7. *Don't use fact mastery as a prerequisite for calculator use.* Requiring that students master the basic facts before they can use a calculator has no foundation. Calculator use should be based on the instructional goals of the day. If your lesson goal is for students to discover the pattern (formula) for the perimeter of rectangles, then using a calculator can quicken the computation in this lesson and keep the focus on measurement.

MyLab Education Self-Check 9.3

RESOURCES FOR CHAPTER 9

LITERATURE CONNECTIONS

Colomba (2013) describes literature links and activities for each of the multiplications facts. The children's books described in Chapters 7 and 8 are also good choices when working on the basic facts. In addition to those, consider these opportunities to develop and practice basic facts. Here are two additional ideas:

One Less Fish

Toft and Sheather (1998)

This beautiful book with an important environmental message starts with 12 fish and counts back to zero fish. On a page with 8 fish, ask, "How many fish are gone?" and "How did you figure it out?" Encourage students to use the Down over 10 strategy. Any counting-up or counting-back book can be used in this way!

The Twelve Days of Summer

Andrews and Jolliffe (2005)

You will quickly recognize the style of this book with five bumblebees, four garter snakes, three ruffed grouse, and so on. The engaging illustrations and motions make this a wonderful book. Students can apply multiplication facts to figure out how many of each item appear by the end of the book. (For example, three ruffed grouse appear on days 3, 4, 5, and so on.)

RECOMMENDED READINGS

Articles

Baroody, A. J. (2006). Why children have difficulties mastering the basic fact combinations and how to help them. *Teaching Children Mathematics, 13*(1), 22–31.

Baroody suggests that basic facts are developmental in nature and contrasts "conventional wisdom" with a number-sense view. Great activities are included as exemplars.

Boaler, J. (2014). Research suggests that timed tests cause math anxiety. *Teaching Children Mathematics 20*(8), 469–474.

This is a wonderful article to challenge the longstanding practice of timed-tests. Just because we have always done this, doesn't mean it's a good idea!

Buchholz, L. (2016). A license to think on the road to fact fluency. *Teaching Children Mathematics*, 22(9), 557–562.

This is a beautifully written reflection on how this second-grade teacher had departed from her daily strategy focus (due to a new curriculum), realized the error in her ways, and her return to daily strategy use. She describes pragmatic ways she ensures that every student develops strategies and fact fluency. (See also Buchholz, 2004 for great strategy ideas).

Kling, G., & Bay-Williams, J. M. (2014). Assessing basic fact fluency. *Teaching Children Mathematics, 20*(8), 489–497.

This article is a great partner with the Boaler article listed here. Boaler describes the problem with timed tests, and these authors reinforce these concerns and offer a rich collection of alternatives to better assess students' journey to fluency.

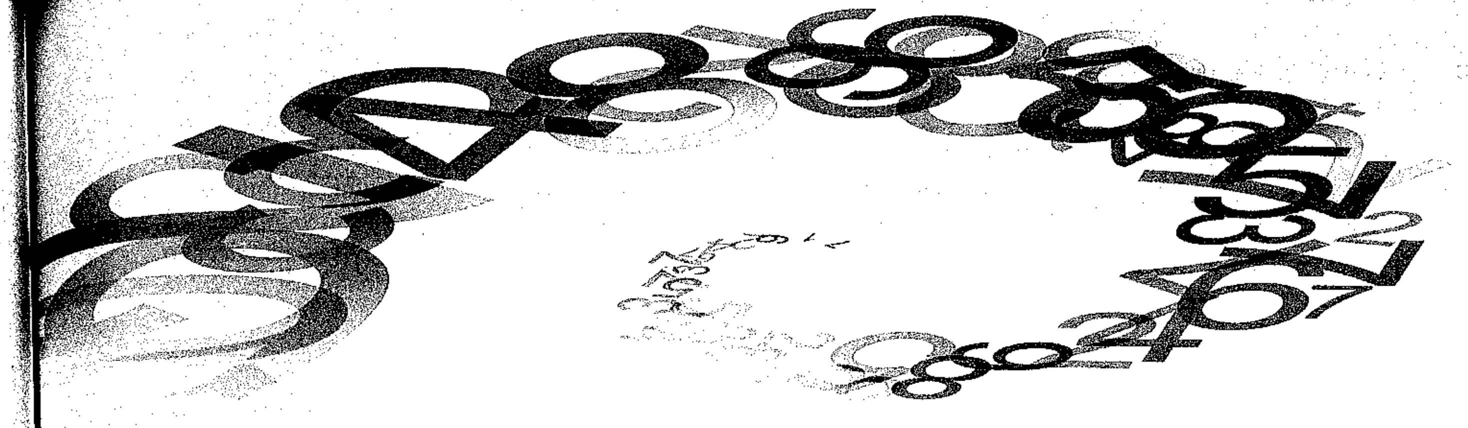
Books

Bay-Williams, J., & Kling, G. (2017). *Quick reference guide: Games and tools for teaching addition facts.* Alexandria, VA: ASCD.

As the title implies, this is a 6-page laminated foldable that offers guidance on teaching and assessing addition facts, six games, and ideas for sharing the vision with families.

O'Connell, S., & SanGiovanni, J. (2011). *Mastering the basic math facts in multiplication and division: Strategies, activities and interventions to move students beyond memorization.* Portsmouth, NH: Heinemann.

Both this book and the partner book about addition and subtraction (same authors and parallel title) are excellent resources for planning and teaching basic facts effectively!



CHAPTER 10 Developing Whole-Number Place-Value Concepts

LEARNER OUTCOMES

After reading this chapter and engaging in the embedded activities and reflections, you should be able to:

10.1 Identify the pre-base-ten understandings based on a count-by-ones approach to quantity.

10.2 Recognize the foundational ideas of place value as an integration of three components: base-ten concepts through groupings and counting, numbers written in place-value notation, and numbers that are spoken aloud.

10.3 Demonstrate how to develop students' skills in place value through the use of base-ten models.

10.4 Explain how students can use grouping activities to deepen their understanding of place-value concepts.

10.5 Explain strategies to support students' ability to write and read numbers.

10.6 Recognize that there are patterns in our number system that provide the foundation for computational strategies.

10.7 Describe how the place-value system extends to large numbers.

Number sense, a rich, relational understanding of number, is linked to a complete understanding of place value and our base-ten number system, including extensions from whole numbers to decimal numeration. In kindergarten and first grade, students count up to 100 and 120 respectively and are exposed to patterns in these numbers. But, importantly, they learn to think about groups of ten objects as a unit. By second grade, these initial ideas of patterns and groups of ten are formally connected to three-digit numbers, and fourth-grade students extend their understanding to numbers up to 1,000,000 in a variety of contexts. In fourth and fifth grades, students generalize place value understanding to see the relationship of the value of the positions of the digits in a number as ten times the value of the previous digit as they move to the left which will soon be linked to decimals as digits in a number are one-tenth the value of the position as they move to the right. This relationship is critical as students consider the powers of ten (NGA Center & CCSSO, 2010).

A significant part of place value development includes students putting numbers together (*composing*) and taking them apart (*decomposing*) in a wide variety of ways as they solve addition and subtraction problems with two- and three-digit numbers. Place value is a way for students to think about larger quantities (Mix, Prather, Smith, & Stockton, 2013) and to enhance their ability to invent their own computation strategies. Without a firm foundation and understanding of place value, students may face chronic low levels of mathematics performance (Chan & Ho, 2010; Moeller, Martignon, Wessolowski, Engel, & Nuerk, 2011). The following big ideas are the foundational concepts that will lead students to a full understanding of place value and its importance in computation.

BIG IDEAS

- Sets of 10 (and tens of tens) can be perceived as single entities or units; for example, three sets of 10 and two singles is base-ten method language to describe 32 single objects.
- The positions of digits in numbers determine what they represent and which size group they count. This is the major organizing principle of place-value numeration and is central for developing number sense.
- There are patterns to the way that numbers are formed. For example, each decade has a symbolic pattern reflective of the 0-to-9 sequence (e.g., 20, 21, 22 . . . 29).
- The groupings of ones, tens, and hundreds can be taken apart in different but equivalent ways. For example, beyond the typical way to decompose 256 of 2 hundreds, 5 tens, and 6 ones, it can be represented as 1 hundred, 14 tens, and 16 ones, or 25 tens and 6 ones. Decomposing and composing multidigit numbers in flexible ways is a necessary foundation for computational estimation and exact computation.
- "Really big" numbers are difficult to conceptualize and are best understood in terms of familiar real-world referents. The number of people who will fill the local sports arena is, for example, a meaningful referent for those who have experienced that crowd.

MyLab Education Video Example 10.1

Watch this video of former NCTM President Francis "Skip" Fennell who is discussing the big ideas of place value.

Pre-Place-Value Understandings

Children know a lot about numbers with two digits (10 to 99) as early as kindergarten. After all, kindergartners learn to count to 100 and count out sets with as many as 20 or more objects. They count students in the room, turn to specific page numbers in their books, and so on. However, initially their understanding is quite different from yours. It is based on a count-by-ones approach to quantity, so the number 18 to them means 18 ones. They are not able to separate the quantity into place-value groups—after counting 18 teddy bears, a young child might tell you that the 1 stands for 1 teddy bear and the 8 stands for 8 teddy bears. Such students have not had enough experiences to realize we are always grouping by tens.

Recall Wright and his colleagues' three levels of understanding: (1) children understand ten as ten ones; (2) children see ten as a unit; and (3) children easily work with units of ten. Let's look at a way to assess where students are in this trajectory.

***FORMATIVE ASSESSMENT* Notes.** In a diagnostic interview, ask first or second graders to count out 53 tiles. Watch closely to note whether they count out the tiles one at a time and push them aside without any type of grouping or if they group them into tens. Have the students write the number that tells how many tiles they just counted. Some may write "35" instead of "53," a simple reversal. You may find that early on students count the tiles one by one and are not yet thinking of ten as a unit (level 1), and are therefore in a pre-place-value stage. The students just described know that there are 53 tiles "because I counted them." Writing the number and saying the number are usually done correctly, but their understanding of 53 derives from the count-by-ones approach. Without your help, students may not easily or quickly develop a meaningful use of groups of ten to represent quantities.

Even if students can tell you that in the numeral 53, the 5 "is in the tens place" or that there are "3 ones," they might just know the name of the positions without understanding that the "tens place" represents how many groups of ten. Similarly, if students use base-ten blocks, they may name a rod of ten as a "ten" and a small cube as a "one" but may not be able to tell how many ones are required to make a ten. Students may attach words to both materials and groups without realizing what the materials or symbols represent.

Students do know that 53 is "a lot" and that it's more than 47 (because you count past 47 to get to 53). But, initially they think of the "53" as a single numeral. In this stage, they do not know that the 5 represents five groups of ten things and the 3 represents three single things (Fuson, 2006). Fuson and her colleagues refer to students' pre-base-ten understanding of number as *unitary*. That is, there are no groupings of ten, even though a two-digit number is associated with the quantity. They initially rely on unitary counts to understand quantities. ■

MyLab Education Self-Check 10.1

Developing Whole-Number Place-Value Concepts

Place-value understanding requires an *integration* of new and sometimes difficult-to-construct concepts of grouping by tens (the base-ten concept) with procedural knowledge of how groups are recorded in our place-value system and how numbers are written and spoken. Importantly, learners must understand the word *grouping*, especially English learners (ELs) who may become confused because the root word *group* is frequently used for instructing students to work together.

Integrating Base-Ten Groupings with Counting by Ones

Once students can count out a set of 53 by ones, help them see that making groupings of tens and leftovers is a way of counting that same quantity. Each of the sets in Figure 10.1 has 53 tiles, yet students move through three distinct grouping stages to construct the idea that all of these sets are the same.

