Risk and Return: Lessons from Market History 10

OPENING CASE

With the S&P 500 Index up about 1.4 percent, and the NASDAQ Composite Index up about 5.7 percent in 2015, overall stock market performance was pretty poor. However, investors in biopharmaceutical company Nymox Pharmaceutical had to be happy about the 720 percent gain that stock delivered, and investors in smartphone marketing company Voltari were advertising the company's 658 percent gain. Of course, not all stocks increased in value during the year. Stock in Chesapeake Energy fell about 76 percent during the year, and stock in Consol Energy also dropped 76 percent.

These examples show that there were tremendous potential profits to be made during 2015, but there was also the risk of losing money, and lots of it. So what should you, as a stock market investor, expect when you invest your own money? In this chapter, we study nine decades of market history to find out.

Please visit us at corecorporatefinance.blogspot.com for the latest developments in the world of corporate finance.

10.1 RETURNS

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Dollar Returns

How did the market do today? Find out at finance.yahoo.com.

Suppose the Video Concept Company has several thousand shares of stock outstanding and you are a shareholder. Further suppose that you purchased some of the shares of stock in the company at the beginning of the year; it is now year-end and you want to figure out how well you have done on your investment. The return you get on an investment in stocks, like that in bonds or any other investment, comes in two forms.

First, over the year most companies pay dividends to shareholders. As the owner of stock in the Video Concept Company, you are a part owner of the company. If the company is profitable, it generally will distribute some of its profits to the shareholders. Therefore, as the owner of shares of stock, you will receive some cash, called a *dividend*, during the year. This cash is the *income component* of your return. In addition to the dividend, the other part of your return is the *capital gain*—or, if it is negative, the *capital loss* (negative capital gain)—on the investment.

For example, suppose we are considering the cash flows of the investment in Figure 10.1 and you purchased 100 shares of stock at the beginning of the year at a price of \$37 per share. Your total investment, then, would be:

$$C_0 = \$37 \times 100 = \$3,700$$

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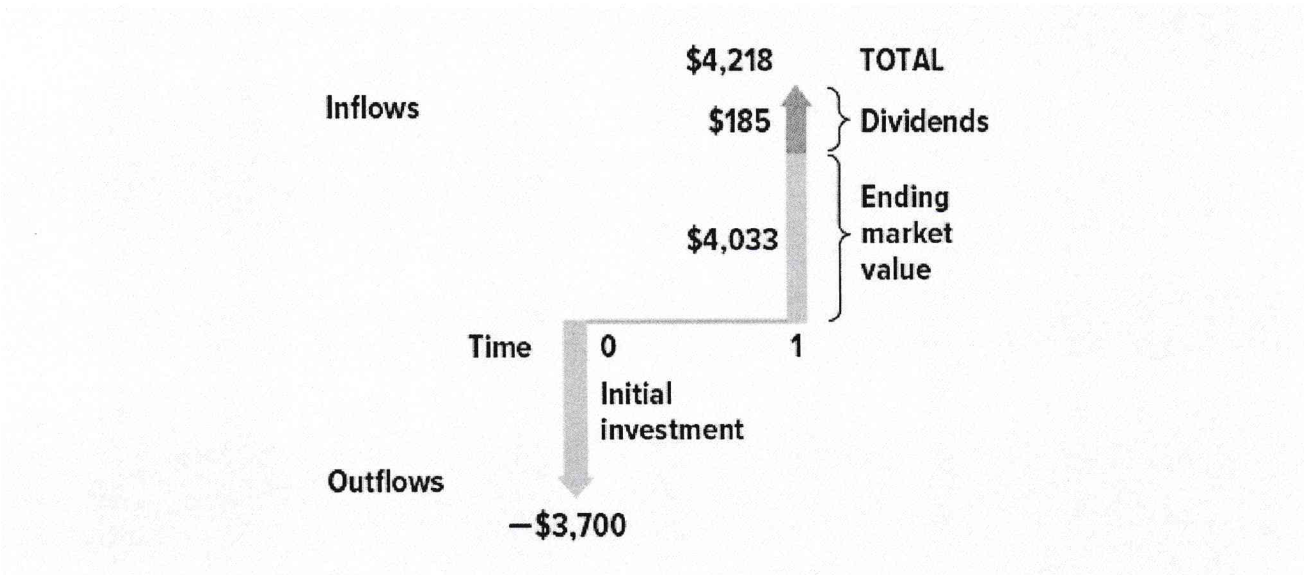
FIGURE 10.1
Dollar Returns



Suppose that over the year the stock paid a dividend of $1.85 per share. During the year, then, you would have received income of:

$$\text{Div} = \$1.85 \times 100 = \$185$$

Suppose, lastly, that at the end of the year the market price of the stock is $40.33 per share. Because the stock increased in price, you have a capital gain of:

$$\text{Gain} = (\$40.33 - 37) \times 100 = \$333$$

The capital gain, like the dividend, is part of the return that shareholders require to maintain their investment in the Video Concept Company. Of course, if the price of Video Concept stock had dropped in value to, say, $34.78, you would have recorded a capital loss of:

$$\text{Loss} = (\$34.78 - 37) \times 100 = -\$222$$

The *total dollar return* on your investment is the sum of the dividend income and the capital gain or loss on the investment:

$$\text{Total dollar return} = \text{Dividend income} + \text{Capital gain (or loss)} \qquad [10.1]$$

(From now on we will refer to *capital losses* as *negative capital gains* and not distinguish them.) In our first example, then, the total dollar return is given by:

Total dollar return = $185 + 333 = $518

Notice that if you sold the stock at the end of the year, your total amount of cash would be the initial investment plus the total dollar return. In the preceding example, then, you would have:

Total cash if stock is sold = Initial investment + Total dollar return
= $3,700 + 518
= $4,218

As a check, notice that this is the same as the proceeds from the sale of stock plus the dividends:

Proceeds from stock sale + Dividends
= $40.33 × 100 + $185
= $4,033 + 185
= $4,218

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Suppose, however, that you hold your Video Concept stock and don't sell it at year-end. Should you still consider the capital gain as part of your return? Does this violate our previous present value rule that only cash matters?

The answer to the first question is a strong yes, and the answer to the second question is an equally strong no. The capital gain is every bit as much a part of your return as is the dividend, and you should certainly count it as part of your total return. That you have decided to hold onto the stock and not sell or *realize* the gain or the loss in no way changes the fact that, if you want to, you could get the cash value of the stock. After all, you could always sell the stock at year-end and immediately buy it back. The total amount of cash you would have at year-end would be the $518 gain plus your initial investment of $3,700. You would not lose this return when you bought back 100 shares of stock. In fact, you would be in exactly the same position as if you had not sold the stock (assuming, of course, that there are no tax consequences and no brokerage commissions from selling the stock).

Percentage Returns

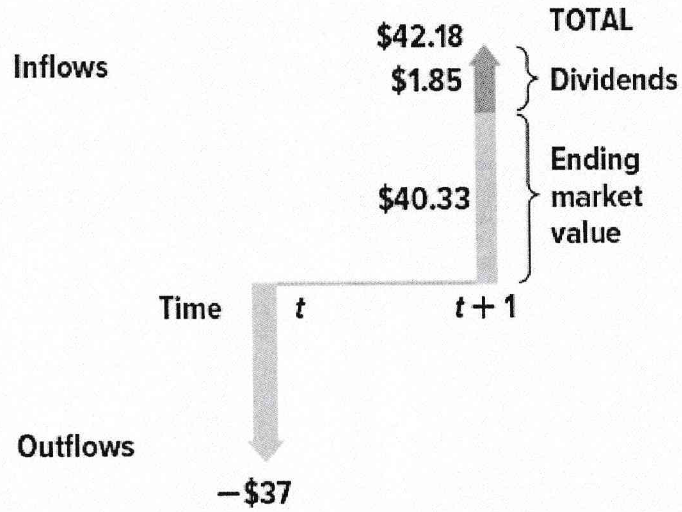
It is more convenient to summarize the information about returns in percentage terms than in dollars, because the percentages apply to any amount invested. The question we want to answer is: How much return do we get for each dollar invested? To find this out, let t stand for the year we are looking at, let P_t be the price of the stock at the beginning of the year, and let Div_{t+1} be the dividend paid on the stock during the year. Consider the cash flows in Figure 10.2.

In our example, the price at the beginning of the year was $37 per share and the dividend paid during the year on each share was $1.85. Hence the percentage income return, sometimes called the *dividend yield*, is:

Go to www.finviz.com/map.ashx for a chart that shows today's returns by market sector.

$$\begin{aligned}\textbf{Dividend yield} &= \text{Div}_{t+1}/P_t \\ &= \$1.85/\$37 \\ &= .05, \text{ or } 5\%\end{aligned} \quad [10.2]$$

FIGURE 10.2
Percentage Returns



$$\text{Percentage return} = \frac{\text{Dividends paid at end of period} + \text{Change in market value over period}}{\text{Beginning market value}}$$

$$1 + \text{Percentage return} = \frac{\text{Dividends paid at end of period} + \text{Market value at end of period}}{\text{Beginning market value}}$$

The **capital gain** (or loss) is the change in the price of the stock divided by the initial price. Letting P_{t+1} be the price of the stock at year-end, the capital gain can be computed:

$$\begin{aligned} \text{Capital gain} &= (P_{t-1} - P_t)/P_t \\ &= (\$40.33 - 37)/\$37 \\ &= \$3.33/\$37 \\ &= .09, \text{ or } 9\% \end{aligned} \qquad [10.3]$$

Combining these two results, we find that the *total return* on the investment in Video Concept stock over the year, which we will label R_{t+1}, was:

$$\begin{aligned} R_{t+1} &= \frac{\text{Div}_{t+1}}{P_t} + \frac{(P_{t+1} - P_t)}{P_t} \\ &= 5\% + 9\% \\ &= 14\% \end{aligned} \qquad [10.4]$$

From now on we will refer to returns in percentage terms.

To give a more concrete example, stock in home improvement store Home Depot began 2015 at \$104.97 per share. The company paid dividends of \$2.36 during 2015, and the stock price at year-end was \$132.25. What was the return for the year? For practice, see if you agree that the answer is 28.24 percent. Of course, negative returns occur as well. For example, in 2015, oil and gas company ExxonMobil's stock price at the beginning of the year was \$92.45 per share, and dividends of \$2.88 were paid. The stock ended the year at \$77.95 per share. Verify that the loss was 12.57 percent for the year.

