Module 12

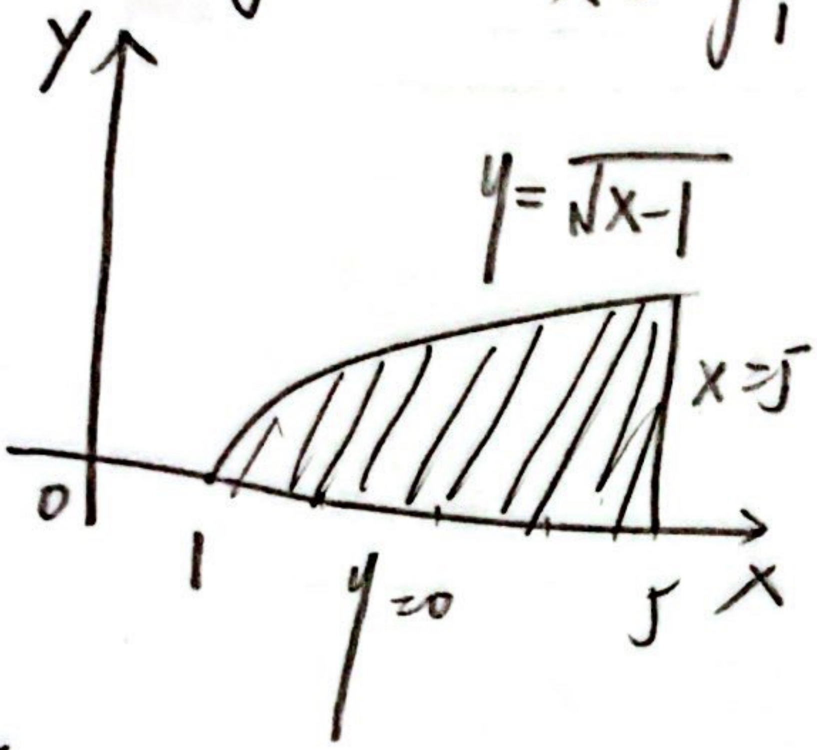
6.2

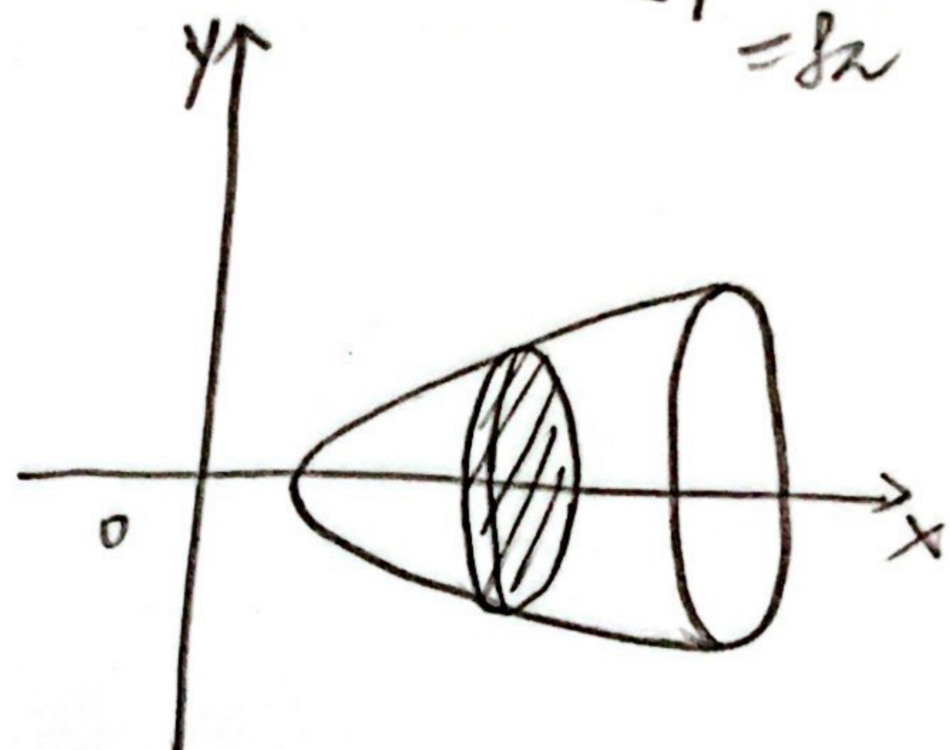
3. $\because$ a cross section is a disk with radius $\sqrt{x-1}$

$\therefore A(x) = \pi(\sqrt{x-1})^2 = \pi(x-1)$

$V = \int_1^5 A(x)\,dx = \int_1^5 \pi(x-1)\,dx = \pi\left[\frac{1}{2}x^2 - x\right]_1^5 = \pi\left[\left(\frac{25}{2}-5\right)-\left(\frac{1}{2}-1\right)\right]$

