Module 6.

1.5

5. First, draw a horizontal line and intersect it with the graph in more than 1 points.
Then, by the Horizontal Line Test, it is not a one-to-one function.

6. First, draw a horizontal line, but it doesnot intersects with the graph more than one point.
Then, by the Horizontal Line Test, it is a one-to-one function.

15. (a) $\because f$ is 1-1, $f(6)=17$ $\therefore f^{-1}(17)=6$

(b) $\because f$ is 1-1, $f^{-1}(3)=2$ $\therefore f(2)=3$

25. $y=f(x)=\ln(x+3)$

$x+3=e^{y}$

$x=e^{y}-3$

$y=e^{x}-3$

$\therefore f^{-1}(x)=e^{x}-3$

36. (a) $\log_5\frac{1}{125}=\log_5\frac{1}{5^3}=\log_5 5^{-3}=-3$ by (7)

(b) $\ln\left(\frac{1}{e^2}\right)=\ln e^{-2}=-2$ by (9)

38. (a) $e^{-\ln 2} = \frac{1}{e^{\ln 2}} = \frac{1}{2}$ by (9)

(b) $e^{\ln(\ln e^3)} = e^{\ln 3}$ [by (9)] $= 3$ by (9)

41. $\frac{1}{3}\ln(x+2)^3 + \frac{1}{2}[\ln x - \ln(x^2+3x+2)^2]$

$= \ln[(x+2)^3]^{\frac{1}{3}} + \frac{1}{2}\ln\frac{x}{(x^2+3x+2)^2}$

$= \ln(x+2) + \ln\frac{\sqrt{x}}{x^2+3x+2}$

$= \ln\frac{(x+2)\sqrt{x}}{(x+1)(x+2)}$

$= \ln\frac{\sqrt{x}}{x+1}$

$\because x>0$ for $\ln x$

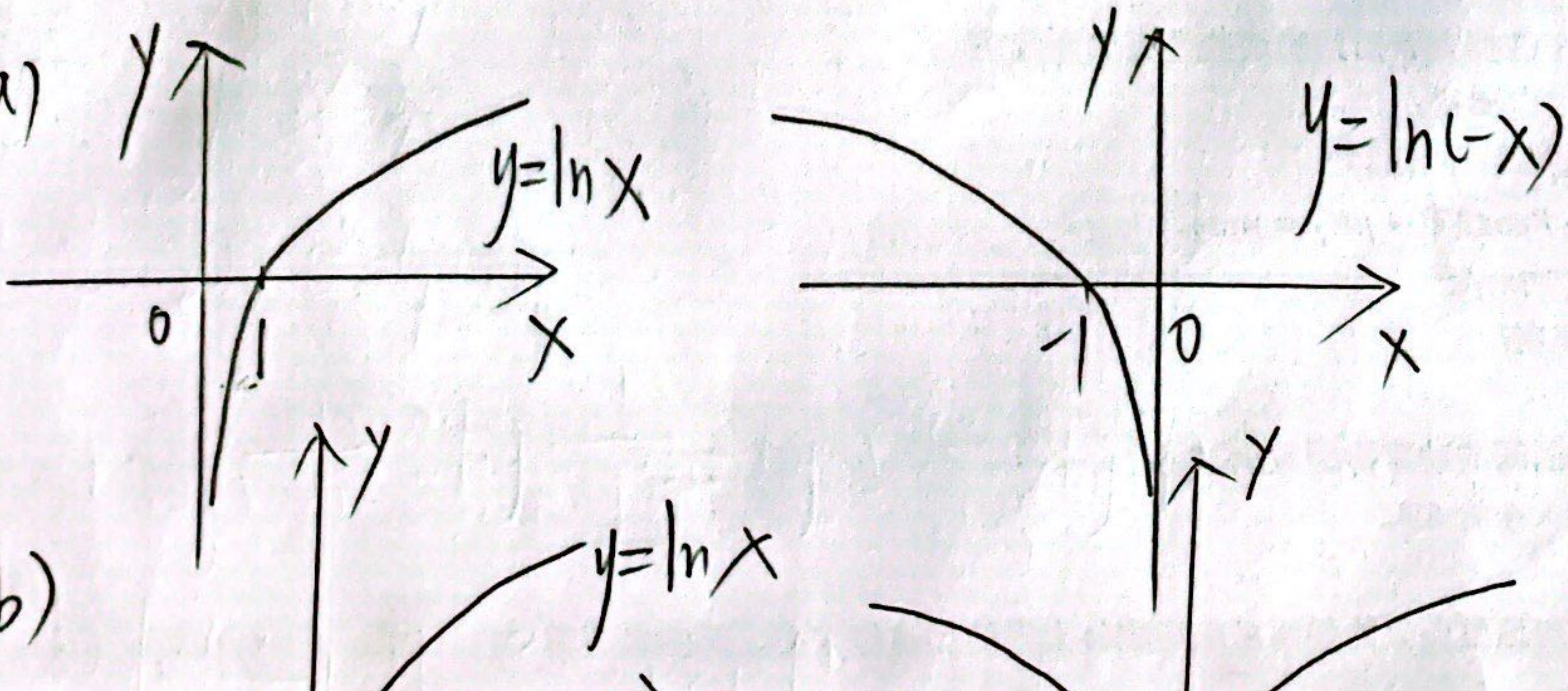
$\therefore$ $x+1$, $x+2$, x^2+3x+2 are positive