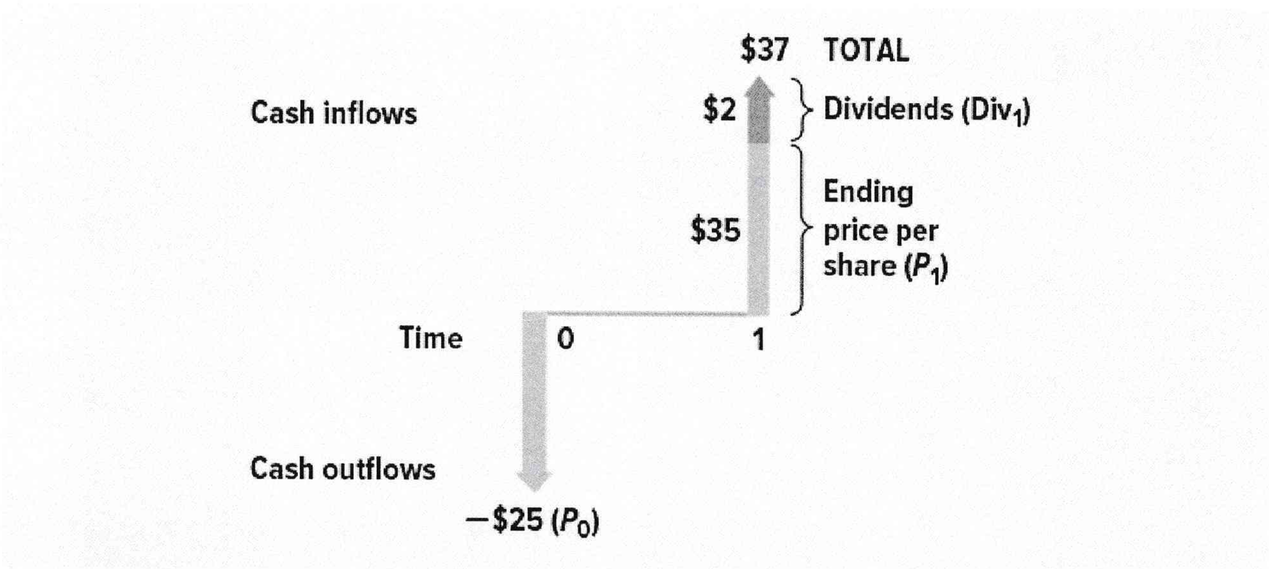
EXAMPLE 10.1

Calculating Returns

Suppose a stock begins the year with a price of \$25 per share and ends with a price of \$35 per share. During the year it paid a \$2 dividend per share. What are its dividend yield, its capital gain, and its total return for the year? We can imagine the cash flows in Figure 10.3.

$$\begin{aligned} R_1 &= \frac{Div_1}{P_0} + \frac{P_1 - P_0}{P_0} \\ &= \frac{\$2}{\$25} + \frac{\$35 - 25}{\$25} = \frac{\$12}{\$25} \\ &= 8\% + \quad 40\% \quad = 48\% \end{aligned}$$

FIGURE 10.3
Cash Flow—An Investment Example



Thus, the stock's dividend yield, its capital gain yield, and its total return are 8 percent, 40 percent, and 48 percent, respectively.

Suppose you had $5,000 invested. The total dollar return you would have received on an investment in the stock is $5,000 × .48 = $2,400. If you know the total dollar return on the stock, you do not need to know how many shares you would have had to purchase to figure out how much money you would have made on the $5,000 investment. You just use the total dollar return.

10.2 HOLDING PERIOD RETURNS

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For more on market history, visit www.globalfinancialdata.com.

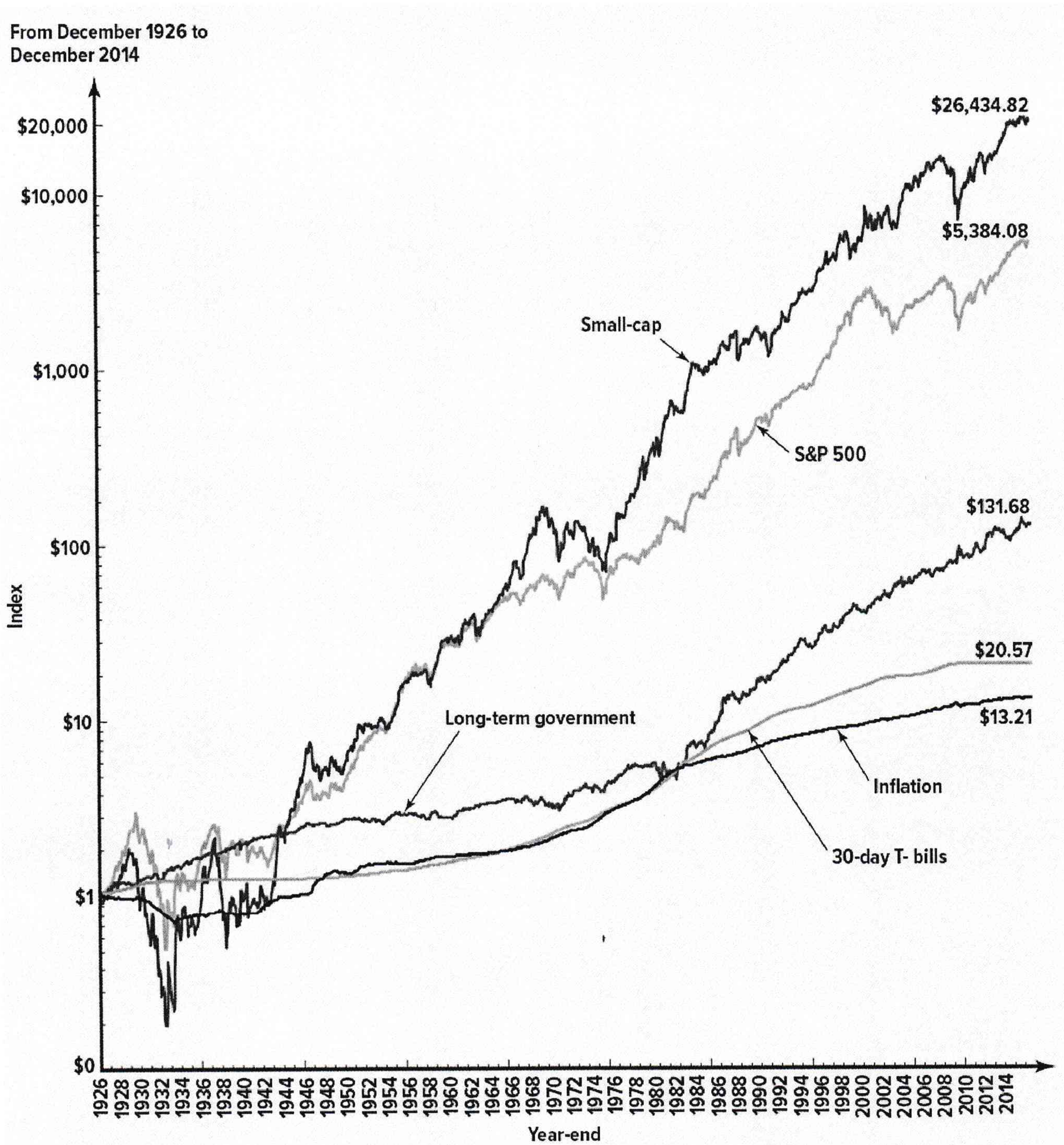
A famous set of studies dealing with rates of return on common stocks, bonds, and Treasury bills was conducted by Roger Ibbotson and Rex Sinquefield.[1] They present year-by-year historical rates of return for the following five important types of financial instruments in the United States:

1. *Large-Company Common Stocks*. The common stock portfolio is based on the Standard & Poor's (S&P) composite index. At present, the S&P composite includes 500 of the largest (in terms of market value) stocks in the United States.
2. *Small-Company Common Stocks*. This is a portfolio corresponding to the bottom fifth of stocks traded on the New York Stock Exchange in which stocks are ranked by market value (i.e., the price of the stock multiplied by the number of shares outstanding).
3. *Long-Term Corporate Bonds*. This is a portfolio of high-quality corporate bonds with a 20-year maturity.
4. *Long-Term U.S. Government Bonds*. This is based on U.S. government bonds with a maturity of 20 years.
5. *U.S. Treasury Bills*. This is based on Treasury bills with a one-month maturity.

None of the returns are adjusted for taxes or transactions costs. In addition to the year-by-year returns on financial instruments, the year-to-year change in the consumer price index is computed. This is a basic measure of inflation. Year-by-year real returns can be calculated by subtracting annual inflation.

