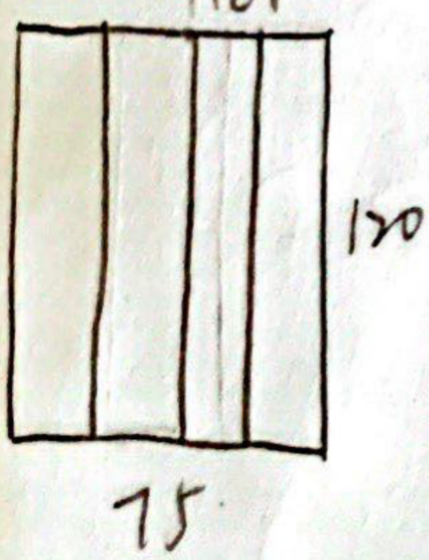
Module 9

4.7

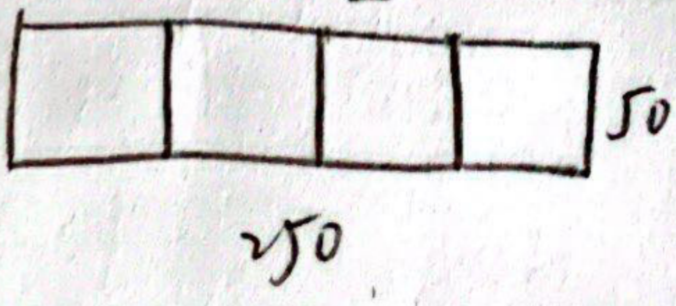
8. if the rectangle has dimensions x and y
its area is $xy = 1000 m^2$ $y = \frac{1000}{x}$
the perimeter $P = 2x + 2y = 2x + \frac{2000}{x}$
to minimize $P(x) = 2x + \frac{2000}{x}$ for $x > 0$
$P'(x) = 2 - \frac{2000}{x^2} = \frac{2}{x^2}(x^2 - 1000)$ the only critical number in the domain of P
is $x = \sqrt{1000}$
$P''(x) = \frac{4000}{x^3} > 0$ P is concave upward throughout its domain and
$P(\sqrt{1000}) = 4\sqrt{1000}$ is an absolute minimum value
the dimensions of the rectangle with minimal perimeter are: $x = y = \sqrt{1000} = 10\sqrt{10}$ m

11. (a)

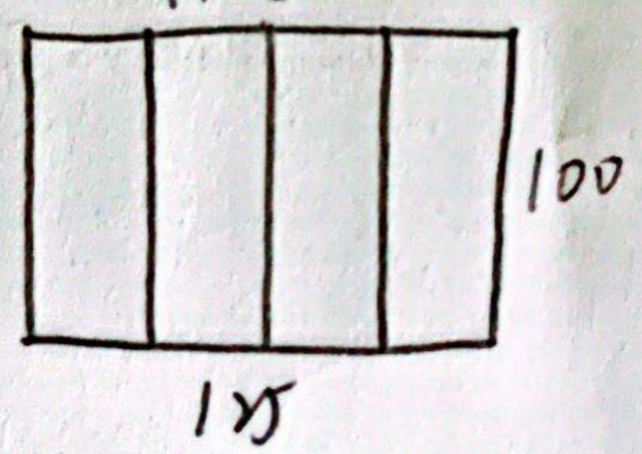
Pic 1



Pic 2



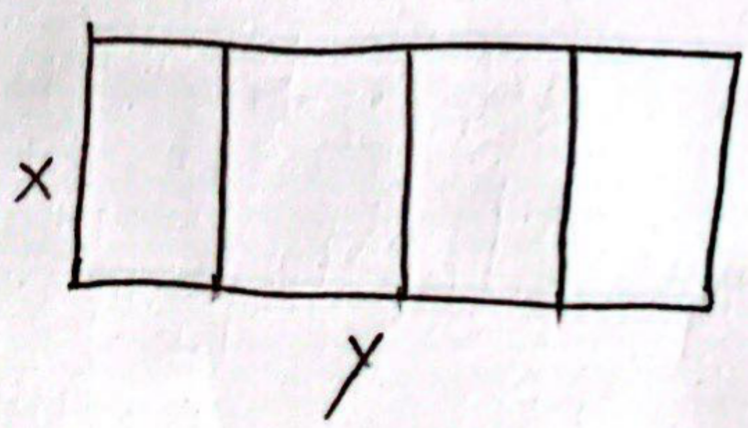
Pic 3



$120 \cdot 75 = 9000$ $\quad 250 \cdot 50 = 12500$ $\quad 125 \cdot 100 = 12500$

a maximum area is at lease 12500 ft^2

(b) Let x denote the length of each two sides and three dividers
— y ——— the other two sides