There is a subtle yet profound difference between students at these stages: Some know that *base-ten* grouping stage is 53 because they understand the idea that 5 groups of ten and 3 more is the same amount as 53 counted by ones; others simply say, "It's 53," because they have been told that when things are grouped this way, it's called 53. The students who understand place value will see no need to count the base-ten grouping by ones. They understand the "fifty-threeness" of the *unitary* and *base-ten* grouping to be the same. The students in the pre-place-value stage may not be sure how many they will get if they count the tiles in the base-ten grouping by ones or if the groups were "ungrouped" how many there would then be.

CCSS Standards for Mathematical Practice

MP2. Construct viable arguments and critique the reasoning of others.

Your foremost objective should be helping students integrate what they already know about numbers from counting by ones with the grouping-by-tens concept. If they only count by ones, ask them, "What will happen if we count these items by groups and singles (or by tens and ones)?" If a set has been grouped into tens and singles and counted, then ask, "How can we be really certain that there are 53 things here?" or "How many do you think we will get if we count by ones?" You cannot just *tell* students that these counts will all be the same and hope that it will make sense to them—it is a relationship they must construct themselves.

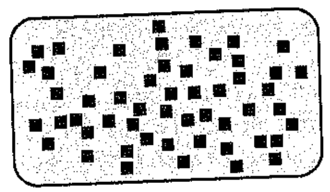
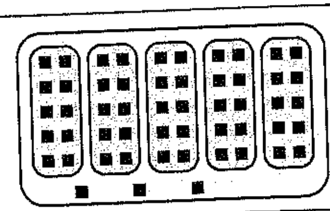
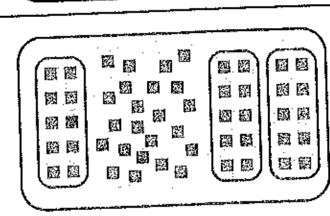
Grouping Stage	Visual Representation	Counting Approach	Students Can:
UNITARY Count by ones		1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, and so on	• Name a quantity or "tell how many" by counting each piece. • Are not yet able to think of 10 as a single unit. • Use counting by ones as the only way they are convinced that different sets have the same amount.
BASE-TEN Count by groups of tens and ones		1, 2, 3, 4, 5 groups of 10 and 1, 2, 3, ones (singles) or 10, 20, 30, 40, 50, 51, 52, 53	• Count a group of 10 objects as a single item (unitizing). • Coordinate the base-ten approach with a count by ones to as a means of telling "how many."
EQUIVALENT Non-standard base-ten		Before counting students would trade and then count 10, 20, 30, 40, 50, 51, 52, 53	• Group the pieces flexibly into versions that include tens and ones but all trades have not been carried out. • Use these alternate groupings to relate to computation by being able to trading or regroup numbers in a variety of ways.

FIGURE 10.1 Three stages of the grouping of 53 objects.

Pause & Reflect

What are some defining characteristics of "pre-place-value" students and students who understand place value? ●

Groupings with fewer than the maximum number of tens are referred to as *equivalent groupings*. Understanding the equivalence of the *base-ten grouping* and the *equivalent grouping* indicates that grouping by tens is not just a rule that is followed, but also that any grouping by tens, including all or some of the singles, can help tell how many. Many computational techniques (e.g., regrouping in addition and subtraction) are based on equivalent representations of numbers.

Integrating Base-Ten Groupings with Words

The way we say a number such as "fifty-three" must also be connected with the grouping-by-tens concept. The counting methods provide a connection. The count by tens and ones results in saying the number of groups and singles separately: "five tens and three." Saying the number of tens and singles separately in this fashion can be called *base-ten language*. Students can associate the base-ten language with the *standard language*: "five tens and three—fifty-three."

There are several variations of the base-ten language for 53: 5 tens and 3, 5 tens and 3 ones, 5 tens and 3 singles, and so on. Each may be used interchangeably with the standard name, "fifty-three." If you have ELs, it is best to select one approach (e.g., 5 tens and 3 ones) and consistently connect it to the standard language. Other languages often use the base-ten format (e.g., 17 in Spanish is *diecisiete*, literally meaning "ten and seven"), so this association can be a good cultural connection.

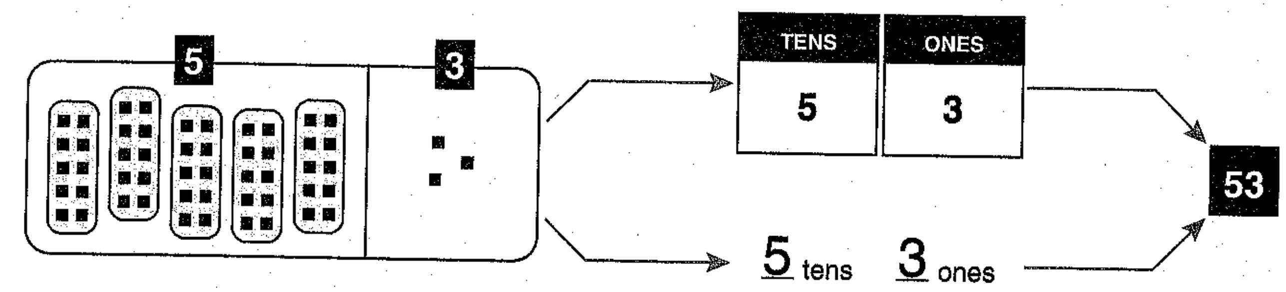


FIGURE 10.2 Groupings by 10 are matched with numerals, recorded in labeled places, and eventually written in standard form.

Be precise in your language. Whenever you refer to a number in the tens, hundreds, or thousands (or beyond), make sure you do not just say "six," but instead refer to it with its place value location, such as 6 tens (or 60). Students are often confused when numbers are discussed as digits rather than describing their actual value.

CCSS Standards for Mathematical Practice
MP6. Attend to precision.

Integrating Base-Ten Groupings with Place-Value Notation

The symbolic scheme that we use for writing numbers (ones on the right, tens to the left of ones, and so on) must be coordinated with the grouping scheme. Activities can be designed so that students physically associate groupings of tens and ones with the correct recording of the individual digits, as Figure 10.2 indicates.

Language again plays a key role in making these connections. The explicit count by groups and singles matches the individual digits as the number is written in the usual left-to-right manner. A similar coordination is necessary for hundreds and other place values. Keep in mind that students will initially find it challenging to see "ten" as both 10 ones and 1 ten.

Figure 10.3 summarizes the ideas of an integrated place-value understanding that have been discussed so far. Note that all three methods of counting shown in the figure support relational understanding when students can flexibly integrate base-ten concepts, written names, and oral names.

MyLab Education Self-Check 10.2

Base-Ten Models for Place Value

When students are learning base-ten concepts, they are combining multiplicative understanding (each place is ten times the value of the place to the right) with a positional system (each place has a value)—something hard to do prior to learning about multiplication! Physical models for base-ten concepts play a key role in helping students develop the idea of "a ten" as both a single entity and as a set of 10 units especially for students with lower ability (Mix, Smith, Stockton, Cheng & Barterian, 2016). Remember, though, that the models do not "show"

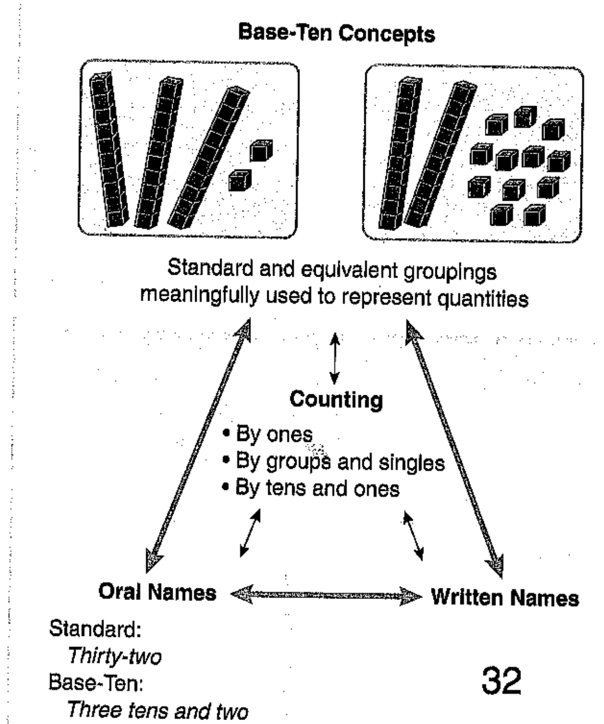


FIGURE 10.3 Relational understanding of place value integrates three components shown as the corners of the triangle.

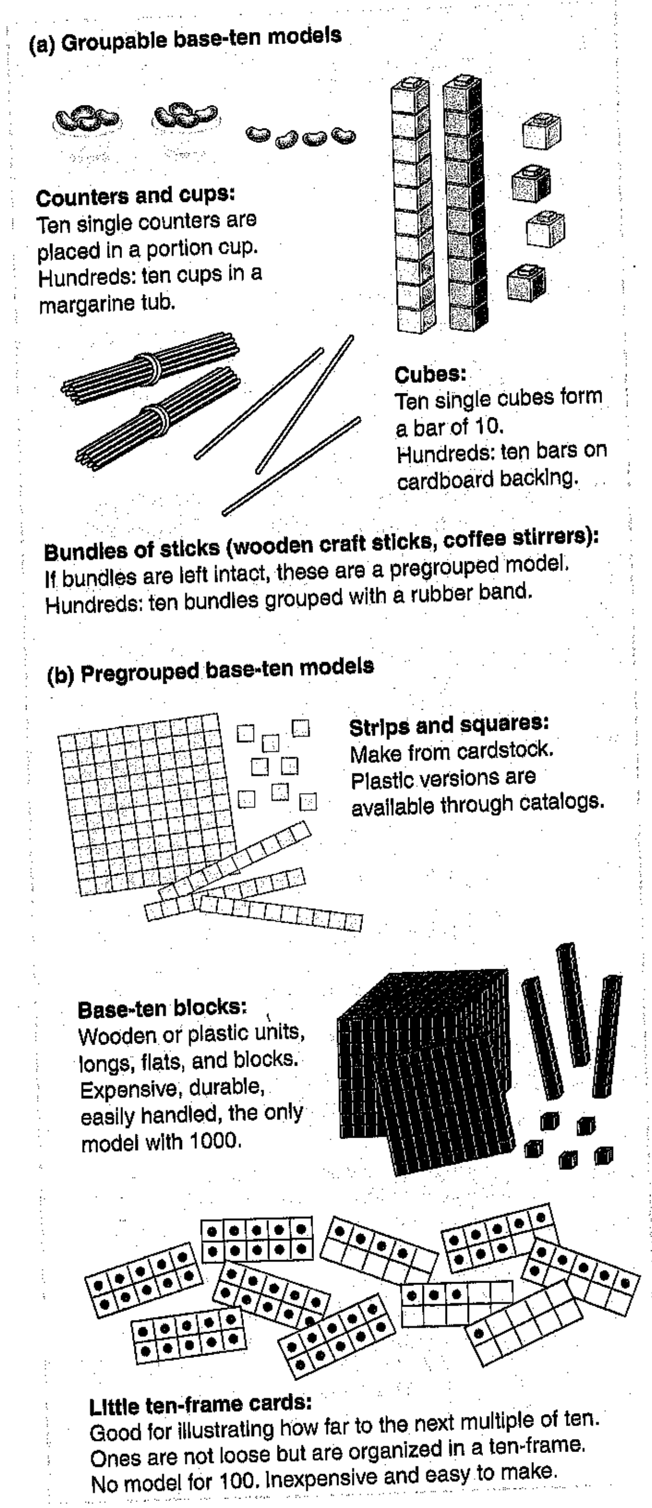


FIGURE 10.4 Groupable and pregrouped base-ten models.

