Module 11

5.4

2. $\int \cos^2 x\,dx = \frac{1}{2}x + \frac{1}{4}\sin 2x + C$

$\frac{d}{dx}(\frac{1}{2}x + \frac{1}{4}\sin 2x + C) = \frac{1}{2} + \frac{1}{4}\cos 2x \cdot 2 + 0 = \frac{1}{2} + \frac{1}{2}\cos 2x$

$= \frac{1}{2} + \frac{1}{2}(2\cos^2 x - 1) = \frac{1}{2} + \cos^2 x - \frac{1}{2} = \cos^2 x$

6. $\int \sqrt[4]{x^5}\,dx = \int x^{\frac{5}{4}}\,dx = \frac{4}{9}x^{\frac{9}{4}} + C$

8. $\int (u^6 - 2u^5 - u^3 + \frac{2}{7})\,du = \frac{1}{7}u^7 - 2\cdot\frac{1}{6}u^6 - \frac{1}{4}u^4 + \frac{2}{7}u + C$

$= \frac{1}{7}u^7 - \frac{1}{3}u^6 - \frac{1}{4}u^4 + \frac{2}{7}u + C$

12. $\int (x^2 + 1 + \frac{1}{x^2+1})\,dx = \frac{x^3}{3} + x + \tan^{-1}x + C$

16. $\int \sec t(\sec t + \tan t)\,dt = \int (\sec^2 t + \sec t\tan t)\,dt = \tan t + \sec t + C$

22. $\int_1^2 (4x^3 - 3x^2 + 2x)\,dx = [x^4 - x^3 + x^2]_1^2 = (16 - 8 + 4) - (1 - 1 + 1) = 12 - 1 = 11$

27. $\int_0^\pi (5e^x + 3\sin x)\,dx = [5e^x - 3\cos x]_0^\pi = [5e^\pi - 3(-1)] - [5\cdot 1 - 3\cdot 1] = 5e^\pi + 1$

28. $\int_1^2 (\frac{1}{x^2} - \frac{4}{x^3})\,dx = \int_1^2 (x^{-2} - 4x^{-3})\,dx = [\frac{x^{-1}}{-1} - \frac{4x^{-2}}{-2}]_1^2 = [-\frac{1}{x} + \frac{2}{x^2}]_1^2$

$= (-\frac{1}{2} + \frac{1}{2}) - (-1 + 2) = -1$

35. $\int_0^1 (x^{10} + 10^x)\,dx = [\frac{x^{11}}{11} + \frac{10^x}{\ln 10}]_0^1 = (\frac{1}{11} + \frac{10}{\ln 10}) - (0 + \frac{1}{\ln 10}) = \frac{1}{11} + \frac{9}{\ln 10}$

37. $\int_0^{\frac{\pi}{4}} \frac{1 + \cos^2\theta}{\cos^2\theta}\,d\theta = \int_0^{\frac{\pi}{4}} (\frac{1}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta})\,d\theta = \int_0^{\frac{\pi}{4}} (\sec^2\theta + 1)\,d\theta$

$= [\tan\theta + \theta]_0^{\frac{\pi}{4}} = (\tan\frac{\pi}{4} + \frac{\pi}{4}) - (0 + 0) = 1 + \frac{\pi}{4}$

51. if $w'(t)$ is the rate of change of weight in pounds per year $w(t)$ represents the weight in pounds of the child at age t

$\int_5^{10} w'(t)\,dt = w(10) - w(5)$ $\therefore$ the integral represents the increase in the child's weight between ages of 5 and 10.

53. $r(t)$ is the rate at which oil leaks
$V(t)$ is the volume of oil at time t $r(t) = -V'(t)$

$$\int_0^{120} r(t)\,dt = -\int_0^{120} V'(t)\,dt = -[V(120) - V(0)] = V(0) - V(120).$$

this represents the number of gallons of oil leaked from the tank in the first 2 hours.

54. By the Net Change Theorem $\int_0^{15} n'(t)\,dt = n(15) - n(0) = n(15) - 100$
this represents the increase in the bee population in 15 weeks
$100 + \int_0^{15} n'(t)\,dt = n(15)$ represents the total bee population after 15 week

55. By the Net Change Theorem $\int_{1000}^{5000} R(x)\,dx = R(5000) - R(1000)$
this represents the increase in revenue when production is increased from 1000 units to 5000 units.