$= 8\pi$

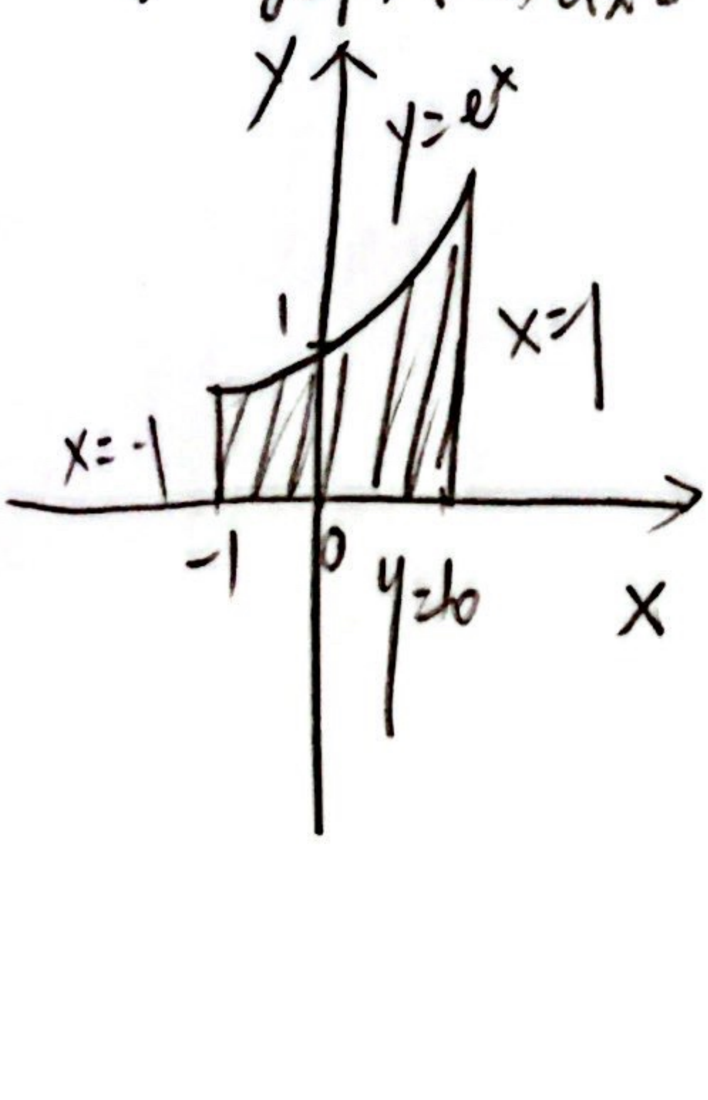


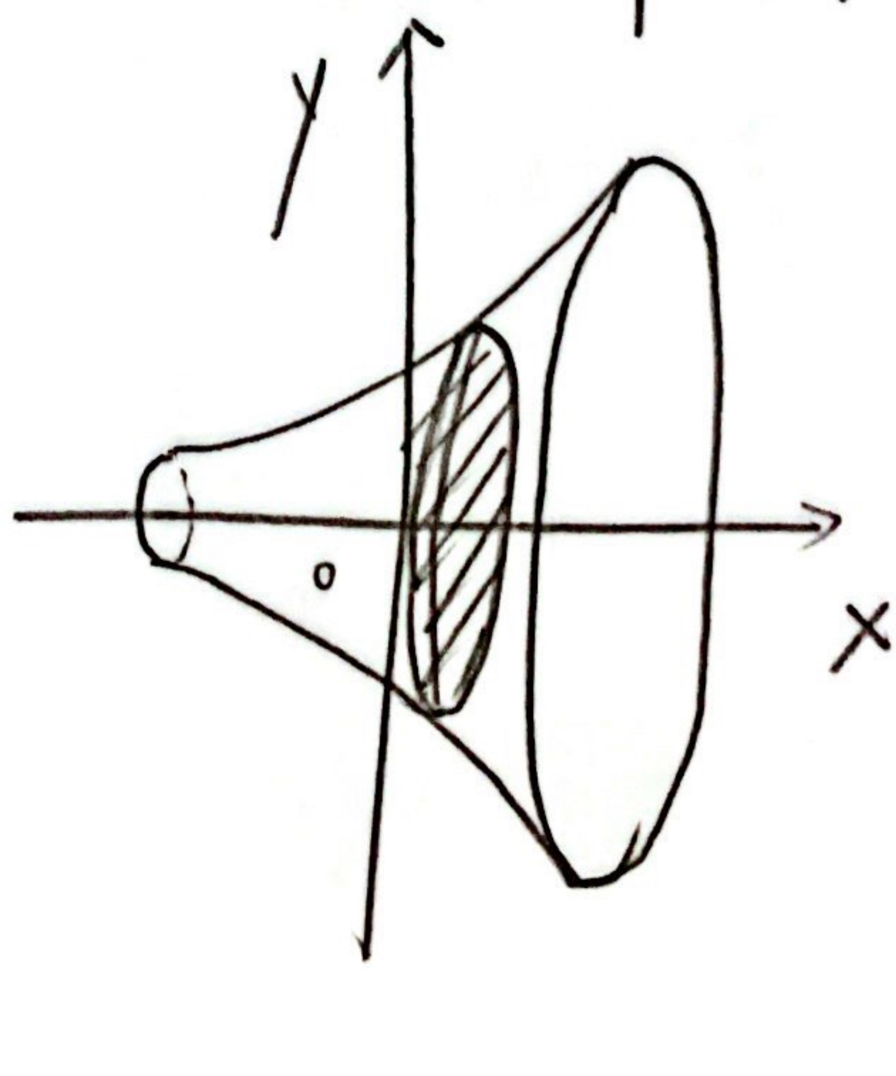


4. $\because$ a cross section is a disk with radius e^x

$\therefore A(x) = \pi(e^x)^2 = \pi e^{2x}$

$V = \int_{-1}^1 A(x)\,dx = \int_{-1}^1 \pi e^{2x}\,dx = \pi\left[\frac{1}{2}e^{2x}\right]_{-1}^1 = \frac{\pi}{2}(e^2 - e^{-2})$





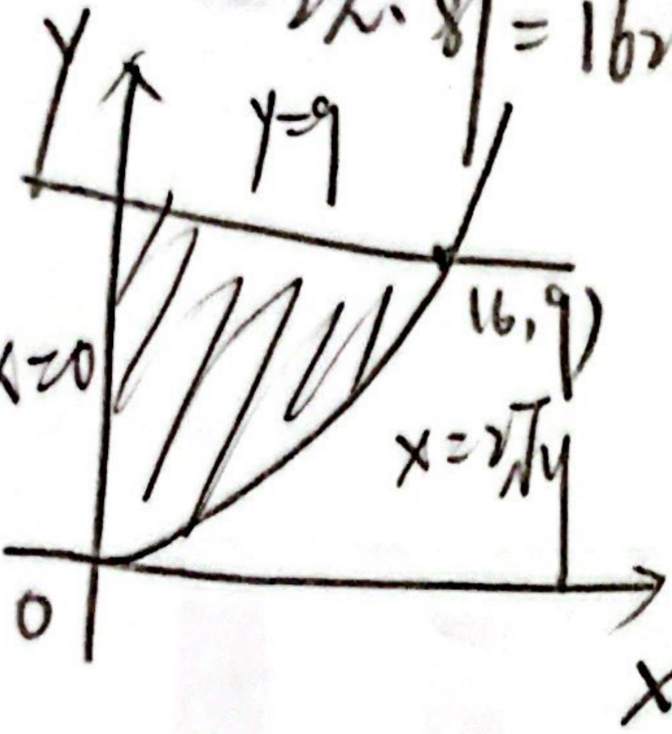
5. ∵ a cross seceton is a disk with radius $2\sqrt{y}$