CCSS Standards for Mathematical Practice
MP7. Attend to precision.

the concept to the students; the students must mentally construct the "ten makes one relationship" and impose it on the model.

An effective base-ten model for ones, tens, and hundreds is one that is *proportional*. That is, a model for ten is physically 10 times larger than the model for a one, and a hundred model is 10 times larger than the ten model. Proportional materials allow students to check that ten of any given piece is equivalent to one piece in the column to the left (10 tens equals 1 hundred, and so on). Base-ten proportional models can be categorized as either *groupable* or *pregrouped*.

Groupable Models

Models that most clearly reflect the relationships of ones, tens, and hundreds are those for which students can build the ten from the single pieces or units and verify its value. When students put 10 beans in a cup, the cup of 10 beans literally *is the same as* the 10 single beans. Bundles of wooden craft sticks or coffee stirrers can be grouped with rubber bands. Plastic connecting cubes can be built into rods of ten and thereby provide a useful transition to pregrouped rods because they form a similar shape. Examples of these groupable models are shown in Figure 10.4(a).

As students begin to make groupings of ten, start introducing the language of "tens" by matching the objects, such as "cups of tens and ones" or "bundles of tens and singles." Then graduate to a general phrase, such as "groups of tens and ones." Eventually you can abbreviate this language simply to "tens" such as "four tens and seven."

When students become more familiar with these models, collections of tens can be made in advance by the students and kept as ready-made tens (e.g., craft sticks can be left bundled, connecting cubes left connected). This approach is a good transition to the pregrouped models described next.

Pregrouped Models

Models that are pregrouped are commonly shown in textbooks and are often used in instructional activities. Pregrouped models, such as those in Figure 10.4(b) and the Base-Ten Materials, cannot be taken apart or put together. When 10 single pieces are accumulated, they must be exchanged or traded for a ten, and likewise, tens must be traded for hundreds. The advantage of these physical models is their ease of use and the efficient way they model large numbers.

MyLab Education **Blackline Master: Base-Ten Materials**

With pregrouped models, make an extra effort to confirm that students understand that a ten piece really is the same as 10 ones. Although there is a pregrouped cube to represent 1000, have students group 10 hundred pieces and attach them together as a cube to show how it is formed. Otherwise, some students may only count the square units they see on the surface of the six faces and may think the cube represents 600.

The Little Ten-Frames effectively link to the familiar ten-frames students used early on to think about numbers, and as such, may initially be more meaningful than base-ten materials made from paper strips and squares. Little ten-frames have the distinct advantage of always

showing the distance to the next decade. For example, when 47 is shown with 4 ten cards and a seven card, a student can see that three more will make five full cards, or 50.

MyLab Education **Blackline Master: Little Ten-Frames**

A significant challenge with using the pregrouped physical models occurs when students have not first had adequate experiences with groupable models. Then there is the potential for students to use them without reflecting on the ten-to-one relationships. For example, if students are just *told* to trade 10 ones for a ten, it is quite possible for them to make this trade without attending to the "ten-ness" of the piece they call a ten. Similarly, students can "make the number 42" by simply picking up 4 tens and 2 ones pieces without understanding that if all the pieces were broken apart there would be 42 ones.

***TECHNOLOGY* Note.** Using electronic versions of base-ten manipulatives, students (including those with disabilities) can place ones, tens, hundreds, or thousands on the screen with simple mouse clicks. Number Pieces (https://www.mathlearningcenter.org/resources/apps/number-pieces) help learners explore place value while helping students visualize operations with multi-digit numbers. Students use the pieces to represent multi-digit numbers, regroup, add, subtract, multiply, and divide. Two versions of the web or app-based tool are available depending on the grade level and concepts to be explored. Students can represent larger numbers and multiplication and division concepts using the regular version of Number Pieces. A simplified version, Number Pieces Basic, is available for use with primary students. Number Pieces Basic has fewer features, putting greater focus on place value, counting, addition, and subtraction.

Compared to real base-ten blocks, these digital materials are free, easily grouped and ungrouped, available in "endless" supply, and can be manipulated by students and displayed by projector or smart board. Remember though, virtual models are no more conceptual than physical models and as such are only a representation for students who understand the relationships involved. ■

CCSS Standards for Mathematical Practice
MP5. Use appropriate tools strategically.

Nonproportional Models

Nonproportional models, or models where the ten is not physically 10 times larger than the one, are *not* used for introducing place-value concepts. They may be used when students already have a conceptual understanding of the numeration system and need additional reinforcement, or by older students who may need to return to place-value concepts because they are struggling with content that requires place-value understanding. Examples of nonproportional models include an abacus that has same-sized beads on wires in different columns, money, or chips that are given different place values by color.

MyLab Education **Self-Check 10.3**

MyLab Education **Application Exercise 10.1: Base-Ten Models for Place Value** Click the link to access this exercise, then watch the video and answer the accompanying questions.

Activities to Develop Base-Ten Concepts

Now that you have a sense of the important place-value concepts, we turn to activities that assist students in developing these concepts. This section focuses on base-ten concepts or grouping by tens (see the top of Figure 10.3). Connecting this important idea with the oral and written names for numbers (the rest of Figure 10.3) is discussed separately to help you focus on how to do each. However, in the classroom, the oral and written names for numbers can and should be developed in concert with conceptual ideas.

Grouping Activities

Reflect for a moment on how strange it must sound to say, "seven ones." Certainly, students have never said they were "seven ones" years old. The use of the word *ten* as a singular group name can be even more mysterious. Consider the phrase "Ten ones makes one ten." The first *ten* carries the usual meaning of 10 things, but the other *ten* is a singular noun, a thing. How can something the student has known for years as the name for a lot of things suddenly become one thing? And if you think this idea is confusing for native speakers, imagine the potential difficulty for ELs.

Because students come to their development of base-ten concepts with a count-by-ones idea of number, you must begin there. You cannot arbitrarily impose grouping by ten on students. Students need to experiment with showing amounts in groups of like size and perhaps come to an agreement that ten is a very useful size to use. The following activity could be done toward the end of first grade as an example of a first effort at developing grouping concepts.

Activity 10.1 CCSS-M: K.NBT.A.1; 1.NBT.B.2a

Counting in Groups

Find a collection of items between 25 and 100 that students might be interested in counting—perhaps the number of shoes in the classroom, a container of cubes, a long chain of plastic links, or the number of crayons in the classroom crayon box. Then pose the question, "How could we count our shoes in some way that would be easier than counting by ones?" Whatever suggestions children suggest, try them. After testing several methods, you can have a discussion of what worked well and what did not. If no one suggests counting by tens, you might casually suggest that as an idea to try.

One teacher challenged her students to find a good way to count all the connecting cubes being held by the students after the children collected a cube for each of their pockets. The first suggestion was to count by sevens. That was tried but did not work very well because none of the students could easily count by sevens. In search of a more efficient way, the next suggestion was to count by twos. This approach did not seem to be much better than counting by ones. Finally, they settled on counting by tens and realized that this method was a good and "fast" way of counting.

This activity and similar ones provide you with the opportunity to suggest that materials actually be arranged into groups of tens before the "fast" way of counting is begun. Remember that students may count "ten, twenty, thirty, thirty-one, thirty-two" but not fully realize the "thirty-two-ness" of the quantity. To connect the count-by-tens method with their understood method of counting by ones, the students need to count both ways and discuss why they get the same result.

The idea in the next activity is for students to make groupings of ten and record or say the amounts. Number words are used so that students will not mechanically match tens and ones with individual digits. It is important that students confront the actual quantity in a manner meaningful to them.

Activity 10.2 CCSS-M: K.NBT.A.1; 1.NBT.B.2a; 1.NBT.B.2b; 1.NBT.B.2c

Groups of Ten

STUDENTS with SPECIAL NEEDS

Prepare bags of different types of objects such as toothpicks, buttons, beans, plastic chips, connecting cubes, craft sticks, or other items. Students should use the Bag of Tens Activity Page a recording sheet similar to the top left of Figure 10.5. The bags can be placed at stations around the room or given to pairs of students. Students empty the bags and count the contents. The amount is recorded as a number word. Then the objects are grouped in as many tens as possible. The groupings are recorded on the form. After returning the objects to the bags, bags are traded, or students move to another station. Note that students with disabilities may initially need to use a ten-frame to support their counting. Then the use of the ten-frame should eventually fade.

MyLab Education **Activity Page: Bag of Tens**

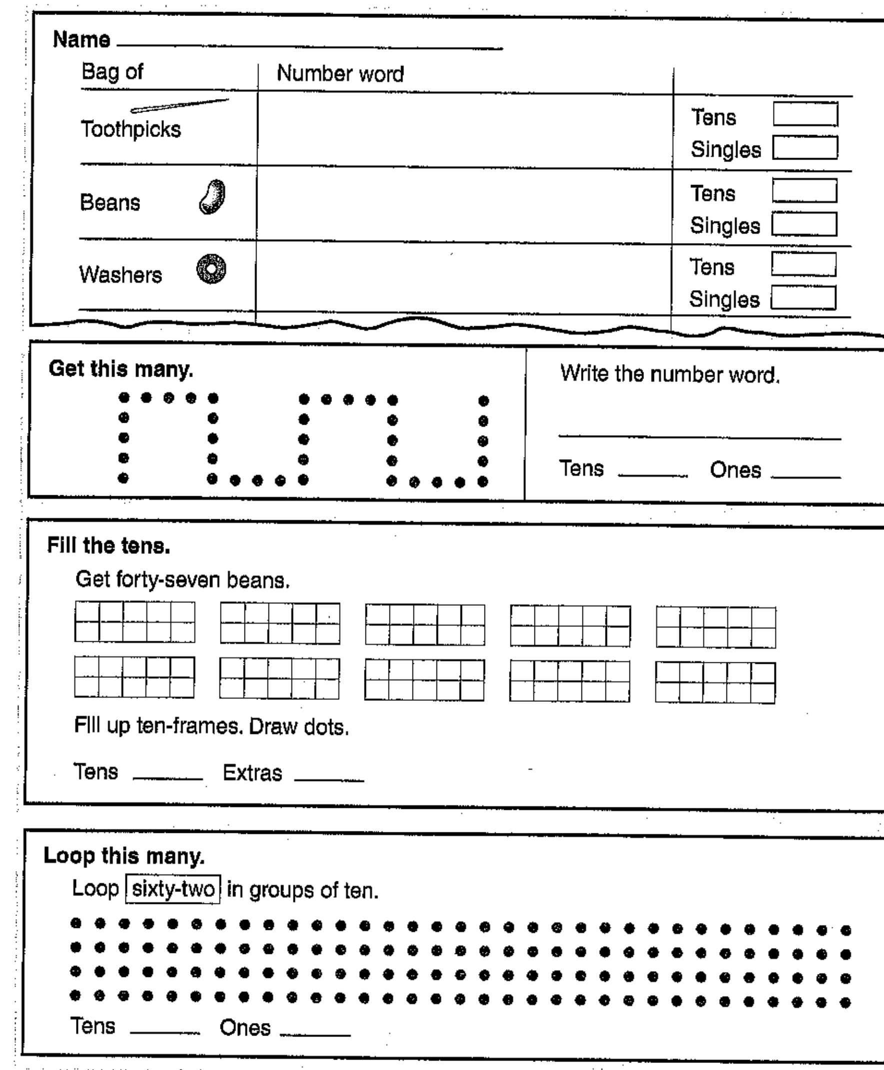


FIGURE 10.5 Activities involving number words and making groups of ten.