64. By the Net Change Theorem

$$\int_0^{10} r(t)\,dt = \int_0^{10} (200 - 4t)\,dt = [200t - 2t^2]_0^{10} = (2000 - 200) - 0 = 1800 \text{ liters}$$

this represents the amount of water that flows from the tank during the first 10 minutes

67. By the Net Change Theorem

$$C(4000) - C(2000) = \int_{2000}^{4000} C'(x)\,dx$$

$$\int_{2000}^{4000} C'(x)\,dx = \int_{2000}^{4000} (3 - 0.01x + 0.000006x^2)\,dx$$

$$= [3x - 0.005x^2 + 0.000002x^3]_{2000}^{4000}$$

this represents the increase in cost if the production level is raised from 2000 yards to 4000 yards.

5.5

3. $u = x^3+1 \quad du = 3x^2dx \quad x^2dx = \frac{1}{3}du$

$$\int x^2\sqrt{x^3+1}\,dx = \int \sqrt{u}\left(\frac{1}{3}du\right) = \frac{1}{3}\frac{u^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{1}{3}\cdot\frac{2}{3}u^{\frac{3}{2}} + C = \frac{2}{9}(x^3+1)^{\frac{3}{2}} + C$$

4. $u = \sin\theta \quad du = \cos\theta\,d\theta$

$$\int \sin^2\theta\cos\theta\,d\theta = \int u^2du = \frac{1}{3}u^3 + C = \frac{1}{3}\sin^3\theta + C$$

8. $u = x^3 \quad du = 3x^2dx \quad x^2dx = \frac{1}{3}du$

$$\int x^2e^{x^3}dx = \int e^u\left(\frac{1}{3}du\right) = \frac{1}{3}e^u + C = \frac{1}{3}e^{x^3} + C$$

12. $u = 2\theta \quad du = 2d\theta \quad d\theta = \frac{1}{2}du$

$$\int \sec^2 2\theta\,d\theta = \int \sec^2 u\left(\frac{1}{2}du\right) = \frac{1}{2}\tan u + C = \frac{1}{2}\tan 2\theta + C$$

13. $u = 5-3x \quad du = -3dx \quad dx = -\frac{1}{3}du$

$$\int \frac{dx}{5-3x} = \int \frac{1}{u}\left(-\frac{1}{3}du\right) = -\frac{1}{3}\ln|u| + C = -\frac{1}{3}\ln|5-3x| + C$$

14. $u = 4-y^3 \quad du = -3y^2dy \quad y^2dy = -\frac{1}{3}du$

$$\int y^2(4-y^3)^{\frac{2}{3}}dy = \int u^{\frac{2}{3}}\left(-\frac{1}{3}du\right) = -\frac{1}{3}\cdot\frac{3}{5}u^{\frac{5}{3}} + C = -\frac{1}{5}(4-y^3)^{\frac{5}{3}} + C$$

17. $x = 1-e^u \quad dx = -e^udu \quad e^udu = -dx$

$$\int \frac{e^u}{(1-e^u)^2}du = \int \frac{1}{x^2}(-dx) = -\int x^{-2}dx = -(-x^{-1}) + C = \frac{1}{x} + C = \frac{1}{1-e^u} + C$$

20. $u = z^3+1 \quad du = 3z^2dz \quad \frac{1}{3}du = z^2dz + C$

$$\int \frac{z^2}{z^3+1}dz = \int \frac{1}{u}\left(\frac{1}{3}du\right) = \frac{1}{3}\ln|u| + C = \frac{1}{3}\ln|z^3+1| + C$$

21. $u = \ln x \quad du = \frac{dx}{x}$

$$\int \frac{(\ln x)^2}{x}dx = \int u^2du = \frac{1}{3}u^3 + C = \frac{1}{3}(\ln x)^3 + C$$

23、 $u=\tan\theta \quad du=\sec^2\theta\, d\theta$

$$\int \sec^2\theta \tan^3\theta\, d\theta = \int u^3 du = \frac{1}{4}u^4 + C = \frac{1}{4}\tan^4\theta + C$$

