

QUICK CHECK 3 How many times do you need to integrate by parts to reduce $\int_1^e (\ln x)^6 dx$ to an integral of $\ln x$? \leftarrow

$$\begin{aligned}
 &= \pi \left(\frac{(\ln x)^2 x}{2} \Big|_1^e - \int_1^e \frac{2 \ln x}{x} dx \right) \\
 &= \pi \left(x(\ln x)^2 \Big|_1^e - 2 \int_1^e \ln x dx \right) \\
 &= \pi \left(x(\ln x)^2 \Big|_1^e - 2(x \ln x - x) \Big|_1^e \right) \\
 &= \pi(e(\ln e)^2 - 2e \ln e + 2e - 2) \\
 &= \pi(e - 2) \approx 2.257.
 \end{aligned}$$

SECTION 8.2 EXERCISES

Getting Started

- On which derivative rule is integration by parts based?
- Use integration by parts to evaluate $\int x \cos x dx$ with $u = x$ and $dv = \cos x dx$.
- Use integration by parts to evaluate $\int x \ln x dx$ with $u = \ln x$ and $dv = x dx$.
- How is integration by parts used to evaluate a definite integral?
- What type of integrand is a good candidate for integration by parts?
- How would you choose dv when evaluating $\int x^n e^{ax} dx$ using integration by parts?

19. $\int \frac{\ln x}{x^{10}} dx$

21. $\int x \sin x \cos x dx$

23. $\int x^2 \sin 2x dx$

25. $\int t^2 e^{-t} dt$

27. $\int e^x \cos x dx$

29. $\int e^{-x} \sin 4x dx$

31. $\int e^{3x} \sin e^x dx$

33. $\int_0^{\pi} x \sin x dx$

35. $\int_0^{\pi/2} x \cos 2x dx$

37. $\int_1^{e^2} x^2 \ln x dx$

7–8. Use a substitution to reduce the following integrals to $\int \ln u du$. Then evaluate the resulting integral using the formula for $\int \ln x dx$.

7. $\int (\sec^2 x) \ln(\tan x + 2) dx$ 8. $\int (\cos x) \ln(\sin x) dx$

Practice Exercises

9–40. Integration by parts Evaluate the following integrals using integration by parts.

9. $\int x \cos 5x dx$

10. $\int x \sin 2x dx$

11. $\int t e^{6t} dt$

12. $\int 2x e^{3x} dx$

13. $\int x \ln 10x dx$

14. $\int s e^{-2s} ds$

15. $\int (2w + 4) \cos 2w dw$

16. $\int \theta \sec^2 \theta d\theta$

17. $\int x 3^x dx$

18. $\int x^9 \ln x dx$

39. $\int_0^1 \sin^{-1} y dy$

41. Evaluate the integral

a. $\int \tan^{-1} x dx$

42–47. Volume when the given

42. The region between the axes is