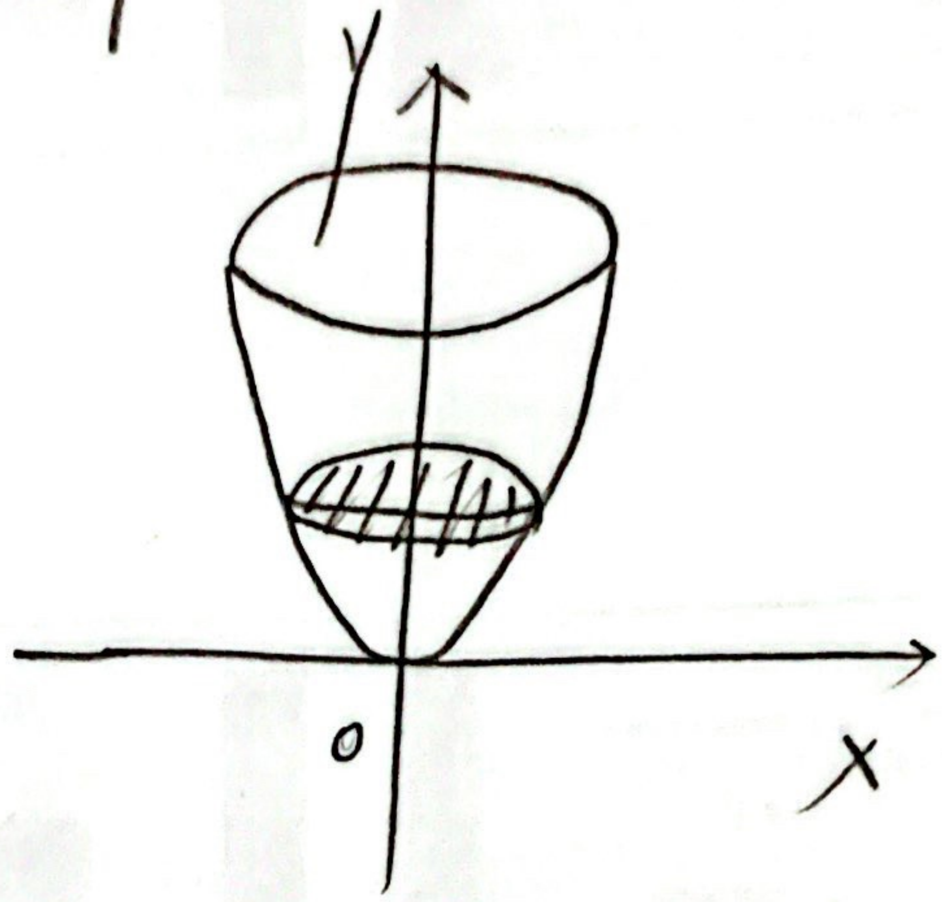
∴ $A(y) = \pi(2\sqrt{y})^2$

$V = \int_0^9 A(y)dy = \int_0^9 \pi(2\sqrt{y})^2 dy$

$= 4\pi\int_0^9 y\,dy = 4\pi\left[\frac{1}{2}y^2\right]_0^9$

$= 2\pi \cdot 81 = 162\pi$

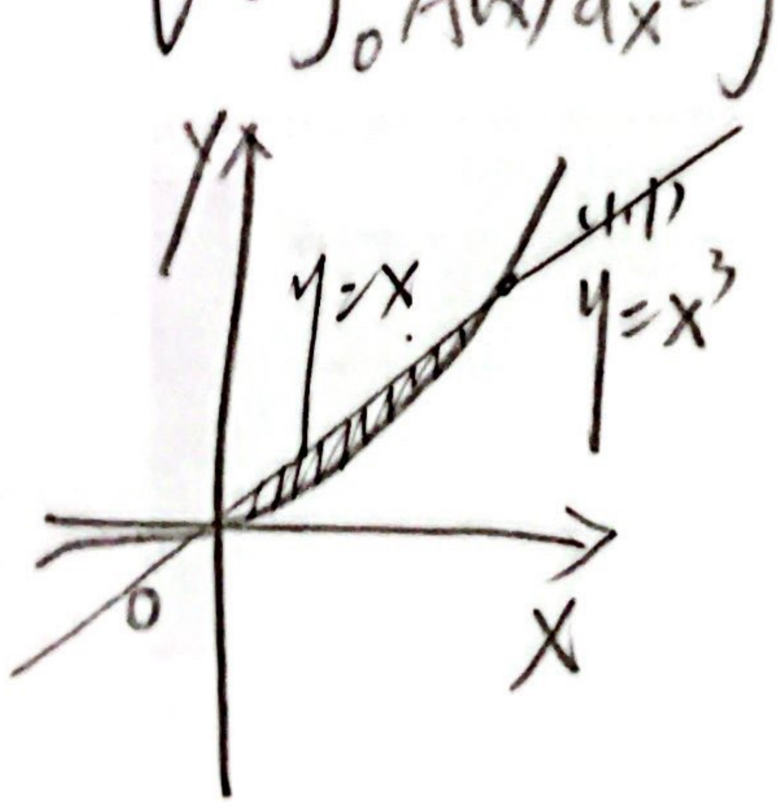


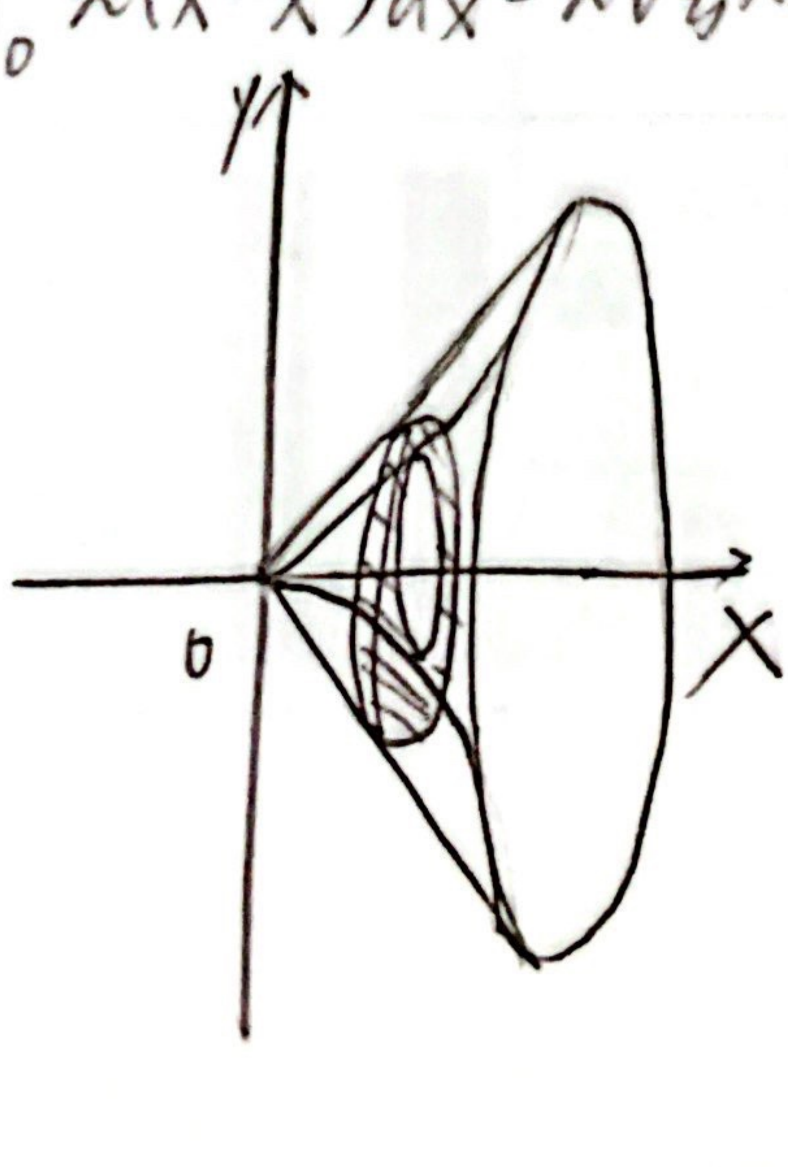


7. ∵ a cross section is a washer with inner radius x^3 and outer radius x

$A(x) = \pi(x)^2 - \pi(x^3)^2 = \pi(x^2 - x^6)$

$V = \int_0^1 A(x)\,dx = \int_0^1 \pi(x^2 - x^6)\,dx = \pi\left[\frac{1}{3}x^3 - \frac{1}{7}x^7\right]_0^1 = \pi\left(\frac{1}{3} - \frac{1}{7}\right) = \frac{4}{21}\pi$