26、 $u=ax+b \quad du=a\,dx \quad dx=\frac{1}{a}du$

$$\int \frac{dx}{ax+b} = \int \frac{\frac{1}{a}du}{u} = \frac{1}{a}\int \frac{1}{u}du = \frac{1}{a}\ln|u| + C = \frac{1}{a}\ln|ax+b| + C$$

28. $u=\cos t \quad du=-\sin t\, dt \quad \sin t\, dt = -du$

$$\int e^{\cos t}\sin t\, dt = \int e^u(-du) = -e^u + C = -e^{\cos t} + C$$

44. $u=x^2 \quad du=2x\,dx$

$$\int \frac{x}{1+x^4}dx = \int \frac{\frac{1}{2}du}{1+u^2} = \frac{1}{2}\tan^{-1}u + C = \frac{1}{2}\tan^{-1}(x^2) + C$$

67. $u=x-1 \quad u+1=x \quad du=dx$

when $x=1$ $u=0$ when $x=2$ $u=1$

$$\int_1^2 x\sqrt{x-1}\,dx = \int_0^1 (u+1)\sqrt{u}\,du = \int_0^1 (u^{\frac{3}{2}}+u^{\frac{1}{2}})\,du = \left[\frac{2}{5}u^{\frac{5}{2}}+\frac{2}{3}u^{\frac{3}{2}}\right]_0^1$$
$$=\frac{2}{5}+\frac{2}{3}=\frac{16}{15}$$

6.1

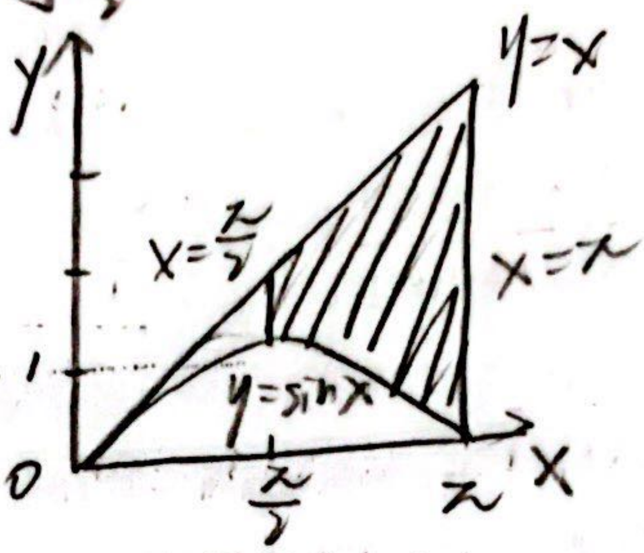
3. $$A=\int_{y=-1}^{y=1}(x_R-x_L)\,dy = \int_{-1}^1[e^y-(y^2-2)]\,dy = \int_{-1}^1(e^y-y^2+2)\,dy$$
$$=\left[e^y-\frac{1}{3}y^3+2y\right]_{-1}^1 = (e-\frac{1}{3}+2)-(e^{-1}+\frac{1}{3}-2) = e-\frac{1}{e}+\frac{10}{3}$$

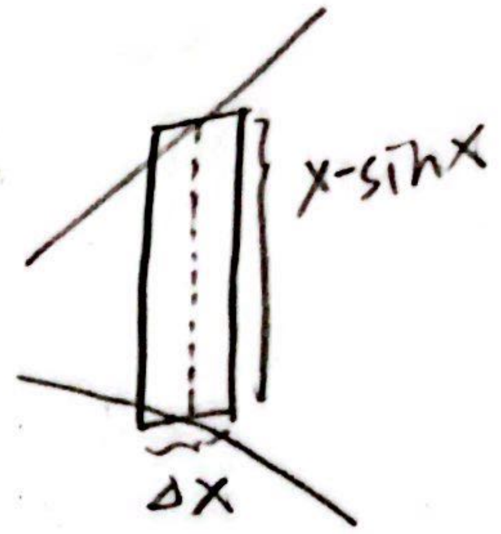
4. $$A=\int_0^3[(2y-y^2)-(y^2-4y)]\,dy = \int_0^3(-2y^2+6y)\,dy = \left[-\frac{2}{3}y^3+3y^2\right]_0^3$$
$$=(-18+27)-0=9$$

