

$\sin 2x + \frac{1}{32} \sin 4x + C$   
found in Example 4b.

$\int \sec^2 x = \frac{1}{4} \sec^2 x + C$

The apparent difference in the two solutions given here is reconciled by using the identity  $1 + \tan^2 x = \sec^2 x$  to transform the second result into the first, the only difference being an additive constant, which is part of  $C$ .

b. In this case, we write the even power of  $\tan x$  in terms of  $\sec x$ :

$$\int \tan^2 x \sec x \, dx = \int (\sec^2 x - 1) \sec x \, dx$$

$$= \int \sec^3 x \, dx - \int \sec x \, dx$$

reduction formula 4

$$= \frac{1}{2} \sec x \tan x + \frac{1}{2} \int \sec x \, dx - \int \sec x \, dx$$

Use reduction formula

$$= \frac{1}{2} \sec x \tan x - \frac{1}{2} \ln |\sec x + \tan x| + C.$$

Add secant integral Table 8.1 in Section 8.1

Table 8.3 summarizes the methods used to integrate  $\int \tan^m x \sec^n x \, dx$ . Analogous techniques are used for  $\int \cot^m x \csc^n x \, dx$ .

Table 8.3	Strategy
$\int \tan^m x \sec^n x \, dx$ $n$ even and positive, $m$ real	Split off $\sec^2 x$ , rewrite the remaining even power of $\sec x$ in terms of $\tan x$ , and use $u = \tan x$ .
$m$ odd and positive, $n$ real	Split off $\sec x \tan x$ , rewrite the remaining even power of $\tan x$ in terms of $\sec x$ , and use $u = \sec x$ .
$m$ even and positive, $n$ odd and positive	Rewrite the even power of $\tan x$ in terms of $\sec x$ to produce a polynomial in $\sec x$ ; apply reduction formula 4 to each term.

### Practice Exercises

9–61. Trigonometric integrals Evaluate the following integrals.

used to integrate  $\sin^2 x$  and  $\cos^2 x$ .  
identities.  
integrate  $\sin^3 x$ .  
integrate  $\sin^m x \cos^n x$ , for  $m$  even and  
 $\int \sin^3 x \, dx$ ?  
 $\int \sec^2 x \, dx$ ?  
 $\int \tan x \, dx$ ?

- 9.  $\int \cos^3 x \, dx$
- 10.  $\int \sin^3 x \, dx$
- 11.  $\int \sin^2 3x \, dx$
- 12.  $\int \cos^4 2\theta \, d\theta$
- 13.  $\int \sin^5 x \, dx$
- 14.  $\int \cos^3 20x \, dx$
- 15.  $\int \sin^3 x \cos^2 x \, dx$
- 16.  $\int \sin^2 \theta \cos^5 \theta \, d\theta$
- 17.  $\int \cos^3 x \sqrt{\sin x} \, dx$
- 18.  $\int \sin^3 \theta \cos^{-2} \theta \, d\theta$
- 19.  $\int_0^{\pi/3} \sin^5 x \cos^{-2} x \, dx$
- 20.  $\int \sin^{-3/2} x \cos^3 x \, dx$