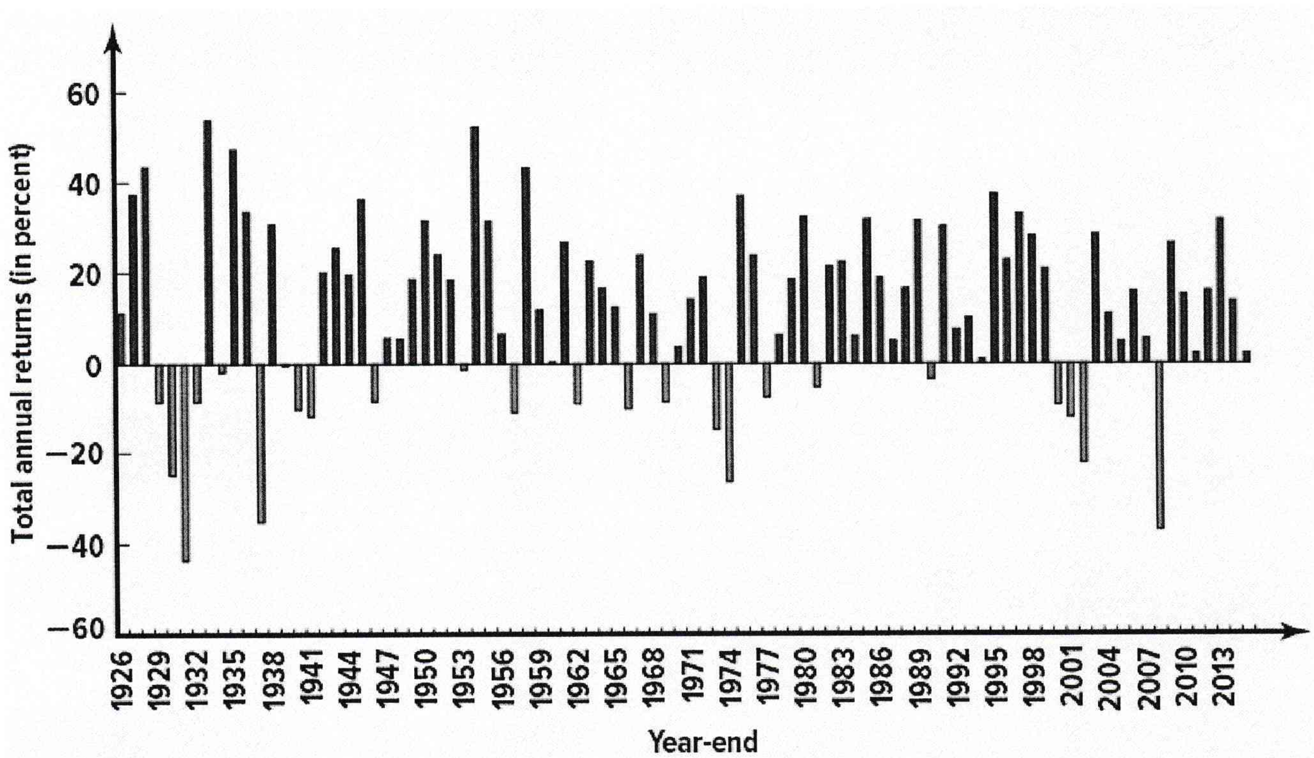
Before looking closely at the different portfolio returns, we graphically present the returns and risks available from U.S. capital markets in the 90-year period from 1926 to 2015. Figure 10.4 shows the growth of $1 invested at the beginning of 1926. Notice that the vertical axis is logarithmic, so that equal distances measure the same percentage change. The figure shows that if $1 were invested in large-company common stocks and all dividends were reinvested, the dollar would have grown to $5,384.08 by the end of 2015. The biggest growth was in the small stock portfolio. If $1 were invested in small stocks in 1926, the investment would have grown to $26,434.82. However, when you look carefully at Figure 10.4, you can see great variability in the returns on small stocks, especially in the earlier part of the period. A dollar in long-term government bonds was very stable as compared with a dollar in common stocks. Figures 10.5 to 10.8 plot each year-to-year percentage return as a vertical bar drawn from the horizontal axis for large-company common stocks, small-company stocks, long-term government bonds and Treasury bills, and inflation, respectively.

FIGURE 10.4
Wealth Indexes of Investments in the U.S. Capital Markets (Year-End 1925 = $1.00)



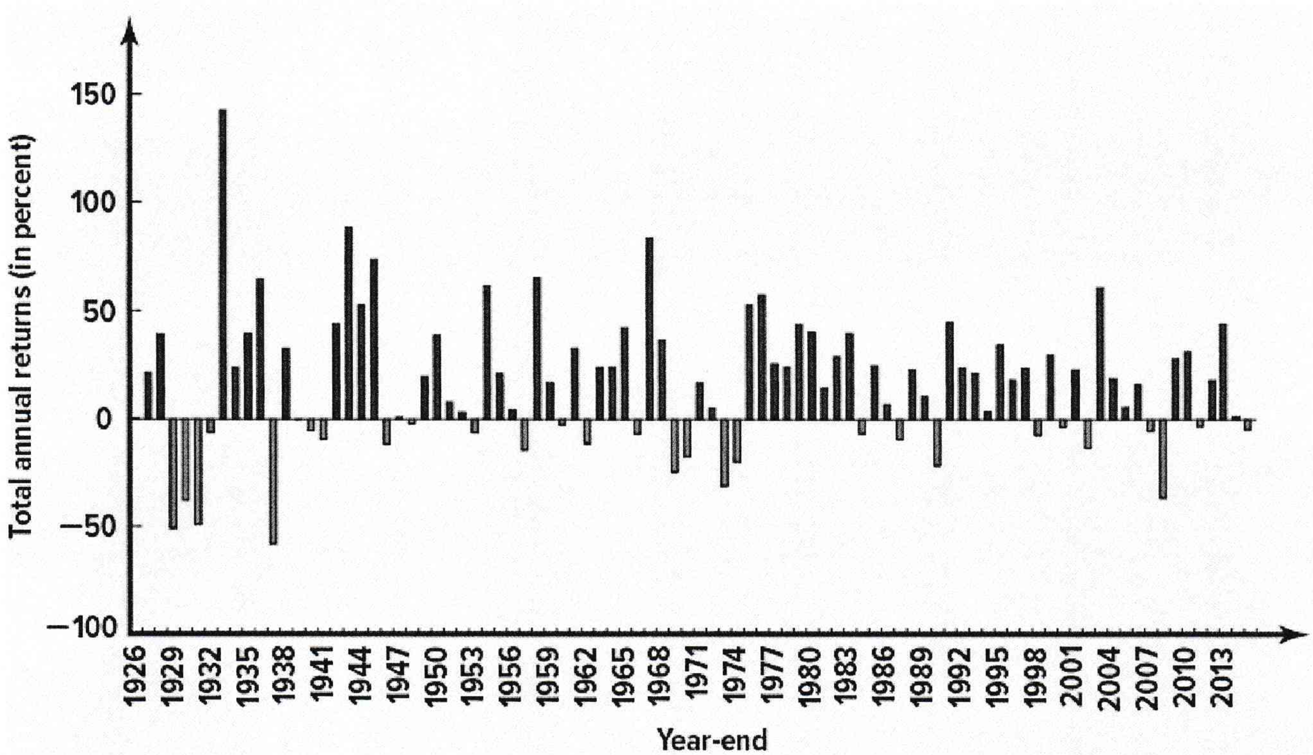
Source: Morningstar, 2016.

FIGURE 10.5
Year-by-Year Total Returns on Large-Company Common Stocks



Source: Morningstar, 2016.