6. $A=\int_{\frac{\pi}{2}}^{\pi}(x-\sin x)dx=\left[\frac{x^2}{2}+\cos x\right]_{\frac{\pi}{2}}^{\pi}$

$=(\frac{\pi^2}{2}-1)-(\frac{\pi^2}{8}+0)$

$=\frac{3\pi^2}{8}-1$





12. $4x+x^2=12$

$(x+6)(x-2)=0$

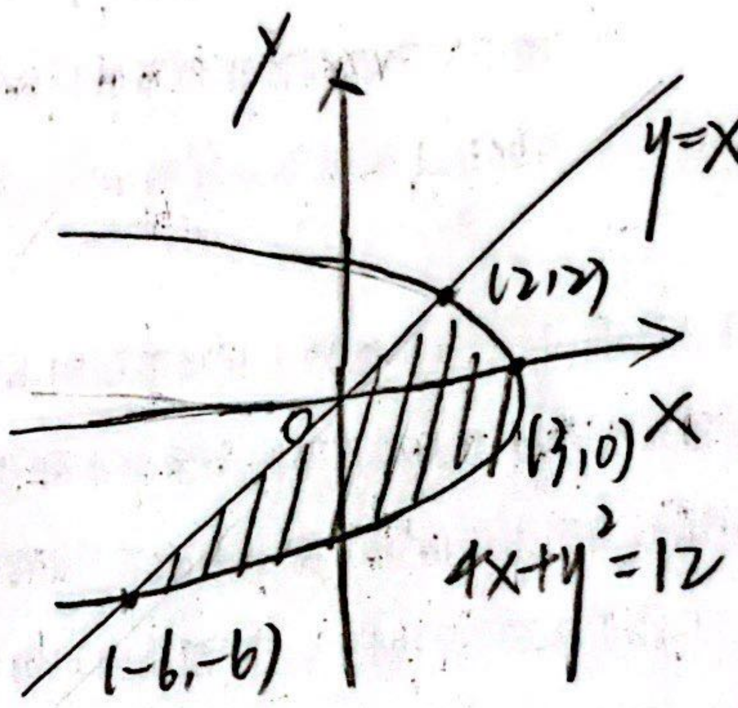
$x=-6/2$ $y=-6/2$

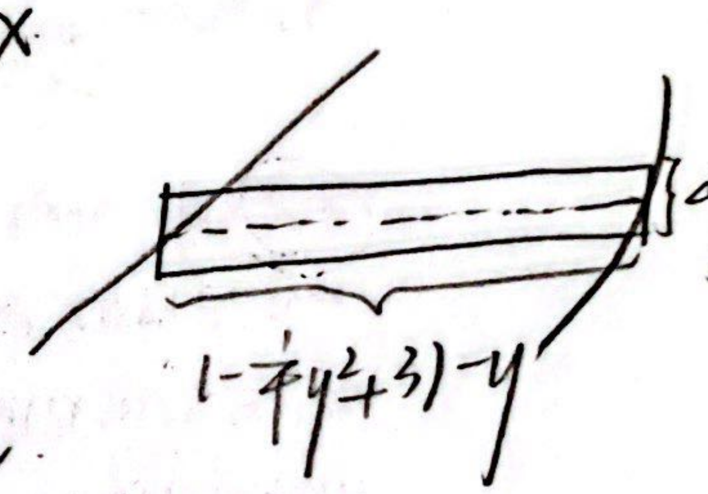
$A=\int_{-6}^{2}[(-\frac{1}{4}y^2+3)-y]dy$