Variations of the "Groups of Ten" activity are suggested by the three other recording sheets in Figure 10.5. On Get This Many Activity Page, students count the dots and then count out the corresponding number of counters. Provide small cups to put the groups of ten in. Notice that the activity requires students to first count the set in a way they understand (e.g., count by ones), record the amount in words, and then make the place-value groupings. The Fill the Tens and Loop This Many Activity Pages begin with a verbal name (number word), and students must count the indicated amount and then make groups.

MyLab Education **Activity Page: Get This Many**

MyLab Education **Activity Page: Fill the Tens**

MyLab Education **Activity Page: Loop This Many**

Activity 10.3 CCSS-M: K.NBT.A.1; 1.NBT.B.2a; 1.NBT.B.2b

Estimating Groups of Tens and Ones

Give students a length that they are going to measure—for example, the length of a child lying down or the distance around a sheet of newspaper. At one end of the length, line up 10 units (e.g., 10 linking cubes, toothpicks, rods, or blocks). On a recording sheet (see Figure 10.6 and the How Long? Activity Page), students record an estimate of how many groups of 10 and ones they think will match the length. Next they find the actual measure, placing units along the full length. These units are counted by ones and grouped in tens. Both results are recorded. Estimating the groups of ten requires children to pay attention to the ten as a group or unit. Notice that all three place-value components from Figure 10.3 are included. Click below to see the Expanded Lesson: Estimating Groups of Tens and Ones.

MyLab Education **Activity Page: How Long?**
MyLab Education **Expanded Lesson: Estimating Groups of Tens and Ones**

***FORMATIVE ASSESSMENT* Notes.** Use a Class Observational Checklist to record observations about how students do these activities. For example, how do students count out the objects? Do they make groupings of ten? Do they count to 10 and then start again at 1? Students who count in these ways are already using the base-ten structure. But what you may see early on is students counting a full set without stopping at tens and without any effort to organize the materials in groups. If you notice that behavior, use a diagnostic interview and ask the student to count a container of beans (with between 30 and 50 beans) and record the number. Ask the student, "If you were to place each group of 10 beans in a small cup, how many cups would you need?" If the student has no idea or makes random guesses, what would you know about the student's knowledge of place value? ■

MyLab Education **Blackline Master: Class Observational Checklist**

Grouping Tens to Make 100

In second grade, numbers up to 1000 become important. Here the issue is not just connecting a count-by-ones concept to a group of 100, but rather seeing the number in multiple ways, including as 100 single objects, as 10 tens, and as a singular thing. In textbooks, this connection is often presented on one page showing how 10 rods of ten can be put together to make 1 hundred piece. This quick demonstration may be lost on many students. Additionally, the word *hundred* is equally strange and can get even less attention. These word names are not as simple as they seem!

NAME Jessica

ITEM	ESTIMATE	ACTUAL
straws	5 TENS 6 ONES	3 TENS 2 ONES thirty-two Number Word
______	____ TENS ____ ONES	____ TENS ____ ONES ______ Number Word

FIGURE 10.6 Recording sheet for estimating groups of tens and ones.

To reinforce the idea that a hundred is a group of 10 tens and also 100 singles, consider the estimation activity Too Many Tens.

MyLab Education **Activity Page: Too Many Tens**

Activity 10.4 CCSS-M: 2.NBT.A.1

Too Many to Count?

Show students any quantity with 250 to 1000 items. For example, you might use a container of lima beans, a long chain of connecting links or paper clips, a box of pennies, or a grocery bag full of straws. First, have students make and record estimates of how many beans, for example, are in the container. Discuss how students determined their estimates. Then distribute portions of the beans to pairs or triads of students to put into cups of 10 beans. Collect leftover beans and put them into groups of ten as well. Now ask, "How can we use these groups of ten to tell how many beans we have? Can we make new groups from the groups of ten? What is 10 groups of ten called?" Be prepared with some larger containers or baggies into which 10 cups (or other collections of 10 tens) can be placed. When all groups are made, count the hundreds, the tens, and the ones separately. Record the total as 4 hundreds + 7 tens + 8 ones. This activity can be extended to third graders with amounts more than 1000.

In this activity, it is important to use a groupable model so that students can see how the 10 groups are the same as the 100 individual items. At first you may think this activity will take too much time. But this activity helps cement the connection that is often lost in the rather simple display of a hundreds piece or a paper hundreds square in the pregrouped base-ten models.

MyLab Education **Video Example 10.2**

Watch a video of a student playing a race to 100 game with a teacher. This game will also help students carry out the equal trades that reinforce the use of pregrouped base-ten models.

Equivalent Representations

An important variation of the grouping activities is aimed at the equivalent representations of numbers. For example, pose the following task to students who have just completed the Groups of Ten Activity 10.2.

What is another way you can show 42 besides 4 groups of ten and 2 singles? Let's see how many ways you can find.

Interestingly, most students will go next to 42 singles. The following activities focus on creating other equivalent representations.

Activity 10.5 CCSS-M: 1.NBT.B.2; 1.NBT.C.5

Can You Make the Link?

Show a collection of materials that are only partly grouped in sets of ten. For example, you may have 5 chains of 10 links and 17 additional unconnected links. Be sure the students understand that each chain has 10 links. Have students count the number of chains and the number of singles in any way they wish to count. Ask, "How many in all?" Record all responses and discuss how they got their answers. Next, before their very eyes, change the groupings (make a ten from the singles, or break apart one of the tens) and repeat. Do not change the total number of links from one time to the next. Once students begin to understand that the total does not change, ask in what other ways the items could be grouped if you use tens and ones.

If you are teaching in second grade, equivalent representations for hundreds as groups of tens can help with the concept of a hundred as 10 tens. The next activity is similar to "Can You Make the Link?" but is done using pregrouped materials and includes hundreds.

Activity 10.6 CCSS-M: 2.NBT.A.1; 2.NBT.A.3; K.CC.B.5

Three Other Ways

Students work in groups or pairs. First, they show 463 on their desks with base-ten materials in the standard representation (4 hundreds pieces, 6 tens and 3 ones). Next, they find and record at least three other ways of representing this quantity. A variation is to challenge students to find a way to show an amount with a specific number of pieces. "Can you show 463 with 31 pieces?" (There is more than one way to do this.)

When students have sufficient experiences with pregrouped materials, a semi-concrete square-line-dot notation can be used for recording ones, tens, and hundreds (see Figure 10.7 and Square Line Dot Activity Page). Use the drawings to reinforce the notion of unitizing (i.e., one ten, one hundred) and as a suggestion for how students can record their thinking and their results.

MyLab Education **Activity Page: Square Line Dot**

The next activity begins to incorporate oral language with equivalent representation ideas.

Activity 10.7 CCSS-M: 1.NBT.A.1; 2.NBT.A.1; 2.NBT.A.3

Base-Ten Riddles

Base-ten riddles can be presented orally or in written form (see Base-Ten Riddle Cards). In either case, students should use base-ten materials to help solve the riddles. The examples here illustrate a variety of different levels of difficulty. Have students write new riddles when they complete these.

I have 23 ones and 4 tens. Who am I?

I have 4 hundreds, 12 tens, and 6 ones. Who am I?

I have 30 ones and 3 hundreds. Who am I?

I am 45. I have 25 ones. How many tens do I have?

I am 341. I have 22 tens. How many hundreds do I have?

I have 13 tens, 2 hundreds, and 21 ones. Who am I?

If you put 3 more tens with me, I would be 115. Who am I?

I have 17 ones. I am between 40 and 50. Who am I? How many tens do I have?

MyLab Education **Activity Page: Base-Ten Riddle Cards**

MyLab Education **Self-Check 10.4**

Reading and Writing Numbers

In this section, we focus on helping students connect oral and written names for numbers (see bottom of Figure 10.3) with their emerging base-ten concepts of using groups of 10 or 100 as efficient methods of counting. Note that the ways we say and write numbers are conventions, not concepts. Students must learn these conventions by being told rather than through problem-based activities. Remember that for EL students, the convention or pattern in our English number words is probably not the same as it is in their native language especially for the numbers 11 to 19.

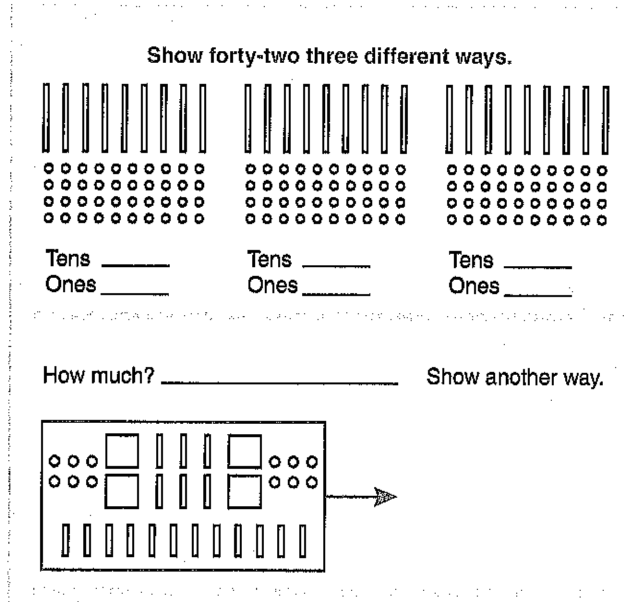


FIGURE 10.7 Equivalent representations using square-line-dot pictures.

Two-Digit Number Names

In kindergarten and first grade, students need to connect the base-ten concepts with the oral number names they have repeatedly used. They know the words but have not thought of them in terms of tens and ones. In fact, early on they may want to write twenty-one as 201.

When teaching oral names, you will want to use base-ten models. Initially, rather than using standard number words, use the more explicit base-ten language (e.g., "4 tens and 7 ones" instead of "forty-seven"). Base-ten language is rarely misunderstood. When it seems appropriate, begin to pair base-ten language with standard language. Emphasize the teens as exceptions to the pattern. Acknowledge that they are formed "backward" and do not fit the patterns. The next activity helps introduce oral names for numbers.

Activity 10.8 CCSS-M: 1.NBT.B.2c

Counting Rows of Ten

ENGLISH LEARNERS

Project the 10 × 10 Multiplication Array of dots. Cover up all but two rows, as shown in Figure 10.8(a). "How many tens? [2.] Two tens is called twenty." Have the class repeat. Show another row. "Three tens is called thirty. Four tens is forty. Five tens could have been fivety but is just fifty. How many tens does sixty have?" The names sixty, seventy, eighty, and ninety all fit the pattern. Slide the cover up and down the array, asking how many tens and the name for that many. ELs may not initially hear the difference between fifty and fifteen, sixty and sixteen, and so on, so explicitly compare these words and clearly enunciate—even overemphasize the word endings.

Use the same 10 × 10 multiplication array to work on names for tens and ones. Show, for example, four full lines: "forty." Next, expose one dot in the fifth row. "Four tens and one. Forty-one." Add more dots one at a time. "Four tens and two. Forty-two." "Four tens and three. Forty-three." This visual is shown in Figure 10.8(b). When that pattern is established, repeat with other decades from 20 through 90. Eventually connect the array to a Hundreds Chart to connect the oral name and the written numeral. For example, children can locate 43 as three rows of ten down and over to the right three ones.

MyLab Education **Blackline Master: 10 × 10 Multiplication Array**
MyLab Education **Blackline Master: Hundreds Chart**

The next activity shows how this basic approach might be done with other base-ten models.