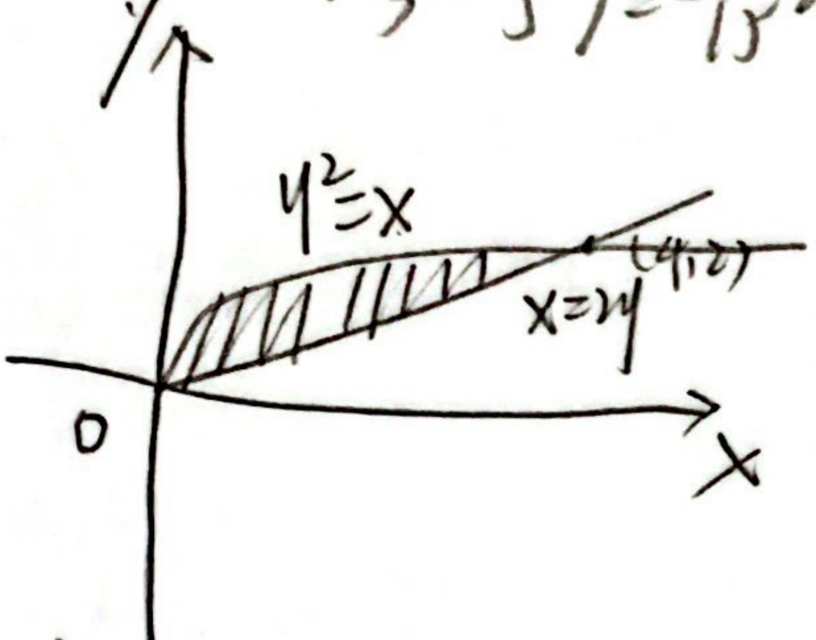


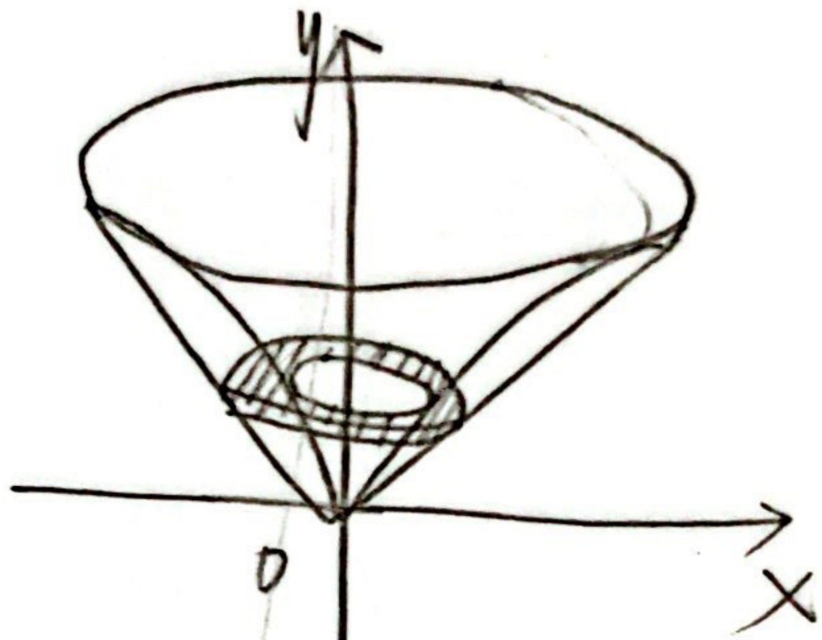
9. ∵ a cross section is a washer with inner radius y^2 and outer radius $2y$

∴ $A(y) = \pi(2y)^2 - \pi(y^2)^2 = \pi(4y^2 - y^4)$

$V = \int_0^2 A(y)\,dy = \pi\int_0^2 (4y^2 - y^4)\,dy = \pi\left[\frac{4}{3}y^3 - \frac{1}{5}y^5\right]_0^2$

$= \pi\left(\frac{32}{3} - \frac{32}{5}\right) = \frac{64}{15}\pi$





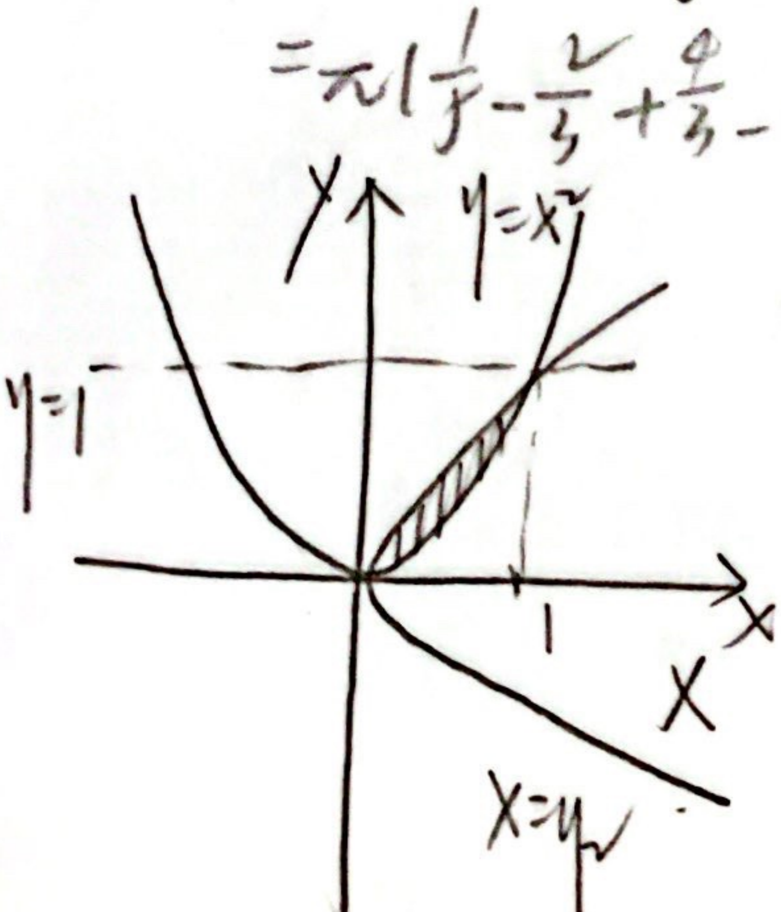
11. ∵ a cross section is a washer with inner radius $1-\sqrt{x}$ and outer radius $1-x^2$

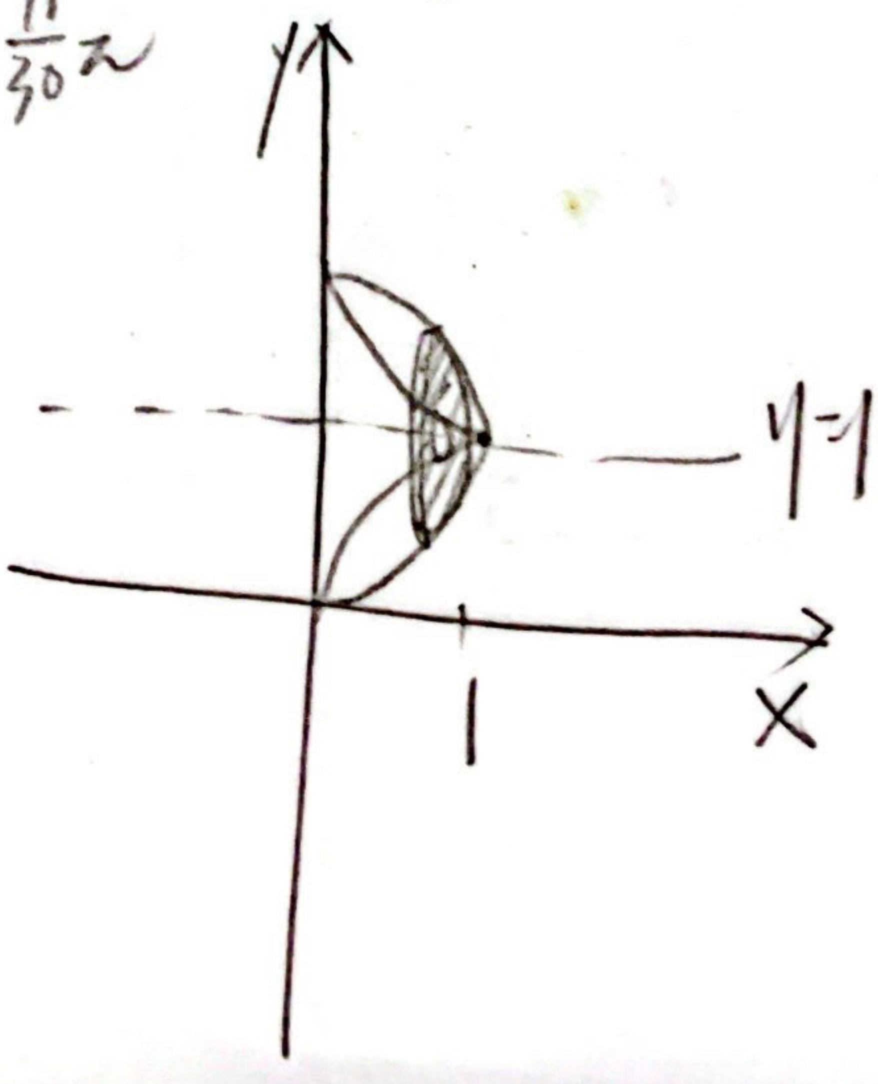
∴ $A(x) = \pi[(1-x^2)^2 - (1-\sqrt{x})^2] = \pi[(1-2x^2+x^4) - (1-2\sqrt{x}+x)]$

