

"Case Study" Solutions

(1) $\bar{x} = 60.29 \text{ g.}$, $s =$

(2) $H_0: \mu = 60$

$H_A: \mu \neq 60$

(3) Type I: Decide the machine correctly ($\mu \neq 60$), working fine.

Type II: Decide the machine

⑤ Answer depends on α -level!

if... $\alpha = 0.01$

↳ 99% conf. interval: $\bar{x} \pm t_{\frac{\alpha}{2}} \left(\frac{s}{\sqrt{n}} \right)$

* $df = 11$

* $\alpha = 0.01$

* $\frac{\alpha}{2} = 0.005$

* $t_{\frac{\alpha}{2}} = 3.106$

$$= 60.29 \pm 3.106 \left(\frac{0.315}{\sqrt{12}} \right)$$

$$= 60.29 \pm 0.282$$

$$= (60.008, 60.572)$$

if... $\alpha = 0.05$

↳ 95% conf. interval: $\bar{x} \pm t_{\frac{\alpha}{2}} \left(\frac{s}{\sqrt{n}} \right)$

* $df = 11$

* $\alpha = 0.05$

* $\frac{\alpha}{2} = 0.025$

* $t_{\frac{\alpha}{2}} = 2.201$

$$= 60.29 \pm 2.201 \left(\frac{0.315}{\sqrt{12}} \right)$$

$$= 60.29 \pm 0.200$$

$$= (60.09, 60.49)$$

if... $\alpha = 0.10$

↳ 90% conf. interval: $\bar{x} \pm t_{\frac{\alpha}{2}} \left(\frac{s}{\sqrt{n}} \right)$

* $df = 11$

* $\alpha = 0.10$

* $\frac{\alpha}{2} = 0.05$

* $t_{\frac{\alpha}{2}} = 1.796$

$$= 60.29 \pm 1.796 \left(\frac{0.315}{\sqrt{12}} \right)$$

$$= 60.29 \pm 0.163$$

$$= (60.127, 60.453)$$

(c) if $\alpha = 0.01$ Critical Value:
 $H_A: \neq$
 (cut α in half $\rightarrow \alpha = 0.01$
 since $H_A: \neq$) $\rightarrow \alpha/2 = 0.005$
 $df = 11$

Test Stat: $t = \frac{\bar{X} - \mu_0}{\left(\frac{S}{\sqrt{n}}\right)} = \frac{60.29 - 60}{\left(\frac{0.315}{\sqrt{12}}\right)} = \underline{3.189}$

Decision: Test Stat is in the rejection region $\rightarrow$ Reject H_0

It appears the machine parts are not averaging and it is out of calibration!

if $\alpha = 0.05$ Critical Value:
 $\alpha = 0.05$
 (cut in half $\rightarrow \alpha/2 = 0.025$
 since $H_A: \neq$) $\rightarrow \alpha/2 = 0.025$
 $df = 11$

Test Stat: $t = 3.189$ (same as above)

Decision: Reject H_0 (same as above)

if $\alpha = 0.10$ Critical Value:
 $\alpha = 0.10$
 (cut in half $\rightarrow \alpha/2 = 0.05$
 since $H_A: \neq$) $\rightarrow \alpha/2 = 0.05$
 $df = 11$

Test Stat: $t = 3.189$ (same as above)

Decision: Reject H_0 (same as above)

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On the t-table!

* Test Stat: $t = 3.189$

(calculated in part 6)

$df = 11$

df
11

0.005

p-value between these 2 values

0.0025

3.106

3.497

3.189 is between these two values

$0.0025 < P\text{-value} < 0.005$

BUT... ($H_A: \mu \neq$) so double p-value

$0.005 < p\text{-value} < 0.01$

← this is p-value for one tailed test

← you must double p-value for two-tailed test

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Since the p-value is less than the α -level and the test statistic is beyond the critical value in the rejection region, Reject H_0 !

The machine is not calibrated correctly.