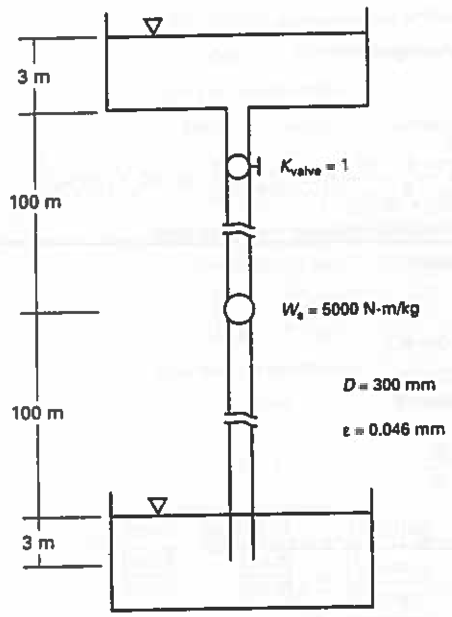


FIGURE 1–16 Schematic for Example 1-4.

Parallel Systems

Parallel piping systems, such as that indicated in Fig. 1–9(b), can be analyzed by applying two principles: (1) the pressure drops across lines in loops with common inlet and exit manifolds must be equal, and (2) the total flow rate is the sum of the individual flow rates in each line of the commonly manifolded lines. For example, in Fig. 1–9(b) the pressures at points A and B are common to lines 1 through 4, and the pressure drops across each individual line must be equal. The total flow rate in the system is the sum of the individual flow rates in lines 1 through 4. Parallel systems are analyzed by considering equal pressure drop with additive flow rates, whereas series systems are analyzed by considering cumulative pressure drops with a constant flow rate. The sample parallel system shown is governed by

$$\Delta P_1 = \Delta P_2 = \Delta P_3 = \Delta P_4 \tag{1-31}$$

$$Q_T = Q_1 + Q_2 + Q_3 + Q_4 \tag{1-32}$$

Parallel systems are often used to reduce the pumping power required for process-control systems. Consider a system composed of four devices, each with a pressure drop of ΔP. Two alternative systems, each with a total flow rate of Q, are considered: configuration 1 [shown in Fig. 1–18(a)] is parallel and configuration 2 [shown in Fig. 1–18(b)] is series. The parallel configuration has a pressure drop ΔP and a flow rate of $Q/4$ through each device. The power required for each device is then $Q\,\Delta P/4$, and the total for the parallel arrangement is $4(Q\,\Delta P/4) = Q\,\Delta P$. For the series system the power required for each device is $Q\,\Delta P$, and for the four devices in series the total power for the arrangement is $4(Q\,\Delta P) = 4Q\,\Delta P$. In the series system the total flow rate must be pumped

TITLE: PIPES implementation of Example 1-4.

ORIGIN ≡ 1 Set origin for counters to 1 from the default value of 0.

Input the pipe geometry:

Diameter in mm: $D := \begin{pmatrix}300\\300\end{pmatrix}\cdot mm$ Length in m: $L := \begin{pmatrix}100\\103\end{pmatrix}\cdot m$ Roughness in mm: $\varepsilon := \begin{pmatrix}0.046\\0.046\end{pmatrix}\cdot mm$

Input the system boundary (initial and end) conditions:

Pressures in Pa: $\begin{pmatrix}P_a\\P_b\end{pmatrix} := \begin{pmatrix}0\\0\end{pmatrix}\cdot Pa$ Elevations in m: $\begin{pmatrix}Z_a\\Z_b\end{pmatrix} := \begin{pmatrix}0\\-203\end{pmatrix}\cdot m$

Input the loss coefficients:

K factor: $K := \begin{pmatrix}1.78\\1\end{pmatrix}$ Equivalent length: $C := \begin{pmatrix}0\\0\end{pmatrix}$ Number of pipes: $N := \mathrm{length}(D)$

Input the fluid properties:

Density in kg/m³: $\rho := 1000\cdot\frac{kg}{m^3}$ Kinematic Viscosity in m²/s: $\nu := 1.14\cdot 10^{-6}\,\frac{m^2}{sec}$

Input the flow rate in cms: $Q := 1\cdot\frac{m^3}{sec}$ Initial guess on flow rate. Increase in head of the pump: $W_s := 5000\cdot newton\cdot\frac{m}{kg}$

Define constants and adjust units for consistency:

$g := 9.806\cdot\frac{m}{sec^2}$ $g_c := 1\cdot\frac{m\cdot kg}{newton\cdot sec^2}$

Define the functions for Reynolds number, fully-rough friction factor, and friction factor:

$Re(q,d) := \frac{4\cdot q}{\pi\cdot d\cdot\nu}$ $f_T(d,\varepsilon) := \frac{0.3086}{\left[\log\left[\left(\frac{\varepsilon}{3.7\cdot d}\right)^{1.11}\right]\right]^2}$

$$f(q,d,\varepsilon) := \begin{cases}\dfrac{0.3086}{\left[\log\left[\dfrac{6.9}{Re(q,d)}+\left(\dfrac{\varepsilon}{3.7\cdot d}\right)^{1.11}\right]\right]^2} & \text{if } Re(q,d)>2300\\ \dfrac{64}{Re(q,d)} & \text{otherwise}\end{cases}$$

FIGURE 1–17 MathCad solution of Example 1-4.

through each device, while for the parallel system only $Q/4$ goes through each device. This simple example thus demonstrates one very important reason for designing parallel rather than series systems.

The previous paragraph examined the behavior of series and parallel systems with devices with specified pressure drops. A more realistic situation is for the pressure drop across a device to be a function of the flow rate through the device; for instance, in turbulent flow the pressure drop is usually quadratic with the flow rate. Thus, $\Delta P = \rho K Q^2$, The generalized energy equation is:

Given

$$W_s \cdot \frac{g_c}{g} = \frac{P_b - P_a}{\rho \cdot g} \cdot g_c + Z_b - Z_a + \sum_{i=1}^{N} \frac{8}{\pi^2} \cdot \frac{Q^2}{(D_i)^4 \cdot g} \cdot \left(f(Q, D_i, \epsilon_i) \cdot \frac{L_i}{D_i} + K_i + C_i \cdot f_T(D_i, \epsilon_i) \right)$$

$q := \text{Find}(Q)$

$q = 2.445 \cdot m^3 \cdot sec^{-1}$ $\quad q = 1.467 \cdot 10^5 \cdot \frac{liter}{min}$

Pump power (input to fluid): $\text{Power} := q \cdot \rho \cdot W_s$

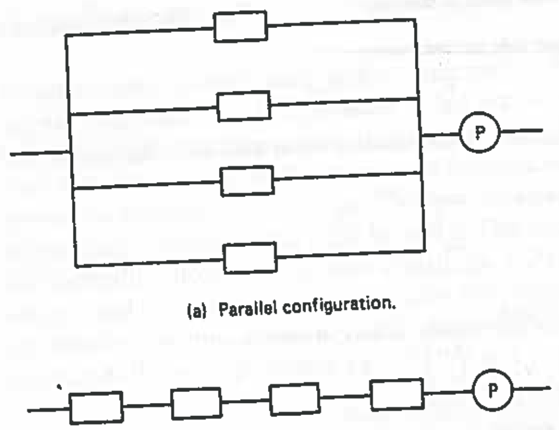
$\text{Power} = 1.222 \cdot 10^4 \cdot kW$

Additional output of useful quantities:

$i := 1..N$ $\quad V(q, D) := \frac{4 \cdot q}{\pi \cdot D^2}$

D_i	$V(q, D_i)$	$Re(q, D_i)$	$f(q, D_i, \epsilon_i)$	$f_T(D_i, \epsilon_i)$
0.3·m	34.584·m·sec⁻¹	$9.101 \cdot 10^6$	0.013	0.013
0.3·m	34.584·m·sec⁻¹	$9.101 \cdot 10^6$	0.013	0.013

FIGURE 1–17 *(continued)*



(a) Parallel configuration.