- 21.  $\int \tan^2 x \, dx$
- 22.  $\int \cos^4 x \, dx$
- 23.  $\int 20 \tan^2 x \, dx$
- 24.  $\int 10 \tan^2 x \sec^2 x \, dx$
- 25.  $\int \tan x \sec^3 x \, dx$
- 26.  $\int \frac{\sec^4(\ln \theta)}{\theta} \, d\theta$
- 27.  $\int_{-\pi/3}^{\pi/3} \sqrt{\sec^2 \theta - 1} \, d\theta$
- 28.  $\int_0^{\pi/4} \sec^7 x \sin x \, dx$
- 29.  $\int \tan^3 4x \, dx$
- 30.  $\int \sec^2 x \tan^{1/2} x \, dx$
- 31.  $\int \frac{\csc^4 x}{\cot^2 x} \, dx$
- 32.  $\int_{\pi/20}^{\pi/10} \csc^2 5w \, dw$
- 33.  $\int (\csc^2 x + \csc x) \, dx$
- 34.  $\int_0^{\pi/4} \sec^4 \theta \, d\theta$
- 35.  $\int_0^{\pi} (1 - \cos \theta) \, d\theta$
- 36.  $\int_0^{\pi/2} \sin^2 \theta \, d\theta$
- 37.  $\int_0^{\pi/4} \sin^2 \theta \, d\theta$
- 38.  $\int_0^{\pi/4} \cos^2 \theta \, d\theta$
- 39.  $\int_0^{\pi/4} \sin^2 \theta \cos^2 \theta \, d\theta$
- 40.  $\int_0^{\pi/4} \sin^4 \theta \, d\theta$
- 41.  $\int_0^{\pi/4} \cos^4 \theta \, d\theta$
- 42.  $\int_0^{\pi/4} \sin^2 \theta \cos^4 \theta \, d\theta$
- 43.  $\int_0^{\pi/4} \cos^2 \theta \sin^4 \theta \, d\theta$
- 44.  $\int_0^{\pi/4} \sin^4 \theta \cos^2 \theta \, d\theta$
- 45.  $\int_0^{\pi/4} \cos^4 \theta \sin^2 \theta \, d\theta$
- 46.  $\int_0^{\pi/4} \sin^4 \theta \cos^4 \theta \, d\theta$
- 47.  $\int_0^{\pi/4} \sin^6 \theta \cos^2 \theta \, d\theta$
- 48.  $\int_0^{\pi/4} \cos^6 \theta \sin^2 \theta \, d\theta$
- 49.  $\int_0^{\pi/4} \sin^8 \theta \cos^2 \theta \, d\theta$
- 50.  $\int_0^{\pi/4} \cos^8 \theta \sin^2 \theta \, d\theta$
- 51.  $\int_0^{\pi/4} \sin^2 \theta \cos^4 \theta \, d\theta$
- 52.  $\int_0^{\pi/4} \cos^2 \theta \sin^4 \theta \, d\theta$
- 53.  $\int_0^{\pi/4} \sin^4 \theta \cos^4 \theta \, d\theta$
- 54.  $\int_0^{\pi/4} \sin^6 \theta \cos^4 \theta \, d\theta$
- 55.  $\int_{\pi/6}^{\pi/3} \cot^3 \theta \, d\theta$
- 56.  $\int_0^{\pi} (1 - \cos \theta) \, d\theta$
- 57.  $\int_0^{\pi/2} \sin^2 \theta \, d\theta$
- 58.  $\int_0^{\pi/2} \cos^2 \theta \, d\theta$
- 59.  $\int_0^{\pi/2} \sin^4 \theta \, d\theta$
- 60.  $\int_0^{\pi/2} \cos^4 \theta \, d\theta$
- 61.  $\int_0^{\pi/4} \sin^2 \theta \, d\theta$
- 62.  $\int_0^{\pi/4} \cos^2 \theta \, d\theta$

$$21. \int_0^1 \cos^3 x \sqrt{\sin^3 x} dx$$

$$22. \int_{\pi/4}^{\pi/2} \sin^2 2x \cos^3 x dx$$

$$23. \int \sin^2 x \cos^2 x dx$$

$$24. \int \sin^3 x \cos^5 x dx$$

$$25. \int \sin^2 x \cos^4 x dx$$

$$26. \int \sin^3 x \cos^{3/2} x dx$$

$$27. \int \tan^2 x dx$$

$$28. \int 6 \sec^4 x dx$$

$$29. \int \cot^4 x dx$$

$$30. \int \tan^3 \theta d\theta$$

$$31. \int 20 \tan^6 x dx$$

$$32. \int \cot^5 3x dx$$

$$33. \int 10 \tan^9 x \sec^2 x dx$$

$$34. \int \tan^9 x \sec^4 x dx$$

$$35. \int \tan x \sec^3 x dx$$

$$36. \int \tan 4x \sec^3 x dx$$

$$37. \int \frac{\sec^4(\ln \theta)}{\theta} d\theta$$

$$38. \int \tan^5 \theta \sec^4 \theta d\theta$$

$$39. \int_{-\pi/3}^{\pi/3} \sqrt{\sec^2 \theta - 1} d\theta$$

$$40. \int_0^{\pi/6} \tan^5 2x dx$$

$$41. \int_{\pi/4}^{\pi/2} \sec^7 x \sin x dx$$

$$42. \int \sqrt{\tan x} \sec^2 x dx$$