Module 10.

Appendix E

2. $\sum_{i=1}^{6} \frac{1}{i+1} = \frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}$

4. $\sum_{i=4}^{6} i^3 = 4^3+5^3+6^3$

10. $\sum_{i=1}^{n} f(x_i)\Delta x_i = f(x_1)\Delta x_1 + f(x_2)\Delta x_2 + f(x_3)\Delta x_3 + \cdots + f(x_n)\Delta x_n$

12. $\sqrt{3}+\sqrt{4}+\sqrt{5}+\sqrt{6}+\sqrt{7} = \sum_{i=3}^{7}\sqrt{i}$

14. $\frac{3}{7}+\frac{4}{8}+\frac{5}{9}+\frac{6}{10}+\cdots+\frac{23}{27} = \sum_{i=3}^{23}\frac{i}{i+4}$

22. $\sum_{i=3}^{6} i(i+2) = 3\cdot 5+4\cdot 6+5\cdot 7+6\cdot 8 = 15+24+35+48 = 122$

26. $\sum_{i=1}^{100} 4 = \underbrace{4+4+4+\cdots+4}_{(100 \text{ summands})} = 100\cdot 4 = 400$

31. $\sum_{i=1}^{n}(i^2+3i+4) = \sum_{i=1}^{n} i^2 + 3\sum_{i=1}^{n} i + \sum_{i=1}^{n} 4 = \frac{n(n+1)(2n+1)}{6} + \frac{3n(n+1)}{2} + 4n$

$= \frac{1}{6}[(2n^3+3n^2+n)+(9n^2+9n)+24n] = \frac{1}{6}(2n^3+12n^2+34n)$

$= \frac{1}{3}n(n^2+6n+17)$

45. $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{2}{n}\left[\left(\frac{2i}{n}\right)^3+5\left(\frac{2i}{n}\right)\right] = \lim_{n\to\infty}\sum_{i=1}^{n}\left[\frac{16}{n^4}i^3+\frac{20}{n^2}i\right] = \lim_{n\to\infty}\left[\frac{16}{n^4}\sum_{i=1}^{n}i^3+\frac{20}{n^2}\sum_{i=1}^{n}i\right]$

$= \lim_{n\to\infty}\left[\frac{16}{n^4}\frac{n^2(n+1)^2}{4}+\frac{20}{n^2}\frac{n(n+1)}{2}\right]$

$= \lim_{n\to\infty}\left[\frac{4(n+1)^2}{n^2}+\frac{10n(n+1)}{n^2}\right] = \lim_{n\to\infty}\left[4\left(1+\frac{1}{n}\right)^2+10\left(1+\frac{1}{n}\right)\right]$

$= 4\cdot 1+10\cdot 1 = 14$

5.2

4. (a) $f(x)=\frac{1}{x}$ $1 \le x \le 2$ $\Delta x=\frac{b-a}{n}=\frac{2-1}{4}=\frac{1}{4}$