(b) Series configuration.

FIGURE 1–18 Schematic representation of process system.

where Q is the flow rate through the device and K is a constant for a given device. Now consider the arrangements of Fig. 1-18 if $\Delta P = \rho K Q^2$. For the parallel arrangement the flow through each device is $Q/4$ with the resulting pressure drop of $\Delta P = \rho K(Q/4)^2 = \rho K Q^2/16$. The power required is still $Q\,\Delta P$ across the parallel network, so that power required is $\rho K Q^3/16$. For the series arrangement the pressure drop across each device is $\Delta P = \rho K Q^2$, since the flow rate through each device is Q, and the total pressure drop across the series arrangement is $\Delta P = (4\rho K Q^2)$. The corresponding power required is $4\rho K Q^3$ for the series system.

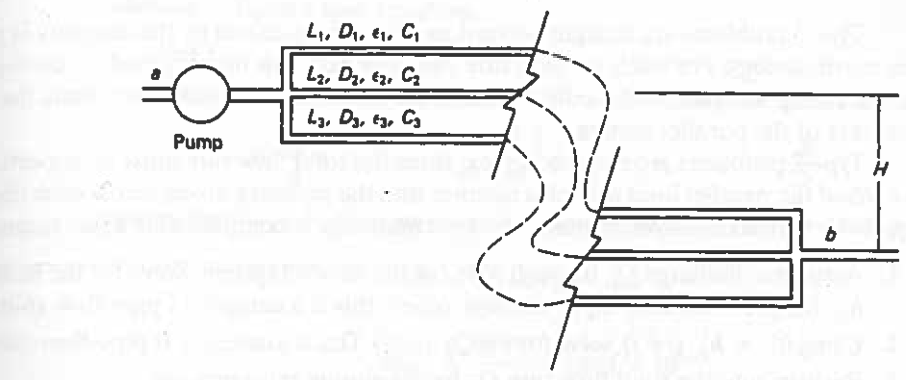


FIGURE 1–19 PIPEP program schematic.

Consider a generalized parallel system such as that schematically illustrated in Fig. 1–19. The increase in head across the pump, W_s, is such that the pressures at a and b are equal. Recall from the previous paragraph that the changes in pressures (or heads) are equal for each leg of a parallel arrangement; hence, W_s is also the change in head across each leg required for $P_a = P_b$. If Q_T is the total flow rate, then conservation of mass yields

$$Q_T = Q_1 + Q_2 + Q_3 \tag{1-30}$$

The energy equation for line i, under the assumptions of $P_a = P_b$ and $V_a = V_b$, can be expressed as

$$W_s \frac{g_c}{g} = z_b - z_a + \frac{8}{\pi^2} \frac{Q_i^2}{g D_i^4} \left[f_i \frac{L_i}{D_i} + C_i f_{T_i} + K_i \right] \tag{1-31}$$

Analysis or design calculations for a parallel piping require the use of conservation of mass, Eq. (1-30), and an energy equation, Eq. (1-31), for each line. For example, for a system composed of two parallel lines the required expressions are

$$Q_T = Q_1 + Q_2 \tag{1-32a}$$

$$W_s \frac{g_c}{g} = z_b - z_a + \frac{8}{\pi^2} \frac{Q_1^2}{g D_1^4} \left[f_1 \frac{L_1}{D_1} + C_1 f_{T_1} + K_1 \right] \tag{1-32b}$$

$$W_s \frac{g_c}{g} = z_b - z_a + \frac{8}{\pi^2} \frac{Q_2^2}{g D_2^4} \left[f_2 \frac{L_2}{D_2} + C_2 f_{T_2} + K_2 \right] \tag{1-32c}$$

together with the usual friction-factor and Reynolds-number definitions.

Two types of parallel-system analysis problems are evident in Eqs. (1-32): (1) given W_s, find Q_T, Q_1, and Q_2; and (2) given Q_T, find W_s, Q_1, and Q_2. We shall examine solution of parallel piping problems from two viewpoints: (1) "manual" calculations, and (2) MathCad software element PIPEP (for PIPEs in Parallel). Consider first the manual approaches to the two types of problems.

Type 1 problems are straightforward, as they can be solved by the category II problem methodology. For each parallel line the flow rate can be obtained, since the line pressure drop is known, and the flow rates of the individual lines added to obtain the total flow rate of the parallel system.

Type 2 problems are more complex, since the total flow rate must be apportioned to each of the parallel lines in such a manner that the pressure drops across each line are equal. We can list a simple sequence to systematically accomplish this apportionment:

1. Assume a discharge Q_i' through pipe i of the parallel system. Solve for the head loss h_{f_i}' (or pressure drop $\Delta p_i'$) through pipe i; this is a category I pipe-flow solution.
2. Using $h_{f_j}' = h_{f_i}'$ $(i \neq j)$, solve for the Q_j' $(i \neq j)$. This is a category II pipe-flow solution.
3. Redistribute the total flow rate Q_T by the simple ratio process

$$Q_k = \frac{Q_k'}{\sum_l Q_l'} Q_T \qquad (k = 1, K, l = 1, K)$$

where K is the total number of pipes.

4. Check the h_{f_k} $(k = 1, K)$ for equality using the Q_k $(k = 1, K)$ obtained from step 3. Repeat steps 1 through 4 until convergence is obtained.

EXAMPLE 1-5

Consider the parallel flow network of Fig. 1–20 with the following specifications:

$L_1 = 3{,}000$ ft	$P_A = 80$ psia	$L_2 = 3{,}000$ ft
$D_1 = 1$ ft	$Z_A = 100$ ft	$D_2 = 8$ in.
$\varepsilon_1 = 0.001$ ft	$Z_B = 80$ ft	$\varepsilon_2 = 0.0001$ ft
$\frac{\varepsilon_1}{D_1} = 0.001$		$\frac{\varepsilon_2}{D_2} = 0.00015$

Find Q_1, Q_2, and P_B.

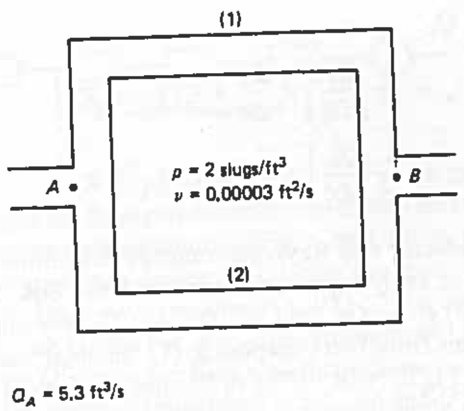


FIGURE 1–20 Parallel flow network for Example 1-5.

Solution. This is a type 2 problem.