$= \pi(x^4 - 2x^2 + 2\sqrt{x} - x)$

$V = \int_0^1 A(x)\,dx = \int_0^1 \pi(x^4 - 2x^2 + 2x^{\frac{1}{2}} - x)\,dx = \pi\left[\frac{1}{5}x^5 - \frac{2}{3}x^3 + \frac{4}{3}x^{\frac{3}{2}} - \frac{1}{2}x^2\right]_0^1$

$= \pi\left(\frac{1}{5} - \frac{2}{3} + \frac{4}{3} - \frac{1}{2}\right) = \frac{11}{30}\pi$





14. $\because$ a cross section is a washer with inner radius $\sin x - (-1)$ and outer radius $\cos x - (-1)$

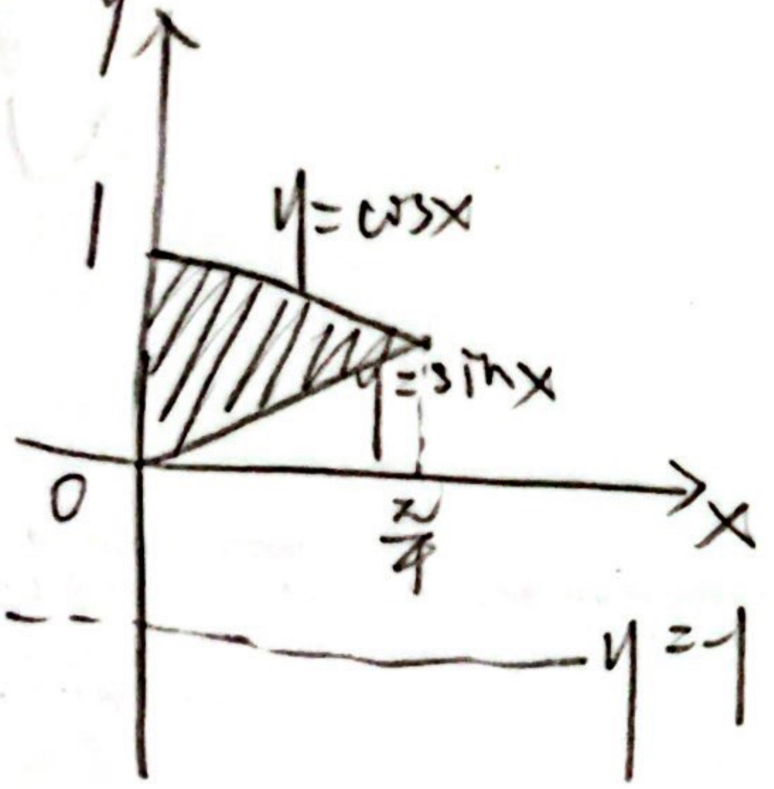
$$\therefore A(x) = \pi\left[(\cos x+1)^2 - (\sin x+1)^2\right] = \pi(\cos^2 x + 2\cos x - \sin^2 x - 2\sin x)$$

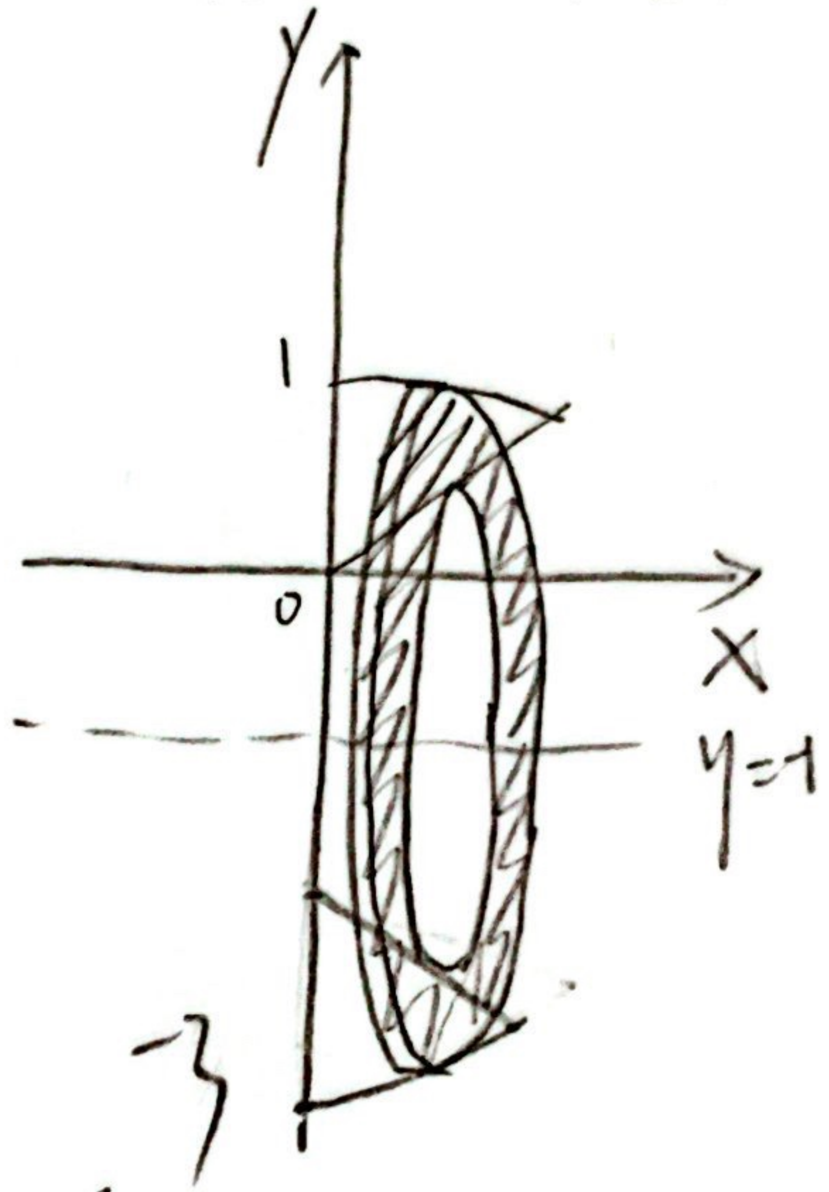
$$= \pi(\cos 2x + 2\cos x - 2\sin x)$$

$$V = \int_0^{\frac{\pi}{4}} A(x)\,dx = \int_0^{\frac{\pi}{4}} \pi(\cos 2x + 2\cos x - 2\sin x)\,dx$$

$$= \pi\left[\tfrac{1}{2}\sin 2x + 2\sin x + 2\cos x\right]_0^{\frac{\pi}{4}} = \pi\left[\left(\tfrac{1}{2} + \sqrt{2} + \sqrt{2}\right) - (0+0+2)\right]$$