use right endpoints $x_i^*=x_i$

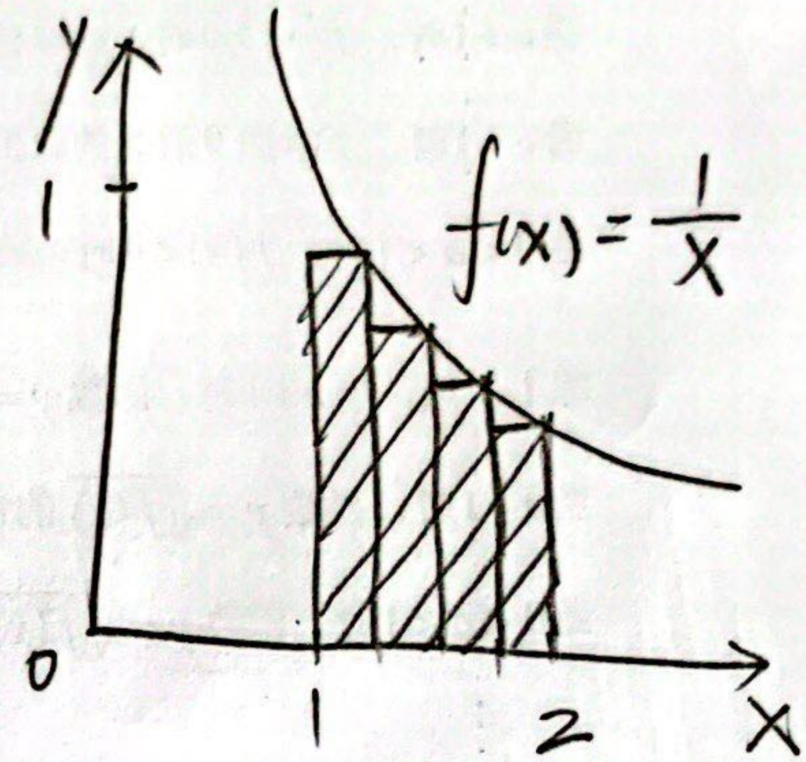
$R_4=\sum_{i=1}^{4} f(x_i)\Delta x$

$=(\Delta x)[f(x_1)+f(x_2)+f(x_3)+f(x_4)]$

$=\frac{1}{4}[f(\frac{5}{4})+f(\frac{6}{4})+f(\frac{7}{4})+f(\frac{8}{4})]$

$=\frac{1}{4}(\frac{4}{5}+\frac{2}{3}+\frac{4}{7}+\frac{1}{2})$

≈ 0.634524



The Riemann sum representes the sum of the areas of the four rectangles.

(b) use midpoints $x_i^*=\bar{x}_i=\frac{1}{2}(x_{i-1}+x_i)$

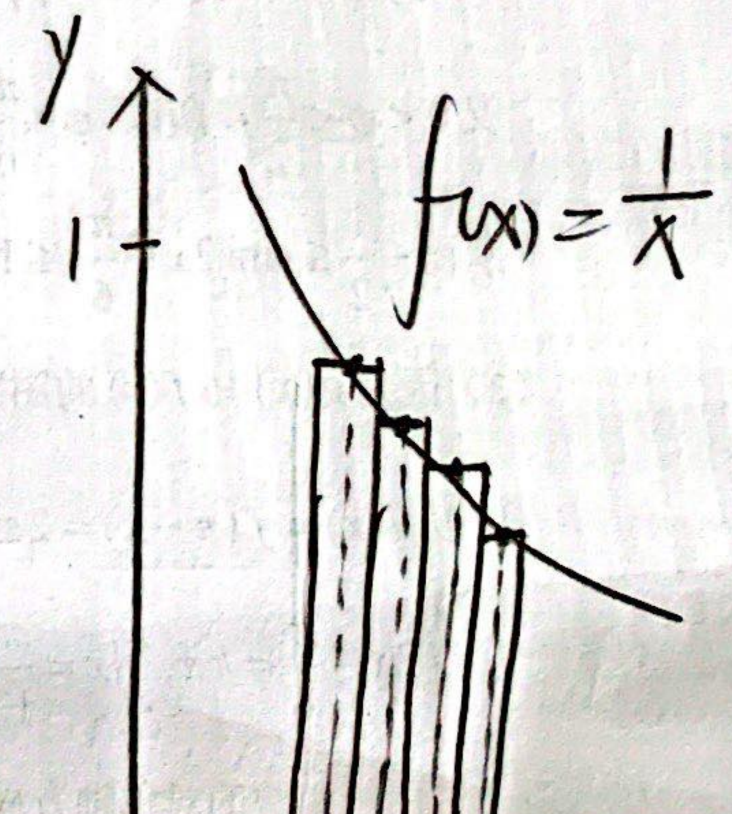
$M_4=\sum_{i=1}^{4} f(\bar{x}_i)\Delta x$