Activity 10.9 CCSS-M: 1.NBT.B.2; 1.NBT.C.5

ENGLISH LEARNERS

STUDENTS with SPECIAL NEEDS

Counting with Base-Ten Models

Show some tens pieces on a projector, or just place them on the carpet in a mixed arrangement. Ask, "How many tens?" Add or remove a ten and repeat the question. Next, add some ones. Always have students give the base-ten name and the standard name. For many students, including ELs and students with disabilities, it is helpful to post examples of base-ten names and the corresponding standard names on the math word wall. Continue to make changes in the materials displayed by adding or removing 1 or 2 tens and by adding and removing ones (or have students create problems). Avoid the standard left-to-right order for tens and ones; the emphasis is on the names of the materials, not the order they are in.

Reverse the activity by having students use Base-Ten Materials at their desks. For example, say, "Make seventy-eight." The students make the number with the models and then give the base-ten name (7 tens and 8 ones) and standard name (78). Students can also record their work (see Figure 10.9).

MyLab Education Blackline Master: Base-Ten Materials

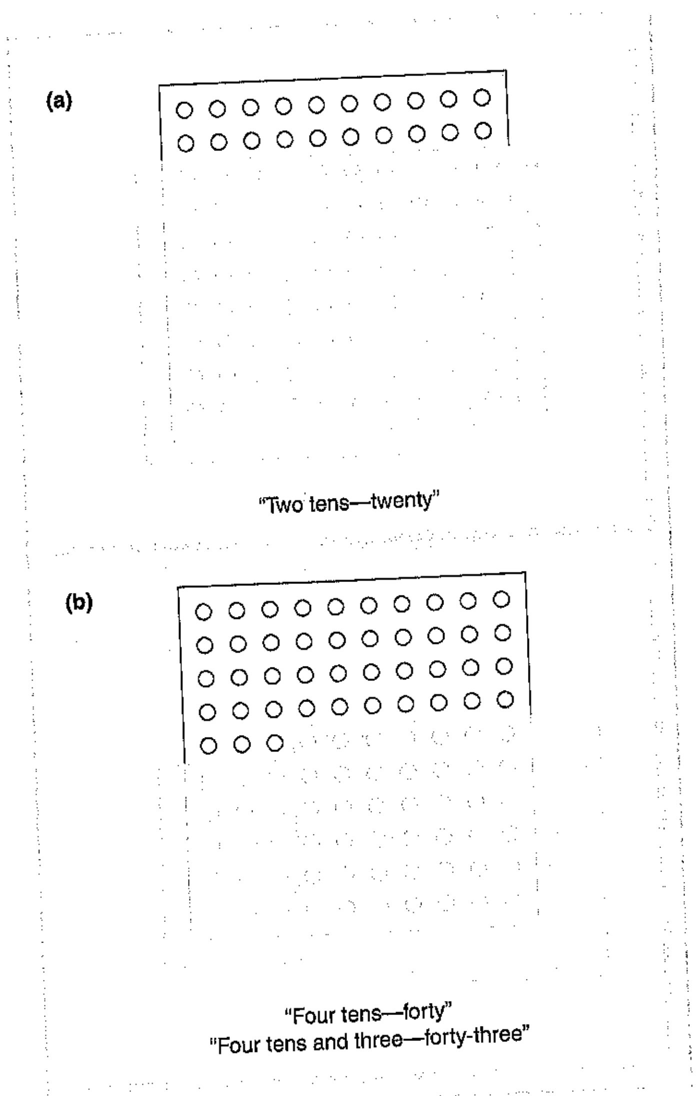


FIGURE 10.8 10 × 10 dot arrays are used to model sets of tens and ones.

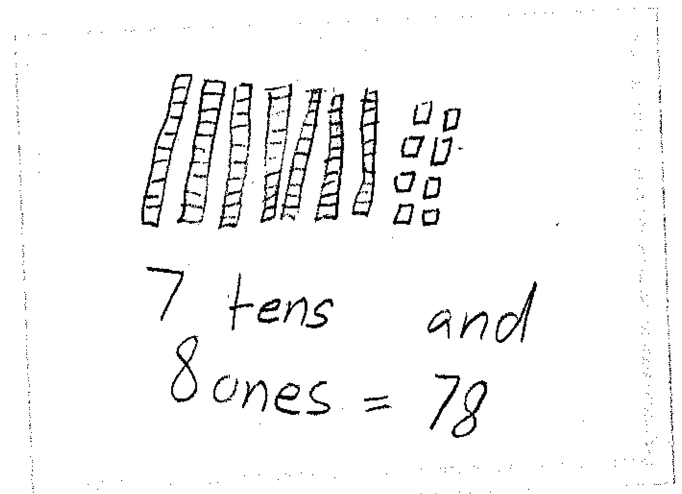


FIGURE 10.9 A student's recording of 78 with models and the base-ten name.

Activity 10.10 CCSS-M: 1.NBT.B.2a; 1.NBT.B.2b; 1.NBT.B.2c

Tens, Ones, and Fingers

Ask your class, "Can you show 6 fingers [or any amount less than 10]?" Then ask, "How can you show 37 fingers?" Some students will figure out that at least four students are required. Line up four students, and have three students hold up 10 fingers while the last student holds up 7 fingers. Have the class count the fingers by tens and ones. Ask for other students to show different numbers. Emphasize the number of sets of ten fingers and the single fingers (base-ten language), and pair this model with standard language.

Activities 10.8, 10.9, and 10.10 will be enhanced by having students explain their thinking. If you don't require students to reflect on their responses, they soon learn how to give the response you want, matching number words to models without actually thinking about the total quantities.

Three-Digit Number Names

The approach to three-digit number names starts by showing mixed arrangements of base-ten materials and have students give the base-ten name (4 hundreds, 3 tens, and 8 ones) and the standard name (438). Vary the arrangement from one example to the next by changing only one type of piece; that is, add or remove only ones or only tens or only hundreds. It is important for students with disabilities to see counterexamples, so purposely point out that some (anonymous!) students wrote 200803 for two hundred eighty-three, and ask them whether that is correct and explain their reasoning. The connection between oral and written numbers is not straightforward, with some researchers suggesting that an early milestone on the route to full understanding is this early (incorrect) expanded form of writing numbers (Byrge, Smith, & Mix, 2013). These discussions allow students to explore their initial ideas and clear up any misunderstandings.

CCSS Standards for Mathematical Practice

MP3. Construct viable arguments and critique the reasoning of others.

The major challenge with three-digit numbers is with numbers involving no tens, such as 702 (or later with numbers such as 1046). As noted earlier, the use of base-ten language is quite helpful here. The difficulty of zero-tens (or more generally the internal zero) is more pronounced when writing numerals. For example, students frequently incorrectly write 7,002 for seven hundred two. Emphasizing meaning in the oral base-ten language will be a significant help. At first, students do not see the importance of zero in place value and do not understand that zero helps us distinguish between such numbers as 203, 23, and 230 (Dougherty, Flores, Louis, & Sophian, 2010). Carefully avoid calling zero a "placeholder," because it is a number with a value. ELs may need additional time to think about how to say and write the numerals, because they are translating all the terms involved with the number.

Researchers note that there are significantly more errors with four-digit number names than three-digit numbers, so do not think that students will easily generalize to larger numbers without actually exploring additional examples and tasks (Cayton & Brizuela, 2007).

Written Symbols

Place-value mats are simple mats divided into two or three sections to hold ones and tens or ones, tens, and hundreds pieces, as shown in Figure 10.10. You can suggest to your students that the mats are a good way to organize their materials when working with base-ten blocks. Explain that the standard way to use a place-value mat is with the space for the ones on the right and the tens and hundreds places to the left.

Although it is not commonly seen in textbooks, it is strongly recommended that two ten-frames be drawn in the ones place as shown on the Place-Value Mat. That way, the amount of ones on the ten-frames is always clearly evident, eliminating the need for repeatedly counting the ones. The ten-frame also makes it very clear how many additional ones are needed to make the next set of ten. If students are modeling two numbers at the same time, one ten-frame could be used for each number.

MyLab Education Blackline Master: Place-Value Mat

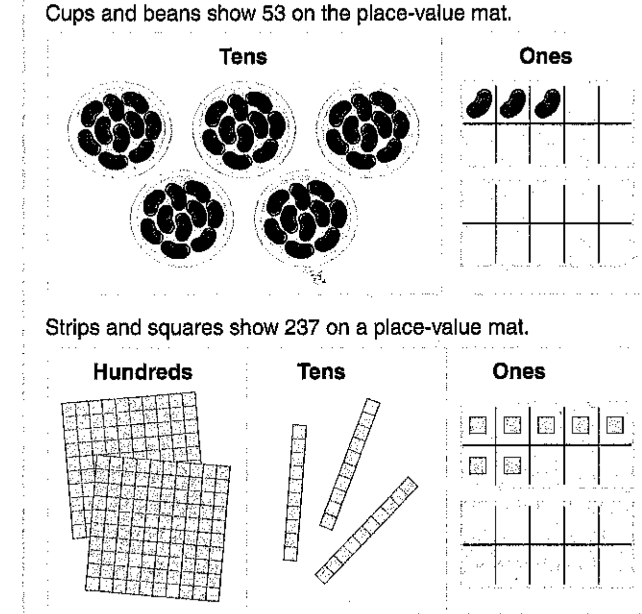


FIGURE 10.10 Place-value mats with two ten-frames in the ones place promote the concept of groups of ten.

As students use their place-value mats, they can be shown how the left-to-right order of the pieces is also the way that numbers are written. To show how the numbers are "built," have a set of Place-Value Cards—one for each of the hundreds

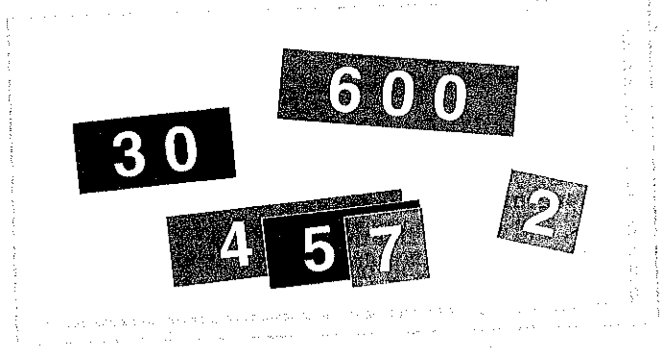


FIGURE 10.11 Building numbers with a set of place-value cards.

(100–900), one for each of the tens (10–90), and ones cards for 1 through 9 (see Figure 10.11). Notice that the cards are made so that the tens card is twice as long as the ones card and the hundreds card is three times as long as the ones card.

MyLab Education **Activity Page: Place-Value Cards**

As students place the materials for a number (e.g., 457) on the mat, have them also place the matching cards (e.g., 400, 50, and 7) below the materials. Then, starting with the hundreds card, layer the others on top, right aligned. This approach will show how the number is built while allowing the students to see the individual place value parts of the number which is especially helpful if there are zero tens. The place-value mat and the matching cards demonstrate the important link between the base-ten models and the written form of the numbers.

The next two activities are designed to help students make connections between models, oral language (base-ten and standard), and written forms. The activities can be done with two- or three-digit numbers, depending on students' needs.

Activity 10.11 CCSS-M: 2.NBT.A.1a; 2.NBT.A.1b; 2.NBT.A.3

ENGLISH LEARNERS

Say It/Press It

Display models of ones, tens, hundreds (and thousands, if appropriate) in a mixed arrangement. Use a projector, virtual manipulatives, or simply draw on the board using the square-line-dot method. Students say the amount shown in base-ten language ("four hundreds, one ten, and five ones") and then in standard language ("four hundred fifteen"); next, student enter it on their calculators or use paper and pencil to respond. Have someone share his or her display and defend it. Make a change in the materials and repeat. You can also do this activity as "Show It/Press It" by saying the standard name for a number and then having students use base-ten materials to show that number and enter it on their calculators (or write it). Again, pay attention to numbers with components in the teens (e.g., 317) and numbers with internal zeros (e.g., 408). ELs may need additional time to think of the words that go with the numbers, especially as the numbers get larger.

