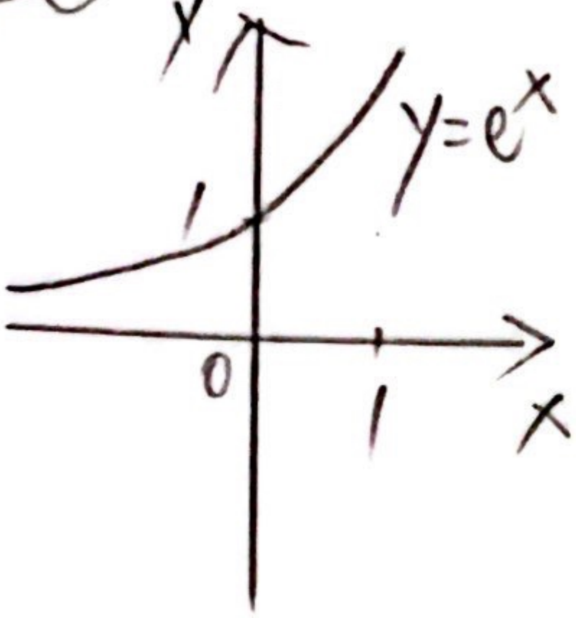
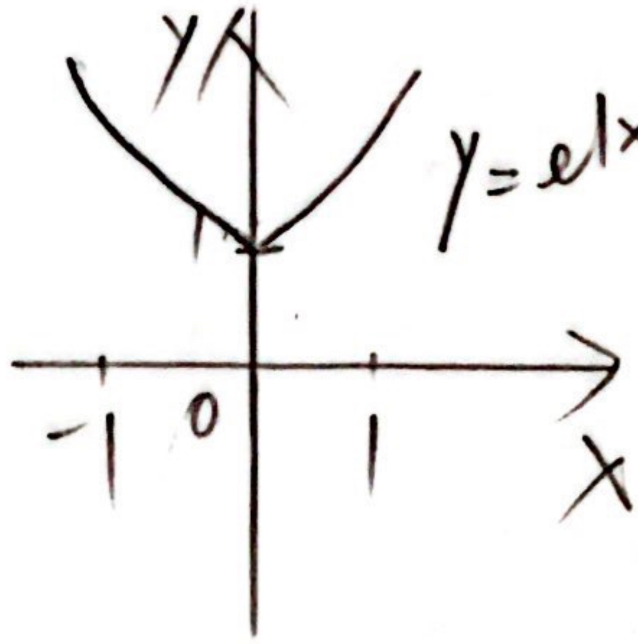
Module 4

L4

14.





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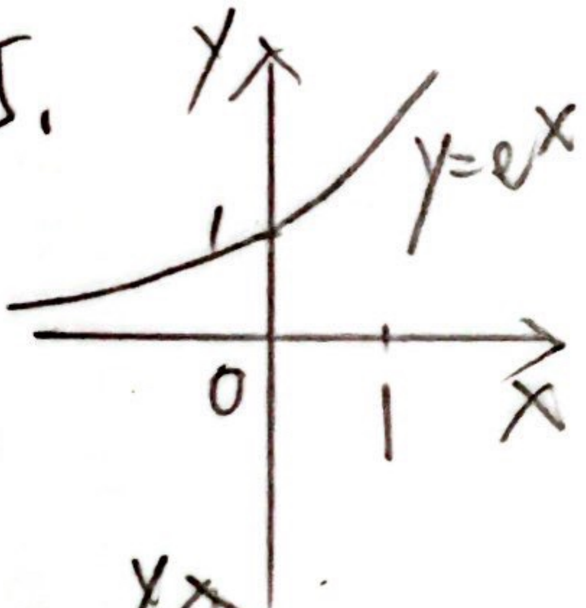

