# HW4 Answers

① $\int \frac{x^2\,dx}{6x-x^2} = \frac{27}{2}\sin^{-1}\left(\frac{x-3}{3}\right)+C$

② $\int \frac{dx}{\sqrt{x^2-2x-8}} = \ln\left(x-1+\sqrt{x^2-2x-8}\right)+C$

IBP [FYI $\int \sin^n dx = -\frac{1}{n}\sin^{n-1}(x)\cos x + \frac{n-1}{n}\int \sin^{n-2}(x)dx$]

③ $\int x\ln x\,dx = \frac{1}{2}x^2\ln x - \frac{1}{4}x^2 + C$

④ $\int e^x \sin x\,dx = \frac{1}{2}e^x(\sin x - \cos x)+C$

⑤ $\int \ln(a^2+x^2)dx = x\ln(a^2+x^2) - 2x + 2a\tan^{-1}\left(\frac{x}{a}\right)+C$

⑥ $\int x^2\ln x\,dx = \frac{x^3}{3}\ln x - \frac{1}{9}x^3 + C$

⑦ – ⑪

Show that the following reduction formulas are true

⑦ $\int (\ln x)^n dx = x(\ln x)^n - n\int (\ln x)^{n-1}dx$

⑧ $\int x^n \cos x\,dx = x^n \sin x - n\int x^{n-1}\sin x\,dx$

⑨ $\int x^n \sin x\,dx = -x^n\cos x + n\int x^{n-1}\cos x\,dx$

⑩ $\int x^n e^x dx = x^n e^x - n\int x^{n-1}e^x dx$

⑪ $\int \cos^n x\,dx = \frac{1}{n}\cos^{n-1}(x)\sin x + \frac{n-1}{n}\int \cos^{n-2}(x)\,dx$

Hint: set $u = \cos^{n-1}(x)$ and $dv = \cos x\,dx$

⑫ – ⑭ ✓✓✓

Use the reduction formulas to find

⑫ $\int (\ln x)^5 dx = x(\ln x)^5 - 5x(\ln x)^4 + 20x(\ln x)^3 - 60x(\ln x)^2 + 120x(\ln x) - 120x + C$

⑬ $\int x^4 \sin x\,dx = (-x^4 + 12x^2 - 24)\cos x + (4x^3 - 24x)\sin x + C$

⑭ $\int \cos^4 x\,dx = \frac{1}{4}\cos^3 x\sin x + \frac{3}{8}\cos x\sin x + \frac{3}{8}x + C$