Module 5

1.5

63. (a) $\because \pi$ is in the range of $\cos^{-1}$ $[0,\pi]$

$\cos\pi = -1$

$\therefore \cos^{-1}(-1) = \pi$

(b) $\because \frac{\pi}{6}$ is in the range of $\sin^{-1}$ $[-\frac{\pi}{2},\frac{\pi}{2}]$

$\sin\frac{\pi}{6} = 0.5$

$\therefore \sin^{-1}(0.5) = \frac{\pi}{6}$

65. (a) $\because \frac{\pi}{4}$ is in the range of $\csc^{-1}$ $(0,\frac{\pi}{2}]\cup(\pi,\frac{3\pi}{2}]$

$\csc\frac{\pi}{4} = \sqrt{2}$

$\therefore \csc^{-1}\sqrt{2} = \frac{\pi}{4}$

(b) $\because \frac{\pi}{2}$ is in the range of arcsin $[-\frac{\pi}{2},\frac{\pi}{2}]$

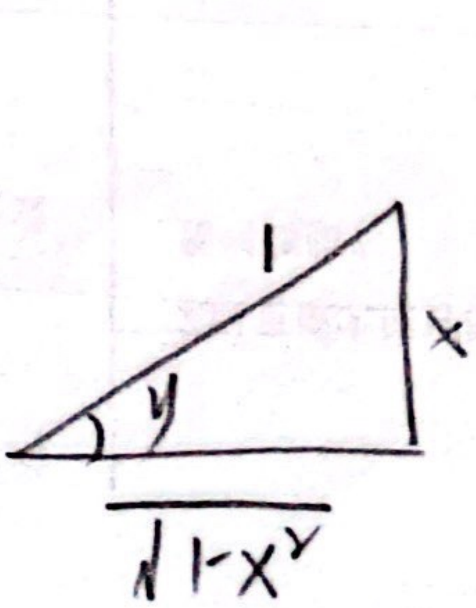
$\sin\frac{\pi}{2} = 1$

$\therefore \arcsin 1 = \frac{\pi}{2}$

70. Let $y = \sin^{-1}x$

$\sin y = x$

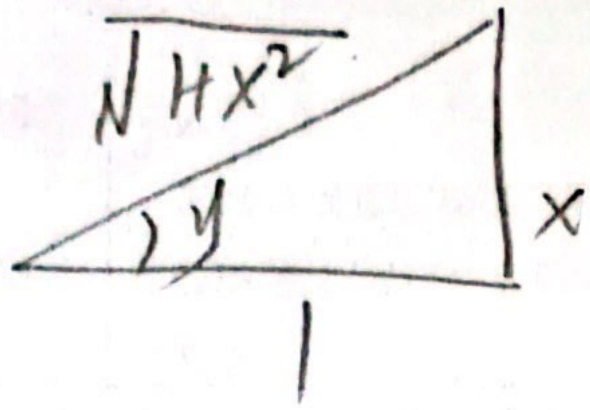
$\tan(\sin^{-1}x) = \tan y = \frac{x}{\sqrt{1-x^2}}$



71. Let $y = \tan^{-1} x$

$\tan y = x$

$\sin(\tan^{-1} x) = \sin y = \dfrac{x}{\sqrt{1+x^2}}$



3.4

2. Let $u = g(x) = 2x^3+5$

$y = f(u) = u^4$

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = 4u^3 \cdot 6x^2 = 24x^2(2x^3+5)^3$$

7. $F(x) = (5x^6+2x^3)^4$

$$F'(x) = 4(5x^6+2x^3)^3 \cdot \frac{d}{dx}(5x^6+2x^3)$$
$$= 4(5x^6+2x^3)^3(30x^5+6x^2)$$
$$= 24x^{11}(5x^3+2)^3(5x^3+1)$$

8. $F(x) = (1+x+x^2)^{99}$

$$F'(x) = 99(1+x+x^2)^{98} \cdot \frac{d}{dx}(1+x+x^2)$$
$$= 99(1+x+x^2)^{98}(1+2x)$$

9. $f(x) = \sqrt{5x+1}$

$$= (5x+1)^{\frac{1}{2}}$$

$$f'(x) = \frac{1}{2}(5x+1)^{-\frac{1}{2}} \times 5$$
$$= \frac{5}{2\sqrt{5x+1}}$$