To support students struggling with reading a number with an internal zero you may want to show (or say) 7 hundreds and 4 ones. Then the class says, "Seven hundreds, zero tens, and four ones, which is—seven hundred (*slight pause*) four." The pause and the base-ten language support the correct reading of the three-digit number.

The next activity is also a good assessment to see whether students really understand the value of digits in two-, three- or four-digit numbers.

Activity 10.12 CCSS-M: 1.NBT.B.2; 1.NBT.C.5; 1.NBT.C.6; 2.NBT.A.1; 2.NBT.A.3; 2.NBT.B.5; 2.NBT.B.8

STUDENTS with SPECIAL NEEDS

Digit Change

Have students enter a specific two-, three-, or four-digit number on the calculator. The task is to change one of the digits in the number without simply entering the new number. For example, change 48 to 78. Change 315 to 305 or to 295. Changes can be made by adding or subtracting an appropriate amount. Students should write or discuss explanations for their solutions. Students with disabilities may at first need the visual support of having cards that say "add ten" or "add one." They may also need support with Base-Ten Materials to be able to conceptualize the number and then move to more abstract work using only the calculator.

MyLab Education **Blackline Master: Base-Ten Materials**

***FORMATIVE ASSESSMENT* Notes.** Students are often able to disguise their lack of place value understanding by following directions, using the tens and ones pieces in prescribed ways, and using the language of place value.

The diagnostic interviews presented here are designed to help you look more closely at students' understanding of the integration of the three components of place value. Designed as interviews rather than full-class activities, these tasks have been used by several researchers and are adapted primarily from Labinowicz (1985), Kamii (1985), and Ross (1986).

The first interview is referred to as the Digit Correspondence Task. Take out 36 blocks. Ask the student to count the blocks, and then have the student write the number that tells how many there are. Circle the 6 in 36 and ask, "Does this part of your 36 have anything to do with how many blocks there are?" Then circle the 3 and repeat the question. As with all diagnostic interviews, do not give clues. Based on responses to the task, Ross (1989, 2002) has identified five distinct levels of understanding of place value:

1. *Single numeral.* The student writes 36 but views it as a single numeral. The individual digits 3 and 6 have no meaning by themselves.
2. *Position names.* The student correctly identifies the tens and ones positions but still makes no connections between the individual digits and the blocks.
3. *Face value.* The student matches 6 blocks with the 6 and 3 blocks with the 3.
4. *Transition to place value.* The 6 is matched with 6 blocks and the 3 with the remaining 30 blocks but not as 3 groups of 10.
5. *Full understanding.* The 3 is correlated with 3 groups of ten blocks and the 6 with 6 single blocks.

For the second interview, write the number 342. Have the student read the number. Then have the student write the number that is 1 more. Next, ask for the number that is 10 more. You may wish to explore further with models. One less and 10 less can be checked the same way. Observe whether the student is counting on or counting back or whether the student immediately knows that ten more is 352. This interview can also be done with a two-digit number.

A third interview can also provide interesting evidence of depth of understanding. Ask the student to write the number that represents 5 tens, 2 ones, and 3 hundreds. Note that the task does not give the places in order. What do you think will be a common misunderstanding? If the student doesn't write 352, then ask the students to show you the number with base-ten materials, and to say what number they have with the materials. Compare to what they wrote previously, if different. What information can you obtain from the results of this interview? ■

MyLab Education **Video Example 10.3**

Watch Zenaida and consider what she knows about place value based on her responses to the interviewer's questions.

MyLab Education **Self-Check 10.5**

Place Value Patterns and Relationships—A Foundation for Computation

In this section, the focus will be the relationships of numbers to important special numbers called *benchmark numbers* and ten-structured thinking—that is, flexibility in using the structure of tens in our number system. These ideas begin to provide a basis for computation as students simultaneously strengthen their understanding of number relationships and place value.

The Hundreds Chart

The Hundreds Chart (see Figure 10.12) deserves special attention in the development of place-value concepts. K–2 classrooms should have a hundreds chart displayed prominently and used often.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

FIGURE 10.12 A hundreds chart.

MyLab Education **Blackline Master: Hundreds Chart**

A useful version of a hundreds chart can be made of transparent pockets into which each of the 100 numeral cards can be inserted. You can cover numbers, give students numbers to place on the chart or highlight patterns such as skip counting by 2 (even numbers), 5, and 10. You can also have students skip count by threes and fours and color in each number they count. Discuss the pattern shown on the chart as well as the patterns in the numbers.

In kindergarten and first grade, students can count and recognize two-digit numbers on the hundreds chart including the decade numbers from 10 to 100. Additionally, first graders can use the hundreds chart to develop a base-ten understanding of adding two-digit numbers with multiples of 10, noticing that jumps up or down are jumps of ten, while already recognizing that jumps to the right or left are jumps of one.

There are lots of patterns on the hundreds chart, and during discussions, different students will describe the same pattern in several ways. Accept all ideas. Here are some of the important place-value-related patterns students may point out:

- The numbers in a column all end with the same number, which is the same as the number at the top of the chart.
- In a row, the first number (tens digit) stays the same and the "second" number (ones digit) counts 1, 2, 3, . . . 9, 0) changes as you move across.
- In a column, the first number (tens digit) "counts" or goes up by ones as you move down.
- You can count by tens going down the far right-hand column.
- Starting at 11 and moving down on the diagonal, you can find numbers with the same digit in the tens and ones (e.g., 11, 22, 33, 44, and so on).

For students, these patterns are not obvious or trivial. For example, one student may notice the pattern in the column under the 4—every number ends in a 4. Two minutes later, another student will "discover" the parallel pattern in the column headed by 7. That this pattern is in every column may not be completely obvious.

Once you've discussed some of the patterns, try the next activity.

Activity 10.13 CCSS-M: K.CC.A.1; K.NBT.A.1; 1.NBT.A.1; 1.NBT.B.2; 1.NBT.C.5

Missing Numbers

STUDENTS with SPECIAL NEEDS

Provide students with a hundreds chart on which some of the number cards have been removed. The students' task is to replace the missing numbers. To begin, have only a random selection of individual numbers removed. Later, remove sequences of several numbers from three or four different rows. Finally, remove all but one or two rows or columns. Eventually, challenge students to replace all of the numbers on the Blank Hundreds Chart. For students with disabilities, model the placement of a missing number using a "think-aloud" to describe how you make your decision and what key features of the number you think about as you place the number properly on the chart.

MyLab Education **Blackline Master: Blank Hundreds Chart**

***FORMATIVE ASSESSMENT* Notes.** Replacing the number cards on a blank chart is a good station activity for two students to try. By listening to how students determine the correct places for numbers, you can assess how well they have constructed an understanding of the 1-to-100 sequence and whether they recognize and purposefully use patterns in our number system to recreate the hundreds chart. ■

Activity 10.14 CCSS-M: K.CC.A.1; K.NBT.A.1; 1.NBT.A.1; 1.NBT.B.2; 1.NBT.C.5

Finding Neighbors on the Hundreds Chart

Begin with a blank or nearly Blank Hundreds Chart (projecting it on a screen or giving copies to individual students). Then, circle a particular missing number. Students should fill in the designated number and its "neighbors," the numbers to the left, right, above, and below. After students become comfortable naming the neighbors of a number, ask what they notice about the neighboring numbers. The numbers to the left and right are one less and one more than the given number. Those above and below are ten less and ten more, respectively. How do numbers differ on the diagonal? By discussing these relationships on the chart, students begin to see how the sequence of numbers is related to numerical relationships.

MyLab Education **Blackline Master: Blank Hundreds Chart**

In the following activity, number relationships are made more explicit by modeling the numbers with base-ten materials and associating those models with the hundreds chart.

Activity 10.15 CCSS-M: K.CC.A.1; K.NBT.A.1; 1.NBT.A.1; 1.NBT.B.2; 1.NBT.C.5

Models with the Hundreds Chart

Use either Base-Ten Materials or the Little Ten-Frame Cards to model two-digit numbers with which the students are familiar.

- Give students one or more numbers to first make with the models and then find on the hundreds chart. Use groups of two or three numbers in either the same row or the same column. Ask students how are the numbers alike and how are they different.
- Indicate a number on the chart. What would you have to change to make each of its neighbors (the numbers to the left, to the right, above, and below)?

As a first step in moving to larger numbers, continue your hundreds charts to 200 and extend the same questions to the larger numbers. Then extend the hundreds chart to a thousands chart.

MyLab Education **Blackline Master: Base-Ten Materials**
MyLab Education **Blackline Master: Little Ten-Frame Cards**

***TECHNOLOGY* Note.** Several web-based resources include hundreds charts that allow students to explore patterns. Learning about Number Relationships is an example from NCTM's Illuminations (https://illuminations.nctm.org) that has a calculator and hundreds chart and allows for a variety of explorations. (There are extensions to thousands charts, too.) Students can skip-count by any number and also begin their counts at any number. ABCya's (http://www.abcya.com) Interactive Number Chart (0–99 or 1–100) allows students to find and document patterns by coloring the squares that contain the numbers. ■

Activity 10.16

CCSS-M: 2.NBT.A.1; 2.NBT.A.2; 2.NBT.A.3; 2.NBT.B.8

The Thousands Chart

Provide students with several Blank Hundreds Charts. Assign groups of three or four students the task of creating a 1-to-1000 chart. The chart is made by taping 10 hundreds charts together in a long vertical strip. Students should decide how they will divide up the task, with different students taking different parts of the chart. The thousands chart should be discussed as a class to examine how numbers change as you count from one hundred to the next, what the patterns are, and so on. All of the earlier hundreds chart activities can be extended to thousands charts.

MyLab Education **Blackline Master: Blank Hundreds Chart**

Relative Magnitude Using Benchmark Numbers

Number sense also includes having a grasp on the size of numbers. Relative magnitude refers to the size relationship one number has with another—is it much larger, much smaller, close or about the same? How students think about these comparisons is supported by the use of models and by the development of benchmark numbers that can be used as signposts for a number's location (as on a number line).

A valuable feature of both the hundreds chart and the little ten-frame cards is how clearly they illustrate the distance to the next multiple of 10—the number of spaces to the end of the row on the chart or the blank spaces on the ten-frame card. Multiples of 10, 100, and occasionally other numbers, such as multiples of 25, are referred to as *benchmark* numbers. Students learn to use this term when they work with informal methods of computation. When finding the difference between 74 and 112, a student might say, "First, I added 6 onto 74 and that equals 80, which is a benchmark number. Then I added 2 tens onto 80 to get to 100 because that's another benchmark number." Whatever terminology is used, understanding how numbers are related to these special numbers is an important step in students' development of number sense and place value understanding.

In addition to the hundreds chart, the number line is an excellent way to explore how one number is related to another and is a predictor of future mathematics performance (Dietrick, Huber, Dackermann, Moeller & Fischer, 2016; van den Bos, et al. 2015). The next two activities are suggestions for using the Number Lines Activity Page as a student recording sheet.