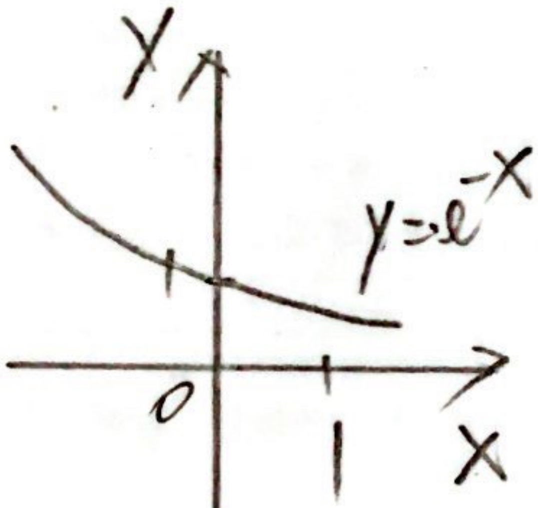

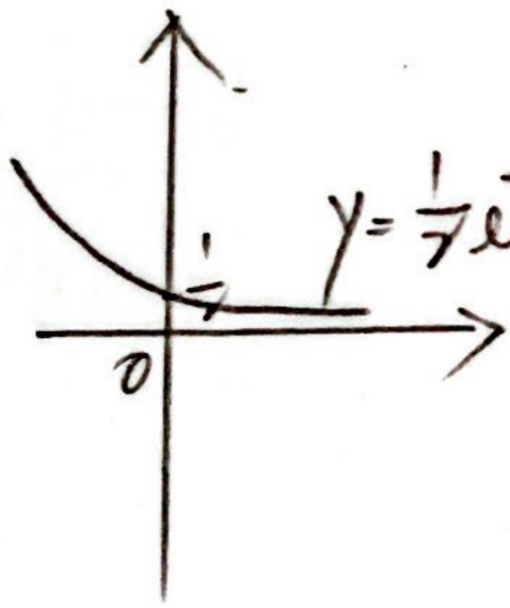

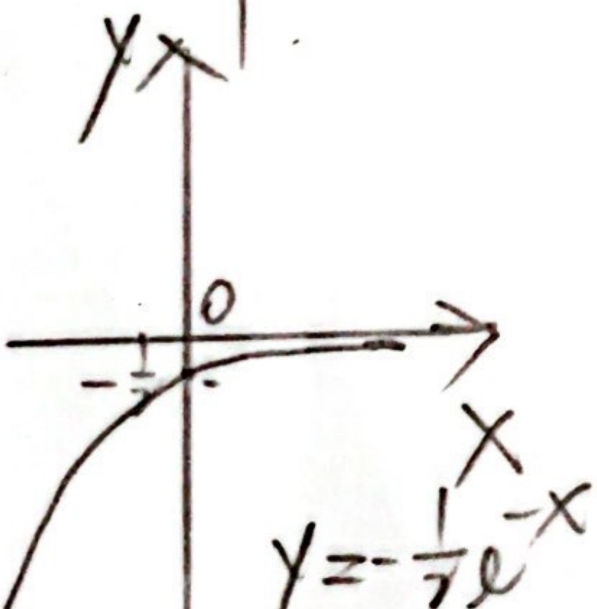

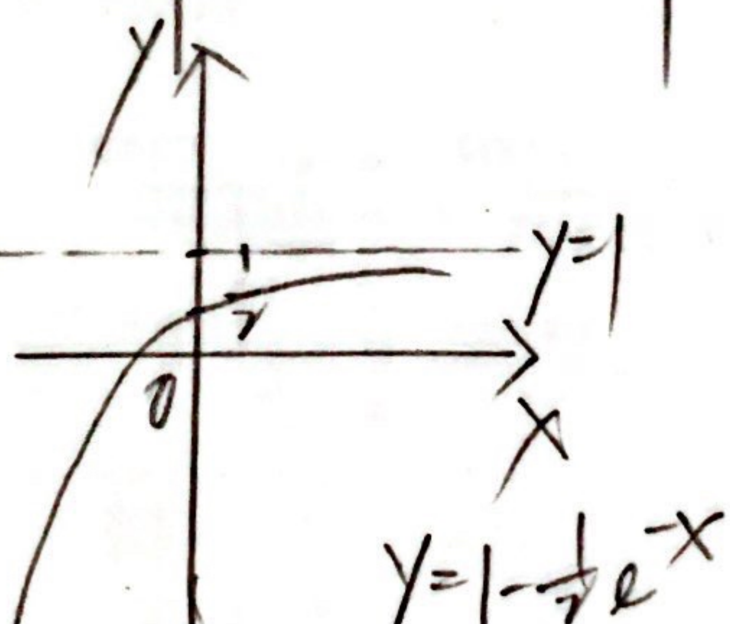

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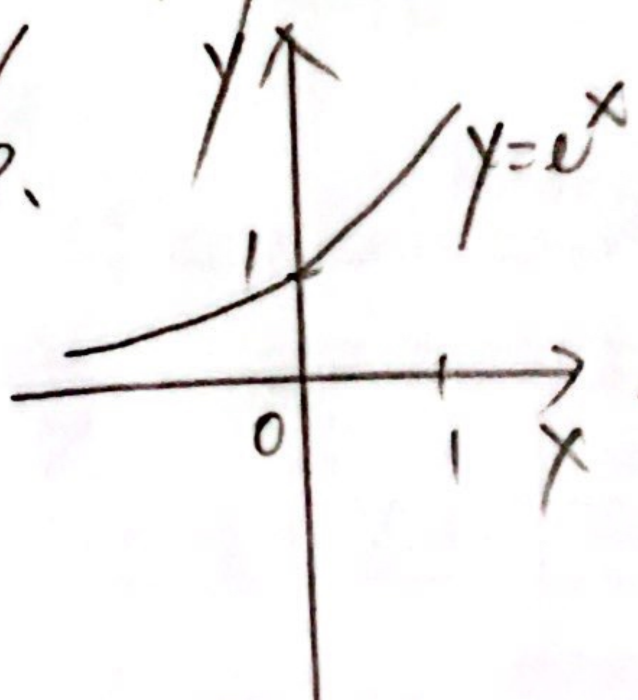