$$= \left(2\sqrt{2} - \tfrac{3}{2}\right)\pi$$

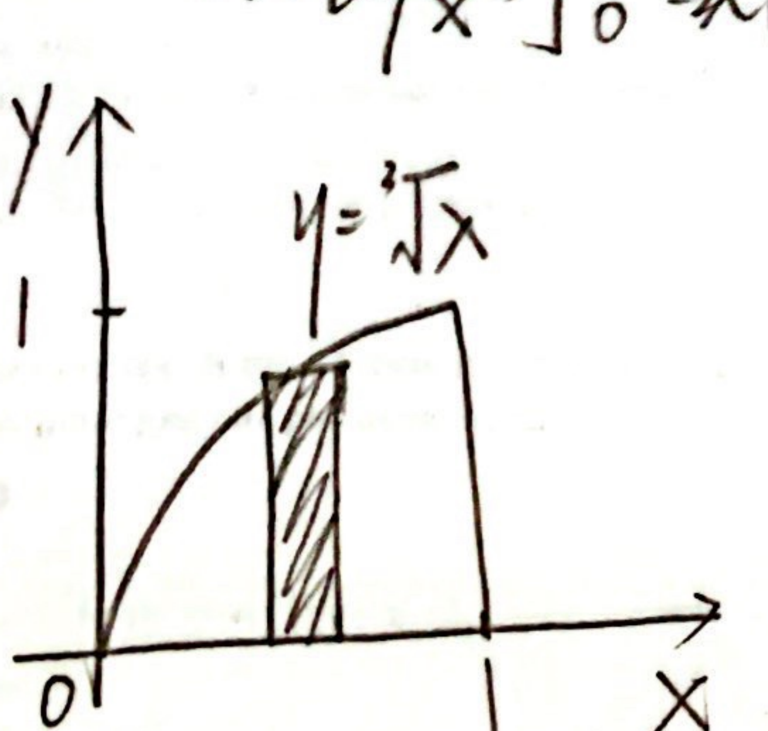


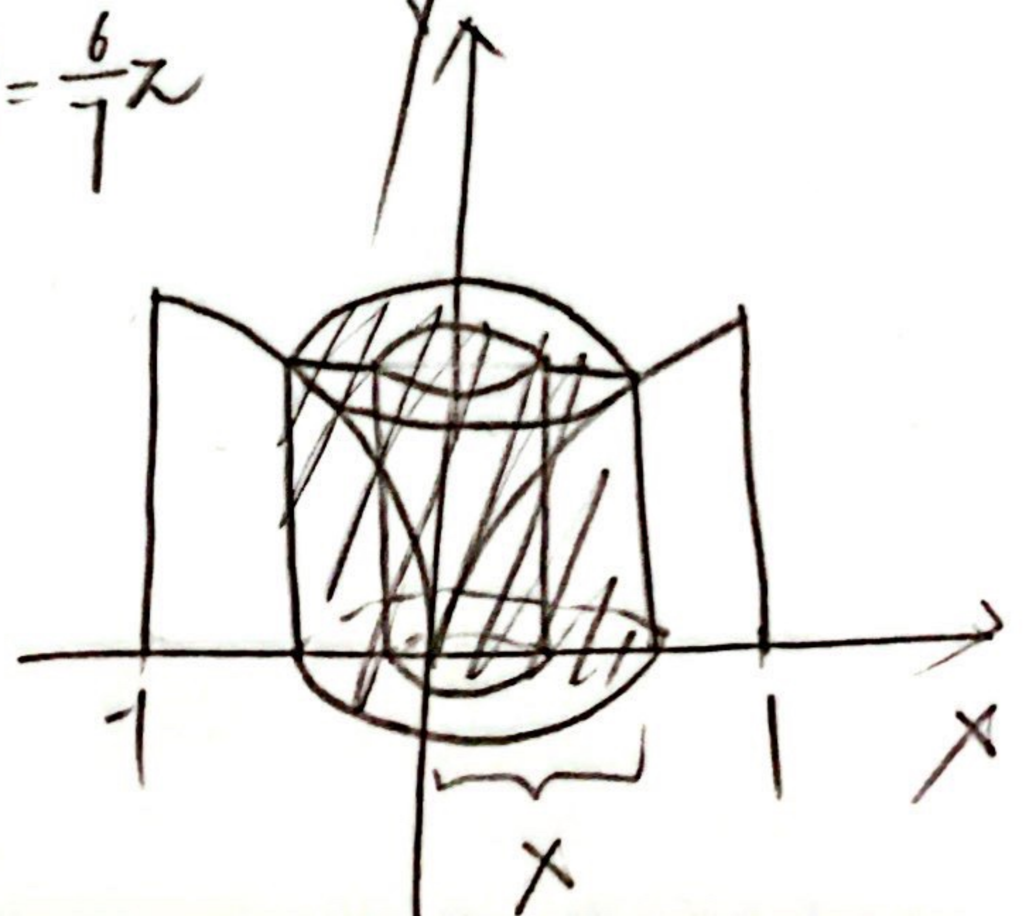


6.3

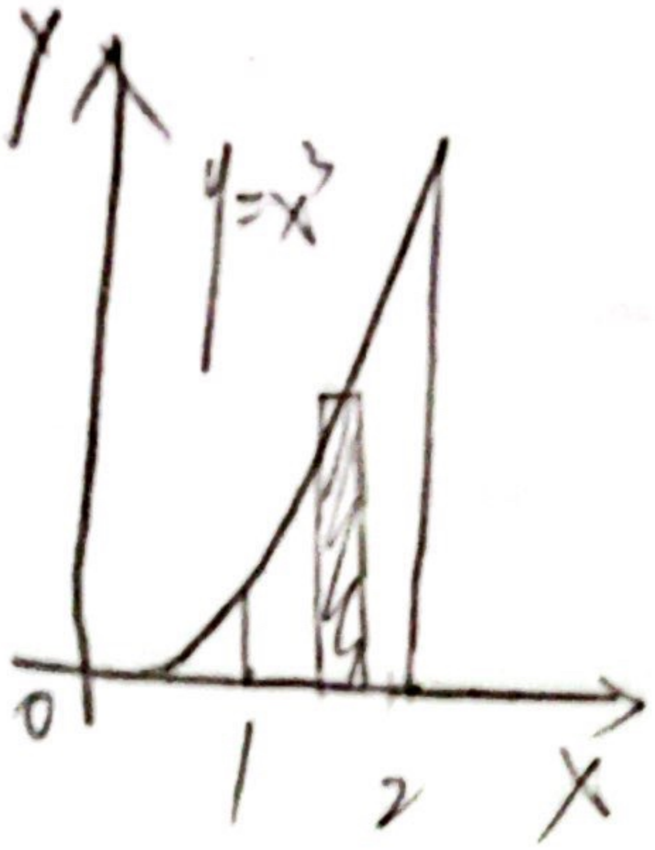
3. $$V = \int_0^1 2\pi x \sqrt[3]{x}\,dx = 2\pi \int_0^1 x^{\frac{4}{3}}\,dx$$