MyLab Education **Activity Page: Number Lines**

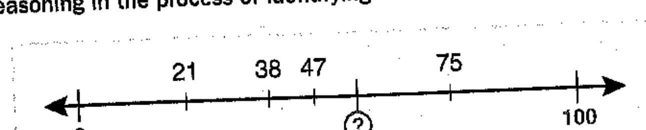
Activity 10.17

CCSS-M: 1.NBT.A.1; 1.NBT.B.2; 1.NBT.B.3; 2.NBT.A.1; 2.NBT.A.2; 2.NBT.A.4

STUDENTS with SPECIAL NEEDS

Who Am I?

Draw a long line (or use cash register tape) and label 0 and 100 at opposite ends. Mark a point with a "?" (on a sticky note) that corresponds to your secret number. Have students try to guess your secret number. For each guess, place and label a mark at that number on the line until your secret number is discovered. Have students explain how they are making their estimations including highlighting any use of benchmark numbers. As a variation, use the Who Am I? Activity Page where the end points are different such as 0 and 1000, 200 and 300, or 500 and 800. For students with disabilities, mark the guesses that have occurred and where they are located. Labeling those numbers at their actual locations will support students' reasoning in the process of identifying the secret number.



MyLab Education **Activity Page: Number Lines**
MyLab Education **Activity Page: Who Am I?**

Activity 10.18

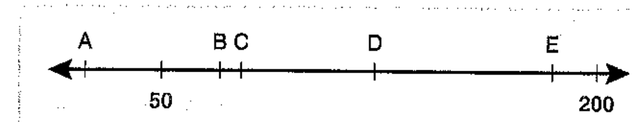
CCSS-M: 1.NBT.A.1; 1.NBT.B.2; 1.NBT.B.3; 2.NBT.A.1; 2.NBT.A.2; 2.NBT.A.4

STUDENTS with SPECIAL NEEDS

ENGLISH LEARNERS

Who Could They Be?

Label two points on a Number Line (not necessarily the ends) with benchmark numbers. Show students different points labeled with letters and ask what numbers they might be and why they think that. In the example shown here, B and C are less than 100 but probably more than 60. E could be about 180. You can also ask where 75 might be or where 400 is located. About how far apart are A and D? Why do you think D is more than 100? For ELs, and children with disabilities say as well as write the numbers on a note card, or ask students to both write and say the numbers. Use the Who Could They Be? Activity Page for more examples.



MyLab Education **Activity Page: Number Lines**
MyLab Education **Activity Page: Who Could They Be?**

The next activity has students apply some of the same ideas about benchmark numbers that we have been exploring.

Activity 10.19

CCSS-M: 1.NBT.A.1; 1.NBT.B.2; 1.NBT.B.3; 2.NBT.A.1; 2.NBT.A.2; 2.NBT.A.4

ENGLISH LEARNERS

STUDENTS with SPECIAL NEEDS

Close, Far, and In Between

Put any three numbers on the board that are appropriate for your students. With these three numbers as referents, ask questions such as the following, encouraging discussion of all responses:

219 364 457

- Which two numbers are closest? How do you know?
- Which is closest to 300? To 250?
- Name a number between 457 and 364.
- Name a multiple of 10 between 219 and 364.
- Name an even number that is greater than all of these numbers.
- About how far apart are 219 and 500? 219 and 5000?
- If these are "big numbers," what are some small numbers? Numbers that are about the same? Numbers that make these seem small?

For ELs, this activity can be modified by using prompts that are similar (rather than changing the prompts each time, which increases the linguistic demand). Also, ELs and students with disabilities will benefit from using a visual, such as a number line, and from writing the numbers rather than just hearing/saying them.

Look at the corresponding Expanded Lesson: Close, Far, and In Between, where students estimate the relative size of a number between 0 and 100, and strengthen their conceptual understanding of number size and place value.

MyLab Education **Expanded Lesson: Close, Far, and In Between**

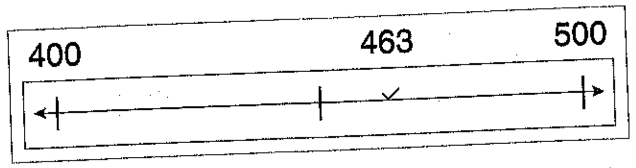


FIGURE 10.13 An empty number line can be labeled in different ways to help students round numbers.

Approximate Numbers and Rounding

The most familiar form of computational estimation is rounding, which is a way of changing the numbers in a problem to others that are easier to compute mentally. The *Common Core State Standards* say that students in third grade are expected to use place-value understanding to round numbers to the nearest 10 or 100 and students in fourth grade should be able to round any multidigit whole number to any place value. At third and fourth grade students use rounding to assess the reasonableness of answers. Fifth graders will be using place value understanding to round decimals to any place.

To be useful in estimation, rounding should be flexible and conceptually well-understood. To round a number simply means to select a compatible number. (Note that the term *compatible* is not a mathematical term. It refers to numbers that would make the problem easier to compute mentally.) The compatible number can be any close number and need not be a multiple of 10 or 100, but in many cases students are asked to round to one of these places.

A number line with benchmark numbers highlighted can be useful in helping students select compatible numbers. An empty number line like the one shown in Figure 10.13 can be made using strips of poster board taped end to end or cash register tape. Labels are written above the line. The ends can be labeled 0 and 100, 100 and 200, . . . 900 and 1000. Indicate the location of a number above the line that you want to round. Discuss the locations of numbers that are close. Teach students the convention that if a number that is being rounded has a 5 in the place being considered, although it is halfway between two numbers, they round up. The number line is a powerful tool for these discussions.

Connections to Real-World Ideas

As students study place-value concepts, encourage them to notice numbers in the world around them. Students K–1 should be thinking of numbers to 100 and 120 respectively while students in the second grade should be thinking about numbers up to 1000 (NGA Center & CCSSO, 2010). Where are these numbers found in the school? You might use the number of children in the third grade, the number of minutes devoted to mathematics each week, or the number of days since school has started. There are measurements, numbers discovered on a field trip, numbers in books in other subject areas, and so on. What do you do with these numbers? Turn them into interesting graphs, write stories using them, and make up problems. For example, how many cartons of chocolate and plain milk are served in the cafeteria each month? Can students estimate how many cartons will be sold in a year? Collecting data and then grouping into tens and hundreds (or thousands) will help cement the value of grouping in situations when you need to count and compare quantities.

The particular way you bring number and the real world together in your class is up to you. But do not underestimate the value of connecting the real world to the classroom.

MyLab Education **Self-Check 10.6**

Numbers Beyond 1000

For students to have good concepts of numbers beyond 1000, the place-value ideas that have been carefully developed must be extended. This development is sometimes difficult to do because physical models for thousands are not readily available, or you may just have one large cube to show. At the same time, number-sense ideas must also be developed. In many ways, connecting very large numbers to real amounts is just as important as connecting smaller numbers to real quantities.

Extending the Place-Value System

Two important ideas developed for three-digit numbers should be extended to larger numbers as students move to thinking about 1,000,000 in fourth grade (NGA Center & CCSSO, 2010).

First, the multiplicative structure of the number system should be generalized. That is, ten in any position makes a single thing (group) in the next position to the left, and vice versa. Second, the oral and written patterns for numbers in three digits are duplicated in a clever way for every three digits to the left. These two related ideas are not as easy for students to understand as adults seem to believe. Because models for large numbers are so difficult to demonstrate or visualize, textbooks must deal with these ideas in a predominantly symbolic manner. That is not sufficient!

Activity 10.20 **CCSS-M: 2.NBT.A.1; 2.NBT.A.3**

What Comes Next?

Use Base-Ten Materials where the unit or ones piece is a 1-cm square the tens piece is a 10-cm × 1-cm strip and the hundreds piece is a square, 10 cm × 10 cm. What is next? Ten hundreds is called a thousand. What shape would a thousand be? Tape together a long strip made of 10 paper hundreds squares. What comes next? (Reinforce the idea of "10 makes 1" that has progressed to this point.) Ten one-thousand strips would make a square measuring 1 meter (m) on each side, making a paper 10,000 model. Once the class has figured out the shape of each piece, the problem posed to them is, "What comes next?" Let small groups work on the dimensions of a 100,000 piece (they will likely need space in the hallway to do this!).

MyLab Education **Blackline Master: Base-Ten Materials**

How far you want to extend this square-strip–square-strip sequence depends on your class. The idea that 10 in one place makes 1 in the next can be brought home dramatically and memorably. It is quite possible with older students to make the next 10-m × 10-m square using chalk lines on the playground. The next strip is 100 m × 10 m. This model can be measured out on a large playground with students marking the corners. By this point, the payoff includes an appreciation of both the increase in size of each successive amount and the 10-makes-1 progression (powers of ten). The 10-m × 10-m square models 1 million, and the 100 m × 10 m strip is the model for 10 million. The difference between 1 million and 10 million is dramatic. Even the concept of 1 million tiny centimeter squares is impressive.

Try the "What Comes Next?" discussion in the context of three-dimensional models. The first three shapes are distinct: a *cube*, a *long*, and a *flat*. What comes next? Stack 10 flats and they make a cube—the same shape as the first one, only 1000 times larger. What comes next? (See Figure 10.14.) Ten cubes make another long. What comes next? Ten big longs make a big flat. The first three shapes have now repeated! Ten big flats will make an even bigger cube, and the trio of shapes begins again. The pattern of "10 of these makes 1 of those" is "infinitely extendable" (Thomas, 2004, p. 305). Note that students with disabilities may have difficulty interpreting spatial information, which plays into their challenges with interpreting the progression of place-value materials (Geary & Hoard, 2005). Although we are using the terms *cube*, *long*, and *flat* to describe the shape of the materials, students will see the shape pattern made as each piece gets 10 times larger. In fact, it is still critical to call these representations "ones, tens, and hundreds," particularly for students with disabilities. We need to consistently name them by the number they represent rather than their shape. This language reinforces conceptual understanding and is less confusing for students who may struggle with these concepts.

Each cube has a name. The first one is the *unit* cube, the next is a *thousand*, the next a *million*, then a *billion*, and so on. Each long is 10 cubes: 10 units, 10 thousands, and 10 millions. Similarly, each flat shape is 100 cubes.

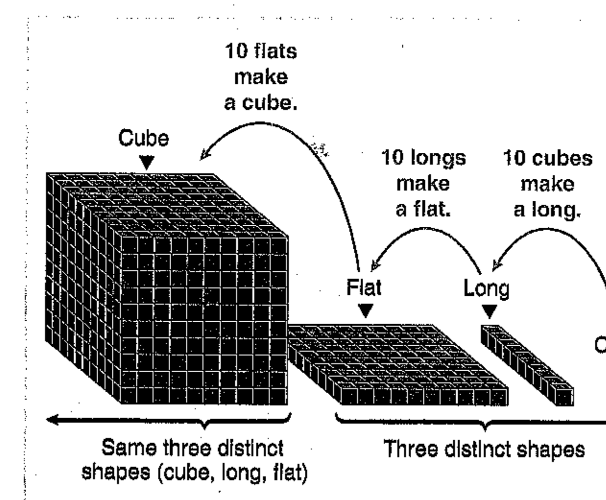


FIGURE 10.14 With every three places, the shapes repeat. Each cube represents a 1, each long represents a 10, and each flat represents a 100.