Step 1:

Assume $Q_1' = 3$ ft³/s. Apply the energy equation along line 1 from A to B to obtain

$$W_s \frac{g_c}{g} = z_b - z_a + \frac{8}{\pi^2} \frac{Q_1'^2}{g D_1^4} \left[f_1 \frac{L_1}{D_1} \right] = z_b - z_a + h_{f_1} \frac{g_c}{g}$$

assuming that $V_A = V_B$. Finding h_{f_1}' with Q_1' specified is a category I problem, and we have

$$V_1' = \frac{Q_1'}{A_1} = 3.82 \quad ft/s$$

$$\mathrm{Re}_{D_1} = \frac{V_1' D_1}{\nu_1} = 1.273 \times 10^5$$

So $f_1' = 0.022$, and

$$h_{f_1}' = \frac{8}{\pi^2} \frac{Q_1'^2}{g D_1^4} \left[f_1' \frac{L_1}{D_1} \right]$$

$$= 14.97 \frac{\text{ft-lbf}}{\text{lbm}}$$

Step 2:

The loss for pipe 2 must then become

$$h_{f_2}' = h_{f_1}' = 14.97 \frac{\text{ft-lbf}}{\text{lbm}}$$

Finding Q_2' for $h_{f_2}' = 14.97$ ft-lbf/lbm is a category II problem. We shall use V_2' as the iteration variable. The results are given in Table 1–4.

TABLE 1–4 Category II Problem for PIPE 2

V_2' (ft/s)	Re_D	f_2 (–)	h_{f_2}' (ft-lbf/lbm)
1	22,233	0.0265	1.859
4	88,932	0.0192	21.47
3.27	72,702	0.0200	14.94

Step 3:

From step 2, $Q_2' = V_2' A_2 = 1.141$ ft³/s and $\sum_{l=1}^{2} Q_l' = 1.141 + 3 = 4.141$ ft³/s. We find the corrected values by using

$$Q_1 = \frac{3}{4.141} 5.3 = 3.84 \text{ ft}^3/\text{s}$$

$$Q_2 = \frac{1.141}{4.141} 5.3 = 1.46 \text{ ft}^3/\text{s}$$

Step 4:

Using $Q_1 = 3.84$ ft³/s and $Q_2 = 1.46$ ft³/s, we compute $V_1, V_2, \mathrm{Re}_{D_1}, \mathrm{Re}_{D_2}, f_1$, and f_2, to find

$$h_{f_1} = 23.85\ \frac{\text{ft-lbf}}{\text{lbm}}$$

$$h_{f_2} = 23.98\ \frac{\text{ft-lbf}}{\text{lbm}}$$

The head losses in the pipes then agree to 0.54 percent, which is sufficient.

With the flow rates in each line and the head loss for the parallel segments known, W_s is computed from the first equation in step 1.

$$W_s\frac{g_c}{g} = z_b - z_a + h_{f_1}\frac{g_c}{g} = 80\text{ ft-}100\text{ft} + 23.91\ \frac{\text{ft-lbf}}{\text{lbm}}\ \frac{32.174\ \dfrac{\text{ft-lbm}}{\text{lbf-s}^2}}{32.174\ \dfrac{\text{ft}}{\text{s}^2}}$$

$$= -20\text{ ft} + 23.91\text{ ft} = 3.91\text{ ft}$$

$$W_s = 3.91\ \frac{\text{ft-lbf}}{\text{lbm}}$$

Thus, a pump with an increase in head of 3.91 ft-lbf/lbm would be required for the system to pass 5.3 ft³/s while maintaining $P_A = P_B$.

Generalized parallel piping system program. The generalized parallel system MathCad software element PIPEP solves both type 1 and type 2 problems. The input format of PIPEP is quite similar to that of PIPES and, like PIPES, it makes use of MathCad's solve block capability. For a given parallel system, the PIPEP solve block contains one equation defining conservation of mass [Eq. (1-30)] and one energy equation [Eq. (1-31)] for each parallel line. Consider the following example.

EXAMPLE 1-6

Work Example 1-5 using PIPEP.

Solution. The parallel piping system is illustrated in Fig. 1-20. For a two-pipe parallel system, conservation of mass and the energy equations become as in Eq. (1-32).

$$Q_T = Q_1 + Q_2 \tag{1-32a}$$

$$W_s\frac{g_c}{g} = z_b - z_a + \frac{8}{\pi^2}\frac{Q_1^2}{gD_1^4}\left[f_1\frac{L_1}{D_1} + C_1 f_{T_1} + K_1\right] \tag{1-32b}$$

$$W_s\frac{g_c}{g} = z_b - z_a + \frac{8}{\pi^2}\frac{Q_2^2}{gD_2^4}\left[f_2\frac{L_2}{D_2} + C_2 f_{T_2} + K_2\right] \tag{1-32c}$$

The above expressions for the system under consideration reduce to the following:

$$5.3 = Q_1 + Q_2$$

$$W_s\frac{g_c}{g} = -20 + \frac{8}{\pi^2}\frac{Q_1^2}{gD_1^4}\left[f_1\frac{L_1}{D_1}\right]$$

TITLE: PIPEP implementation of Example 1-6.

ORIGIN ≡ 1 Set origin for counters to1 from default value of 0.

Input the pipe geometry and the elevation difference:

$D := \begin{pmatrix}12\\8\end{pmatrix}\cdot\text{in}$ $L := \begin{pmatrix}3000\\3000\end{pmatrix}\cdot\text{ft}$ $\varepsilon := \begin{pmatrix}0.001\\0.0001\end{pmatrix}\cdot\text{ft}$ $\begin{pmatrix}Z_a\\Z_b\end{pmatrix} := \begin{pmatrix}100\\80\end{pmatrix}\cdot\text{ft}$

Input the loss coefficients:

K factor	Equivalent length	Number of pipes
$K := \begin{pmatrix}0\\0\end{pmatrix}$	$C := \begin{pmatrix}0\\0\end{pmatrix}$	$N := \text{length}(D)$

Input the fluid properties:

Density in lbm/ft³	Viscosity in lbm/ft-s
$\rho := 64.35\cdot\dfrac{\text{lb}}{\text{ft}^3}$	$\mu := 0.00193\cdot\dfrac{\text{lb}}{\text{ft}\cdot\text{sec}}$

Define constants and adjust units for consistency:

$g := 32.174\cdot\dfrac{\text{ft}}{\text{sec}^2}$ $g_c := 32.174\cdot\dfrac{\text{ft}\cdot\text{lb}}{\text{lbf}\cdot\text{sec}^2}$

Define the functions for Reynolds number and the friction factors:

$\text{Re}(q,D) := \dfrac{4\cdot\rho\cdot q}{\pi\cdot D\cdot\mu}$ $f_T(D,\varepsilon) := \dfrac{0.3086}{\left[\log\left[\left(\dfrac{\varepsilon}{3.7\cdot D}\right)^{1.11}\right]\right]^2}$

$$f(q,D,\varepsilon) := \begin{cases}\dfrac{0.3086}{\left[\log\left[\dfrac{6.9}{\text{Re}(q,D)} + \left(\dfrac{\varepsilon}{3.7\cdot D}\right)^{1.11}\right]\right]^2} & \text{if } \text{Re}(q,D)>2300\\[2ex] \dfrac{64}{\text{Re}(q,D)} & \text{otherwise}\end{cases}$$

Setup solve block by defining specified inputs and guessed values:

$Q_T := 5.3\cdot\dfrac{\text{ft}^3}{\text{sec}}$ $Q1 := \dfrac{Q_T}{N}$ $Q2 := \dfrac{Q_T}{N}$ $W_s := 1\cdot\text{ft}\cdot\dfrac{\text{lbf}}{\text{lb}}$

Given

$Q_T = Q1 + Q2$

FIGURE 1–21 MathCad solution of Example 1-6.

$$W_s\frac{g_c}{g} = -20 + \frac{8}{\pi^2}\frac{Q_2^2}{gD_2^4}\left[f_2\frac{L_2}{D_2}\right]$$