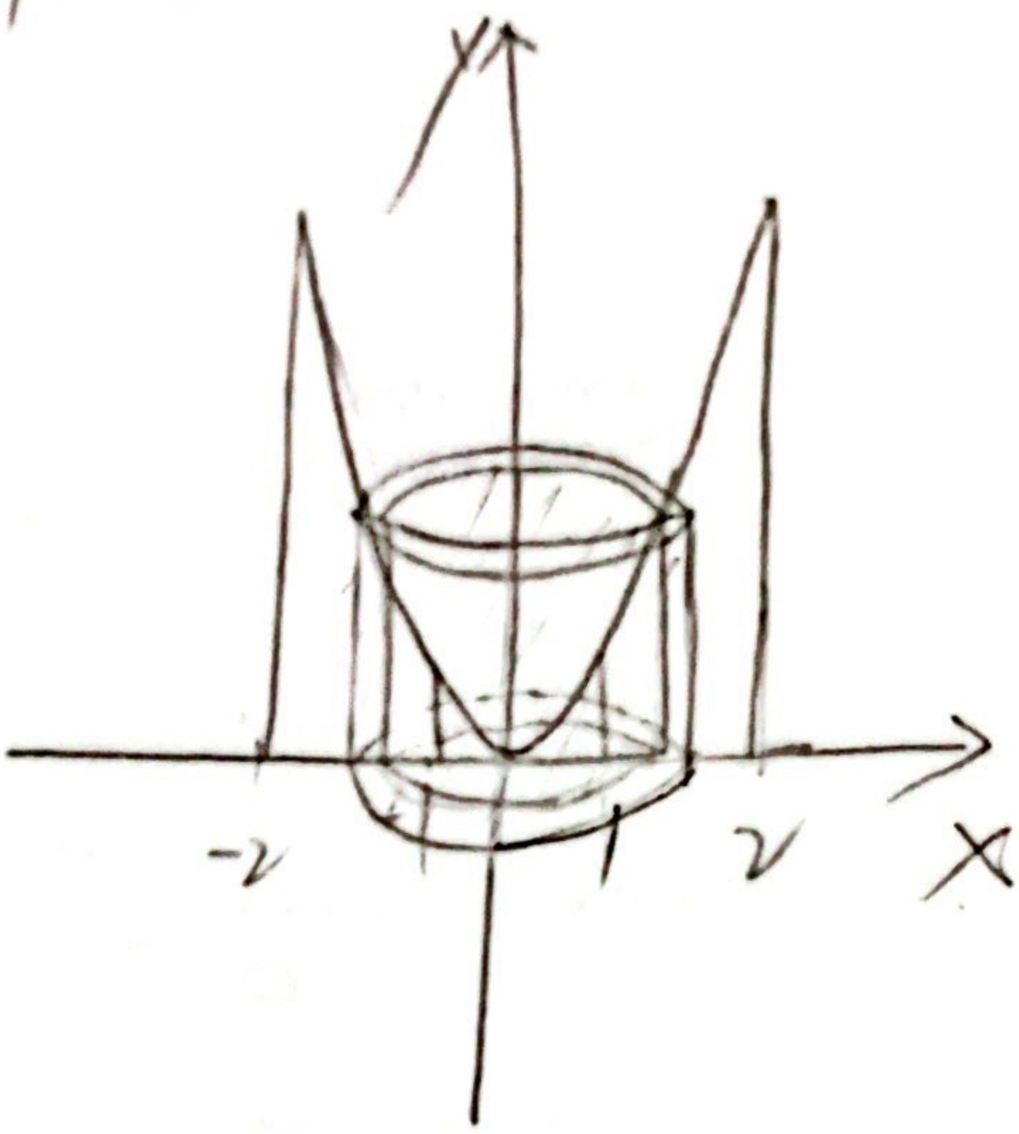
$$= 2\pi\left[\tfrac{3}{7}x^{\frac{7}{3}}\right]_0^1 = 2\pi\left(\tfrac{3}{7}\right) = \tfrac{6}{7}\pi$$





7. $V=\int_1^2 2\pi x\cdot x^3dx=2\pi\int_1^2 x^4dx=2\pi[\frac{1}{5}x^5]_1^2=2\pi(\frac{32}{5}-\frac{1}{5})=\frac{62}{5}\pi$





