P.S. 6 (part 1!)

(1) Find the length of the spiral $r = a\theta^2$ where $0 \le \theta \le 2\pi$

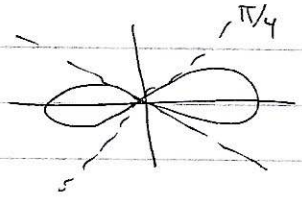
Answer: $\frac{a}{3}\left[\left(4 + 4\pi^2\right)^{3/2} - 8\right]$

(2) Find the surface area of the volume generated by rotating the circle $r = 2a\cos\theta$ about the y-axis

Answer: $4\pi^2 a^2$

(3) Find the area enclosed by the lemniscate

$r^2 = 2a^2 \cos 2\theta$



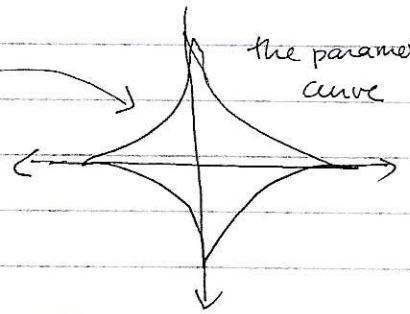
Note that this curve is symmetric about both the x and the y axis and that

$$A = \int_0^{2\pi} dA = 4\int_0^{\pi/4} dA$$

Ans: $2a^2$

(4) Say $x = a\cos^3 t$ and $y = a\sin^3 t$

Find the area of the surface of revolution obtained by rotating the astroid curve → around the x-axis



Ans: $\frac{12\pi a^2}{5}$

Evaluate

(1) $\lim_{x \to 0^+} \frac{\ln(\sin^2 x)}{\ln x}$

(2) $\lim_{x \to \infty} x \sin(1/x)$

(3) $\lim_{x \to \infty} (x^2 - 1) e^{-x^2}$

(4) $\lim_{x \to 0^+} \sin x \ln x$

(5) $\lim_{x \to \infty} \left(\frac{x^2}{x-1} - \frac{x^2}{x+1} \right)$

(6) $\lim_{x \to 0} x^{\tan x}$

(7) $\lim_{x \to \infty} (1 + e^{ax})^{1/x}$

(8)

(1) $\int_1^{\infty} \frac{1}{x^2} \sin\left(\frac{1}{x}\right) dx$

(2) $\int_0^2 \frac{\ln x \, dy}{\sqrt{x}}$

(3) $\int_2^{\infty} e^{-3x} dx$

(4) $\int_4^{\infty} \frac{dx}{x\sqrt{x}}$

(5) $\int_0^{\infty} \frac{x \, dx}{x^4+1}$

(6) $\int_2^{\infty} \frac{dx}{4+x^2}$

(7) $\int_0^{\infty} \frac{x^2 dx}{e^{x^3}}$

(8) $\int_e^{\infty} \frac{dx}{x \ln x \sqrt{\ln x}}$

(9) $\int_0^3 \frac{x \, dx}{\sqrt{9-x^2}}$