficient portfolios.

We stated earlier that, under homogeneous expectations, Point *A* in Figure 11.8 becomes the market portfolio. In this situation, Line *II* is referred to as the **capital market line (CML).**

SUMMARY AND CONCLUSIONS

This chapter sets forth the fundamentals of modern portfolio theory. Our basic points are these:

1. This chapter shows us how to calculate the expected return and variance for individual securities, and the covariance and correlation for pairs of securities. Given these statistics, the expected return and variance for a portfolio of two securities, *A* and *B*, can be written as:

$$\text{Expected return on portfolio} = X_A \mathrm{E}(R_A) + X_B \mathrm{E}(R_B)$$
$$\text{Variance of portfolio return} = X_A^2\sigma_A^2 + 2X_AX_B\sigma_{AB} + X_B^2\sigma_B^2$$

2. In our notation, *X* stands for the proportion of a security in one's portfolio. By varying *X*, one can trace out the efficient set of portfolios. We graphed the efficient set for the two-asset case as a curve, pointing out that the degree of curvature or bend in the graph reflects the diversification effect: The lower the correlation between the two securities, the greater the bend. The same general shape of the efficient set holds in a world of many assets.
3. A diversified portfolio can eliminate only some, not all, of the risk associated with individual securities. The reason is that part of the risk with an individual asset is unsystematic, meaning essentially unique to that asset. In a well-diversified portfolio, these unsystematic risks tend to cancel out. Systematic, or market, risks are not diversifiable.
4. The efficient set of risky assets can be combined with riskless borrowing and lending. In this case, a rational investor will always choose to hold the portfolio of risky securities represented by Point *A* in Figure 11.8. Then he can either borrow or lend at the riskless rate to achieve any desired point on Line *II* in the figure.
5. The contribution of a security to the risk of a large, well-diversified portfolio is proportional to the covariance of the security's return with the market's return. This contribution, when standardized, is called the beta. The beta of a security can also be interpreted as the responsiveness of a security's return to that of the market.
6. The CAPM states that:

$$\mathrm{E}(R) = R_F + \beta[\mathrm{E}(R_M) - R_F]$$

In other words, the expected return on a security is positively (and linearly) related to the security's beta.

CONCEPT QUESTIONS

1. **Diversifiable and Nondiversifiable Risks** In broad terms, why is some risk diversifiable? Why are some risks nondiversifiable? Does it follow that an investor can control the level of unsystematic risk in a portfolio, but not the level of systematic risk?
2. **Information and Market Returns** Suppose the government announces that, based on a just-completed survey, the growth rate in the economy is likely to be 2 percent in the coming year,

as compared to 5 percent for the year just completed. Will security prices increase, decrease, or stay the same following this announcement? Does it make any difference whether or not the 2 percent figure was anticipated by the market? Explain.

3. **Systematic versus Unsystematic Risk** Classify the following events as mostly systematic or mostly unsystematic. Is the distinction clear in every case?

   a. Short-term interest rates increase unexpectedly.

   b. The interest rate a company pays on its short-term debt borrowing is increased by its bank.

   c. Oil prices unexpectedly decline.

   d. An oil tanker ruptures, creating a large oil spill.

   e. A manufacturer loses a multimillion-dollar product liability suit.

   f. A Supreme Court decision substantially broadens producer liability for injuries suffered by product users.