P.S. 8 p+2

Find the power series expansion for the following functions and determine the values of x for which these expansions are valid

(1) $\dfrac{1}{(1+x)^2}$

(2) $\dfrac{1}{(1+x)^3}$

(3) Find the sum of the following series:

(3) $x + \frac{x^3}{3} + \frac{x^5}{5} + \cdots + \frac{x^{2n+1}}{2n+1} + \cdots$

(4) $1 + \frac{x}{2!} + \frac{x^2}{3!} + \frac{x^3}{4!} + \cdots + \frac{x^{n-1}}{n!} + \cdots$

(5) $x + 2x^2 + 3x^3 + \cdots + nx^n + \cdots$

(6) $x + 2x^3 + 3x^5 + \cdots + n\,x^{2n-1}$

By looking at the first ~~five~~ five or six terms show that it is plausible that the Bessel function $J_0(x)$ is a solution to the differential equation $xy'' + y' + xy = 0$ (Let $y = J_0(x)$ and show the equation makes sense for the first 5 or 6 terms)

$$J_0(x) = 1 - \frac{x^2}{2^2} + \frac{x^4}{2^2 \cdot 4^2} - \frac{x^6}{2^2 \cdot 4^2 \cdot 6^2} + \frac{x^8}{2^2 \cdot 4^2 \cdot 6^2 \cdot 8^2} - \frac{x^{10}}{2^2 \cdot 4^2 \cdot 6^2 \cdot 8^2 \cdot 10^2} + \cdots$$