(c) $A = xy$

(d) Length of fencing:
$5x + 2y = 750$

(e) $y = 375 - \frac{5}{2}x$ $\quad A(x) = (375 - \frac{5}{2}x)x = 375x - \frac{5}{2}x^2$.

(f) $A'(x) = 375 - 5x = 0 \quad x = 75$.

$\because A''(x) = -5 < 0$ there is an absolute maximum when $x = 75$

$y = \frac{375}{2} = 187.5$

the largest area is $75 \cdot 187.5 = 14062.5 \; ft^2$

these value of x any y are between the value of Pic 2 and Pic 3 in (a)

the original estimate is low

14.

Let b be the length of the base and h be the height.

volume: $b^2 h = 32000 \quad h = \frac{32000}{b^2}$

the surface of the open box: $S = b^2 + 4hb = b^2 + 4\left(\frac{32000}{b^2}\right)b = b^2 + \frac{4 \cdot 32000}{b}$

$S'(b) = 2b - \frac{4 \cdot 32000}{b^2} = \frac{2(b^3 - 64000)}{b^2} = 0 \quad b = \sqrt[3]{64000} = 40$

$\because S'(b) < 0$ if $0 < b < 40 \quad S'(b) > 0$ if $b > 40$

there is an absolute minimum

the box should be $40 \cdot 40 \cdot 20$

15. Let b be the length of the base and h be the height.

the surface: $b^2 + 4hb = 1200 \quad h = \frac{1200 - b^2}{4b}$

the volume: $V = b^2 h = b^2 \frac{1200 - b^2}{4b} = 300b - \frac{b^3}{4}$

$V'(b) = 300 - \frac{3}{4}b^2 \quad V'(b) = 0 \quad b = \sqrt{400} = 20$

$\because V'(b) > 0$ if $0 < b < 20 \quad V'(b) < 0$ if $b > 20$

there is an absolute maximum when $b = 20$

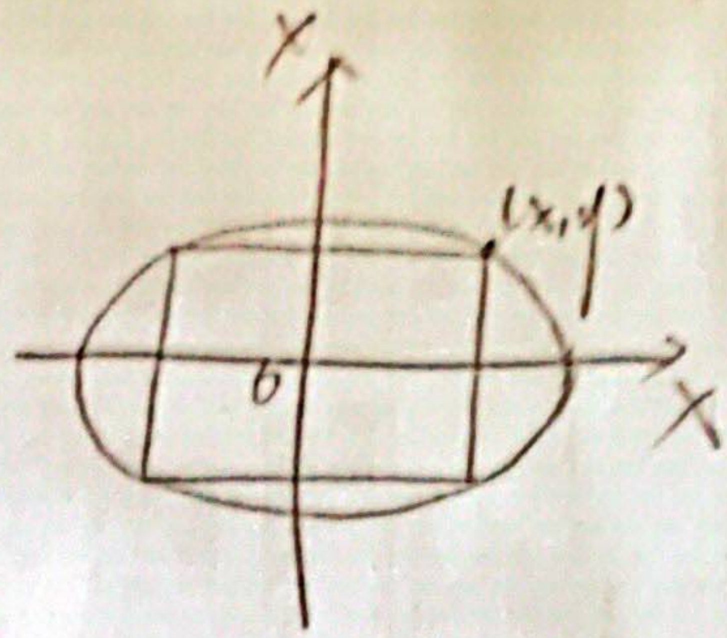
if $b = 20$, $h = \frac{1200 - 20^2}{4 \cdot 20} = 10$ the largest possible volum is

$b^2 h = 20^2 \cdot 10 = 4000 \; cm^3$

$200+6x^{\frac{1}{2}} = \frac{16000}{x} + 200 + 4x^{\frac{1}{2}}$

26. area of rectangle $2x \cdot 2y = 4xy$

$\because a=3 \quad b=2 \quad \frac{x^2}{9}+\frac{y^2}{4}=1 \quad y=\frac{2}{3}\sqrt{9-x^2}$