17. $f(x)=(2x-3)^4(x^2+x+1)^5$

$f'(x)=(2x-3)^4\cdot 5(x^2+x+1)^4(2x+1)+(x^2+x+1)^5\cdot 4(2x-3)^3\cdot 2$

$=(2x-3)^3(x^2+x+1)^4[(2x-3)\cdot 5(2x+1)+(x^2+x+1)\cdot 8]$

$=(2x-3)^3(x^2+x+1)^4(20x^2-20x-15+8x^2+8x+8)$

$=(2x-3)^3(x^2+x+1)^4(28x^2-12x-7)$

18. $g(x)=(x^2+1)^3(x^2+2)^6$

$g'(x)=(x^2+1)^3\cdot 6(x^2+2)^5\cdot 2x+(x^2+2)^6\cdot 3(x^2+1)^2\cdot 2x$

$=6x(x^2+1)^2(x^2+2)^5[2(x^2+1)+(x^2+2)]$

$=6x(x^2+1)^2(x^2+2)^5(3x^2+4)$

19. $h(t)=(t+1)^{\frac{2}{3}}(2t^2-1)^3$

$h'(t)=(t+1)^{\frac{2}{3}}\cdot 3(2t^2-1)^2\cdot 4t+(2t^2-1)^3\cdot\frac{2}{3}(t+1)^{-\frac{1}{3}}$

$=\frac{2}{3}(t+1)^{-\frac{1}{3}}(2t^2-1)^2[18t(t+1)+(2t^2-1)]$

$=\frac{2}{3}(t+1)^{-\frac{1}{3}}(2t^2-1)^2(20t^2+18t-1)$

20. $F(t)=(3t-1)^4(2t+1)^{-3}$

$F'(t)=(3t-1)^4(-3)(2t+1)^{-4}\cdot 2+(2t+1)^{-3}\cdot 4(3t-1)^3\cdot 3$

$=6(3t-1)^3(2t+1)^{-4}[-(3t-1)+2(2t+1)]$

$=6(3t-1)^3(2t+1)^{-4}(t+3)$

24. $f(t) = 2^{t^3}$

$$f'(t) = 2^{t^3} \ln 2 \frac{d}{dt}(t^3)$$

$$= 3(\ln 2) t^2 2^{t^3}$$

28. $f(z) = e^{\frac{z}{z-1}}$

$$f'(z) = e^{\frac{z}{z-1}} \cdot \frac{d}{dz} \cdot \frac{z}{z-1}$$

$$= e^{\frac{z}{z-1}} \frac{(z-1)\cdot 1 - z\cdot 1}{(z-1)^2}$$

$$= -\frac{e^{\frac{z}{z-1}}}{(z-1)^2}$$

36. $y = x^2 e^{\frac{-1}{x}}$

$$y' = x^2 e^{\frac{-1}{x}}\left(\frac{1}{x^2}\right) + e^{\frac{-1}{x}} \cdot 2x$$

$$= e^{\frac{-1}{x}} + 2x e^{\frac{-1}{x}}$$

$$= e^{\frac{-1}{x}}(1+2x)$$

50. $y = e^{e^x}$

$$y' = e^{e^x} \cdot (e^x)'$$

$$= e^{e^x} \cdot e^x$$

$$y'' = e^{e^x} \cdot (e^x)' + e^x \cdot (e^{e^x})'$$

$$= e^{e^x} \cdot e^x + e^x \cdot e^{e^x} \cdot e^x$$

$$= e^{e^x} \cdot e^x (1+e^x)$$

57. $y=\sqrt{1+x^3}=(1+x^3)^{\frac{1}{2}}$

$y'=\frac{1}{2}(1+x^3)^{-\frac{1}{2}}\cdot 3x^2$

$=\frac{3x^2}{2\sqrt{1+x^3}}$

At $(2,3)$ $y'=\frac{3\times 4}{2\sqrt{9}}=2$

the equation of the tangent line: $y=2x-1$

63. (a) $h(x)=f(g(x))$

$h'(x)=f'(g(x))\cdot g'(x)$

$h'(1)=f'(g(1))\cdot g'(1)=f'(2)\cdot 6=5\times 6=30$

(b) $H(x)=g(f(x))$

$H'(x)=g'(f(x))\cdot f'(x)$

$H'(1)=g'(f(1))\cdot f'(1)=g'(3)\cdot 4=9\times 4=36$

(a) $4^x\ln(4)$ (b) $7^x\ln(7)$ (c) $2\cdot 3^{2x}\ln(3)$ (d) $6\cdot 5^{6x}\ln(5)$