$\therefore$ the logarithms are defined

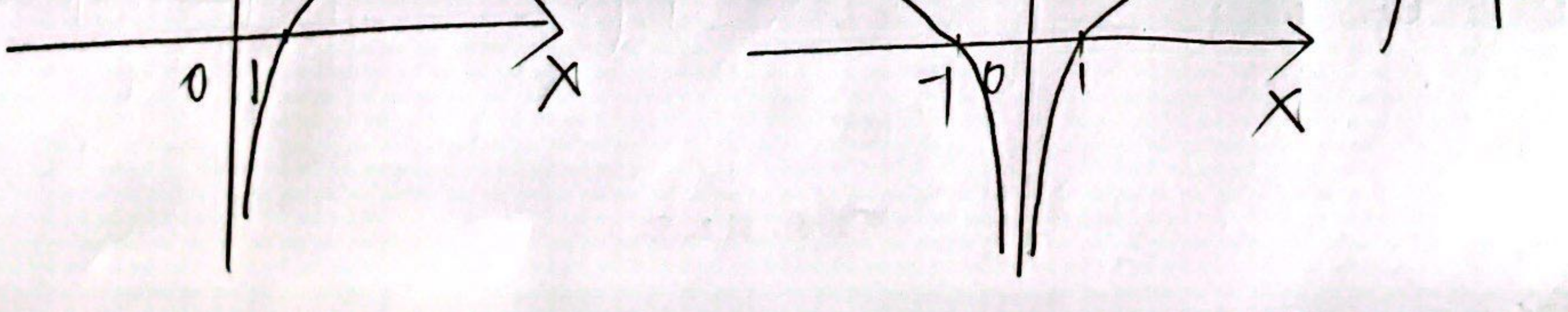
42. (a) $\log_5 10 = \frac{\ln 10}{\ln 5}$ [by (10)] ≈ 1.430677

(b) $\log_3 57 = \frac{\ln 57}{\ln 3}$ [by (10)] ≈ 3.680144

48. (a)



(b)



51. (a) $e^{7-4x}=6$

$7-4x=\ln 6$

$7-\ln 6=4x$

$x=\frac{1}{4}(7-\ln 6)$

(b) $\ln(3x-10)=2$

$3x-10=e^2$

$3x=e^2+10$

$x=\frac{1}{3}(e^2+10)$

3.6

2. $f(x)=x\ln x-x$

$f'(x)=x\cdot\frac{1}{x}+\ln x\cdot 1-1=\ln x$

4. $f(x)=\ln(\sin^2 x)=\ln(\sin x)^2=2\ln|\sin x|$

$f'(x)=2\cdot\frac{1}{\sin x}\cdot\cos x=2\cot x$

5. $f(x)=\ln\frac{1}{x}$

$f'(x)=\frac{1}{\frac{1}{x}}\frac{d}{dx}\left(\frac{1}{x}\right)=x\left(-\frac{1}{x^2}\right)=-\frac{1}{x}$

6. $y=\frac{1}{\ln x}=(\ln x)^{-1}$

$y'=-1(\ln x)^{-2}\cdot\frac{1}{x}=-\frac{1}{x(\ln x)^2}$

12. $h(x)=\ln(x+\sqrt{x^2-1})$

$h'(x)=\frac{1}{x+\sqrt{x^2-1}}\cdot\left(1+\frac{x}{\sqrt{x^2-1}}\right)=\frac{1}{\sqrt{x^2-1}}$