FIGURE 10.6
Year-by-Year Total Returns on Small-Company Stocks

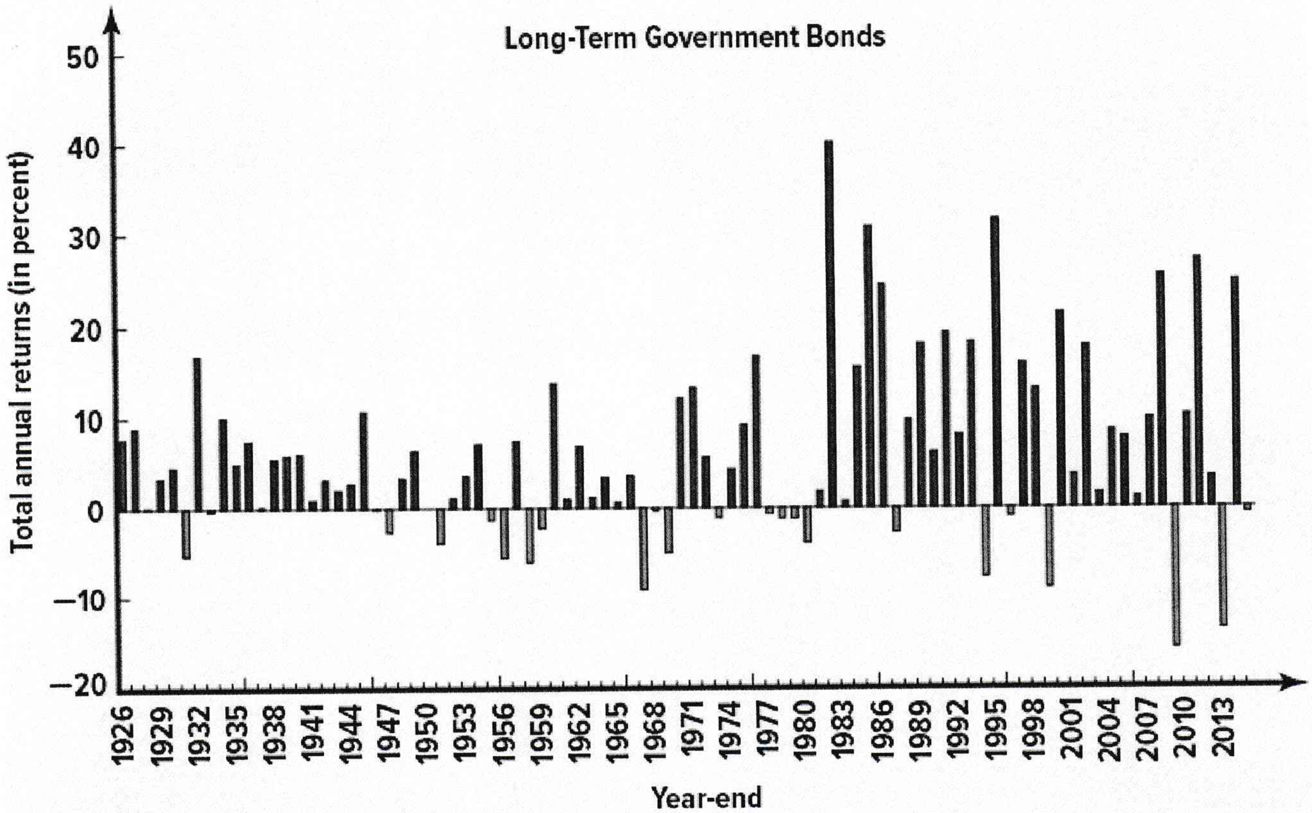


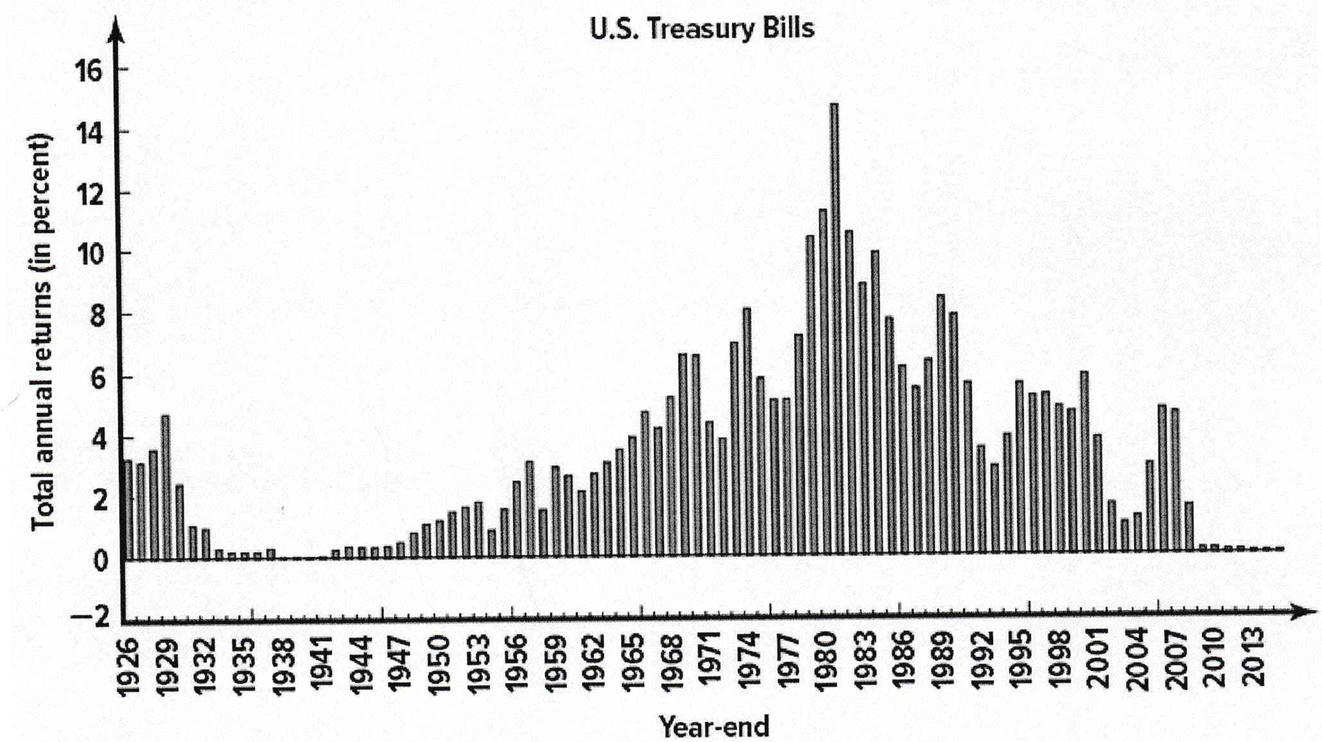
Source: Morningstar, 2016.

Figure 10.4 gives the growth of a dollar investment in the stock market from 1926 through 2015. In other words, it shows what the worth of the investment would have been if the dollar had been left in the stock market and if each year the dividends from the previous year had been reinvested in more stock. If R_t is the return in Year t (expressed in decimals), the value you would have at the end of Year T is the product of 1 plus the return in each of the years:

$$(1 + R_1) \times (1 + R_2) \times \ldots \times (1 + R_t) \times \ldots \times (1 + R_T)$$

FIGURE 10.7
Year-by-Year Total Returns on Bonds and Bills





Source: Morningstar, 2016.

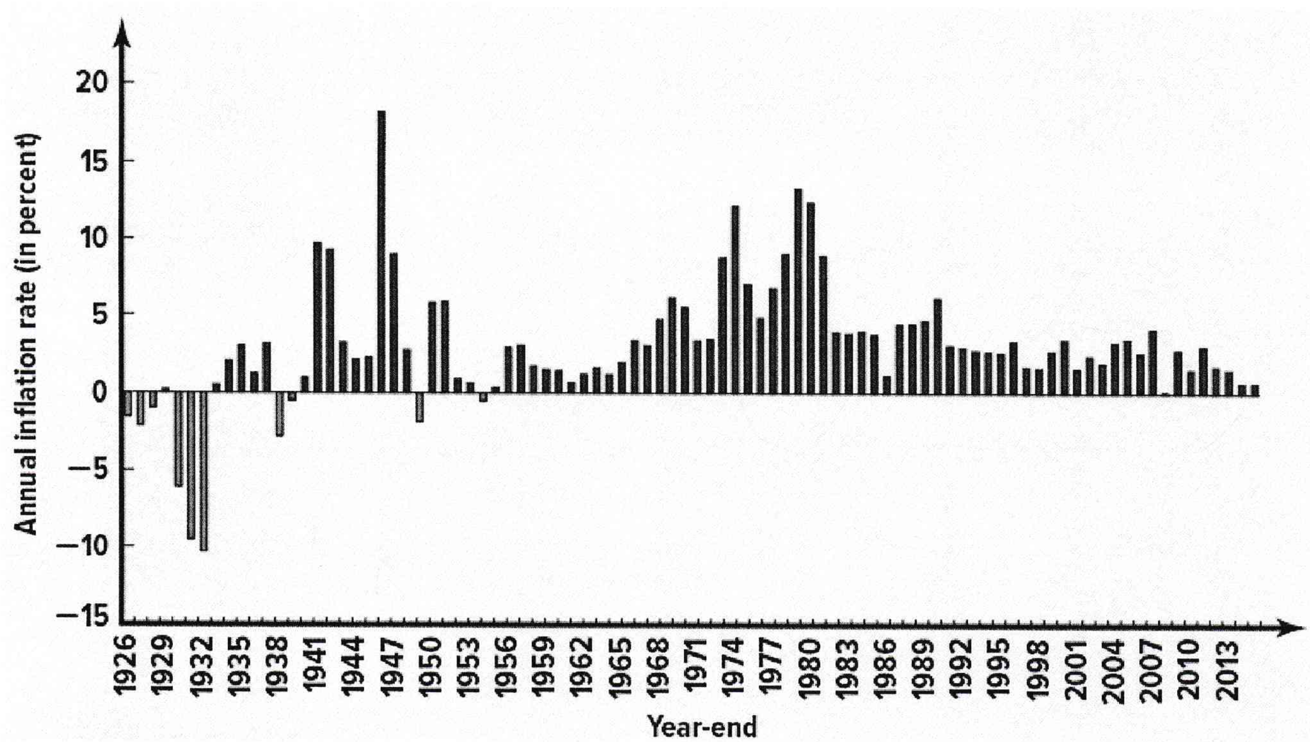
For example, if the returns were 11 percent, −5 percent, and 9 percent in a three-year period, an investment of $1 at the beginning of the period would be worth:

$$\$1 \times (1 + R_1) \times (1 + R_2) \times (1 + R_3) = (1 + .11) \times (1 - .05) \times (1 + .09)$$
$$= 1.11 \times .95 \times 1.09$$
$$= \$1.15$$

Go to bigcharts.marketwatch.com to see both intraday and long-term charts.

at the end of the three years. Notice that .15 or 15 percent is the total return and that it includes the return from reinvesting the first-year dividends in the stock market for two more years and reinvesting the second-year dividends for the final year. The 15 percent is called a three-year **holding period return**. Table 10.1 gives the annual returns each year for selected investments from 1926 to 2015. From this table, you can determine holding period returns for any combination of years.

FIGURE 10.8
Year-by-Year Inflation



Source: Morningstar, 2016 bls.gov, and author calculations.

TABLE 10.1 Year-by-Year Total Returns, 1926–2015

YEAR	LARGE-COMPANY STOCKS	LONG-TERM GOVERNMENT BONDS	U.S. TREASURY BILLS	CONSUMER PRICE INDEX
1926	11.14%	7.90%	3.30%	−1.12%
1927	37.13	10.36	3.15	−2.26
1928	43.31	−1.37	4.05	−1.16
1929	−8.91	5.23	4.47	.58
1930	−25.26	5.80	2.27	−6.40
1931	−43.86	−8.04	1.15	−9.32
1932	− 8.85	14.11	.88	−10.27
1933	52.88	.31	.52	.76
1934	−2.34	12.98	.27	1.52
1935	47.22	5.88	.17	2.99
1936	32.80	8.22	.17	1.45
1937	−35.26	−.13	.27	2.86

1938	33.20	6.26	.06	−2.78
1939	−.91	5.71	.04	.00
1940	−10.08	10.34	.04	.71
1941	−11.77	−8.66	.14	9.93
1942	21.07	2.67	.34	9.03
1943	25.76	2.50	.38	2.96
1944	19.69	2.88	.38	2.30
1945	36.46	5.17	.38	2.25
1946	−8.18	4.07	.38	18.13
1947	5.24	−1.15	.62	8.84
1948	5.10	2.10	1.06	2.99
1949	18.06	7.02	1.12	−2.07
1950	30.58	−1.44	1.22	5.93
1951	24.55	−3.53	1.56	6.00
1952	18.50	1.82	1.75	.75
1953	−1.10	−.88	1.87	.75
1954	52.40	7.89	.93	−.74

1955	31.43	−1.03	1.80	.37
1956	6.63	−3.14	2.66	2.99
1957	−10.85	5.25	3.28	2.90
1958	43.34	−6.70	1.71	1.76
1959	11.90	−1.35	3.48	1.73
1960	.48	7.74	2.81	1.36
1961	26.81	3.02	2.40	.67
1962	−8.78	4.63	2.82	1.33
1963	22.69	1.37	3.23	1.64
1964	16.36	4.43	3.62	.97
1965	12.36	1.40	4.06	1.92
1966	−10.10	−1.61	4.94	3.46
1967	23.94	−6.38	4.39	3.04
1968	11.00	5.33	5.49	4.72
1969	−8.47	−7.45	6.90	6.20
1970	3.94	12.24	6.50	5.57
1971	14.30	12.67	4.36	3.27
1972	18.99	9.15	4.23	3.41
1973	−14.69	−12.66	7.29	8.71
1974	−26.47	−3.28	7.99	12.34
1975	37.23	4.67	5.87	6.94
1976	23.93	18.34	5.07	4.86
1977	−7.16	2.31	5.45	6.70
1978	6.57	−2.07	7.64	9.02
1979	18.61	−2.76	10.56	13.29
1980	32.50	−5.91	12.10	12.52
1981	−4.92	.16	14.60	8.92
1982	21.55	49.99	10.94	3.83

1983	22.56	−2.11	8.99	3.79
1984	6.27	16.53	9.90	3.95
1985	31.73	39.03	7.71	3.80
1986	18.67	32.51	6.09	1.10
1987	5.25	−8.09	5.88	4.43
1988	16.61	8.71	6.94	4.42
1989	31.69	22.15	8.44	4.65
1990	−3.10	5.44	7.69	6.11
1991	30.46	20.04	5.43	3.06
1992	7.62	8.09	3.48	2.90
1993	10.08	22.32	3.03	2.75
1994	1.32	−11.46	4.39	2.67
1995	37.58	37.28	5.61	2.54
1996	22.96	−2.59	5.14	3.32
1997	33.36	17.70	5.19	1.70
1998	28.58	19.22	4.86	1.61
1999	21.04	−12.76	4.80	2.68
2000	−9.10	22.16	5.98	3.39
2001	−11.89	5.30	3.33	1.55

2002 −22.10 14.08 1.61 2.38 2003 28.68
1.62 1.03 1.88 2004 10.88 10.34 1.43
3.26 2005 4.91 10.35 3.30 3.42 2006
15.79 .28 4.97 2.54 2007 5.49 10.85
4.52 4.08 2008 −37.00 19.24 1.24 .09 2009
26.46 −9.49 .15 2.72 2010 15.06
7.73 .14 1.50 2011 2.11 35.75 .06 2.96
2012 16.00 1.80 .08 1.74 2013 32.39
−14.69 .05 1.50 2014 13.70 12.90 .03 .80
2015 1.48 −2.15 .04 .70

Source: Global Financial Data (www.globalfinancialdata.com) copyright 2016, www.stlouisfed.org,bls.gov.