Maximize $A(x) = 4 \cdot \frac{2}{3} x \sqrt{9-x^2}$

$A'(x) = \frac{8}{3}[x \cdot \frac{1}{2}(9-x^2)^{-\frac{1}{2}}(-2x) + (9-x^2)^{\frac{1}{2}} \cdot 1]$

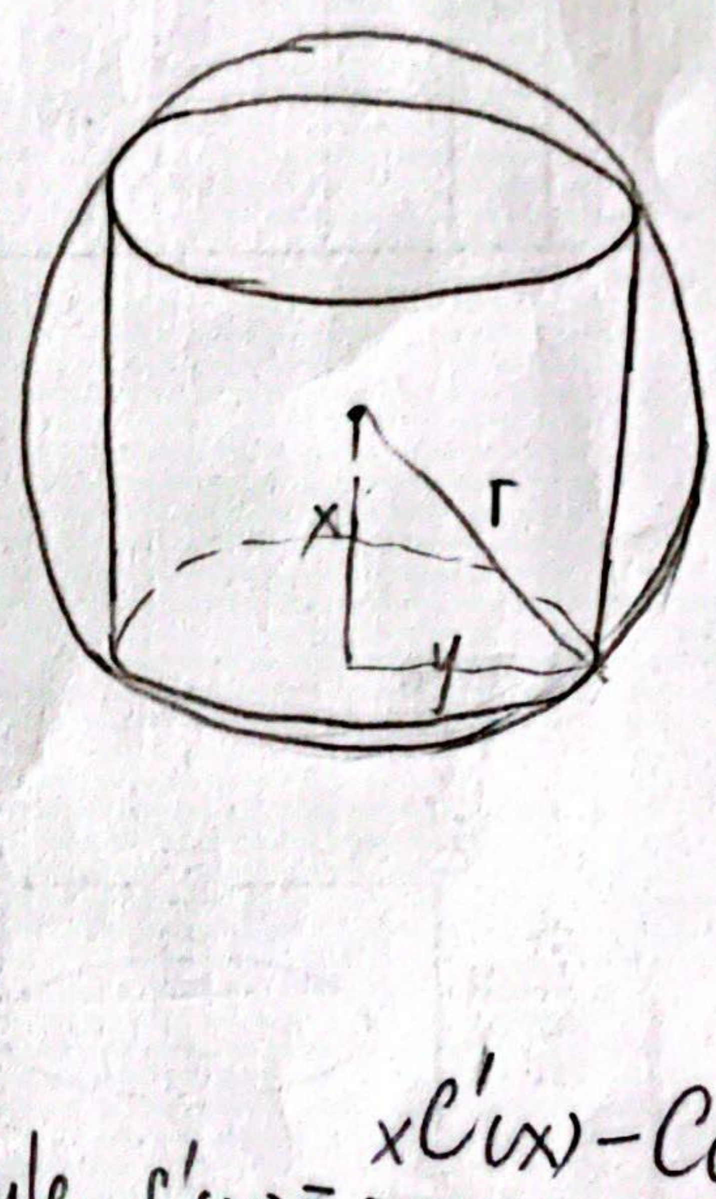
$= \frac{8}{3\sqrt{9-x^2}}[9-2x^2]$

the critical number is $x = \frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$ it gives maximum

$\because y = \frac{2}{\sqrt{2}} = \sqrt{2}$ $\therefore$ the maximum area is $4 \cdot \frac{3\sqrt{2}}{2} \cdot \sqrt{2} = 12$

31. the volume: $V = \pi y^2 \cdot 2x$

$x^2+y^2=r^2 \quad y^2=r^2-x^2$



$V(x) = \pi(r^2-x^2)2x = 2\pi(r^2x-x^3)$

when $0 \le x \le r$

$V'(x) = 2\pi(r^2-3x^2)=0 \quad x=\frac{r}{\sqrt{3}}$

$V(0)=V(r)=0$

there is a maximum when $x=\frac{r}{\sqrt{3}}$

$V(\frac{r}{\sqrt{3}}) = \pi(r^2-\frac{r^2}{3})\frac{2r}{\sqrt{3}} = \frac{4\pi r^3}{3\sqrt{3}}$