$=[-\frac{1}{12}y^3-\frac{1}{2}y^2+3y]_{-6}^{2}$

$=(-\frac{2}{3}-2+6)-(18-18-18)$

$=22-\frac{2}{3}=\frac{64}{3}$





13. $12-x^2=x^2-6$

$2x^2=18$

$x=\pm3$

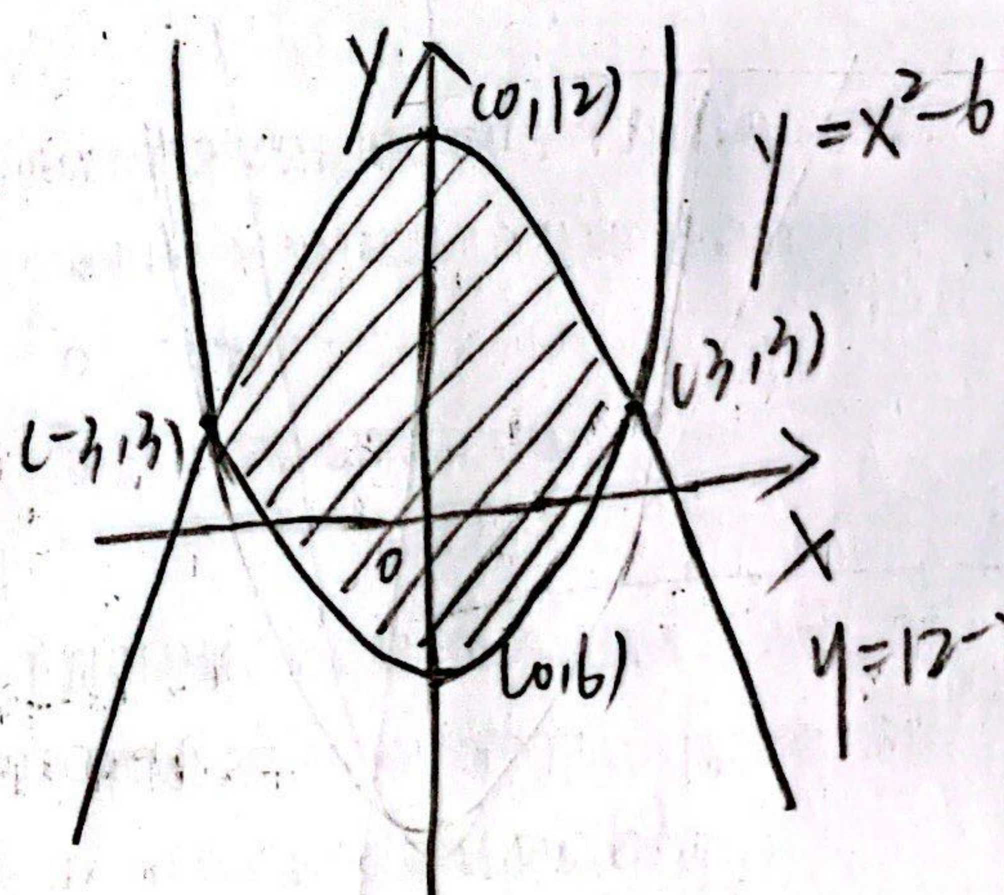
$A=\int_{-3}^{3}[(12-x^2)-(x^2-6)]dx$

$=2\int_{0}^{3}(18-2x^2)dx$

$=2[18-x-\frac{2}{3}x^3]_{0}^{3}$