10.3 RETURN STATISTICS

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The history of capital market returns is too complicated to be handled in its undigested form. To use the history, we must first find some manageable ways of describing it, dramatically condensing the detailed data into a few simple statements.

This is where two important numbers summarizing the history come in. The first and most natural number is some single measure that best describes the past annual returns on the stock market. In other words, what is our best estimate of the return that an investor could have realized in a particular year over the 1926 to 2015 period? This is the *average return.*

Figure 10.9 plots the histogram of the yearly stock market returns. This plot is the **frequency distribution** of the numbers. The height of the graph gives the number of sample observations in the range on the horizontal axis.

Given a frequency distribution like that in Figure 10.9, we can calculate the **average**, or **mean**, of the distribution. To compute the average of the distribution, we add up all of the values and divide by the total (T) number (90 in our case because we have 90 years of data). The bar over the R is used to represent the mean, and the formula is the ordinary formula for the average:

$$\text{Mean} = \bar{R} = \frac{(R_1 + \ldots + R_T)}{T} \qquad [10.5]$$

The mean return of the portfolio of large-company stocks from 1926 to 2015 is 11.9 percent.

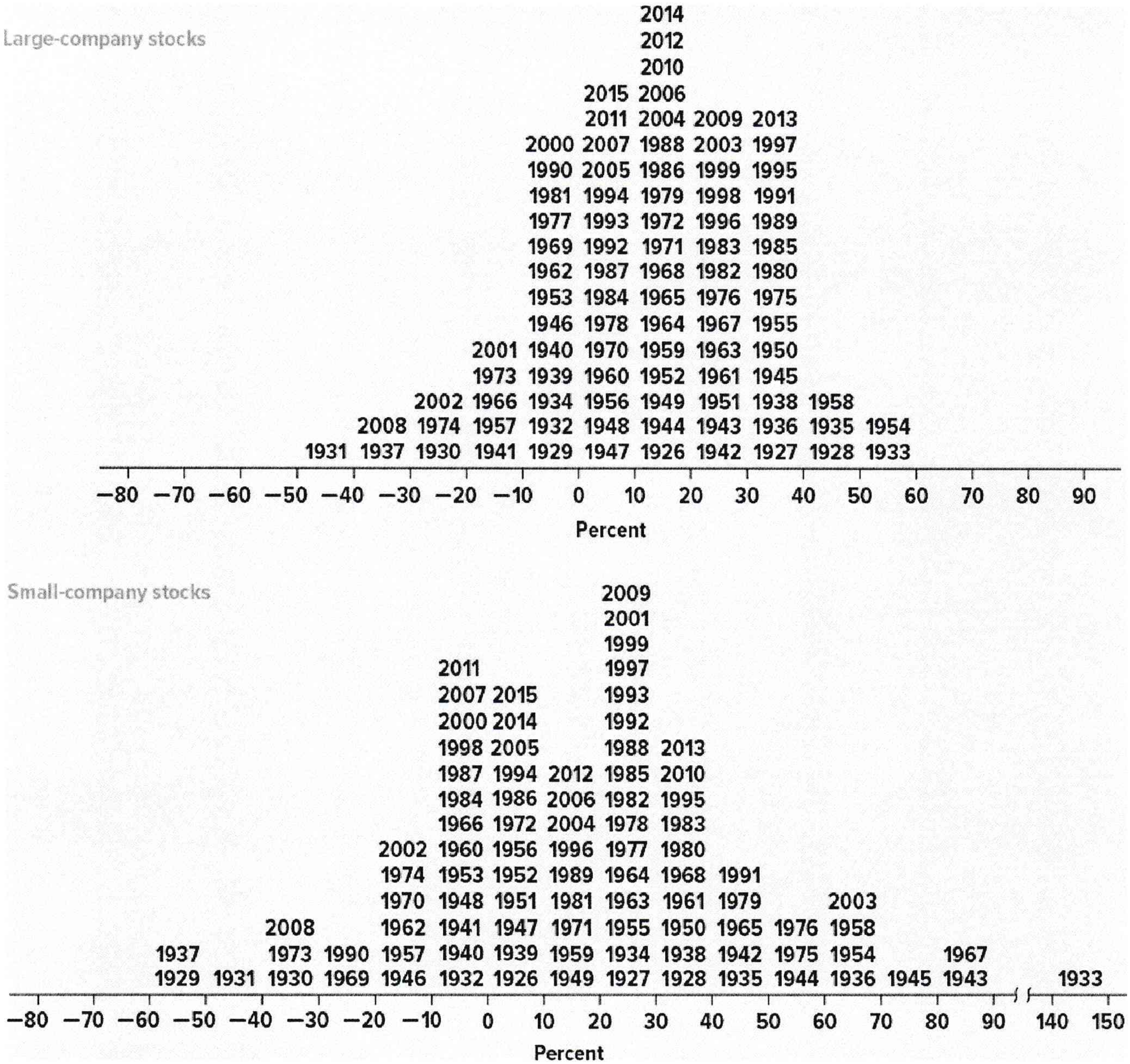
EXAMPLE 10.2

Calculating Average Returns

Suppose the returns on common stock over a four-year period are .1370, .3580, .4514, and —.0888, respectively. The average, or mean, return over these four years is:

$$\bar{R} = \frac{.1370 + .3580 + .4514 - .0888}{4} = .2144, \text{ or } 21.44\%$$

FIGURE 10.9
Histogram of Returns on Common Stocks, 1926–2015



Source: Morningstar, 2016. Author calculations.

10.4 AVERAGE STOCK RETURNS AND RISK-FREE RETURNS

Now that we have computed the average return on the stock market, it seems sensible to compare it with the returns on other securities. The most obvious comparison is with the low-variability returns in the government bond market. These are free of most of the volatility we see in the stock market.

The government borrows money by issuing bonds, which the investing public holds. As we discussed in an earlier chapter, these bonds come in many forms, and the ones we will look at here are called *Treasury bills*, or *T-bills*. Once a week the government sells some bills at an auction. A typical bill is a pure discount bond that will mature in a year or less.

finance, and Chapter 11 is devoted entirely to this. However, part of the answer can be found in the variability of the various types of investments. We see in Table 10.1 many years when an investment in T-bills achieved higher returns than an investment in large-company common stocks. Also, we note that the returns from an investment in common stocks are frequently negative whereas an investment in T-bills never produces a negative return. So, we now turn our attention to measuring the variability of returns and an introductory discussion of risk.

We first look more closely at Table 10.2. We see that the standard deviation of T-bills is substantially less than that of common stocks. This suggests that the risk of T-bills is less than that of common stocks. Because the answer turns on the riskiness of investments in common stock, we next turn our attention to measuring this risk.

10.5 RISK STATISTICS

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The second number that we use to characterize the distribution of returns is a measure of the risk in returns. There is no universally agreed-upon definition of risk. One way to think about the risk of returns on common stock is in terms of how spread out the frequency distributions in Figure 10.9 are. The spread, or dispersion, of a distribution is a measure of how much a particular return can deviate from the mean return. If the distribution is very spread out, the returns that will occur are very uncertain. By contrast, a distribution whose returns are all within a few percentage points of each other is tight, and the returns are less uncertain. The measures of risk we will discuss are variance and standard deviation.

Variance

The **variance** and its square root, the **standard deviation**, are the most common measures of variability or dispersion. We will use Var and σ^2 to denote the variance and SD and s to represent the standard deviation. σ is, of course, the Greek letter sigma.

For an easy-to-read review of basic stats, check out www.robertniles.com/stats.

EXAMPLE 10.3

Volatility

Suppose the returns on common stocks over a four-year period are (in decimals) .1370, .3580, .4514, and —.0888, respectively. The variance of this sample is computed as:

$$\text{Var} = \frac{1}{T-1}[(R_1 - \bar{R})^2 + (R_2 - \bar{R})^2 + (R_3 - \bar{R})^2 + (R_4 - \bar{R})^2] \qquad [10.6]$$

$$.0582 = \frac{1}{3}[(.1370 - .2144)^2 + (.3580 - .2144)^2$$

$$+ (.4514 - .2144)^2 + (-.0888 - .2144)^2]$$

$$SD = \sqrt{.0582} = .2413, \text{ or } 24.13\%$$

This formula tells us just what to do: Take the T individual returns (R_1, R_2, . . .) and subtract the average return R, square the result, and add them up. Finally, this total must be divided by the number of returns less one ($T - 1$). The standard deviation is always just the square root of the variance.

Using the stock returns for the 90-year period from 1926 through 2015 in the above formula, the resulting standard deviation of large-company stock returns is 20.0 percent. The standard deviation is the standard statistical measure of the spread of a sample, and it will be the measure we use most of the time. Its interpretation is facilitated by a discussion of the normal distribution.