59. (a) $c(x) = \frac{C(x)}{x}$ by the Quotient Rule $c'(x) = \frac{xC'(x)-C(x)}{x^2}$

$c'(x)=0$ when $xC'(x)-C(x)=0$ this gives $C'(x) = \frac{C(x)}{x} = c(x)$

$\therefore$ the marginal cost = the average cost

(b) (i) $C(x) = 16000+200x+4x^{\frac{3}{2}}$ $C(1000) = 16000+200000+40000\sqrt{10}$

≈ 342491 dollars

$c(x) = \frac{C(x)}{x} = \frac{16000}{x} + 200 + 4x^{\frac{1}{2}}$ $c(1000) \approx \$342.49$/unit

$C'(x) = 200+6x^{\frac{1}{2}}$ $C'(1000) = 200+6\sqrt{10} \approx \$389.74$/unit

(ii) $C'(x) = c(x)$

$200 + 6x^{\frac{1}{2}} = \frac{16000}{x} + 200 + 4x^{\frac{1}{2}}$

$2x^{\frac{3}{2}} = 16000$

$x = 8000^{\frac{2}{3}} = 400$ units

$c'(x) = \frac{-16000}{x^2} + \frac{2}{\sqrt{x}}$

$= \frac{2}{x^2}(x^{\frac{3}{2}} - 8000)$

this is negative for $x < 8000^{\frac{2}{3}} = 400$

0 at $x = 400$ and positive for $x > 400$

∴ c is decreasing on $(0, 400)$

increasing on $(400, \infty)$

c has an absolute minimum at $x = 400$

(iii) the minimum average cost: $c(400) = 40 + 200 + 80 = \$320$ unit

60. (a) total profit: $P(x) = R(x) - C(x)$

to maximize profit, we need the critical numbers of P, which is the numbers where the marginal profit is 0.

if $P'(x) = R'(x) - C'(x) = 0$, $R'(x) = C'(x)$

if the profit is a maximum, the marginal revenue = the marginal cost

(b) $C(x) = 16000 + 500x - 1.6x^2 + 0.004x^3$ $p(x) = 1700 - 7x$

$R(x) = xp(x) = 1700x - 7x^2$

if the profit is a maximum $R'(x) = C'(x)$

$1700 - 14x = 500 - 3.2x + 0.012x^2$

$0.012x^2 + 10.8x - 1200 = 0$

$x^2 + 900x - 100000 = 0$

$(x + 1000)(x - 100) = 0$

$x = 100$ ∵ $x > 0$

the profit is maximized when $P''(x) < 0$

∵ $P''(x) = R''(x) - C''(x)$ ∴ we can check $R''(x) < C''(x)$

$R''(x) = -14 < -3.2 + 0.024x = C''(x)$ for $x > 0$

there is a maximum at $x = 100$

61 (a) ∵ the demand function p is linear and $p(27000)=10$
$p(33000)=8$

∴ the slope is $\frac{10-8}{27000-33000} = -\frac{1}{3000}$

the equation is $y-10=(-\frac{1}{3000})(x-27000)$

$y=p(x)=-\frac{1}{3000}x+19$

(b) the revenue: $R(x)=xp(x)=19x-\frac{x^2}{3000}$

$R'(x)=19-\frac{x}{1500}=0 \quad x=28500$

∵ $R''(x)=-\frac{1}{1500}<0$ the maximum revenue occurs when $x=28500$

the price is $p(28500)=\$9.50$

4.8

6. $f(x)=2x^3-3x^2+2 \quad f'(x)=6x^2-6x$

$x_{n+1}=x_n-\frac{2x_n^3-3x_n^2+2}{6x_n^2-6x_n}$

$x_1=-1 \quad x_2=-1-\frac{2(-1)^3-3(-1)^2+2}{6(-1)^2-6(-1)}=-1-\frac{-3}{12}=-\frac{3}{4}$

$x_3=-\frac{3}{4}-\frac{2(-\frac{3}{4})^3-3(-\frac{3}{4})^2+2}{6(-\frac{3}{4})^2-6(-\frac{3}{4})}=-\frac{3}{4}-\frac{\frac{17}{32}}{\frac{63}{8}}=-\frac{43}{63}\approx-0.6825$