$=2[(54-18)-0]$

$=2\cdot36=72$

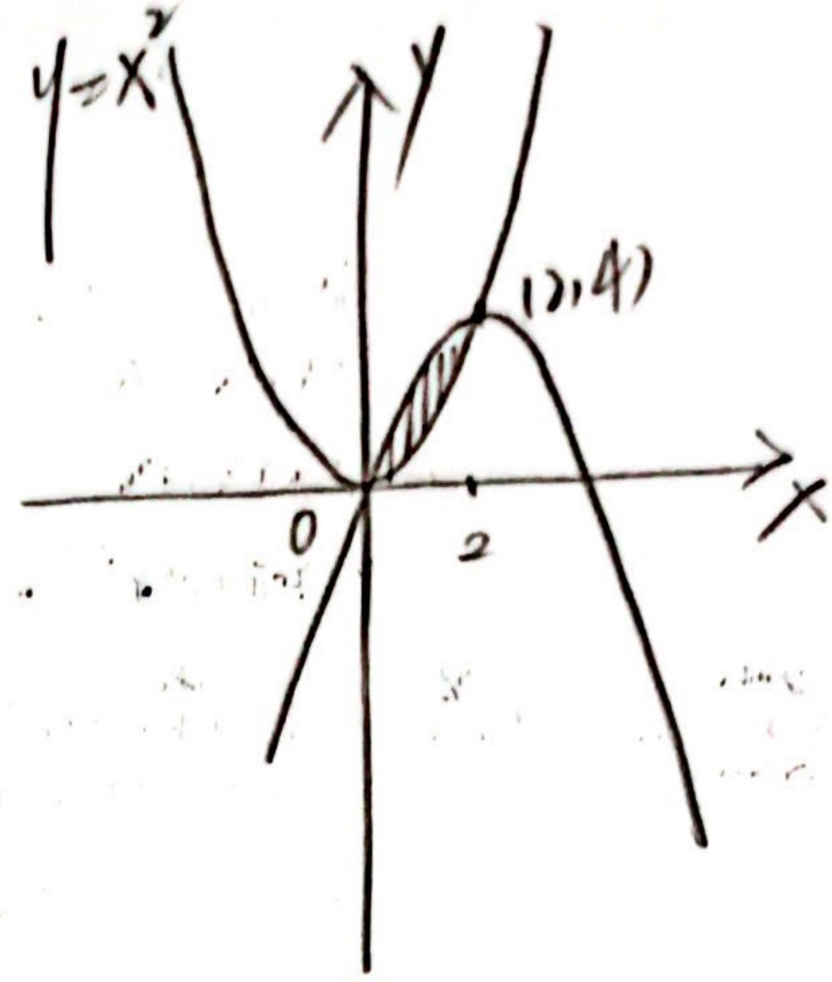


14. $x^2=4x-x^2$

$\Rightarrow 2x^2-4x=0$

$\Rightarrow 2x(x-2)=0$

$x=0/2$



$A=\int_0^2[(4x-x^2)-x^2]dx$

$=\int_0^2(4x-2x^2)dx$

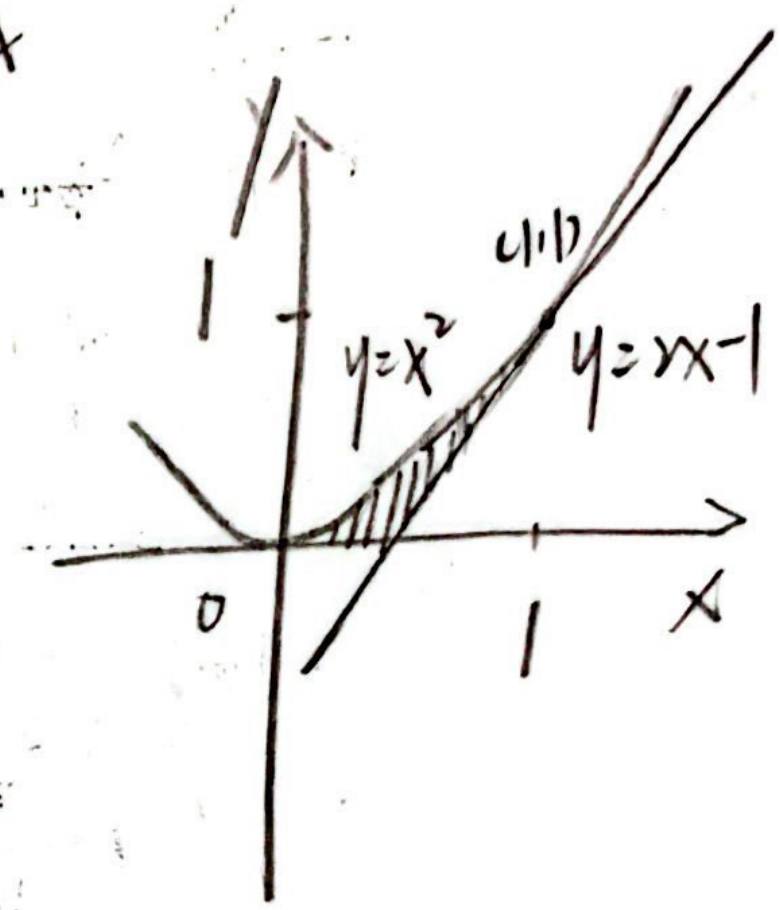
$=[2x^2-\frac{2}{3}x^3]_0^2=8-\frac{16}{3}=\frac{8}{3}$