The MathCad worksheet for the solution to this parallel flow system is presented in Fig. 1–21. The answers are very close to those of Example 1-5, the differences being due to reading the Moody diagram for the friction factor in Example 1-5 and using the Haaland equation in Example 1-6. Since W_s represents the required increase in head across a pump to maintain $P_a = P_b$, the change in head at b without a pump in the system would be −3.393 ft-lbf/lbm.

$$W_s \cdot \frac{g_c}{g} = Z_b - Z_a + \frac{8}{\pi^2} \cdot \frac{(Q1)^2}{g \cdot (D_1)^4} \cdot \left(f(Q1, D_1, \varepsilon_1) \cdot \frac{L_1}{D_1} + K_1 + C_1 \cdot f_T(D_1, \varepsilon_1) \right)$$

$$W_s \cdot \frac{g_c}{g} = Z_b - Z_a + \frac{8}{\pi^2} \cdot \frac{(Q2)^2}{g \cdot (D_2)^4} \cdot \left(f(Q2, D_2, \varepsilon_2) \cdot \frac{L_2}{D_2} + K_2 + C_2 \cdot f_T(D_2, \varepsilon_2) \right)$$

$$\begin{pmatrix} W_s \\ Q1 \\ Q2 \end{pmatrix} = \text{Find}(W_s, Q1, Q2)$$

$$W_s = 3.393 \cdot \frac{\text{ft} \cdot \text{lbf}}{\text{lb}} \qquad Q1 = 3.824 \cdot \text{ft}^3 \cdot \text{sec}^{-1} \qquad Q2 = 1.476 \cdot \text{ft}^3 \cdot \text{sec}^{-1}$$

Additional output of useful quantities:

$$i := 1 .. N \qquad V(q, D) := \frac{4 \cdot q}{\pi \cdot (D)^2} \qquad Q_1 := Q1 \qquad Q_2 := Q2$$

D_i	$V(Q_i, D_i)$	$Re(Q_i, D_i)$	$f(Q_i, D_i, \varepsilon_i)$	$f_T(D_i, \varepsilon_i)$
1 ft	4.869 ft·sec^{-1}	$1.623 \cdot 10^5$	0.021	0.02
0.667 ft	4.229 ft·sec^{-1}	$9.4 \cdot 10^4$	0.019	0.013

FIGURE 1–21 *(continued)*

An interesting variation to Example 1-6 is to consider the system with an additional 8-in. pipe in parallel with the original two pipes. Figure 1–22 is the output of PIPEP for such a three-pipe system. The solution shows that the addition of the third pipe would result in an increase in the pressure at point *b*, since $W_s = -5.253$ ft-lbf/lbm. The friction loss between the two-pipe and three-pipe parallel systems is such that the two-pipe system head loss due to friction is greater than the decrease in the elevation, while the head loss for the three-pipe system is less than the decrease in the elevation; hence, the difference in sign of the W_s term for the two cases.

EXAMPLE 1-7

Work Example 1-5 if a pump with an increase in head of 50 ft-lbf/lbm is placed in the system.

Solution. The parallel piping system is illustrated in Fig. 1–20, and the conditions with the pump are presumed to result in $P_A = P_B$. Example 1-6 is a type 2 problem in which the total system flow rate Q_T is given and the flow rates in the individual lines as well as the pump/turbine increase/decrease in head are required. This problem is type 1, in which the pump increase in head is given and the total system flow rate as well as the individual line flow rates are required. Equations (1-32) still model the system, and the first part of PIPEP, Fig. 1–21, does not require any changes. The only changes required are in the solve block, where the three unknowns are identified as Q_T, Q_1, and Q_2. The addition of the 50 ft-lbf/lbm pump results in a flow rate of 9.396 ft^3/s. This example illustrates the generality and ease of use of PIPEP for different circumstances.

Series-Parallel Networks

By extending the concepts previously developed for series and parallel piping systems to the more complex series-parallel *networks*, we can develop analysis techniques for

TITLE: Pipe addition to Example 1-6.

ORIGIN := 1 Set origin for counters to 1 from default value of 0.

Input the pipe geometry and the elevation difference:

$$D := \begin{pmatrix} 12 \\ 8 \\ 8 \end{pmatrix} \cdot \text{in} \qquad L := \begin{pmatrix} 3000 \\ 3000 \\ 3000 \end{pmatrix} \cdot \text{ft} \qquad \varepsilon := \begin{pmatrix} 0.001 \\ 0.0001 \\ 0.0001 \end{pmatrix} \cdot \text{ft} \qquad \begin{pmatrix} Z_a \\ Z_b \end{pmatrix} := \begin{pmatrix} 100 \\ 80 \end{pmatrix} \cdot \text{ft}$$

Input the loss coefficients:

K factor / Equivalent length / Number of pipes

$$K := \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \qquad C := \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \qquad N := \text{length}(D)$$

Input the fluid properties:

Density in lbm/ft^3 / Viscosity in lbm/ft-s

$$\rho := 64.35 \cdot \frac{\text{lb}}{\text{ft}^3} \qquad \mu := 0.00193 \cdot \frac{\text{lb}}{\text{ft} \cdot \text{sec}}$$

Define constants and adjust units for consistency:

$$g := 32.174 \cdot \frac{\text{ft}}{\text{sec}^2} \qquad g_c := 32.174 \cdot \frac{\text{ft} \cdot \text{lb}}{\text{lbf} \cdot \text{sec}^2}$$

Define the functions for Reynolds number and the friction factors:

$$Re(q, D) := \frac{4 \cdot \rho \cdot q}{\pi \cdot D \cdot \mu} \qquad f_T(D, \varepsilon) := \frac{0.3086}{\left[\log\left[\left(\frac{\varepsilon}{3.7 \cdot D} \right)^{1.11} \right] \right]^2}$$

$$f(q, D, \varepsilon) := \begin{cases} \dfrac{0.3086}{\left[\log\left[\dfrac{6.9}{Re(q, D)} + \left(\dfrac{\varepsilon}{3.7 \cdot D} \right)^{1.11} \right] \right]^2} & \text{if } Re(q, D) > 2300 \\ \dfrac{64}{Re(q, D)} & \text{otherwise} \end{cases}$$

Set up solve block by defining specified inputs and guessed values:

$$Q_T := 5.3 \cdot \frac{\text{ft}^3}{\text{sec}} \qquad Q1 := \frac{Q_T}{N} \qquad Q2 := \frac{Q_T}{N} \qquad Q3 := \frac{Q_T}{N} \qquad W_s := 1 \cdot \text{ft} \cdot \frac{\text{lbf}}{\text{lb}}$$

FIGURE 1–22 The addition of another pipe to Example 1-6.

very complex series-parallel networks. Most analysis methods for series-parallel networks are devised around loops and nodes. A *loop* is defined as a series of pipes forming a closed path, a *node* as a point where two or more lines are joined. The sign conventions typically used are that the head loss h_f in a line is taken as positive for flow in the counterclockwise direction around a loop and that flow toward a node is positive. Loops and nodes are important for the two concepts we adopt from series and parallel systems:

Given

$$Q_T = Q1 + Q2 + Q3$$

$$W_s \cdot \frac{g_c}{g} = Z_b - Z_a + \frac{8}{\pi^2} \cdot \frac{(Q1)^2}{g \cdot (D_1)^4} \cdot \left(f(Q1, D_1, \varepsilon_1) \cdot \frac{L_1}{D_1} + K_1 + C_1 \cdot f_T(D_1, \varepsilon_1) \right)$$

$$W_s \cdot \frac{g_c}{g} = Z_b - Z_a + \frac{8}{\pi^2} \cdot \frac{(Q2)^2}{g \cdot (D_2)^4} \cdot \left(f(Q2, D_2, \varepsilon_2) \cdot \frac{L_2}{D_2} + K_2 + C_2 \cdot f_T(D_2, \varepsilon_2) \right)$$

$$W_s \cdot \frac{g_c}{g} = Z_b - Z_a + \frac{8}{\pi^2} \cdot \frac{(Q3)^2}{g \cdot (D_3)^4} \cdot \left(f(Q3, D_3, \varepsilon_3) \cdot \frac{L_3}{D_3} + K_3 + C_3 \cdot f_T(D_3, \varepsilon_3) \right)$$

$$\begin{pmatrix} W_s \\ Q1 \\ Q2 \\ Q3 \end{pmatrix} := \mathrm{Find}(W_s, Q1, Q2, Q3)$$

$$W_s = -5.253 \cdot \frac{\mathrm{ft \cdot lbf}}{\mathrm{lb}}$$

$Q1 = 3.011 \cdot \mathrm{ft}^3 \cdot \mathrm{sec}^{-1}$ $\quad Q2 = 1.145 \cdot \mathrm{ft}^3 \cdot \mathrm{sec}^{-1}$ $\quad Q3 = 1.145 \cdot \mathrm{ft}^3 \cdot \mathrm{sec}^{-1}$

Additional output of useful quantities:

$i := 1..N \quad V(q, D) := \frac{4 \cdot q}{\pi \cdot (D)^2} \quad Q_1 := Q1 \quad Q_2 := Q2 \quad Q_3 := Q3$

D_i	$V(Q_i, D_i)$	$Re(Q_i, D_i)$	$f(Q_i, D_i, \varepsilon_i)$	$f_T(D_i, \varepsilon_i)$
1·ft	3.834·ft·sec^{-1}	1.278·10^5	0.022	0.02
0.667·ft	3.279·ft·sec^{-1}	7.288·10^4	0.02	0.013
0.667·ft	3.279·ft·sec^{-1}	7.288·10^4	0.02	0.013

FIGURE 1–22 *(continued)*

1. Conservation of mass at a node, which for node α is

$$\sum_{\beta=1} Q_\beta = 0 \tag{1-33a}$$

That is, the node cannot accumulate mass.

2. The pressure at a node must be single valued, which, for the *i*th loop, becomes

$$\sum_{j=1} h_{f_j} = 0 \tag{1-33b}$$

That is, the sum of the pressure drops around a loop must be zero.

These concepts, taken with the method developed for category II problems, are in principle all that is needed to analyze any series-parallel system. Such an approach to complex series-parallel networks is very awkward and can easily lead to ill-posed sys-

Set up solve block by defining specified inputs and guessed values:

$Q_T := 5.3 \cdot \frac{\mathrm{ft}^3}{\mathrm{sec}} \quad Q1 := \frac{Q_T}{N} \quad Q2 := \frac{Q_T}{N} \quad W_s := 50 \cdot \mathrm{ft} \cdot \frac{\mathrm{lbf}}{\mathrm{lb}}$

Given

$$Q_T = Q1 + Q2$$

$$W_s \cdot \frac{g_c}{g} = Z_b - Z_a + \frac{8}{\pi^2} \cdot \frac{(Q1)^2}{g \cdot (D_1)^4} \cdot \left(f(Q1, D_1, \varepsilon_1) \cdot \frac{L_1}{D_1} + K_1 + C_1 \cdot f_T(D_1, \varepsilon_1) \right)$$

$$W_s \cdot \frac{g_c}{g} = Z_b - Z_a + \frac{8}{\pi^2} \cdot \frac{(Q2)^2}{g \cdot (D_2)^4} \cdot \left(f(Q2, D_2, \varepsilon_2) \cdot \frac{L_2}{D_2} + K_2 + C_2 \cdot f_T(D_2, \varepsilon_2) \right)$$

$$\begin{pmatrix} Q_T \\ Q1 \\ Q2 \end{pmatrix} := \mathrm{Find}(Q_T, Q1, Q2) \qquad W_s = 50 \cdot \frac{\mathrm{ft \cdot lbf}}{\mathrm{lb}}$$

$Q_T = 9.396 \cdot \mathrm{ft}^3 \cdot \mathrm{sec}^{-1}$ $\quad Q1 = 6.711 \cdot \mathrm{ft}^3 \cdot \mathrm{sec}^{-1}$ $\quad Q2 = 2.684 \cdot \mathrm{ft}^3 \cdot \mathrm{sec}^{-1}$

Additional output of useful quantities:

$i := 1..N \quad V(q, D) := \frac{4 \cdot q}{\pi \cdot (D)^2} \quad Q_1 := Q1 \quad Q_2 := Q2$

D_i	$V(Q_i, D_i)$	$Re(Q_i, D_i)$	$f(Q_i, D_i, \varepsilon_i)$	$f_T(D_i, \varepsilon_i)$
1·ft	8.545·ft·sec^{-1}	2.849·10^5	0.021	0.02
0.667·ft	7.69·ft·sec^{-1}	1.709·10^5	0.017	0.013

FIGURE 1–23 MathCad solution of Example 1-7.

tems of equations. For these reasons, we shall develop a systematic approach to complex fluid flow networks. This approach, called the Hardy-Cross method, has been extensively used for the analysis of fluid-conveying networks. Because of its importance, it will be covered as a primary topic in this chapter.

1-5 HARDY-CROSS METHOD

The *Hardy-Cross* formulation is an iterative method for obtaining the steady-state solution for any generalized series-parallel flow network. Its great advantage is systematicness. The Hardy-Cross method can be systematically applied to any fluid flow network, and, if the guidelines are followed, a converged solution will always be obtained. The approach is readily adapted to the computer; in fact, a number of software firms have generalized Hardy-Cross programs available for sale.

The basis for any Hardy-Cross analysis technique is the same as for any series-parallel flow network analysis: (1) conservation of mass at a node and (2) uniqueness of pressure at a given point in the loop. Category I problems always arise in this application; and, to reduce the numerical evaluations necessary per step, a slightly modified version of the Darcy-Weisbach expression is developed. This modified version of the head-loss representation is called the *Hazen-Williams* expression. Since flow rate rather than velocity is usually of primary interest, the conventional head-loss expression is written in terms of the flow rate as

$$h_f = \frac{fL}{D}\frac{1}{2g_c}\frac{Q^2}{A^2} \tag{1-34}$$

For circular pipes, this becomes

$$h_f = \frac{16}{2\pi^2 g_c}\frac{fL}{D^5}Q^2 \tag{1-35}$$

or, with $K_1 = 8/(g_c\pi^2)$,

$$h_f = K_1\frac{fL}{D^5}Q^2 \tag{1-36}$$

The friction factor f is a function of the Reynolds number and, hence, of the flow rate, diameter, and relative roughness. Hazen and Williams suggested that for a general fluid the head loss could then be written as

$$h_f = KQ^n \tag{1-37}$$

where K and n are determined either by experiment or by curve fits using the Moody diagram. For water flow through pipes, the head loss is usually written as

$$h_f = \frac{k_1 L}{C^{1.852}D^{4.8704}}Q^{1.852} \tag{1-38}$$

where C is a dimensionless number that is indicative of the roughness of the pipe and is called the *Hazen-Williams coefficient*. Values of the Hazen-Williams coefficient for various pipe surfaces are given in Table 1–5. The smoother the pipe is, the larger will be

TABLE 1–5 Hazen-Williams Coefficients

Types of pipe	C
Extremely smooth and straight pipes	140
New, smooth cast iron pipes	130
Average cast iron, new riveted steel pipes	110
Vitrified sewer pipes	110
Cast iron pipes, some years in service	100
Cast iron pipes, in bad condition	80

Used with permission, from R.V. Giles, *Fluid Mechanics and Hydraulics*, Schaum's Outline Series, McGraw-Hill, 1962.