13. $G(y)=\ln\frac{(2y+1)^5}{\sqrt{y^2+1}}=\ln(2y+1)^5-\ln(y^2+1)^{\frac{1}{2}}=5\ln(2y+1)-\frac{1}{2}\ln(y^2+1)$

$G'(y)=5\cdot\frac{1}{2y+1}\cdot 2-\frac{1}{2}\cdot\frac{1}{y^2+1}\cdot 2y=\frac{10}{2y+1}-\frac{y}{y^2+1}$

20. $H(z)=\ln\sqrt{\dfrac{a^2-z^2}{a^2+z^2}}=\ln\left(\dfrac{a^2-z^2}{a^2+z^2}\right)^{\frac{1}{2}}=\dfrac{1}{2}\ln\left(\dfrac{a^2-z^2}{a^2+z^2}\right)$

$=\frac{1}{2}\ln(a^2-z^2)-\frac{1}{2}\ln(a^2+z^2)$

$H'(z)=\dfrac{1}{2}\cdot\dfrac{1}{a^2-z^2}\cdot(-2z)-\dfrac{1}{2}\cdot\dfrac{1}{a^2+z^2}\cdot 2z=\dfrac{z(z^2+a^2)-z(z^2-a^2)}{(z^2-a^2)(z^2+a^2)}$

$=\dfrac{z^3+za^2-z^3+za^2}{(z^2-a^2)(z^2+a^2)}=\dfrac{2za^2}{z^4-a^4}$

39. $y=(x^2+2)^2(x^4+4)^4$

$\ln y=\ln[(x^2+2)^2(x^4+4)^4]$

$\ln y=2\ln(x^2+2)+4\ln(x^4+4)$

$y'=\left[2\cdot\dfrac{1}{x^2+2}\cdot 2x+4\cdot\dfrac{1}{x^4+4}\cdot 4x^3\right]y$

$y'=y\left(\dfrac{4x}{x^2+2}+\dfrac{16x^3}{x^4+4}\right)$

$y'=(x^2+2)^2(x^4+4)^4\left(\dfrac{4x}{x^2+2}+\dfrac{16x^3}{x^4+4}\right)$

43. $y=x^x$

$\ln y=\ln x^x$

$\ln y=x\ln x$

$y'=\left(x\cdot\dfrac{1}{x}+\ln x\cdot 1\right)y$

$y'=y(1+\ln x)$

$y'=x^x(1+\ln x)$

45. $y=x^{\sin x}$

$\ln y=\sin x\ln x$

$y'=\left(\sin x\cdot\dfrac{1}{x}+\ln x\cdot\cos x\right)y$

$y'=x^{\sin x}\left(\dfrac{\sin x}{x}+\ln x\cos x\right)$

3.8

3. (a) $P(t) = P(0)e^{kt} = 100e^{kt}$

$P(1) = 100e^{k(1)} = 420$

$e^k = 420/100$

$k = \ln 4.2$

$P(t) = 100e^{(\ln 4.2)t} = 100(4.2)^t$

(b) $P(3) = 100(4.2)^3 = 7408.8 \approx 7409$ bacteria

(c) $\frac{dP}{dt} = kP$

$P'(3) = k \cdot P(3) = \ln 4.2 \cdot 100(4.2)^3$

(d) $P(t) = 100(4.2)^t = 10{,}000$

$(4.2)^t = 100$

$t = \frac{\ln 100}{\ln 4.2} \approx 3.2h$

5. (a) $P(t) = P(1750)e^{k(t-1750)}$

$P(1800) = 980 = 790e^{k(1800-1750)}$

$e^{k(50)} = \frac{980}{790}$

$50k = \ln\frac{980}{790}$

$k = \frac{1}{50}\ln\frac{980}{790} \approx 0.0043104$

$P(1900) = 790e^{k(1900-1750)} \approx 1508$ million

$P(1950) = 790e^{k(1950-1750)}$

≈ 1871 million

$\therefore$ Both are too low.

