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Calculus III, Math 123
Exam 1

1. Verify that the given functions are solutions of the differential equation $y'' - 3y' + 2y = 0$

$$y = e^x, \quad y = e^{2x}$$

2. Solve the given differential equations by the method of separation of variables:

(a) $\dfrac{dy}{dx} = (1+y)x^3$ (b) $y' = 4y^3\cos(2x);\ y(\pi/8) = 1$

3. Solve the following differential equations using the method of first-order linear equations:

(a) $\dfrac{dy}{dx} + 5y = e^{-x}$ (b) $\dfrac{dy}{dx} + y - \dfrac{1}{1+2e^x} = 0$

4. Find the general term of the sequence, starting with n = 1; determine whether the sequence converges and if so find its limit (provide details of analysis)

(a) $\dfrac{1}{3}, -\dfrac{2}{5}, \dfrac{3}{7}, -\dfrac{4}{9}, \ldots$ (b) $\dfrac{3}{2^2-1^2}, \dfrac{4}{3^2-2^2}, \dfrac{5}{4^2-3^2}, \ldots$

5. Use the difference $a_{n+1} - a_n$ to show that the given sequence $\{a_n\}$ is strictly increasing or strictly decreasing (show all details).

(a) $\{n - n^2\}_{n=1}^{+\infty}$ (b) $\left\{\dfrac{n}{2n+1}\right\}_{n=1}^{+\infty}$

6. Use the ratio a_{n+1}/a_n to show that the given sequence $\{a_n\}$ is strictly increasing or strictly decreasing (show all details).

(b) $\left\{\dfrac{n}{2n+1}\right\}_{n=1}^{+\infty}$ (b) $\left\{\dfrac{20^n}{(2n)!}\right\}_{n=1}^{+\infty}$

7. Use differentiation to show that the given sequence is strictly increasing or strictly decreasing

(a) $\left\{\dfrac{\ln(n+2)}{n+2}\right\}_{n=1}^{+\infty}$ (b) $\{ne^{-2n}\}_{n=1}^{+\infty}$

8. Determine whether the series converges, and if so find its sum

(a) $\sum\limits_{k=1}^{+\infty}\left(\dfrac{2}{3}\right)^{k+2}$ (b) $\sum\limits_{k=1}^{+\infty}\dfrac{1}{k(k+1)}$

9. Apply ONLY the divergence test and state what it tells you about the series

(a) $\sum\limits_{k=1}^{+\infty}\dfrac{k}{4k^3+1}$ (b) $\sum\limits_{k=1}^{+\infty}\dfrac{k^2}{\sqrt{k^4}+1}$

10. Use the integral test to analyze the following series for convergence

(a) $\sum\limits_{k=1}^{+\infty}\dfrac{k}{k^2+1}$ (b) $\sum\limits_{k=1}^{+\infty}\dfrac{1}{5k+2}$