(1 point)
Find parametric equations for line that is tangent to the curve $x = \cos t,\ y = \sin t,\ z = t$ at the point $(\cos(\frac{-5\pi}{6}), \sin(\frac{-5\pi}{6}), \frac{-5\pi}{6})$.

Parametrize the line so that it passes through the given point at t=0. All three answers are required for credit.

$x(t)$ =

$y(t)$ =

$z(t)$ =

(1 point)

Find a vector parametrization of the line through $P = (7, -9, -7)$ in the direction $\mathbf{v} = \langle -2, 4, -5 \rangle$

$\mathbf{r}(t) = ($ ______ $+$ ______ $t\,)\,\mathbf{i} +$

$($ ______ $+$ ______ $t\,)\,\mathbf{j} +$

$($ ______ $+$ ______ $t\,)\,\mathbf{k}$

(1 point) The function $\mathbf{r}(t)$ traces a circle. Determine the radius, center, and plane containing the circle

$$\mathbf{r}(t) = 4\mathbf{i} + (6\cos(t))\,\mathbf{j} + (6\sin(t))\,\mathbf{k}$$

Plane : $x =$ ______

Circle's Center : (______ , ______ , ______)

Radius : ______

(1 point)

Evaluate the limit:

$$\lim_{h \to 0} \frac{\mathbf{r}(t+h) - \mathbf{r}(t)}{h} \text{ for } \mathbf{r}(t) = \langle t^{-4}, \sin t, 8 \rangle$$

$\mathbf{r}'(t) = \langle$ ______ , ______ , ______ $\rangle$

(1 point) Use the appropriate Product Rule to evaluate the derivative, where

$\mathbf{r}_1(t) = \langle 10t, 5, -t^5 \rangle, \mathbf{r}_2(t) = \langle 8, e^t, -5 \rangle$

$\frac{d}{dt}(\mathbf{r}_1(t) \cdot \mathbf{r}_2(t)) =$ ______

(1 point)

Evaluate $\frac{d}{dt}\mathbf{r}(g(t))$ using the Chain Rule:

$$\mathbf{r}(t) = \langle e^t, e^{6t}, -7 \rangle, \quad g(t) = 6t + 9$$

$\frac{d}{dt}\mathbf{r}(g(t)) = \langle$ ______ , ______ , ______ $\rangle$

(1 point) Let $\mathbf{r}(t) = \left\langle t^2, 1 - t, 4t \right\rangle$. Calculate the derivative of $\mathbf{r}(t) \cdot \mathbf{a}(t)$ at $t = 3$, assuming that $\mathbf{a}(3) = \langle -3, -9, 9 \rangle$ and $\mathbf{a}'(3) = \langle -8, -6, 6 \rangle$

$$\frac{d}{dt}\mathbf{r}(t) \cdot \mathbf{a}(t)\Big|_{t=3} =$$

(1 point)

Evaluate the integral:

$$\int_0^t (7s\mathbf{i} + 3s^2\mathbf{j} + 9\mathbf{k})\, ds$$

Answer : ______ $\mathbf{i}+$ ______ $\mathbf{j}+$ ______ $\mathbf{k}$

(1 point)

Find the solution $\mathbf{r}(t)$ of the differential equation with the given initial condition:

$$\mathbf{r}'(t) = \langle \sin 6t, \sin 5t, 9t \rangle, \mathbf{r}(0) = \langle 2, 7, 6 \rangle$$

$\mathbf{r}(t) = \langle$ ______ , ______ , ______ $\rangle$

(1 point)

Compute the length of the curve $\mathbf{r}(t) = \langle -8t, 6t+2, -8t+3 \rangle$ over the interval $0 \leq t \leq 3$

$L =$

(1 point)

Compute the length of the curve $\mathbf{r}(t) = \langle 2t, \ln t, t^2 \rangle$ over the interval $1 \leq t \leq 9$

$L =$

(1 point)

Compute the length of the curve $\mathbf{r}(t) = 2t\,\mathbf{i} + 7t\,\mathbf{j} + (t^2 - 6)\,\mathbf{k}$ over the interval $0 \le t \le 6$.

HINT: use the formula

$$\int \sqrt{t^2 + a^2}\,dt = \frac{1}{2}t\sqrt{t^2 + a^2} + \frac{1}{2}a^2 \ln\left(t + \sqrt{t^2 + a^2}\right) + C$$

$L =$

(1 point)

Find the speed at the given value of t:

$\mathbf{r}(t) = \left\langle e^{t-8}, -1, 8t^{-1} \right\rangle, t = 8$

$v(8) =$

(1 point)

Find an arc length parametrization $\mathbf{r}_1(s)$ of $\mathbf{r}(t) = \langle e^t \sin(t)\,, e^t \cos(t)\,, 5e^t \rangle$.
Assume $t(s) = 0$ when $s = 0$, and $t'(0) > 0$.

$\mathbf{r}_1(s) = \langle$ ______ , ______ , ______ $\rangle$

(1 point)

Calculate $\mathbf{r}'(t)$, $\mathbf{T}(t)$, and $\mathbf{T}(3)$ where

$$\mathbf{r}(t) = \langle t^3, t^2 \rangle.$$

$\mathbf{r}'(t) = \langle$ ______ , ______ $\rangle$.

$\mathbf{T}(t) = \langle$ ______ , ______ $\rangle$.

$\mathbf{T}(3) = \langle$ ______ , ______ $\rangle$.

(1 point)

Calculate the velocity and acceleration vectors, and speed for

$$\mathbf{r}(t) = \langle \cos(4t)\,, \cos(t)\,, \sin(t) \rangle$$

when $t = \frac{5\pi}{4}$.

Velocity:

Acceleration:

Speed:

Usage: To enter a vector, for example $\langle x, y, z \rangle$, type "$<$ x, y, z $>$"

(1 point)

Find $\mathbf{v}(t)$ given the acceleration

$$\mathbf{a}(t) = 2\boldsymbol{i}$$

and initial condition

$$\mathbf{v}(0) = \boldsymbol{k}.$$

$\mathbf{v}(t) =$ [ ]

Usage: To enter a vector, for example $\langle x, y, z \rangle$, type "< x, y, z >"

(1 point)

Find $\mathbf{r}(t)$ and $\mathbf{v}(t)$ given acceleration

$$\mathbf{a}(t) = t^2\boldsymbol{k},$$

initial velocity

$$\mathbf{v}(0) = -3\boldsymbol{j},$$

and initial position

$$\mathbf{r}(0) = -6\boldsymbol{i}.$$

$\mathbf{v}(t) =$ ______

$\mathbf{r}(t) =$ ______

Usage: To enter a vector, for example $\langle x, y, z\rangle$, type "< x, y, z >"