3.5

5. $\frac{d}{dx}(x^2-4xy+y^2)=\frac{d}{dx}(4)$

$2x-4[xy'+y(1)]+2yy'=0$

$2yy'-4xy'=4y-2x$

$y'(y-2x)=2y-x$

$y'=\frac{2y-x}{y-2x}$

12. $\frac{d}{dx}\cos(xy)=\frac{d}{dx}(1+\sin y)$

$-\sin(xy)(xy'+y\cdot 1)=\cos y\cdot y'$

$-xy'\sin(xy)-\cos y\cdot y'=y\sin(xy)$

$y'[-x\sin(xy)-\cos y]=y\sin(xy)$

$y'=-\frac{y\sin(xy)}{x\sin(xy)+\cos y}$

10. $\frac{d}{dx}(xe^y)=\frac{d}{dx}(x-y)$

$xe^y y'+e^y\cdot 1=1-y'$

$xe^y y'+y'=1-e^y$

$y'(xe^y+1)=1-e^y$

$y'=\frac{1-e^y}{xe^y+1}$

26. $\sin(x+y)=2x-2y$

$\cos(x+y)\cdot(1+y')=2-2y'$

$\cos(x+y)\cdot y'+2y'=2-\cos(x+y)$

$y'[\cos(x+y)+2]=2-\cos(x+y)$

$y'=\frac{2-\cos(x+y)}{\cos(x+y)+2}$.

At (π,π), $y'=\frac{2-1}{1+2}=\frac{1}{3}$

the equation of the tangent line: $y=\frac{1}{3}x+\frac{2\pi}{3}$

11. $\frac{d}{dx}(y\cos x)=\frac{d}{dx}(x^2+y^2)$

$y\cdot(-\sin x)+\cos x\cdot y'=2x+2yy'$

$\cos x\cdot y'-2yy'=2x+y\sin x$

$y'(\cos x-2y)=2x+y\sin x$

$y'=\frac{2x+y\sin x}{\cos x-2y}$

33.(a) $y^2=5x^4-x^2$

$2yy'=5\cdot 4x^3-2x$

$y'=\frac{20x^3-2x}{2y}=\frac{10x^3-x}{y}$

At $(1,2)$, $y'=\frac{10\cdot 1^3-1}{2}=\frac{9}{2}$

the equation of the tangent line: $y=\frac{9}{2}x-\frac{5}{2}$

35. $x^2+4y^2=4$

$2x+8yy'=0$

$y'=\frac{-2x}{8y}=-\frac{x}{4y}$

$y''=-\frac{1}{4}\frac{y\cdot 1-x\cdot y'}{y^2}$

$=-\frac{1}{4}\frac{y-x(-\frac{x}{4y})}{y^2}$

$=-\frac{1}{4}\frac{4y^2+x^2}{4y^3}$

$=-\frac{1}{4y^3}$

50. $y=\tan^{-1}(x^2)$

$y'=\frac{1}{1+(x^2)^2}\cdot\frac{d}{dx}(x^2)$

$=\frac{1}{1+x^4}\cdot 2x$

$=\frac{2x}{1+x^4}$

51. $y=\sin^{-1}(2x+1)$

$y'=\frac{1}{\sqrt{1-(2x+1)^2}}\cdot\frac{d}{dx}(2x+1)$

$=\frac{1}{\sqrt{1-(4x^2+4x+1)}}\cdot 2$

$=\frac{2}{\sqrt{-x^2-x}}$

74. (a) $x^2-xy+y^2=3$

$y'=\frac{y-2x}{2y-x}$

At $(-1,1)$, the slope of the tangent line: $\frac{1-2\cdot(-1)}{2\cdot 1-(-1)}=1$

the slope of the normal line: $-\frac{1}{1}=-1$

the equation: $y=-x$

$\therefore x^2-x\cdot(-x)+(-x)^2=3$

$x^2+x^2+x^2=3$

$3x^2=3$

$x=\pm 1$

the normal line intersect the ellipse a second time at $x=1$

$\because y=-x$

$\therefore$ the other intersection is $(1,-1)$