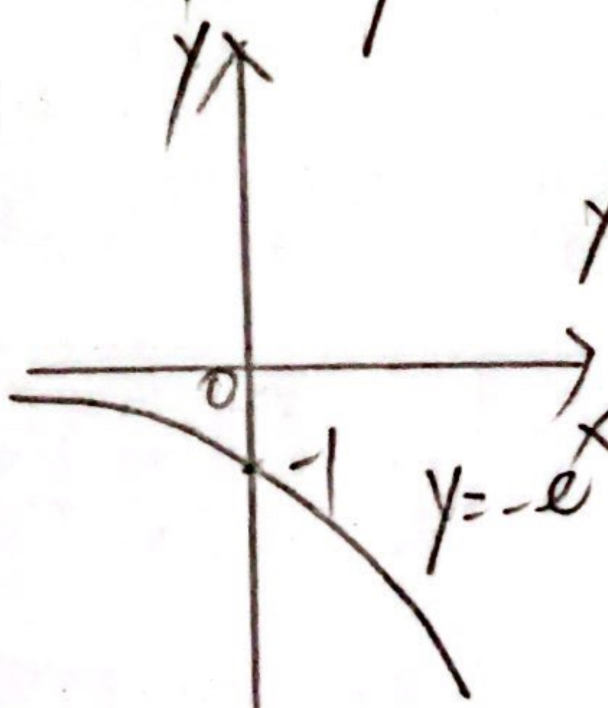
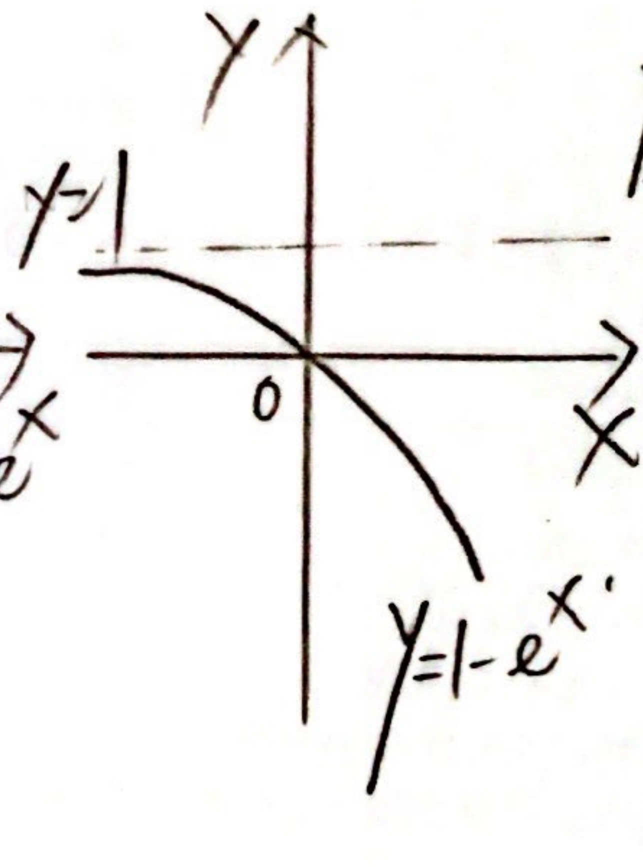
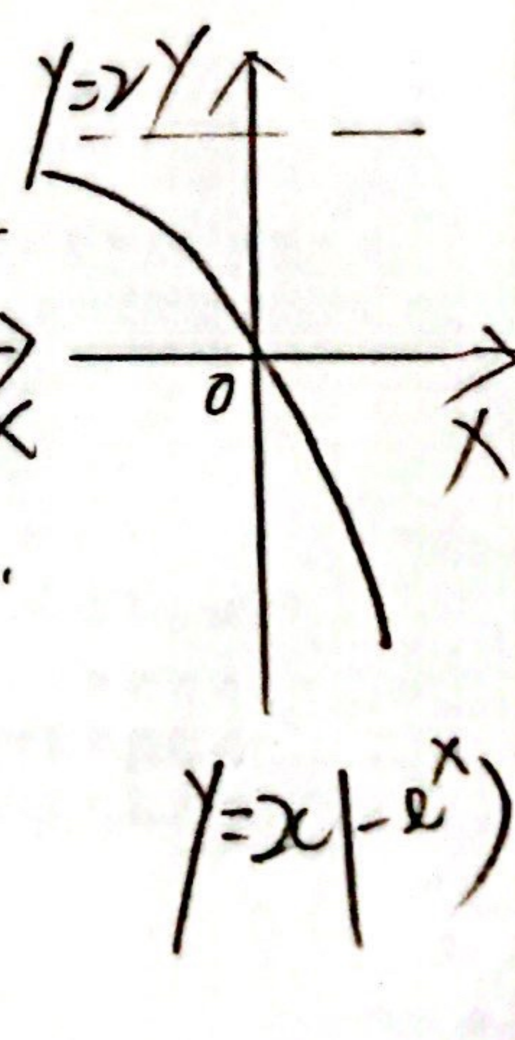








22. ∵ the points $(-1,3)$ $(1,\frac{4}{3})$

∴ $3=Cb^{-1}$ $\quad \frac{4}{3}=Cb^{1}$

$C=3b$ $\quad \frac{4}{3}=3b\times b$

$b^2=\frac{4}{9}$

$b=\frac{2}{3}$ $(b>0)$

$C=3b=3\times\frac{2}{3}=2$

$f(x)=2(\frac{2}{3})^x$

24. Suppose the month is February that has only 28 days.

Payment: $2^{28-1}=2^{27}=134,217,728 > 1,000,000$

∴ I prefer method II

3.