7. $f(x)=\frac{2}{x}-x^2+1 \quad f'(x)=-\frac{2}{x^2}-2x$

$x_{n+1}=x_n-\frac{\frac{2}{x_n}-x_n^2+1}{-\frac{2}{x_n^2}-2x_n}$

$x_1=2 \quad x_2=2-\frac{1-4+1}{-\frac{1}{2}-4}=\frac{14}{9} \quad x_3=\frac{14}{9}-\frac{\frac{2}{\frac{14}{9}}-(\frac{14}{9})^2+1}{\frac{-2}{(\frac{14}{9})^2}-2\cdot\frac{14}{9}}\approx1.5215$

4.9

7. $f(x)=7x^{\frac{2}{5}}+8x^{-\frac{4}{5}}$ $\quad F(x)=7(\frac{5}{7}x^{\frac{7}{5}})+8(5x^{\frac{1}{5}})+C=5x^{\frac{7}{5}}+40x^{\frac{1}{5}}+C$

8. $f(x)=x^{3.4}-2x^{\sqrt{2}-1}$ $\quad F(x)=\frac{x^{4.4}}{4.4}-2(\frac{x^{\sqrt{2}}}{\sqrt{2}})+C=\frac{5}{22}x^{4.4}-\sqrt{2}x^{\sqrt{2}}+C$

10. $f(x)=e^2$ is a constant function. $F(x)=e^2x+C$

12. $f(x)=\sqrt[3]{x^2}+x\sqrt{x}=x^{\frac{2}{3}}+x^{\frac{3}{2}}$ $\quad F(x)=\frac{3}{5}x^{\frac{5}{3}}+\frac{2}{5}x^{\frac{5}{2}}+C$

16. $r(\theta)=\sec\theta\tan\theta-2e^{\theta}$ $\quad R(\theta)=\sec\theta-2e^{\theta}+C_n$ on $(n\pi-\frac{\pi}{2}, n\pi+\frac{\pi}{2})$

17. $h(\theta)=2\sin\theta-\sec^2\theta$ $\quad H(\theta)=-2\cos\theta-\tan\theta+C_n$ on $(n\pi-\frac{\pi}{2}, n\pi+\frac{\pi}{2})$

25. $f''(x)=20x^3-12x^2+6x$ $\quad f'(x)=20(\frac{x^4}{4})-12(\frac{x^3}{3})+6(\frac{x^2}{2})+C$

$=5x^4-4x^3+3x^2+C$

$f(x)=5(\frac{x^5}{5})-4(\frac{x^4}{4})+3(\frac{x^3}{3})+Cx+D=x^5-x^4+x^3+Cx+D$

26. $f''(x)=x^6-4x^4+x+1$ $\quad f'(x)=\frac{1}{7}x^7-\frac{4}{5}x^5+\frac{1}{2}x^2+x+C$

$f(x)=\frac{1}{56}x^8-\frac{2}{15}x^6+\frac{1}{6}x^3+\frac{1}{2}x^2+Cx+D$

28. $f''(x)=\frac{1}{x^2}=x^{-2}$ $\quad f'(x)=\begin{cases}-\frac{1}{x}+C_1 & x<0\\ -\frac{1}{x}+C_2 & x>0\end{cases}$ $\quad f(x)=\begin{cases}-\ln(-x)+C_1x+D_1 & x<0\\ -\ln x+C_2x+D_2 & x>0\end{cases}$

32. $f'(x)=5x^4-3x^2+4$ $\quad f(x)=x^5-x^3+4x+C$

$f(-1)=-1+1-4+C$ $\quad f(-1)=2$ $\quad -1+1-4+C=2$ $\quad C=6$

$f(x)=x^5-x^3+4x+6$

40. $f''(x)=8x^3+5$ $\quad f'(x)=2x^4+5x+C$ $\quad f'(1)=2+5+C$ $\quad f'(1)=8$ $\quad C=1$

$f'(x)=2x^4+5x+1$ $\quad f(x)=\frac{2}{5}x^5+\frac{5}{2}x^2+x+D$ $\quad f(1)=\frac{2}{5}+\frac{5}{2}+1+D$

$=D+\frac{39}{10}$

$f(1)=0$ $\quad D=-\frac{39}{10}$ $\quad f(x)=\frac{2}{5}x^5+\frac{5}{2}x^2+x-\frac{39}{10}$