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Standard deviations are widely reported for mutual funds. For example, the Fidelity Magellan Fund is one of the largest mutual funds in the United States. How volatile is it? To find out, we went to www.morningstar.com, entered the ticker symbol FMAGX, and hit the "Ratings & Risk" link. Here is what we found:

MPT Statistics FMAGX

3-Year | 5-Year | 10-Year | 15-Year

3-Year Trailing	Index	R-Squared	Beta	Alpha	Treynor Ratio	Currency
vs. Best-Fit Index						
FMAGX	Russell 3000 Growth TR USD	96.99	1.02	-0.47	—	USD
vs. Standard Index						
FMAGX	S&P 500 TR USD	93.46	1.06	0.42	12.14	USD
Category: LG	S&P 500 TR USD	85.86	1.03	-0.92	10.69	USD

03/31/2016

Volatility Measures FMAGX

3-Year | 5-Year | 10-Year | 15-Year

3-Year Trailing	Standard Deviation	Return	Sharpe Ratio	Sortino Ratio	Bear Market Percentile Rank
FMAGX	12.44	12.92	1.04	1.96	—
S&P 500 TR USD	11.36	11.82	1.04	1.93	—
Category: LG	12.71	11.07	0.89	1.61	—

03/31/2016

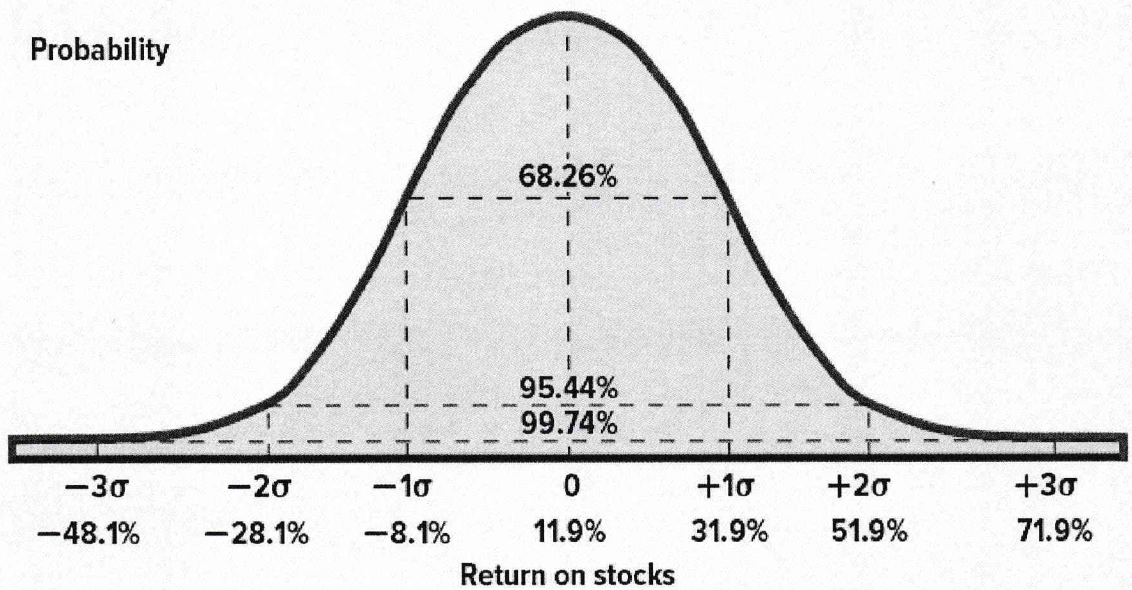
Source: Morningstar, 2016.

Over the last three years, the standard deviation of the return on the Fidelity Magellan Fund was 12.44 percent. When you consider the average stock has a standard deviation of about 50 percent, this seems like a low number, but the Magellan Fund is a relatively well-diversified portfolio, so this is an illustration of the power of diversification, a subject we will discuss in detail later. The mean is the average return, so, over the last three years, investors in the Magellan Fund gained a 12.92 percent return per year. Also under the Volatility Measures section, you will see the Sharpe ratio. The Sharpe ratio is calculated as the risk premium of the asset divided by the standard deviation. As such, it is a measure of return to the level of risk taken (as measured by standard deviation). This ratio is 1.04 for the period covered. The "beta" for the Fidelity Magellan Fund is 1.06. We will have more to say about this number—lots more—in the next chapter.

Normal Distribution and Its Implications for Standard Deviation

A large enough sample drawn from a **normal distribution** looks like the bell-shaped curve drawn in Figure 10.10. As you can see, this distribution is *symmetric* about its mean, not *skewed*, and has a much cleaner shape than the actual distributions of yearly returns drawn in Figure 10.9. Of course, if we had been able to observe stock market returns for 1,000 years, we might have filled in a lot of the jumps and jerks in Figure 10.9 and had a smoother curve.

FIGURE 10.10
The Normal Distribution



In the case of a normal distribution, there is a 68.26 percent probability that a return will be within one standard deviation of the mean. In this example, there is a 68.26 percent probability that a yearly return will be between −8.1 percent and 31.9 percent.

There is a 95.44 percent probability that a return will be within two standard deviations of the mean. In this example, there is a 95.44 percent probability that a yearly return will be between −28.1 percent and 51.9 percent.

Finally, there is a 99.74 percent probability that a return will be within three standard deviations of the mean. In this example, there is a 99.74 percent probability that a yearly return will be between −48.1 percent and 71.9 percent.

In classical statistics, the normal distribution plays a central role, and the standard deviation is the usual way to represent the spread of a normal distribution. For the normal distribution, the probability of having a return that is above or below the mean by a certain amount depends only on the standard deviation. For example, the probability of having a return that is within one standard deviation of the mean of the distribution is approximately .68, or 2/3, and the probability of having a return that is within two standard deviations of the mean is approximately .95.

The 20.0 percent standard deviation we found for the portfolio of large-company stock returns from 1926 through 2015 can now be interpreted in the following way: If stock returns are roughly normally distributed, the probability that a yearly return will fall within 20.0 percent of the mean of 11.9 percent will be approximately 2/3. That is, about 2/3 of the yearly returns will be between −8.1 percent and 31.9 percent. (Note that −8.1% = 11.9% − 20.0% and 31.9% = 11.9% + 20.0%.) The probability that the return in any year will fall within two standard deviations is about .95. That is, about 95 percent of yearly returns will be between −28.1 percent and 51.9 percent.

10.6 THE U.S. EQUITY RISK PREMIUM: HISTORICAL AND INTERNATIONAL PERSPECTIVES

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So far in this chapter we have studied the United States in the period from 1926 to 2015. As we have discussed, the historical U.S. stock market risk premium has been substantial. Of course, anytime we use the past to predict the future, there is a danger that the past period isn't representative of what the future will hold. Perhaps U.S. investors got lucky over this period and earned particularly large returns. Data from earlier years for the United States is available, though it is not of the same quality. With that caveat in mind, researchers have tracked returns back to 1802, and the U.S. equity risk premium in the pre-1926 era was smaller. Using the U.S. return data from 1802, the historical equity risk premium was 5.2 percent.[2]

TABLE 10.3 World Stock Market Capitalization, select markets, 2015

COUNTRY	$ IN TRILLIONS	PERCENT
United States	$25.1	38%
East Asia & Pacific (excluding Japan)	16.7	26
China	8.2	13
European Union	6.3	10
Japan	4.9	7
Canada	1.6	2
Middle East & North Africa	1.3	2
Latin America & Caribbean	1.3	2
	$65.3	100%

Source: www.worldbank.org.

TABLE 10.4 Annualized Equity Risk Premiums and Sharpe Ratios for 17 Countries, 1900–2010

COUNTRY	HISTORICAL EQUITY RISK PREMIUMS (%) (1)	STANDARD DEVIATION (%) (2)	THE SHARPE RATIO (1)/(2)
Denmark	4.6%	20.5%	.22
Switzerland	5.1	18.9	.27
Ireland	5.3	21.5	.25
Spain	5.4	21.9	.25
Belgium	5.5	24.7	.22
Canada	5.6	17.2	.33
Norway	5.9	26.5	.22
United Kingdom	6.0	19.9	.30
Netherlands	6.5	22.8	.29
Sweden	6.6	22.1	.30
United States	7.2	19.8	.36
South Africa	8.3	22.1	.37

Australia	8.3	17.6	.47
France	8.7	24.5	.36
Japan	9.0	27.7	.32
Germany*	9.8	31.8	.31
Italy	9.8	32.0	.31

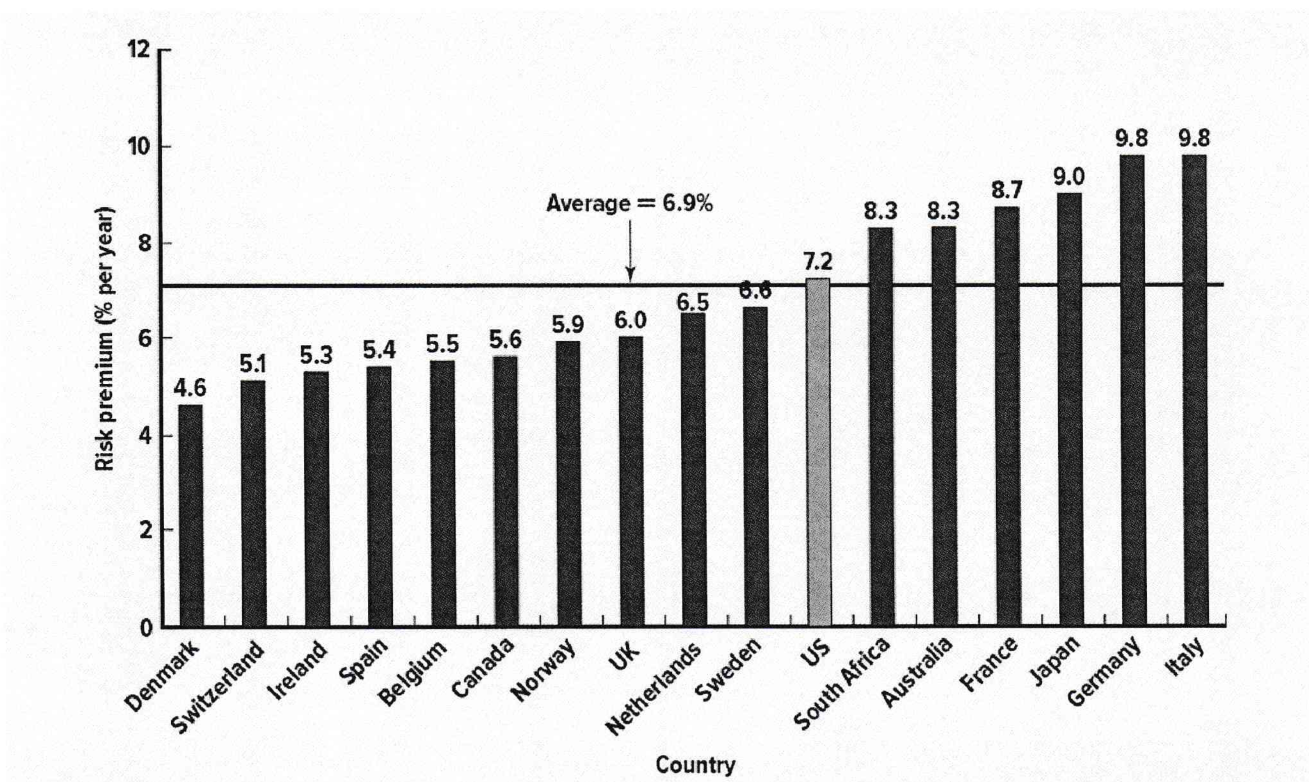
* Germany omits 1922–1923.