To read a number, first mark it off in triples from the right. The triples are then read, stopping at the end of each

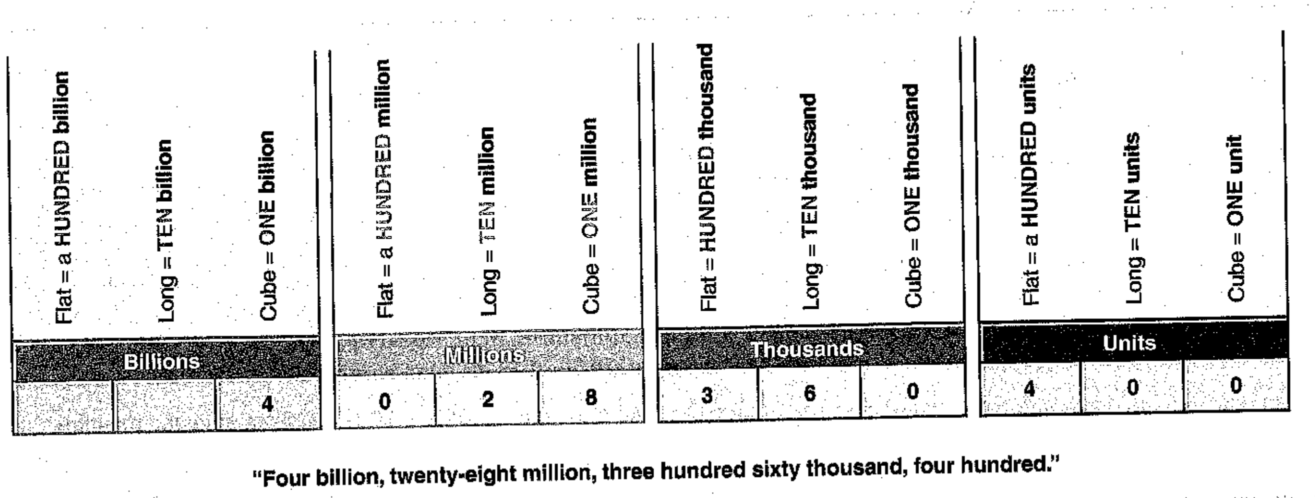


FIGURE 10.15 The triples system for naming large numbers.

to name the unit for that triple (see Figure 10.15). Leading zeros in each triple are ignored. If students can learn to read numbers like 059 (fifty-nine) or 009 (nine), they should be able to read any number. To write a number, use the same scheme. If first mastered orally, the system is quite easy. Remind students *not* to use the word "and" when reading a whole number. For example, 106 should be read as "one hundred six," not "one hundred *and* six." The word "and" will be needed to signify a decimal point. Please make sure you read numbers accurately.

It is important for students to realize that the system does have a logical structure, is not totally arbitrary, and can be understood.

Conceptualizing Large Numbers

The ideas just discussed are only partially helpful in thinking about the actual quantities involved in very large numbers. For example, in extending the paper square-strip-square-strip sequence, some appreciation for the quantities of 1000 or of 100,000 is acquired. But it is hard for anyone to translate quantities of small squares into quantities of other items, distances, or time.

MyLab Education Video Example 10.4

Explore this video of Donny reasoning about 192,000 as 19,200 tens or 1920 hundreds.

These ideas are important to discuss as a class as students need to explore these relationships.

Pause & Reflect

How do you think about 1000 or 100,000? Do you have any real concept of a million? ●

In the following activities, numbers like 1000, 10,000 (see 10,000 Grid Paper), or even 1,000,000 are translated literally or imaginatively into something that is more meaningful or fun to think about. Interesting quantities become lasting reference points or benchmarks for large numbers and thereby add meaning to numbers encountered in real life.

MyLab Education Blackline Master: 10,000 Grid Paper

Activity 10.21 CCSS-M: 2.NBT.A.2; 2.NBT.A.3

Collecting 10,000

As a class or grade-level project, collect some type of object with the objective of reaching some specific quantity—for example, 1,000 or 10,000 bread tabs or soda can pop tops. If you begin aiming for 100,000 or 1,000,000, be sure to think it through. One teacher spent nearly 10 years with her classes before amassing a million bottle caps. It takes a small dump truck to hold that many!

Activity 10.22 CCSS-M: 2.NBT.A.2; 2.NBT.A.3

Showing 10,000

Sometimes it is easier to create large amounts than to collect them. For example, start a project in which students draw 100 or 200 or even 500 dots on a sheet of paper. Each week, different students contribute a specified number. Another idea is to cut up newspapers into pieces the same size as dollar bills to see what a large quantity would look like. Paper chain links can be constructed over time and hung down the hallways with special numbers marked. Let the school be aware of the ultimate goal.

Activity 10.23 CCSS-M: 2.NBT.A.1a; 2.NBT.A.3

How Long?/How Far?

In this activity, talk about real or imagined distances with students by posing investigations for them to consider such as, How long is a million baby steps? Other ideas that address length include a line of toothpicks, dollar bills, or energy bars end to end; students holding hands in a line; blocks or bricks stacked up; or students lying down head to toe. Standard measures—feet, centimeters, meters—can also be used with students noting that larger numbers emerge when the smallest units are used.

Activity 10.24 CCSS-M: 3.MD.A.1

A Long Time

How long is 1000 seconds? How long is a million seconds? A billion? How long would it take to count to 10,000 or 1,000,000? (To make the counts all the same, use your calculator to do the counting. Just press the [=].) How long would it take to do some task like buttoning a button 1000 times?

Activity 10.25 CCSS-M: 2.MD.A.3; 3.MD.A.1; 3.MD.C.5

Really Large Quantities

Ask how many:

- Energy bars would cover the floor of your classroom
- Steps an ant would take to walk around the school building
- Grains of rice would fill a cup or a gallon jug
- Quarters could be stacked in one stack from floor to ceiling
- Pennies can be laid side by side down the entire hallway
- Pieces of notebook paper would cover the gym floor
- Seconds you have lived

Big-number projects need not take up large amounts of class time. They can be explored over several weeks as take-home projects, done as group projects, or, perhaps best of all, translated into great schoolwide estimation events.

MyLab Education Self-Check 10.7

MyLab Education Blackline Master: Base-Ten Materials

Now that you've explored many of the main ideas in this chapter, look at Table 10.1 for some of the possible common challenges and misconceptions your students may face. Suggestions on what you might notice and how to help are included.

TABLE 10.1 COMMON CHALLENGES AND MISCONCEPTIONS IN PLACE VALUE AND HOW TO HELP

Common Challenge or Misconception	What It Looks Like	How to Help
1. Students lose track of the fact that each digit in a multidigit numeral carries a value dependent on its position in the number.	When students are asked to compare the numbers (2) bolded in the two amounts that follow they will say they are the same. **2**357 and 49,99**2**.	• Use Base-Ten Materials and have student show with materials the value of these two numbers. The reading of numbers in addition or subtraction problems as digits (saying 5 instead of 5 tens or fifty) confuses students. • Use the place value cards discussed previously to reinforce how numbers are built. • Students hear numbers like 2357 read as two, three, five, seven—when they should always be read two thousand, three hundred, fifty-seven. • Use the digit correspondence task described in this chapter to identify which of the five levels of understanding matches your student's performance. • Reinforce that the value of an individual digit in a multidigit number is the product of that digit multiplied by the value assigned to its position in the number
2. Student reverse the digits when writing two-digit numbers.	Writes "53" when should write "35."	• Have students use virtual base-ten materials that display the corresponding number to check their answer. • Have the child model both 53 and 35 with base-ten materials and describe how the numbers are similar and different.
3. Student represents a number with base-ten materials using the face value of the digits.	When asked to represent 13 with base-ten materials, the student uses one piece for the "1" and three pieces for the "3" as shown 1 □ 3 □□□	• Have the student use base-ten materials to count out 13 single units. Then ask them to compare that amount to what they previously showed. • Have the student build the number with the place value cards and then use base-ten materials to represent the corresponding amounts. Again, compare to the amount originally shown.
4. Students put the word "and" in a number when they read it aloud.	When reading 1016 students will say "one thousand and sixteen."	• Students must practice reading numbers without using the word "and." The only time the word "and" is used is to represent a decimal point.
5. Students use a form of "expanded number writing" (Byrge, Smith, & Mix, 2013).	Students write "three hundred eighty-five" as something like 300805, 310085 or 3085.	• Provide examples of the actual materials on a place value mat and use the place value cards to show how the matching number is built.
6. When shown a collection of base-ten materials where there is an internal zero the students ignore the zero or misunderstand the zero.	Given 5 hundreds and 8 units in base-ten materials, students will write that number as 85. The student believes that 802 and 8002 represent the same amount.	• Focus on the meaning of a zero in any number by starting with a number like 408 and asking how that would be shown with materials. Explicitly discuss the role of an internal 0 in the number. • Never refer to 0 as a "placeholder." This terminology gives the impression that it is not a numerical value and it is there just as a way to fill a space. • Never read or refer to 0 as oh or zip. Say "zero" as it is a number.
7. If students are given the place values of numbers out of order they write the number as given left to right regardless of the place value.	When students are asked to write the number that represents: 7 ones, 4 tens, 1 thousand, and 3 hundreds. They write 7413.	• Go back to the base-ten materials and use the place value mat to take out the same amount of base-ten materials as in the number. Then have the student write the number of base-ten materials. The student should then compare the two answers to consider which one is accurate
8. Students misinterpret the value of the base-ten materials.	Students think the value of the large 1000 place value block is actually 600 by just calculating the number of squares on each face of the cube.	• Particularly with the 1000 place value block, if students don't see the building of the block (grouping into a unit), they may confuse the value. So, explicitly show the building of the cube by taking ten hundreds blocks and forming a cube with them (holding them together with elastic bands.)

RESOURCES FOR CHAPTER 10

LITERATURE CONNECTIONS

Books that emphasize groups of things, even simple counting books, are a good beginning to the notion of ten things in a single group. Many books have wonderful explorations of large quantities and how the numbers can be composed and decomposed.

100th Day Worries

Cuyler (2005)

The 100th Day of School from the Black Lagoon

Thaler, 2014

Both of these books focus on the 100th day of school, which is one way to recognize the benchmark number of 100. Through a variety of ways to think about 100 (such as collections of 100 items), students will be able to use these stories to think about the relative size of 100 or ways to make 100 using a variety of combinations.

How Much Is a Million?

Schwartz (2004)

If You Made a Million

Schwartz (1994)

On Beyond a Million: An Amazing Math Journey

Schwartz (2001)

The Magic of a Million Activity Book—Grades 2–5

Schwartz & Whitin (1999)

Schwartz authored a collection of entertaining and conceptually sound books about the powers of ten or what makes a million—from visual images of students standing on one another's shoulders in a formation that reaches the moon to various monetary collections. The activity book by Schwartz and Whitin provides activities to help students interpret large numbers.

If I Had a Million Bucks

Johnson (2012)

This story is about Ada, a girl who likes to plan. Ada is thinking about what she could do if she had one million dollars. This planning is a fun way to have students think about what could be purchased with large amounts of money.

RECOMMENDED READINGS

Articles

Burris, J. T. (2013). Virtual place value. *Teaching Children Mathematics, 20*(4), 228–236.

This interesting study explores teaching place value using several activities with technology. Students worked with both concrete base-ten materials and virtual versions to help reinforce conceptual structures, particularly for showing equivalent representations.

Kari, A. R., & Anderson, C. B. (2003). Opportunities to develop place value through student dialogue. *Teaching Children Mathematics, 10*(2), 78–82.

Two teachers describe a first-/second-grade classroom, where students' understanding of two-digit numbers is shown to be quite mistaken, but can be developed conceptually with the aid of discussion. At first, the student in the article is convinced that 11 + 11 + 11 is 60. This article emphasizes the wide range of student ideas and the value of classroom discourse.

Book

Richardson, K. (2003). *Assessing math concepts: Grouping tens.* Bellingham, WA: Mathematical Perspectives.

This book is one of a nine-part series on using diagnostic interviews and other assessment tools (including blackline masters) to understand students' grasp of a concept—in this case, grouping by tens. Tips are shared about conducting observations, with suggestions for instruction.