4. ∵ $f(x)=e^5$ is a constant function

∴ $f'(x)=0$

8. $f(t)=1.4t^5-2.5t^2+6.7$

$f'(t)=1.4\times5t^4-2.5\times2t+0=7t^4-5t$

12. $B(y)=cy^{-6}$

$B'(y)=c\times(-6y^{-7})=-6cy^{-7}$

13. $R(a)=(3a+1)^2$

$R'(a)=9\times2a+6\times1+0=18a+6$

16. $h(t)=\sqrt[4]{t}-4e^t=t^{\frac{1}{4}}-4e^t$

$h'(t)=\frac{1}{4}t^{-\frac{3}{4}}-4e^t$

17. $S(p)=\sqrt{p}-p=p^{\frac{1}{2}}-p$

$S'(p)=\frac{1}{2}p^{-\frac{1}{2}}-1$

18. $y=\sqrt[3]{x}(2+x)=2x^{\frac{1}{3}}+x^{\frac{4}{3}}$

$y'=2\times(\frac{1}{3}x^{-\frac{2}{3}})+\frac{4}{3}x^{\frac{1}{3}}=\frac{2}{3}x^{-\frac{2}{3}}+\frac{4}{3}x^{\frac{1}{3}}$

19. $y=3e^x+\frac{4}{\sqrt[3]{x}}=3e^x+4x^{-\frac{1}{3}}$

$y'=3e^x+4\times(-\frac{1}{3})x^{-\frac{4}{3}}=3e^x-\frac{4}{3}x^{-\frac{4}{3}}$

32. $y=e^{x+1}+1=e^x\cdot e^1+1$

$y'=e^x\cdot e=e^{x+1}$

37. $y=x^4+2e^x$

$y'=4x^3+2e^x$

At $(0,2)$, $y'=2$

the equation of the tangent line: $y=2x+2$

the slope of the normal line: $-\frac{1}{2}$

the equation of the normal line: $y=-\frac{1}{2}x+2$

55. $y=2x^3+3x^2-12x+1$

$y'=6x^2+6x-12$

horizontal tangent: $6x^2+6x-12=0$ $\quad x=-2$ / $x=1$

$2\times(-2)^3+3\times(-2)^2-12\times(-2)+1=21$ $\quad 2\times1^3+3\times1^2-12\times1+1=-6$

$\therefore$ the points: $(-2,21)$ $(1,-6)$

62. $y=f(x)=x^2-1$ $\quad f'(x)=2x$

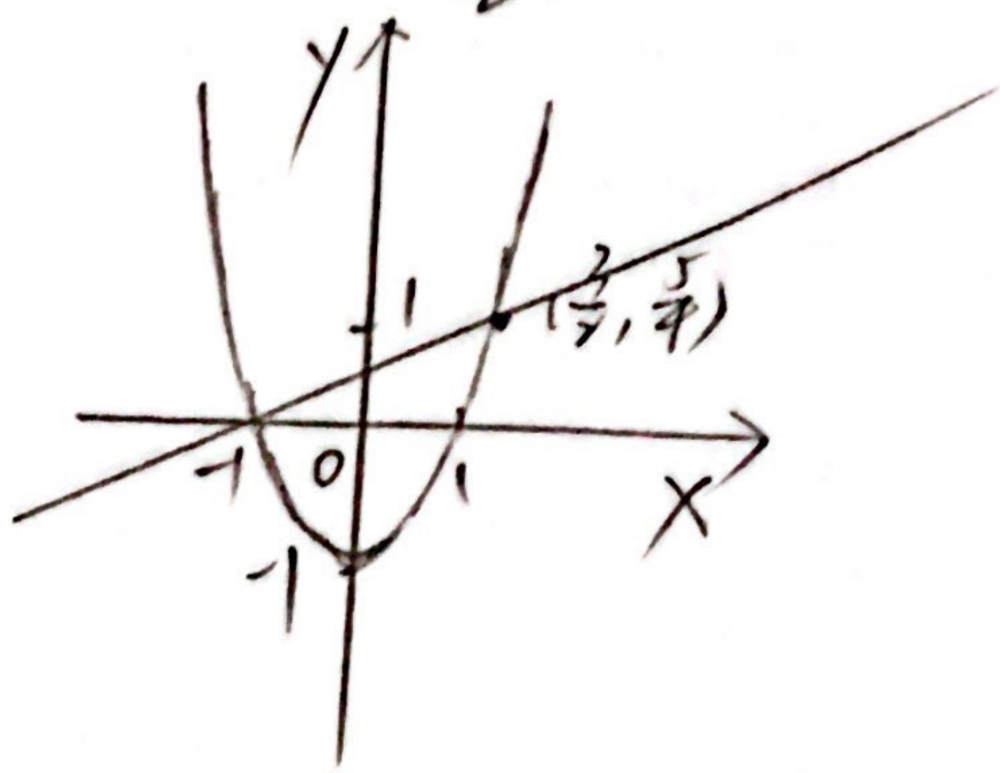
$f'(-1)=-2$ the slope of the normal line : $\frac{1}{2}$