Source: Elroy Dimson, Paul Marsh, and Michael Staunton, "The Worldwide Equity Premium: A Smaller Puzzle," in *Handbook of the Equity Risk Premium,* Rajnish Mehra, ed. (Elsevier, 2007), as updated by the authors.

Also, we have not looked at other major countries. Actually, more than half of the value of tradable stock is not in the United States. From Table 10.3, we can see that while the total world stock market capitalization was $65.3 trillion in 2015, about 38 percent was in the United States. Thanks to Dimson, Marsh, and Staunton, data from earlier periods and other countries are now available to help us take a closer look at equity risk premiums. Table 10.4 and Figure 10.11 show the historical stock market risk premiums for

17 countries around the world in the period from 1900 to 2010. Looking at the numbers, the U.S. historical equity risk premium is the 7th highest at 7.2 percent (which differs from our earlier estimate because of the different time periods examined). The overall world average risk premium is 6.9 percent. It seems clear that U.S. investors did well, but not exceptionally so relative to many other countries. The top-performing countries according to the Sharpe ratio were the United States, Australia, France, and South Africa, while the worst performers were Belgium, Norway, and Denmark. Germany, Japan, and Italy might make an interesting case study because they have the highest stock returns over this period (despite World Wars I and II), but also the highest risk.

FIGURE 10.11
Stock Market Risk Premiums for 17 Countries: 1900–2010



Source: Elroy Dimson, Paul Marsh, and Mike Staunton, "The Worldwide Equity Premium: A Smaller Puzzle," in *Handbook of the Equity Risk Premium,* Rajnish Mehra, ed. (Elsevier, 2007), as updated by the authors.

	AVERAGE RETURNS 1802–2008 (%)
Common stock	9.5
Treasury bills	4.3
Equity risk premium	5.2

Adopted and updated from J. Siegel, Stocks for the Long Run, 4th ed. (New York: McGraw-Hill, 2008).

So what is a good estimate of the U.S. equity risk premium going forward? Unfortunately, nobody can know for sure what investors expect in the future. If history is a guide, the expected U.S. equity risk premium could be 7.2 percent based upon estimates from 1900 to 2010. We should also be mindful that the average world equity risk premium was 6.9 percent over this same period. On the other hand, the more recent periods (1926–2015) suggest higher estimates of the U.S. equity risk premium, and earlier periods going back to 1802 suggest lower estimates.

The standard error (SE) helps with the issue of how much confidence we can have in our historical average of 7.2 percent. The SE is the standard deviation of the historical risk premium and is given the following formula:

$$\text{SE} = \text{SD}(\bar{R}) = \frac{\text{SD}(R)}{\sqrt{\text{The number of observations}}} \qquad [10.7]$$

If we assume that the distribution of returns is normal and that each year's return is independent of all the others, we know there is a 95.4 percent probability that the true mean return is within two standard errors of the historical average.

More specifically, the 95.4 percent confidence interval for the true equity risk premium is the historical average return ± (2 × standard error). From 1900 to 2010, the historical equity risk premium of U.S. stocks was 7.2 percent and the standard deviation was 19.8 percent. Therefore 95.4 percent of the time the true equity risk premium should be within 3.4 and 11 percent:

$$7.2 \pm 2\left(\frac{19.8}{\sqrt{110}}\right) = 7.2 \pm 2\left(\frac{19.8}{10.5}\right) = 7.2 \pm 3.8$$

In other words, we can be 95.4 percent confident that our estimate of the U.S. equity risk premium from historical data is in the range from 3.4 percent to 11 percent.

Taking a slightly different approach, Ivo Welch asked the opinions of 226 financial economists regarding the future U.S. equity risk premium, and the median response was 7 percent.[3]

We are comfortable with an estimate based on the historical U.S. equity risk premium of about 7 percent, but estimates of the future U.S. equity risk premium that are somewhat higher or lower could be reasonable if we have good reason to believe the past is not representative of the future.[4] The bottom line is that any estimate of the future equity risk premium will involve assumptions about the future risk environment as well as the amount of risk aversion of future investors.

10.7 2008: A YEAR OF FINANCIAL CRISIS

2008 entered the record books as one of the worst years for stock market investors in U.S. history. How bad was it? The widely followed S&P 500 Index, which tracks the total market value of 500 of the largest U.S. corporations, decreased 37 percent for the year. Of the 500 stocks in the index, 485 were down for the year.

Over the period 1926–2008, only the year 1931 had a lower return than 2008 (−44 percent versus −37 percent). Making matters worse, the downdraft continued with a further decline of 25.1 percent through March 9, 2009. In all, from November 2007 (when the decline began) through March 9, 2009, the S&P 500 lost 56.8 percent of its value. Fortunately for investors, things turned around dramatically for the rest of the year. From March 9, 2009, to December 31, 2009, the market gained about 65 percent!

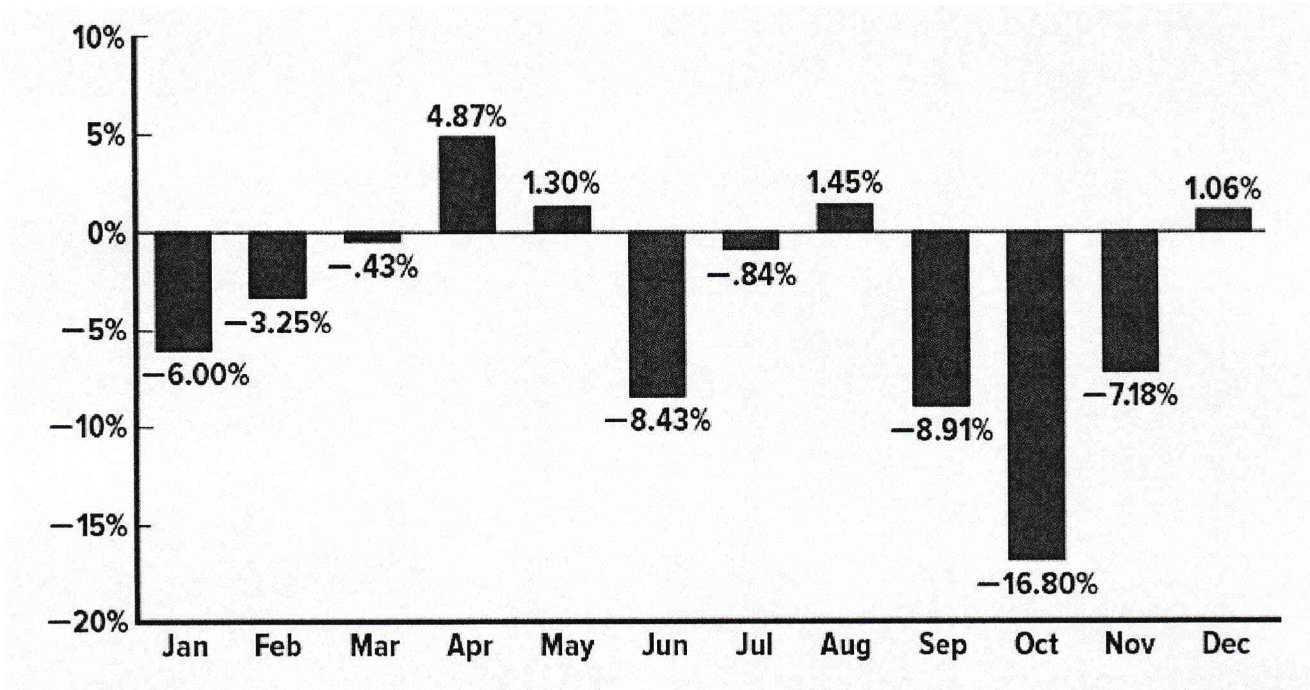
Figure 10.12 shows the month-by-month performance of the S&P 500 during 2008. As indicated, returns were negative in 8 of the 12 months. Most of the decline occurred in the fall, with investors losing almost 17 percent in October alone. Small stocks fared no better. They also fell 37 percent for the year (with a 21 percent drop in October), their worst performance since losing 58 percent in 1937.

As Figure 10.12 suggests, stock prices were highly volatile at the end of the year—more than has been generally true historically. Oddly, the S&P had 126 up days and 126 down days (remember the markets are closed weekends and holidays). Of course, the down days were much worse on average.

The drop in stock prices was a global phenomenon, and many of the world's major markets declined by much more than the S&P. China, India, and Russia, for example, all experienced declines of more than 50 percent. Tiny Iceland saw share prices drop by more than 90 percent for the year.

Trading on the Icelandic exchange was temporarily suspended on October 9. In what has to be a modern record for a single day, stocks fell by 76 percent when trading resumed on October 14.

FIGURE 10.12
S&P 500 Monthly Returns, 2008



Did any types of securities perform well in 2008? The answer is yes because, as stock values declined, bond values increased, particularly U.S. Treasury bonds. In fact, long-term Treasury bonds *gained* 20 percent, while shorter-term Treasury bonds were up 13 percent. Higher-quality long-term corporate bonds did less well but still managed to achieve a positive return of about 9 percent. These returns were especially impressive considering that the rate of inflation, as measured by the CPI, was very close to zero.

Of course, stock prices can be volatile in both directions. From March 2009 through February 2011, a period of about 700 days, the S&P 500 doubled in value. This climb was the fastest doubling since 1936, when the S&P did it in just 500 days.

What lessons should investors take away from this recent bit of capital market history? First, and most obviously, stocks have significant risk! But there is a second, equally important lesson. Depending on the mix, a diversified portfolio of stocks and bonds probably would have suffered in 2008, but the losses would have been much smaller than those experienced by an all-stock portfolio. Finally, because of increased volatility and heightened risk aversion, many have argued that the equity risk premium going forward is probably (at least temporarily) somewhat higher than has been true historically.