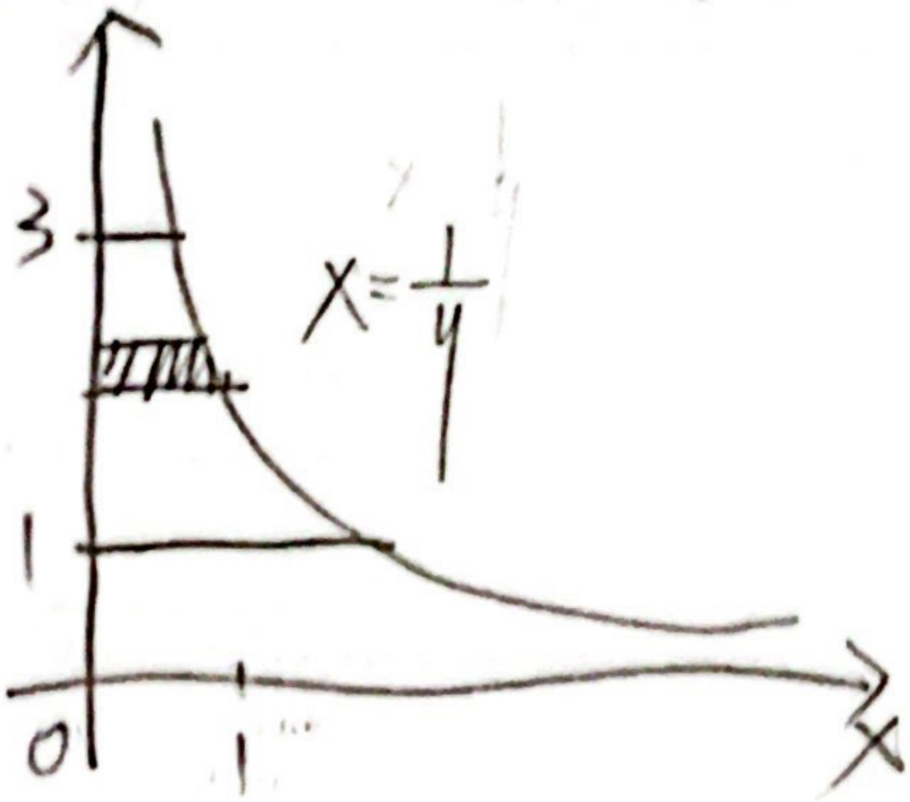
9. $xy=1$ $x=\frac{1}{y}$

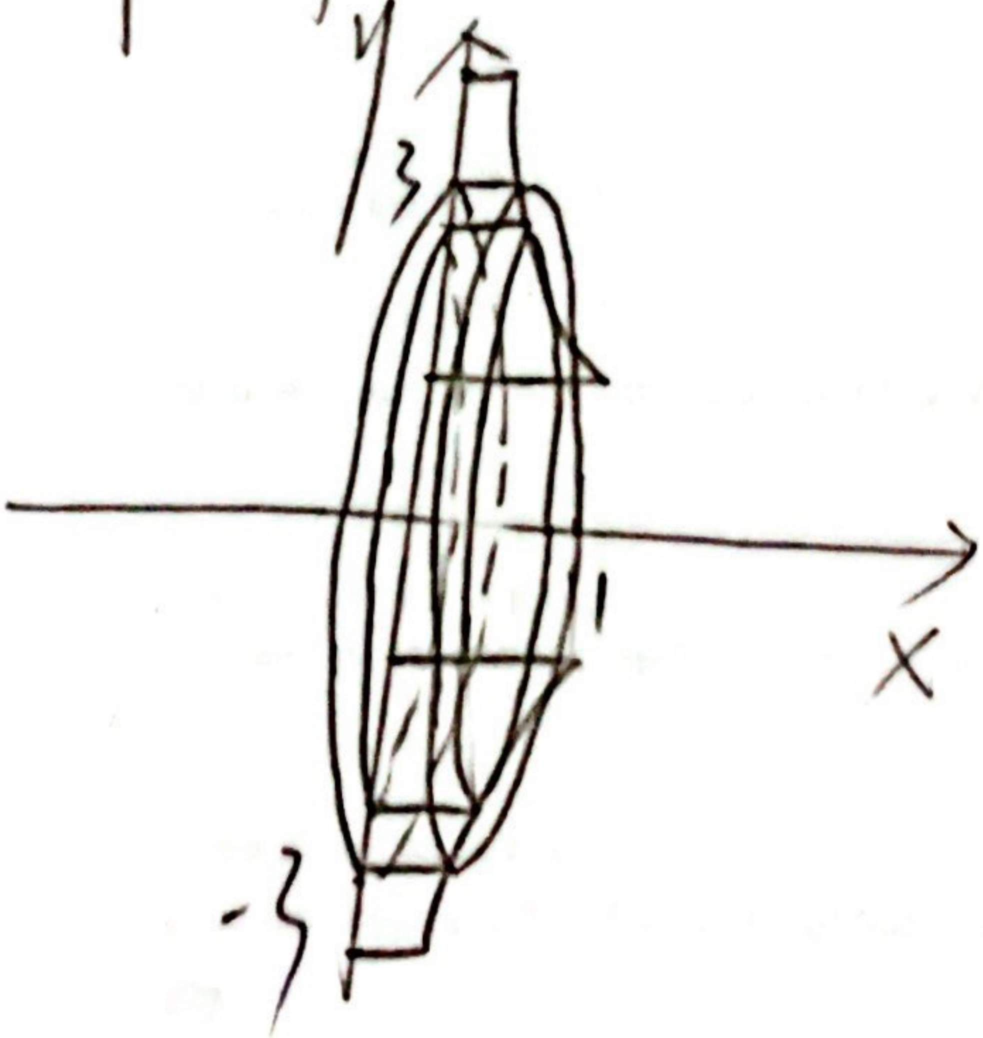
the shell has radius y

circumference $2\pi y$

lerghe $\frac{1}{y}$

$V=\int_1^3 2\pi y(\frac{1}{y})dy=2\pi\int_1^3 dy=2\pi[y]_1^3=2\pi(3-1)=4\pi$

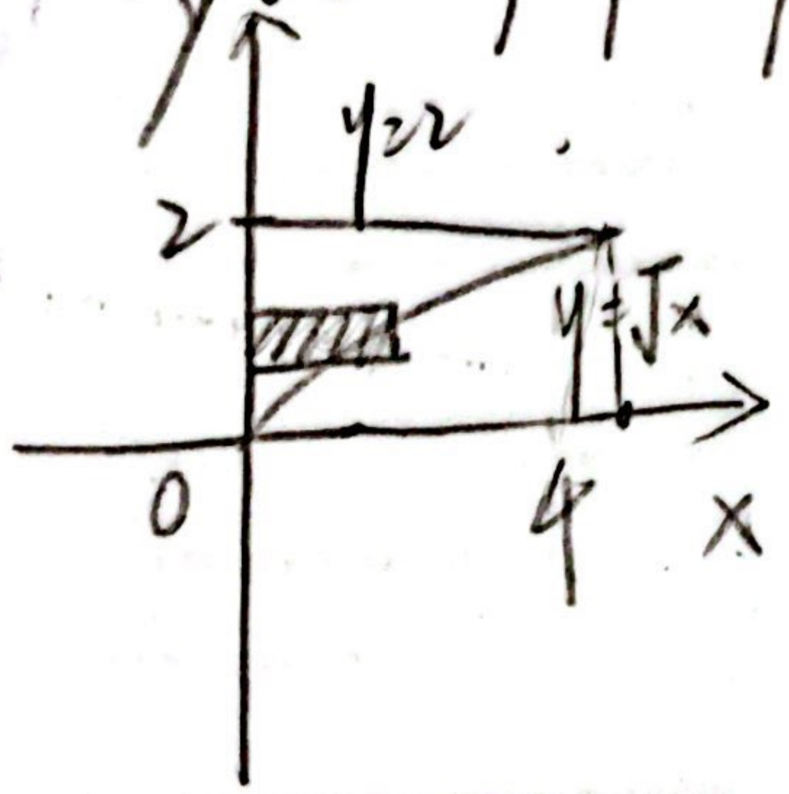


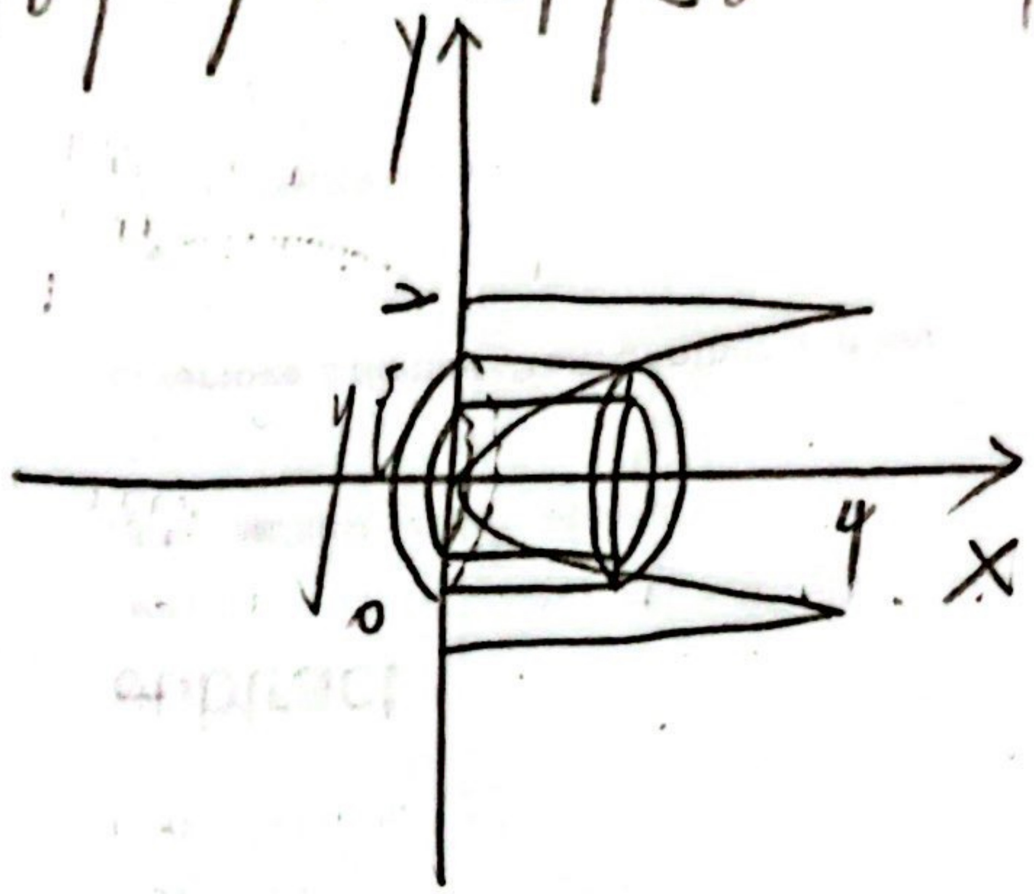


10. $y=\sqrt{x}$ $x=y^2$

the shell has radius y circumference $2\pi y$ Lenght y^2.

$$V=\int_0^2 2\pi y(y^2)dy = 2\pi\int_0^2 y^3dy = 2\pi[\tfrac{1}{4}y^4]_0^2 = 2\pi\cdot 4 = 8\pi$$





17. the shell has radius $x-1$ circumference $2\pi(x-1)$

Lenght $(4x-x^2)-3 = -x^2+4x-3$

$$V=\int_1^3 2\pi(x-1)(-x^2+4x-3)dx = 2\pi\int_1^3(-x^3+5x^2-7x+3)dx$$

$$=2\pi[-\tfrac{1}{4}x^4+\tfrac{5}{3}x^3-\tfrac{7}{2}x^2+3x]_1^3 = 2\pi\cdot\tfrac{4}{3} = \tfrac{8}{3}\pi$$