TABLE 1–6 Values of k_1 for Different Units

Units of Q	k_1
CFS (ft³/s)	4.727
MGD (million gals/day)	10.63
CMS (m³/s)	10.466

Used with permission, from W. Bober and R.A. Kenyon, *Fluid Mechanics*, John Wiley & Sons, Inc., 1980.

the value of the Hazen-Williams coefficient. The constant k_1 is dependent on the dimensions of Q; various values of k_1 are given in Table 1–6. Then, in accordance with the preceding,

$$K = \frac{k_1 L}{C^{1.852}D^{4.8704}} \tag{1-39}$$

For a given line, K is a constant and need be evaluated only once. Situations arise in which the Hazen-Williams coefficient is not known or water at typical temperatures is not the flowing fluid. In either of these cases, the constant K and, if necessary, the exponent n can be estimated by parametrically solving category I problems for a given pipe or set of pipes and curve-fitting the results for the "best" values of K and n in the least-squares sense. The prior determination of the K's and n's is necessary for the Hardy-Cross procedure.

The basic idea of the Hardy-Cross methodology is that conservation of mass at each node can be established initially without consideration of uniqueness of pressure. Uniqueness of pressure can then be used to calculate correction factors for each loop. Thus, the procedure maintains conservation of mass at each node and iterates using uniqueness of pressure to drive the iteration. The fundamental development needed for the Hardy-Cross approach is a systematic scheme for calculating the correction factor for a given loop for a system in which nodal conservation is always enforced.

Let us consider the triangular network of Fig. 1–24. The system is divided into two loops, and each pipe is assigned a number. The flow rate in each pipe is then identified as Q_j, where j the pipe number; thus, pipe 3 has a flow rate identified as Q_3. The initial guesses for all Q_j are made such that conservation of mass is enforced at each node; the superscript 0 denotes the current iterate or the first guess. For Q_j^0 we have forced

$$\sum_\beta Q_\beta^0 = 0 \quad (\text{node } \alpha) \tag{1-40}$$

but in general we have

$$\sum_j h_{f_j}^0 \neq 0 \quad (\text{loop } i) \tag{1-41}$$

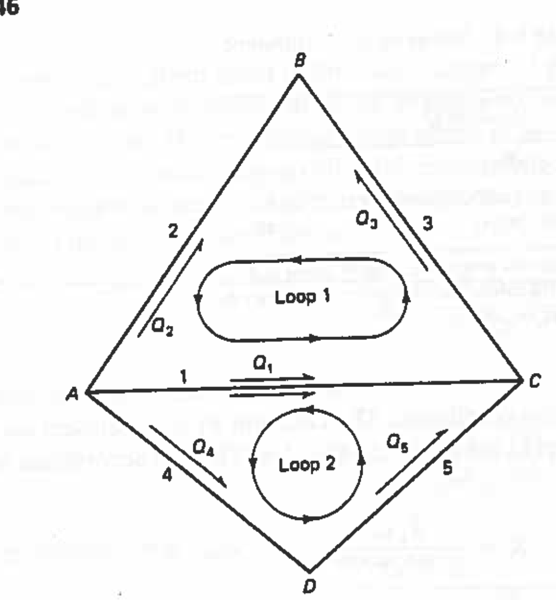


FIGURE 1-24 Network nomenclature for Hardy-Cross development.

We need to develop a method for calculating the correction factor ΔQ_i for each loop such that uniqueness of pressure at a node is obtained.

Since the sign convention yields either a positive or a negative Q_j, some preliminary relations taking these signs into account need to be examined. Consistency suggests that a negative Q_j yield a negative h_{f_j}, so that

$$h_f = \begin{cases} KQ^n, & Q \geq 0 \\ -K(-Q)^n, & Q < 0 \end{cases} \tag{1-42}$$

and then

$$\frac{dh_f}{dQ} = \begin{cases} nKQ^{n-1}, & Q \geq 0 \\ nK(-Q)^{n-1}, & Q < 0 \end{cases} \tag{1-43}$$

A Taylor series can be used to expand the head loss about Q such that

$$h_f(Q + \Delta Q) = h_f(Q) + \frac{dh_f}{dQ}\Delta Q + \frac{d^2 h_f}{dQ^2}\frac{\Delta Q^2}{2!} + \cdots \tag{1-44}$$

As the iteration approaches convergence, ΔQ for a given loop decreases and higher-order terms of the expansion can be neglected, with the result that

$$h_f(Q + \Delta Q) = \begin{cases} K[Q^n + nQ^{n-1}\Delta Q], & Q \geq 0 \\ -K[(-Q)^n - n(-Q)^{n-1}\Delta Q], & Q < 0 \end{cases} \tag{1-45}$$

Equations (1-45) can now be used to evaluate the correction factor ΔQ_1 needed to achieve uniqueness of pressure for any given node in loop 1 of Fig. 1-24. The figure indicates that $Q_1^0 > 0$, $Q_3^0 > 0$, and $Q_2^0 < 0$. The initial guesses Q_j^0 yield

$$h_{f_1}^0 + h_{f_3}^0 - h_{f_2}^0 \neq 0 \tag{1-46}$$

while

$$\pm h_{f_1}^1 \pm h_{f_3}^1 \pm h_{f_2}^1 = 0 \tag{1-47}$$

is desired; the superscript 1 indicates the next iterate. Substituting the results for $h_f(Q + \Delta Q)$ into Eq. (1-47), we have

$$K_1[(Q_1^0)^n + n(Q_1^0)^{n-1}\,\Delta Q_1] + K_3[(Q_3^0)^n + n(Q_3^0)^{n-1}\,\Delta Q_1] - K_2\left[(-Q_2^0)^n - n(-Q_2^0)^{n-1}\,\Delta Q_1\right] = 0 \tag{1-48}$$

Solving for ΔQ_1, we obtain

$$\Delta Q_1 = -\frac{K_1(Q_1^0)^n + K_3(Q_3^0)^n - K_2(-Q_2^0)^n}{n(Q_1^0)^{n-1} + n(Q_3^0)^{n-1} + n(-Q_2^0)^{n-1}} \tag{1-49}$$

or, using absolute values,

$$\Delta Q_1 = -\frac{K_1Q_1^0|Q_1^0|^{n-1} + K_3Q_3^0|Q_3^0|^{n-1} + K_2Q_2^0|Q_2^0|^{n-1}}{n(K_1|Q_1^0|^{n-1} + K_3|Q_3^0|^{n-1} + K_2|Q_2^0|^{n-1})} \tag{1-50}$$

Use of the absolute-value signs in Eq. (1-50) allows the correct sense of the sign on h_{f_i} to be maintained; that is, a negative flow rate yields a negative head loss and a positive flow rate yields a positive head loss. The summation notation for loop 1 permits

$$\Delta Q_1 = -\frac{\sum_{j=1}^{3} K_j Q_j^0 |Q_j^0|^{n-1}}{n\sum_{j=1}^{3} K_j |Q_j^0|^{n-1}} \tag{1-51}$$

and for any loop i with J lines,

$$\Delta Q_i = -\frac{\sum_{j=1}^{J} K_j Q_j^0 |Q_j^0|^{n-1}}{n\sum_{j=1}^{J} K_j |Q_j^0|^{n-1}} \tag{1-52}$$

This equation allows the evaluation of the correction factor for any loop i. The methodology of Bober and Kenyon (2) can then be utilized to analyze any complex parallel-series network.