$\because (-1,0)$ $\therefore y-0=\frac{1}{2}[x-(-1)]$

$y=\frac{1}{2}x+\frac{1}{2}$

$\frac{1}{2}x+\frac{1}{2}=x^2-1$ $\quad x=\frac{3}{2}$ / $x=-1$

$(\frac{3}{2})^2-1=\frac{5}{4}$ $\therefore (\frac{3}{2},\frac{5}{4})$



3.2

4. $g(x)=(x+2\sqrt{x})e^x$

$g'(x)=(x+2\sqrt{x})(e^x)'+e^x(x+2\sqrt{x})'$

$=(x+2\sqrt{x})e^x+e^x(1+2x\frac{1}{2}x^{-\frac{1}{2}})$

$=e^x[(x+2\sqrt{x})+(1+\frac{1}{\sqrt{x}})]$

$=e^x(x+2\sqrt{x}+1+\frac{1}{\sqrt{x}})$

5. $y=\frac{x}{e^x}$

$y'=\frac{e^x\cdot 1-x\cdot e^x}{(e^x)^2}$

$=\frac{e^x(1-x)}{(e^x)^2}$

$=\frac{1-x}{e^x}$

6. $y=\frac{e^x}{1-e^x}$

$y'=\frac{(1-e^x)e^x-e^x(-e^x)}{(1-e^x)^2}$

$=\frac{e^x-e^{2x}+e^{2x}}{(1-e^x)^2}$

$=\frac{e^x}{(1-e^x)^2}$

8. $G(x)=\frac{x^2-2}{2x+1}$

$G'(x)=\frac{(2x+1)\cdot 2x-(x^2-2)\cdot 2}{(2x+1)^2}$

$=\frac{4x^2+2x-2x^2+4}{(2x+1)^2}$

$=\frac{2x^2+2x+4}{(2x+1)^2}$

9. $H(u)=(u-\sqrt{u})(u+\sqrt{u})$

$H'(u)=(u-\sqrt{u})(1+\frac{1}{2\sqrt{u}})+(u+\sqrt{u})(1-\frac{1}{2\sqrt{u}})$

$=u+\frac{1}{2}\sqrt{u}-\sqrt{u}-\frac{1}{2}+u-\frac{1}{2}\sqrt{u}+\sqrt{u}-\frac{1}{2}$