56. the equation of the tangent line: $y'=2x$

the slope of the tangent: $2\cdot 1=2$

the equation: $y-1=2(x-1)$

$y=2x-1$

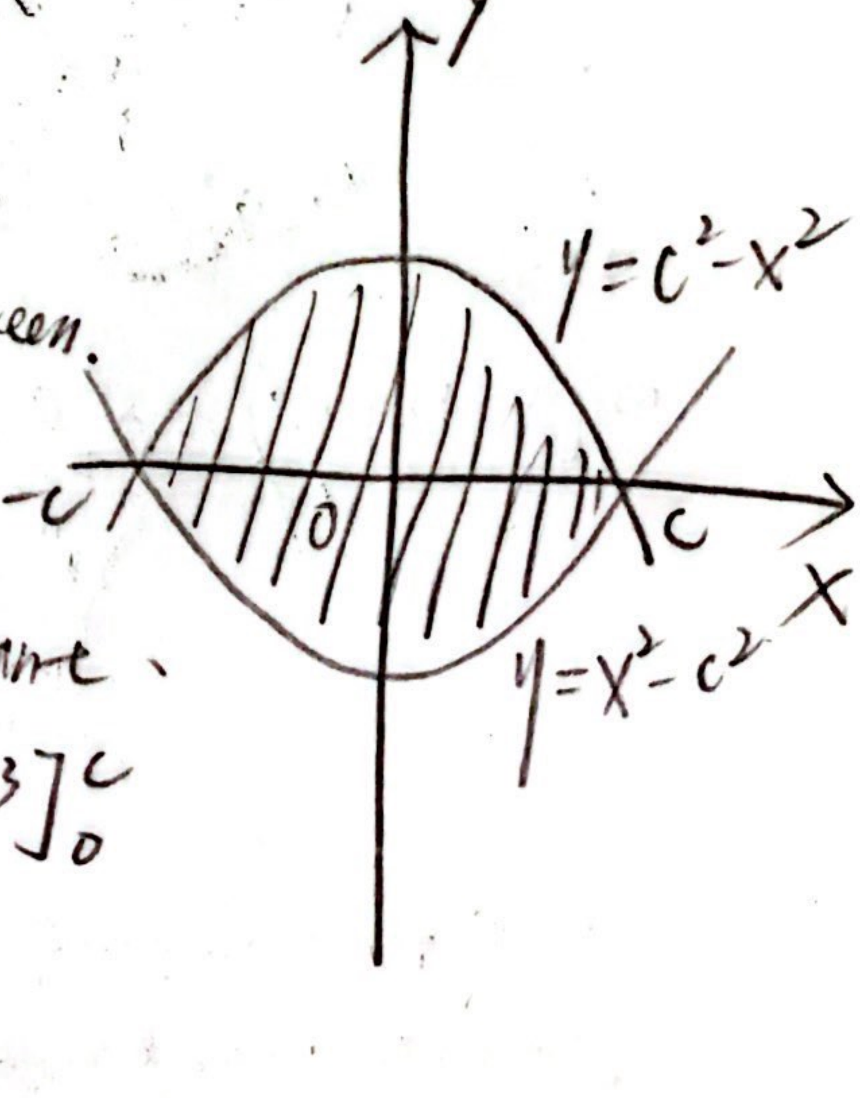


We need 2 integrals to integrate with respect to x, 1 to integrate with respect to y

$A=\int_0^1[\frac{1}{2}(y+1)-\sqrt{y}]dy=[\frac{1}{4}y^2+\frac{1}{2}y-\frac{2}{3}y^{\frac{3}{2}}]_0^1$

$=\frac{1}{4}+\frac{1}{2}-\frac{2}{3}=\frac{1}{12}$

59. the enclosed area A is graphed between $x=-c$ and c



$A=4\cdot$ the area in the first quadrant

$=4\int_0^c(c^2-x^2)dx=4[c^2x-\frac{1}{3}x^3]_0^c$

$=4(c^3-\frac{1}{3}c^3)=4\cdot\frac{2}{3}c^3=\frac{8}{3}c^3$

$A=576=\frac{8}{3}c^3 \quad c=6$

$\because$ the graph is the same, there is another solution which is $c=-6$