The Bober and Kenyon procedure* for implementing the Hardy-Cross method is as follows:

* Adapted from and used with permission of W. Bober and R. A. Kenyon, *Fluid Mechanics* (New York: John Wiley & Sons, Inc., 1980).

1. Subdivide the network into a number of loops. Be sure that all pipes are included in at least one loop.
2. Determine the zeroth estimate for the flow rate Q_β^0 for each line according to the node convention. Let s equal the total number of nodes in the system and r equal the total number of lines. Invariably, r will be greater than s. Write a node equation for each node, using the sign convention for the node rule. Since $r > s$, there will be more unknowns than equations. If we assume $(r - s)$ values of Q_β^0, then the system should reduce to s linear algebraic equations in s unknowns. However, in all instances the system of s linear equations will be linearly dependent, and one additional Q_β^0 assumption must be made. A set of initial values $\{Q_\beta^0\}$ can then be established. Note that if the initial set of values $\{Q_\beta^0\}$ are reasonably accurate, the convergence will be quick.
3. Relabel the set of $\{Q_\beta^0\}$ values obtained in step 2 according to the loop rule, giving a set of values $\{Q_j^0\}$.
4. Now determine a correction factor ΔQ_i for each loop according to Eq. (1-52), i.e.,

$$\Delta Q_i = -\frac{\sum_{j=1}^{J} K_j Q_j^0 |Q_j^0|^{n-1}}{n \sum_{j=1}^{J} K_j |Q_j^0|^{n-1}}$$

where J is the number of lines in the loop and K_j equals the constant given by Eq. (1-39) for the jth line.

5. After obtaining a ΔQ_i for each loop, obtain algebraically a new value for the flow rate in each line; that is,

$$Q_j^1 = Q_j^0 + \Delta Q_i \tag{1-53}$$

for the lines in loop i.

This method is best explained by considering a specific example.

EXAMPLE 1-8

Use the Hardy-Cross method to obtain the flow rates in each of the lines of the network shown in Fig. 1-25(a) ($C = 130$).

Solution

Step 1:

We begin by dividing the network into loops and numbering all pipes and nodes in each loop. Figure 1-25(b) shows the results of step 1. The choices are arbitrary as long as every pipe is included in at least one loop.

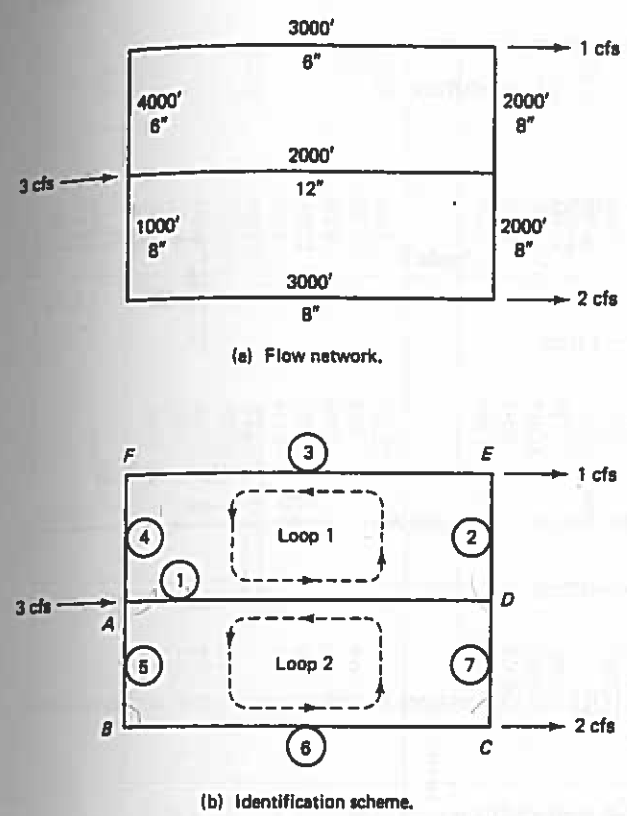
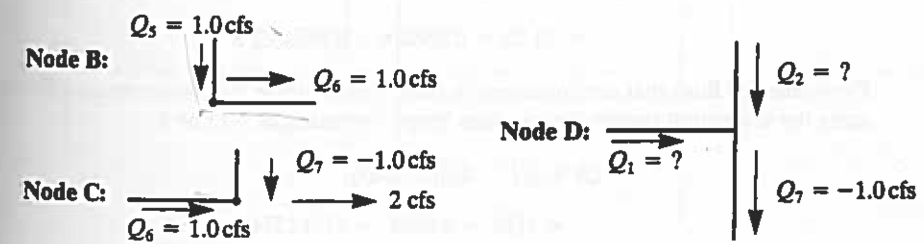


FIGURE 1-25 Flow network for Example 1-8.

Steps 2 and 3:

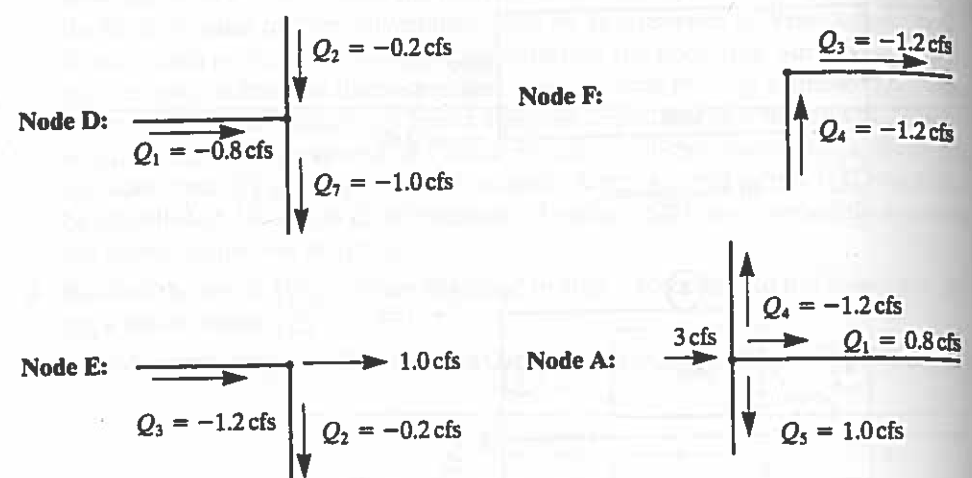
Determine the zeroth estimate. The system has 6 nodes ($s = 6$) and 7 lines ($r = 7$). If we write a conservation equation for each node, we will have 6 equations with 7 unknowns, so that one $(r - s = 1)$ Q_β^0 will have to be assumed. The resulting system of equations will be linearly dependent, and a second Q_β^0 must be assumed. Bober and Kenyon (2) state that $(r - s + 1)$ values of Q_β^0 must be chosen. We shall follow their conclusions and accept 2 (since $7 - 6 + 1 = 2$). An equation will be written at each node. To start the process, we shall assume that $Q_5 = 1.0$. Then, visiting each node in turn, we obtain:



Here we make the second assumption, viz.,

$$Q_1 = -0.8 \text{ cfs}$$

from which we have:



This procedure has produced $\{Q^0_\beta\}$ for the network in such a manner that conservation of mass is satisfied at each node.

Steps 4 and 5:

Determine the correction factor ΔQ_i for each loop.