8. (a) $y(t) = y(0)e^{kt} = 50e^{kt}$

$y(28) = 50e^{k(28)} = 25$

$e^{28k} = \frac{1}{2}$

$28k = \ln\frac{1}{2}$

$k = -\frac{\ln 2}{28}$

$y(t) = 50e^{-\frac{\ln 2}{28}t} = 50 \cdot 2^{-\frac{t}{28}}$

(b) $y(40) = 50 \cdot 2^{-\frac{40}{28}} \approx 18.6\,mg$

(c) $y(t) = 2$

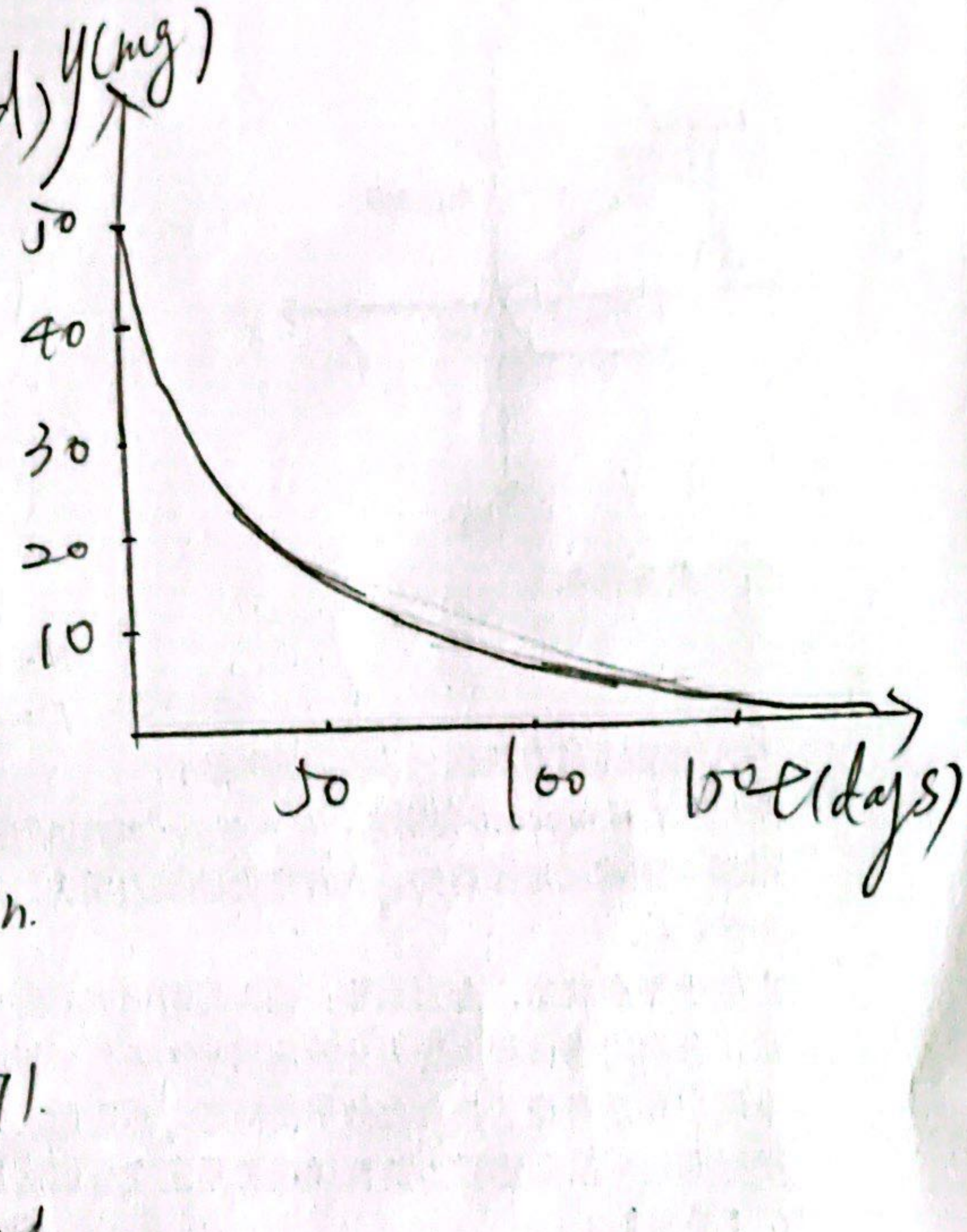
$50 \cdot 2^{-\frac{t}{28}} = 2$

$2^{-\frac{t}{28}} = \frac{2}{50}$

$-\frac{t}{28}\ln 2 = \ln\frac{1}{25}$

$t = \dfrac{-28\ln\frac{1}{25}}{\ln 2} \approx 130\text{ days}$

(d)



20 (a) $A = A_0\left(1+\frac{r}{n}\right)^{nt} = 1000\left(1+\frac{0.08}{n}\right)^{3n}$