10.8 MORE ON AVERAGE RETURNS

Thus far in this chapter, we have looked closely at simple average returns. But there is another way of computing an average return. The fact that average returns are calculated two different ways leads to some confusion, so our goal in this section is to explain the two approaches and also the circumstances under which each is appropriate.

Arithmetic versus Geometric Averages

Let's start with a simple example. Suppose you buy a particular stock for $100. Unfortunately, the first year you own it, it falls to $50. The second year you own it, it rises back to $100, leaving you where you started (no dividends were paid).

What was your average return on this investment? Common sense seems to say that your average return must be exactly zero since you started with $100 and ended with $100. But if we calculate the returns year by year, we see that you lost 50 percent the first year (you lost half of your money). The second year, you made 100 percent (you doubled your money). Your average return over the two years was thus (−50 percent + 100 percent)/2 = 25 percent!

So which is correct, 0 percent or 25 percent? The answer is that both are correct; they just answer different questions. The 0 percent is called the **geometric average return**.

The 25 percent is called the **arithmetic average return**. The geometric average return answers the question, "*What was your average compound return per year over a particular period?*" The arithmetic average return answers the question, "*What was your return in an average year over a particular period?*"

Notice that, in previous sections, the average returns we calculated were all arithmetic averages, so we already know how to calculate them. What we need to do now is (1) learn how to calculate geometric averages and (2) learn the circumstances under which one average is more meaningful than the other.

Calculating Geometric Average Returns

First, to illustrate how we calculate a geometric average return, suppose a particular investment had annual returns of 10 percent, 12 percent, 3 percent, and −9 percent over the last four years. The geometric average return over this four-year period is calculated as $(1.10 \times 1.12 \times 1.03 \times .91)^{1/4} - 1 = .0366$, or 3.66 percent. In contrast, the average arithmetic return we have been calculating is $(.10 + .12 + .03 - .09)/4 = .04$, or 4.0 percent.

In general, if we have T years of returns, the geometric average return over these T years is calculated using this formula:

$$\text{Geometric average return} = [(1 + R_1) \times (1 + R_2) \times \ldots \times (1 + R_T)]^{1/T} - 1 \qquad [10.8]$$

This formula tells us that four steps are required:

1. Take each of the T annual returns $R_1, R_2, \ldots, R_T$ and add 1 to each (after converting them to decimals!).
2. Multiply all the numbers from Step 1 together.
3. Take the result from Step 2 and raise it to the power of $1/T$.
4. Finally, subtract 1 from the result of Step 3. The result is the geometric average return.

EXAMPLE 10.4

Calculating the Geometric Average Return

Calculate the geometric average return for S&P 500 large-cap stocks for a five-year period using the numbers given here.

First, convert percentages to decimal returns, add 1, and then calculate their product:

S&P 500 RETURNS	PRODUCT
.1375	1.1375
.3570	×1.3570
.4508	×1.4508

−.0880		× .9120
−.2513		× .7487
		1.5291

Notice that the number 1.5291 is what our investment is worth after five years if we started with a $1 investment. The geometric average return is then calculated as

$$\text{Geometric average return} = 1.5291^{1/5} - 1 = .0887, \text{ or } 8.87\%$$

Thus, the geometric average return is about 8.87 percent in this example. Here is a tip: If you are using a financial calculator, you can put $1 in as the present value, $1.5291 as the future value, and 5 as the number of periods. Then, solve for the unknown rate. You should get the same answer we did.

One thing you may have noticed in our examples thus far is that the geometric average returns seem to be smaller. It turns out that this will always be true (as long as the returns are not all identical, in which case the two "averages" would be the same). To illustrate, Table 10.5 shows the arithmetic averages and standard deviations from Table 10.2, along with the geometric average returns.

TABLE 10.5 Geometric versus Arithmetic Average Returns: 1926–2015

SERIES	GEOMETRIC AVERAGE	ARITHMETIC AVERAGE	STANDARD DEVIATION
Small-company stocks	12.0%	16.5%	32.0%
Large-company stocks	10.0	11.9	20.0
Long-term corporate bonds	6.0	6.3	8.4
Long-term government bonds	5.6	6.0	10.0
Intermediate-term government bonds	5.2	5.3	5.7
U.S. Treasury bills	3.4	3.5	3.1
Inflation	2.9	3.0	4.1

Source: Morningstar, 2016. Author calculations.

As shown in Table 10.5, the geometric averages are all smaller, but the magnitude of the difference varies quite a bit. The reason is that the difference is greater for more volatile investments. In fact, there is useful approximation. Assuming all the numbers are expressed in decimals (as opposed to percentages), the geometric average return is approximately equal to the arithmetic average return minus half the variance. For example, looking at the large-company stocks, the arithmetic average is 11.9 percent and the standard deviation is .200, implying that the variance is .040. The approximate geometric average is thus 11.9 − 1/2(4.00) = 9.90 percent, which is quite close to the actual value.

EXAMPLE 10.5

More Geometric Averages

Take a look back at Figure 10.4. There, we showed the value of a $1 investment after 90 years. Use the value for the small-company stock investment to check the geometric average in Table 10.5.

In Figure 10.4, the small-company investment grew to $26,434.82 over 90 years. The geometric average return is thus:

$$\text{Geometric average return} = 26{,}434.82^{1/90} - 1 = .120\text{, or } 12.0\%$$

This 12.0% is the value shown in Table 10.5. For practice, check some of the other numbers in Table 10.5 the same way.

Arithmetic Average Return or Geometric Average Return?

When we look at historical returns, the difference between the geometric and arithmetic average returns isn't too hard to understand. To put it slightly differently, the geometric average tells you what you actually earned per year on average, compounded annually. This fact makes the geometric average return a very useful measure of past performance. The arithmetic average tells you what you earned in a typical year. This fact makes the arithmetic average return a very useful measure of expected future yearly returns. You should use whichever one answers the question you want answered.

A somewhat trickier question concerns forecasting the long-run future, and there's a lot of confusion about this point among analysts and financial planners. The problem is this.

If we have *estimates* of both the arithmetic and geometric average yearly returns, then the arithmetic average is probably too high for longer periods and the geometric average is probably too low.[5]

SUMMARY AND CONCLUSIONS

1. This chapter presents returns for a number of different asset classes. The general conclusion is that stocks have outperformed bonds over most of the twentieth century, though stocks have also exhibited more risk.
2. The statistical measures in this chapter are necessary building blocks for the material of the next three chapters. In particular, standard deviation and variance measure the variability of the return on an individual security and on portfolios of securities. In the next chapter, we will argue that standard deviation and variance are appropriate measures of the risk of an individual security if an investor's portfolio is composed of that security only.
3. Both arithmetic and geometric averages are commonly reported. The chapter explains how both are calculated and interpreted.

CONCEPT QUESTIONS

1. **Investment Selection** Given that Nymox Pharmaceutical was up by 720 percent for 2015, why didn't all investors hold Nymox?
2. **Investment Selection** Given that Chesapeake Energy was down by 76 percent for 2015, why did some investors hold the stock? Why didn't they sell out before the price declined so sharply?
3. **Risk and Return** We have seen that over long periods of time stock investments have tended to substantially outperform bond investments. However, it is not at all uncommon to observe investors with long horizons holding their investments entirely in bonds. Are such investors irrational?
4. **Stocks versus Gambling** Critically evaluate the following statement: Playing the stock market is like gambling. Such speculative investing has no social value, other than the pleasure people get from this form of gambling.
5. **Effects of Inflation** Look at Table 10.1 and Figure 10.7 in the text. When were T-bill rates at their highest over the period from 1926 through 2015? Why do you think they were so high during this period? What relationship underlies your answer?
6. **Risk Premiums** Is it possible for the risk premium to be negative before an investment is undertaken? Can the risk premium be negative after the fact? Explain.
7. **Returns** Two years ago, General Materials' and Standard Fixtures' stock prices were the same. During the first year, General Materials' stock price increased by 10 percent while Standard Fixtures' stock price decreased by 10 percent. During the second year, General Materials' stock price decreased by

10 percent and Standard Fixtures' stock price increased by 10 percent. Do these two stocks have the same price today? Explain.

8. **Returns** Two years ago, the Lake Minerals and Small Town Furniture stock prices were the same. The average annual return for both stocks over the past two years was 10 percent. Lake Minerals's stock price increased 10 percent each year. Small Town Furniture's stock price increased 25 percent in the first year and lost 5 percent last year. Do these two stocks have the same price today?

9. **Arithmetic versus Geometric Returns** What is the difference between arithmetic and geometric returns? Suppose you have invested in a stock for the last 10 years. Which number is more important to you, the arithmetic or geometric return?

10. **Historical Returns** The historical asset class returns presented in the chapter are not adjusted for inflation. What would happen to the estimated risk premium if we did account for inflation? The returns are also not adjusted for taxes. What would happen to the returns if we accounted for taxes? What would happen to the volatility?

QUESTIONS AND PROBLEMS

connect

Basic (Questions 1–20)

1. **Calculating Returns** Suppose a stock had an initial price of $84 per share, paid a dividend of $1.65 per share during the year, and had an ending share price of $93. Compute the total percentage return.

2. **Calculating Yields** In problem 1, what was the dividend yield? The capital gains yield?

3. **Calculating Returns** Rework problems 1 and 2 assuming the ending share price is $72.

4. **Calculating Returns** Suppose you bought a bond with a coupon rate of 5.6 percent one year ago for $985. The bond sells for $1,015 today. The bond pays annual coupons.

   a. Assuming a $1,000 face value, what was your total dollar return on this investment over the past year?
   b. What was your total nominal rate of return on this investment over the past year?
   c. If the inflation rate last year was 3.2 percent, what was your total real rate of return on this investment?

5. **Nominal versus Real Returns** What was the arithmetic average annual return on large-company stocks from 1926 through 2015:

   a. In nominal terms?
   b. In real terms?

6. **Bond Returns** What is the historical real return on long-term government bonds? On long-term corporate bonds?

7. **Calculating Returns and Variability** Using the following returns, calculate the average returns, the variances, and the standard deviations for X and Y.

	RETURNS	
YEAR	X	Y
1	23%	34%
2	6	12
3	14	17
4	−16	−23
5	19	17