The equation derived previously for the correction factors is very convenient to use either in a computer program or by hand calculation. Table 1–7 gives some details of the iteration process and provides a useful format for hand calculations. The advantage of the form of the equation is evident. After iteration number 1, the correction factor for loop 1 is $\Delta Q_1 = 0.6326$ cfs and for loop 2 it is $\Delta Q_2 = -0.1574$ cfs. These factors are added to the assumed flow rates for each loop according to the loop sign convention.

Flow rates for lines that are contained in a single loop are corrected simply by

$$Q^1_j = Q^0_j + \Delta Q_i \tag{1-53}$$

For example, for line 3

$$Q^1_3 = Q^0_3 + \Delta Q_1$$

$$= -1.20 + 0.6362 = -0.5674 \text{ CFS}$$

Flow rates for lines that are contained in more than a single loop are corrected by considering the correction factors for common loops. For example for line 1,

$$Q^1_1 = Q^0_1 + \Delta Q_1 - \Delta Q_2$$

$$= 0.80 + 0.6326 - (-0.1574)$$

$$= 1.590 \text{ cfs}$$

TABLE 1-7 Hardy-Cross Iteration Sequence

Loop no.	Pipe no.	Pipe diameter (in.)	Pipe length (ft)	K	Q	$K\|Q\|^{0.852}$	$KQ\|Q\|^{0.852}$	ΔQ
				Iteration 1				
1	1	12	2,000	1.148	0.8	0.949	0.759	
	2	8	2,000	8.269	−0.2	2.099	−0.420	
	3	6	3,000	50.360	−1.2	58.823	−70.587	
	4	6	4,000	67.146	−1.2	78.430	−94.116	
						Sum 140.301	−164.364	0.6326
2	5	8	1,000	4.134	1.0	4.134	4.134	
	6	8	3,000	12.403	1.0	12.403	12.403	
	7	8	2,000	8.269	−1.0	8.269	−8.269	
	1	12	2,000	1.148	−0.8	.949	−0.759	
						Sum 25.755	7.509	−0.1574
				Iteration 2				
1	1			1.148	1.590	1.704	2.710	
	2			8.269	0.433	4.049	1.751	
	3			50.360	−0.567	31.076	−17.633	
	4			67.146	−0.567	41.434	−23.511	
						Sum 78.263	−36.683	0.2531
2	5			4.134	0.843	3.573	3.010	
	6			12.403	0.843	10.719	9.031	
	7			8.269	−1.157	9.365	−10.840	
	1			1.148	−1.590	1.704	−2.710	
						Sum 25.361	−1.509	0.0321
				Iteration 3				
1	1			1.148	1.811	1.904	3.448	
	2			8.269	0.686	5.995	4.111	
	3			50.360	−0.314	18.788	−5.906	
	4			67.146	−0.314	25.050	−7.875	
						Sum 51.737	−6.222	0.0649
2	5			4.134	0.875	3.689	3.226	
	6			12.403	0.875	11.066	9.679	
	7			8.269	−1.125	9.144	−10.290	
	1			1.148	−1.811	1.904	−3.448	
						Sum 25.803	−0.833	0.0174

(Continued)

TABLE 1-7 (Continued)

Loop no.	Pipe no.	Pipe diameter (in.)	Pipe length (ft)	K	Q	$K\|Q\|^{0.852}$	$KQ\|Q\|^{0.852}$	ΔQ
Iteration 4								
1	1			1.148	1.858	1.947	3.618	
	2			8.269	0.751	6.475	4.860	
	3			50.360	−0.249	15.426	−3.847	
	4			67.146	−0.249	20.569	−5.130	
					Sum	44.417	−0.499	0.0061
2	5			4.134	0.892	3.751	3.346	
	6			12.403	0.892	11.253	10.039	
	7			8.269	−1.108	9.023	−10.000	
	1			1.148	−1.858	1.947	−3.618	
					Sum	25.974	−0.233	0.0048
Iteration 5								
1	1			1.148	1.86	1.948	3.623	
	2			8.269	0.757	6.520	4.933	
	3			50.360	−0.243	15.106	−3.676	
	4			67.146	−0.243	20.141	−4.901	
					Sum	43.715	−0.021	0.0003
2	5			4.134	0.897	3.768	3.379	
	6			12.403	0.897	11.304	10.139	
	7			8.269	−1.103	8.990	−9.917	
	1			1.148	−1.860	1.948	−3.622	
					Sum	26.010	−0.021	0.0005
Iteration 6								
1	1			1.148	1.86	1.948	3.622	
	2			8.269	0.757	6.522	4.937	
	3			50.360	−0.243	15.092	−3.669	
	4			67.146	−0.243	20.123	−4.892	
					Sum	43.685	−0.002	0.0000
2	5			4.134	0.897	3.770	3.383	
	6			12.403	0.897	11.309	10.148	
	7			8.269	−1.103	8.987	−9.910	
	1			1.148	−1.860	1.948	−3.622	
					Sum	26.014	−0.009	0.0000

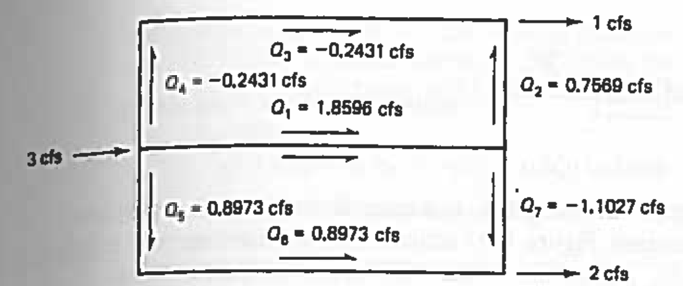


FIGURE 1–26 Converged solution for Hardy-Cross example problem.

The negative sign is required because positive flow rates in one loop are negative flow rates in other common loops. The converged solution is presented in Fig. 1–26. With the flow rates and K_j's known, the pressure drop between any two nodes can be calculated from $K_j Q_j^n$.

The procedures are repeated for each iteration. Table 1–7 gives a summary of each iteration until convergence is reached. Convergence is rapid if reasonable estimates are made for the initial flow rates. Examination of the ΔQ_i's from various iterations illustrates the typical approach to convergence.

The Hardy-Cross method of pipe network analysis is potent for complex networks. It is easily programmed for use on the digital computer. The Hardy-Cross analysis is even adaptable to programmable hand-held calculators. A number of software specialty houses market versions executable on a wide variety of hand-held calculators and personal computers. Most Hardy-Cross software requires as input the initial guesses on all line flow rates, as well as the system geometry (diameters. lengths, and K values).

1-6 GENERALIZED HARDY-CROSS ANALYSIS

The Hardy-Cross analysis with which we have dealt has been restricted to flow networks in which the pipe wall friction represented the only loss (i.e., minor loss has been neglected). If line lengths are short enough so that minor losses are important, the equivalent-length approach can be used to include the losses due to fittings. The *equivalent length* is the additional length of pipe needed to give the same head loss (or pressure drop) as a fitting at a given flow rate. Hence the equivalent length L_{eq} is obtained by equating the loss-coefficient expression to the conventional head-loss expression, i.e.,

$$K\frac{V^2}{2g_c} = f\frac{L_{eq}}{D}\frac{V^2}{2g_c} \tag{1-54}$$

or

$$L_{eq} = K\frac{D}{f} \tag{1-55}$$

However, the flexibility of the Hardy-Cross method makes a generalized formulation possible. The *generalized Hardy-Cross analysis* can be used for piping networks in which the lines can contain devices that result either in additional pressure drop (a heat exchanger or turbine, for example) or in a pressure increase (a pump, for example).