CASE 4.1 SIMPSON'S PARADOX

The results we obtain with conditional probabilities can be quite counterintuitive, even paradoxical. This case is similar to one described in an article by Blyth (1972), and is usually referred to as Simpson's paradox. [Two other examples of Simpson's paradox are described in articles by Westbrooke (1998) and Appleton et al. (1996).] Essentially, Simpson's paradox says that even if one treatment has a better effect than another on *each* of two separate subpopulations, it can have a *worse* effect on the population as a whole.

Suppose that the population is the set of managers in a large company. We categorize the managers as those with an MBA degree (the *B*s) and those without an MBA degree (the $\overline{B}$s). These categories are the two "treatment" groups. We also categorize the managers as those who were hired directly out of school by this company (the *C*s) and those who worked with another company first (the $\overline{C}$s). These two categories form the two "subpopulations." Finally, we use as a measure of effectiveness those managers who have been promoted within the past year (the *A*s).

Assume the following conditional probabilities are given:

$$P(A|B \text{ and } C) = 0.10, P(A|\overline{B} \text{ and } C) = 0.05 \quad \mathbf{(4.12)}$$

$$P(A|B \text{ and } \overline{C}) = 0.35, P(A|\overline{B} \text{ and } \overline{C}) = 0.20 \quad \mathbf{(4.13)}$$

$$P(C|B) = 0.90, P(C|\overline{B}) = 0.30 \quad \mathbf{(4.14)}$$

Each of these can be interpreted as a proportion. For example, the probability $P(A|B \text{ and } C)$ implies that 10% of all managers who have an MBA degree and were hired by the company directly out of school were promoted last year. Similar explanations hold for the other probabilities.

Joan Seymour, the head of personnel at this company, is trying to understand these figures. From the probabilities in Equation (4.12), she sees that among the subpopulation of workers hired directly out of school, those with an MBA degree are twice as likely to be promoted as those without an MBA degree. Similarly, from the probabilities in Equation (4.13), she sees that among the subpopulation of workers hired after working with another company, those with an MBA degree are *almost* twice as likely to be promoted as those without an MBA degree. The information provided by the probabilities in Equation (4.14) is somewhat different. From these, she sees that employees with MBA degrees are three times as likely as those without MBA degrees to have been hired directly out of school.

Joan can hardly believe it when a whiz-kid analyst uses these probabilities to show—correctly—that

$$P(A|B) = 0.125, P(A|\overline{B}) = 0.155 \quad \mathbf{(4.15)}$$

In words, those employees *without* MBA degrees are more likely to be promoted than those with MBA degrees. This appears to go directly against the evidence in Equations (4.12) and (4.13), both of which imply that MBAs have an advantage in being promoted.

Two-way frequency tables for employees with an MBA degree (B) and those without an MBA degree (B') are shown below. These frequencies are comatible with the probabilities presented in equations 4.12–4.15.

MBA (B)	Promoted (A)	Not promoted (A')	
College (C)	18	162	180
Company (C')	7	13	20
	25	175	200

No MBA (B')	Promoted (A)	Not promoted (A')	
College (C)	6	114	120
Company (C')	56	224	280
	62	338	400