$=2u-1$

24. $F(t)=\frac{At}{Bt^2+Ct^3}=\frac{A}{Bt+Ct^2}$

$F'(t)=\frac{(Bt+Ct^2)\times 0-A(B+2Ct)}{(Bt+Ct^2)^2}$

$=\frac{A(B+2Ct)}{-t^2(B+Ct)^2}$

26. $f(x)=\frac{ax+b}{cx+d}$

$f'(x)=\frac{(cx+d)\times a-(ax+b)\times c}{(cx+d)^2}$

$=\frac{acx+ad-acx-bc}{(cx+d)^2}$

$=\frac{ad-bc}{(cx+d)^2}$

34. $y=\frac{2x}{x^2+1}$

$y'=\frac{(x^2+1)\times 2-2x\cdot 2x}{(x^2+1)^2}$

$=\frac{2-2x^2}{(x^2+1)^2}$

At $(1,1)$, $y'=0$

the equation of the tangent line: $y=1$

the slope of the normal line: undefined

the equation of the normal line: $x=1$

46. $\frac{d}{dx}\left(\frac{h(x)}{x}\right)=\frac{xh'\cdot x-hx\cdot 1}{x^2}$

$\frac{d}{dx}\left(\frac{h(x)}{x}\right)\Big|_{x=2}=\frac{2h'\cdot 2-h\cdot 2}{2^2}=\frac{2\cdot(-3)-4}{4}=-2.5$

3.5

2. $f(x)=x\cos x+2\tan x$

$f'(x)=x\cdot(-\sin x)+\cos x\cdot 1+2\sec^2 x$

$=-x\sin x+\cos x+2\sec^2 x$

3. $f(x)=e^x\cos x$

$f'(x)=e^x\cdot(-\sin x)+\cos x\cdot e^x$

$=e^x(\cos x-\sin x)$

4. $y = 2\sec x - \csc x$

$y' = 2 \cdot \sec x \tan x - (-\csc x \cot x)$

$= 2\sec x \tan x + \csc x \cot x$

5. $y = \sec\theta \tan\theta$

$y' = \sec\theta \cdot \sec^2\theta + \tan\theta \cdot \sec\theta \tan\theta$

$= \sec\theta(\sec^2\theta + \tan^2\theta)$

$= \sec\theta(1 + 2\tan^2\theta)$

7. $f(\theta) = \dfrac{\sin\theta}{1+\cos\theta}$.

$f'(\theta) = \dfrac{(1+\cos\theta)\cdot\cos\theta - \sin\theta\cdot(-\sin\theta)}{(1+\cos\theta)^2}$

$= \dfrac{\cos\theta + \cos^2\theta + \sin^2\theta}{(1+\cos\theta)^2}$

$= \dfrac{\cos\theta + 1}{(1+\cos\theta)^2}$

$= \dfrac{1}{1+\cos\theta}$.

12. $y = \dfrac{\cos x}{1-\sin x}$

$y' = \dfrac{(1-\sin x)\cdot(-\sin x) - \cos x\cdot(-\cos x)}{(1-\sin x)^2}$

$= \dfrac{-\sin x + \sin^2 x + \cos^2 x}{(1-\sin x)^2} = \dfrac{-\sin x + 1}{(1-\sin x)^2} = \dfrac{1}{1-\sin x}$

19. $y = \dfrac{\sin t}{1+\tan t}$

$$y' = \frac{(1+\tan t)\cdot \cos t - \sin t \cdot \sec^2 t}{(1+\tan t)^2}$$

$$= \frac{\cos t + \sin t - \frac{\sin t}{\cos^2 t}}{(1+\tan t)^2}$$

$$= \frac{\cos t + \sin t - \tan t \sec t}{(1+\tan t)^2}$$

22. $y = e^x \cos x$

$y' = e^x(-\sin x) + \cos x \cdot e^x = e^x(\cos x - \sin x)$

At $(0,1)$, $y'(0) = \cos 0 - \sin 0 = 1 - 0 = 1$

the equation of the tangent line: $y = x+1$

30. $f(t) = \sec t$

$f'(t) = \sec t \tan t$.

$f''(t) = \sec t \cdot \sec^2 t + \tan t \cdot \sec t \tan t$.

$= \sec^3 t + \sec t \tan^2 t$.

$f''(\frac{\pi}{4}) = (\sqrt{2})^3 + \sqrt{2} \times 1^2 = 3\sqrt{2}$