(i) $n=1$ $A = 1000\left(1+\frac{0.08}{1}\right)^{3\cdot 1} = 1259.71$

(ii) $n=4$ $A = 1000\left(1+\frac{0.08}{4}\right)^{3\cdot 4} = 1268.24$

(iii) $n=12$ $A = 1000\left(1+\frac{0.08}{12}\right)^{3\cdot 12} = 1270.24$

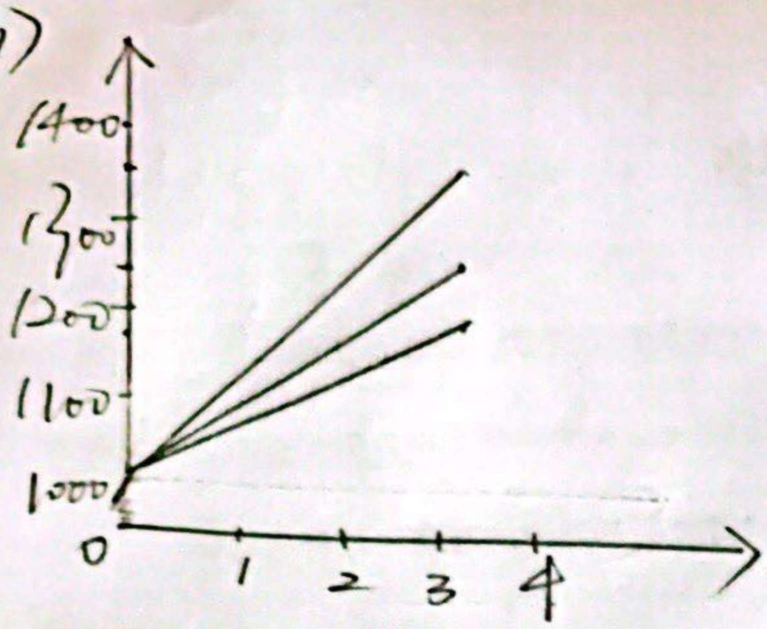
(iv) $n=52$ $A = 1000\left(1+\frac{0.08}{52}\right)^{3\cdot 52} = 1271.01$

(v) $n=365$ $A = 1000\left(1+\frac{0.08}{365}\right)^{3\cdot 365} = 1271.22$

(vi) $n=365.24$ $A = 1000\left(1+\frac{0.08}{365.24}\right)^{3\cdot 365.24} = 1271.25$

(vii) $A = 1000e^{3(0.08)} = 1271.25$

(b)



$A_{0.10}(3) = 1349.86$

$A_{0.08}(3) = 1271.25$

$A_{0.06}(3) = 1197.22$

22. (a) $A_0 e^{0.06t} = 2A_0$

$e^{0.06t} = 2$

$0.06t = \ln 2$

$t = \frac{50}{3}\ln 2 \approx 11.55$

3.11

2. (a) $\tanh 0 = \dfrac{\frac{e^0 - e^{-0}}{2}}{\frac{e^0 + e^{-0}}{2}} = 0$ (b) $\tanh 1 = \dfrac{e^1 - e^{-1}}{e^1 + e^{-1}} = \dfrac{e^2 - 1}{e^2 + 1} \approx 0.76159$

9. $\cosh x + \sinh x = \frac{1}{2}(e^x + e^{-x}) + \frac{1}{2}(e^x - e^{-x}) = \frac{1}{2} \cdot 2e^x = e^x$

15. $\because y = x$

$\therefore \sinh 2x = \sinh(x+x) = \sinh x \cosh x + \cosh x \sinh x = 2\sinh x \cosh x$

37. $y = e^{\cosh 3x}$

$y' = e^{\cosh 3x} \cdot \sinh 3x \cdot 3 = 3e^{\cosh 3x} \sinh 3x$

40. $y = \sinh^{-1}(\tan x)$

$$y' = \frac{1}{\sqrt{1+(\tan x)^2}} \cdot \frac{d}{dx}(\tan x)$$

$$= \frac{\sec^2 x}{\sqrt{\sec^2 x}} = \frac{|\sec^2 x|}{|\sec x|} = |\sec x|$$

41. $y = \cosh^{-1}\sqrt{x}$

$$y' = \frac{1}{\sqrt{(\sqrt{x})^2 - 1}} \frac{d}{dx}(\sqrt{x})$$

$$= \frac{1}{\sqrt{x-1}} \cdot \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x(x-1)}}$$