

Determine if the following sequences are convergent or divergent. If convergent, find their limit

(1) $\sqrt[3]{n}$

(2) $\sin(\pi/5n)$

(3) $\frac{3^n}{2^n + 10^{10}}$

(4) $\ln(n+1) - \ln(n)$

(5) $\frac{n^2}{\sqrt{4n^4 + 5}}$

(6) $\cos(n\pi)$

(7) $n \sin(\pi/n)$

Show the following:

(8) $\left(1 + \frac{1}{2n+3}\right)^{2n+3} \rightarrow e$

(9) $\left(1 + \frac{1}{n^2}\right)^{n^2} \rightarrow e$

(10) $\left(1 + \frac{1}{n^2}\right) \rightarrow 1$

~~(11)~~

Series convergence

Find the values of x for which the series converges.
Express the sum as a function of x

① $ax + ax^3 + ax^5 + \dots$ $a \neq 0$

② $x + \frac{x}{1+x} + \frac{x}{(1+x)^2} + \frac{x}{(1+x)^3} + \dots$

③ $\ln x + (\ln x)^2 + (\ln x)^3 + \dots$

④ Show that
$$x^2 + \frac{x^2}{(1+x^2)} + \frac{x^2}{(1+x^2)^2} + \frac{x^2}{(1+x^2)^3} + \dots$$

converges for all x and find its sum

⑤ Show that $\sum 1/e^n \downarrow$

⑥ Show that $\sum 1/e^{nn} \uparrow$

⑦ For what values of x does $\sum e^{nx} \downarrow$?

Series convergence

Establish C or D for following series using a comparison test

$$(1) \sum \frac{1}{\sqrt{n(n+1)}}$$

$$(14) \sum \frac{n^2}{n^5 - \pi}$$

$$(2) \sum \frac{1}{\sqrt{n^2(n+1)}}$$

$$(15) \sum \ln(1 + 1/n^p) \quad p > 0$$

$$(3) \sum \frac{1}{n^n}$$

$$(4) \sum (\ln(n))^n$$

$$(5) \sum \frac{1}{n^{\ln(n)}}$$

$$(6) \sum \frac{n+1}{n(n-1)}$$

$$(7) \sum \frac{(2n+3)^n}{n^{2n}}$$

By any method, determine C or D

$$(8) \sum \frac{1+3n^2}{n^3+700}$$

$$(9) \sum \cos\left(\frac{1}{n^2}\right)$$

$$(10) \sum \frac{1}{(1+1/n)^n}$$

$$(11) \sum \frac{\ln n}{n}$$

$$(12) \sum \frac{1}{n + \sqrt{n}}$$

$$(13) \sum \frac{1}{n^{10^n}}$$

Series convergence - ratio test

C = convergent AC = abs. convergent

D = divergent CC = conditionally convergent

Use the ratio test to determine the behavior of the following

① $\sum \frac{n^n}{2^n}$

② $\sum \frac{n!}{n^n}$

③ $\sum \frac{(n!)^2}{(2n)!}$

④ $\sum \frac{2^{3n}}{3^{2n}}$

~~14~~ Classify each as CC, AC, or D

⑤ $\sum (-1)^{n+1} \frac{1}{\sqrt{n+10}}$

⑬ $\sum (-1)^{n+1} \frac{1}{\sqrt[5]{n^4}}$

⑥ $\sum (-1)^{n+1} \frac{n}{n+2}$

⑦ $\sum (-1)^{n+1} \frac{2^n}{n!}$

⑧ $\sum (-1)^{n+1} \frac{\sqrt{n}}{\ln n}$

⑨ $\sum (-1)^{n+1} \frac{\ln n}{\sqrt{n}}$

⑩ $\sum (-1)^{n+1} \frac{\sqrt{n}}{n+3}$

⑪ $\sum (-1)^{n+1} \frac{2}{3n^2}$

⑫ $\sum (-1)^{n+1} \frac{\sin^2 n}{n^2}$