$=(\Delta x)[f(\bar{x}_1)+f(\bar{x}_2)+f(\bar{x}_3)+f(\bar{x}_4)]$

$=\frac{1}{4}[f(\frac{9}{8})+f(\frac{11}{8})+f(\frac{13}{8})+f(\frac{15}{8})]$

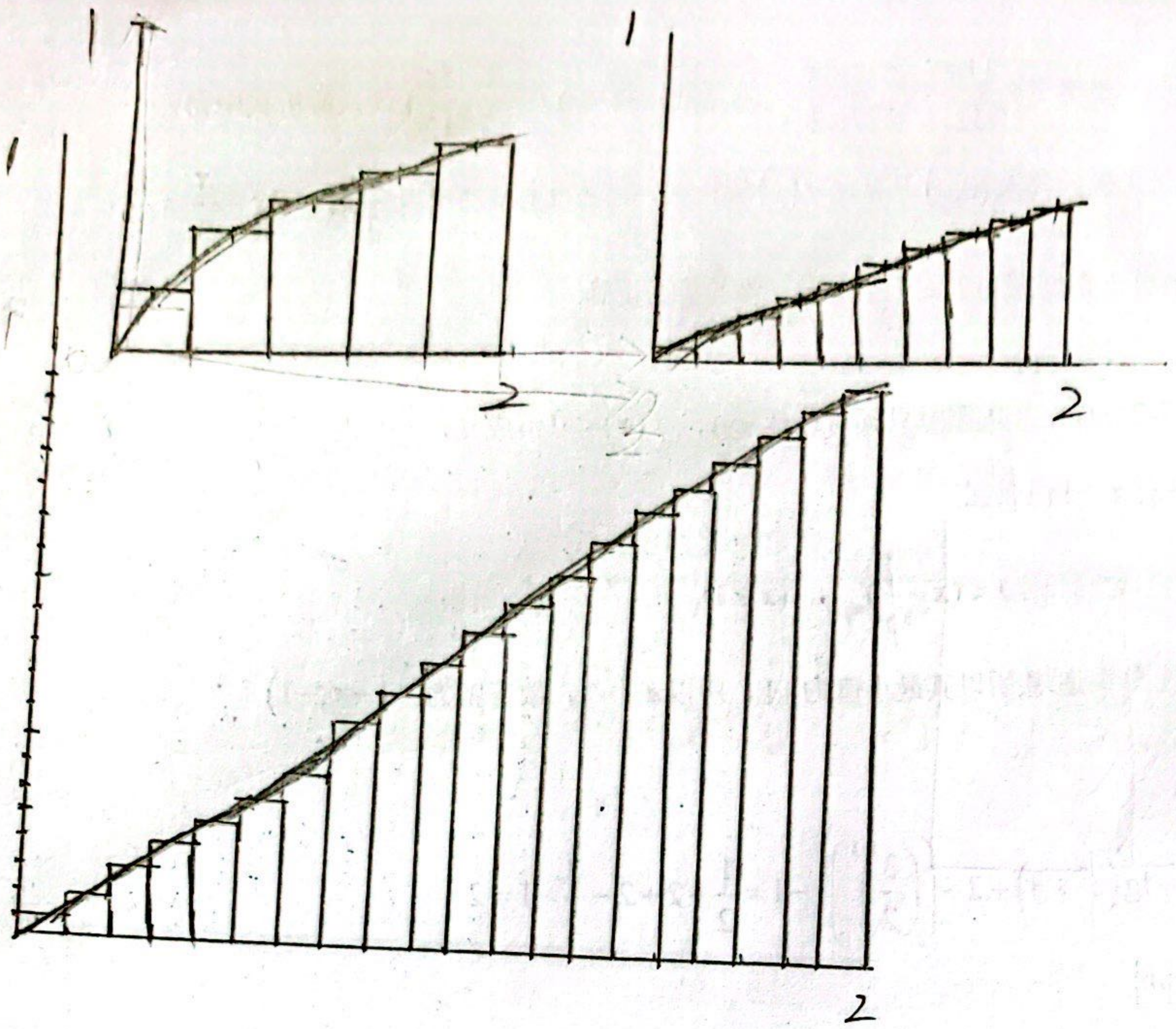
$=\frac{1}{4}(\frac{8}{9}+\frac{8}{11}+\frac{8}{13}+\frac{8}{15})$

≈ 0.691220



The Riemann sum representes the sum of the areas of the four rectangles.

13.



14. For $f(x)=\frac{x}{x+1}$ on $[0,2]$ $L_{100} \approx 0.89409$ $R_{100} \approx 0.90802$

$\because f$ is increasing on $[0,2]$

$\therefore L_{100}$ is an underesetimate of $\int_0^2 \frac{x}{x+1}dx$

R_{100} is an overesetimate of $\int_0^2 \frac{x}{x+1}dx$

$\therefore 0.8946 < \int_0^2 \frac{x}{x+1}dx < 0.9081$

16. $\int_0^2 e^{-x^2}dx$ with $n=5.10.50.100$

n	L_n	R_n
5	1.077467	0.684794
10	0.980007	0.783670
50	0.901705	0.862438
100	0.891896	0.872262

the value of the integral lies between 0.872 and 0.892

$\because f(x)=e^{-x^2}$ is decreasing on $(0,2)$

$\because f$ is increasing on $(-1,0)$

$\therefore$ we cannot make a similar statement for $\int_{-1}^2 e^{-x^2}$

22. $\Delta x = \frac{4-1}{n} = \frac{3}{n}$ $x_i = 1 + i\Delta x = 1 + \frac{3i}{n}$

$$\int_1^4 (x^2-4x+2)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x = \lim_{n\to\infty} \sum_{i=1}^{n} f(1+\frac{3i}{n})\frac{3}{n}$$

$$= \lim_{n\to\infty} \frac{3}{n} \sum_{i=1}^{n} [(1+\frac{3i}{n})^2 - 4(1+\frac{3i}{n}) + 2]$$

$$= \lim_{n\to\infty} \frac{3}{n} \sum_{i=1}^{n} (1+\frac{6i}{n} + \frac{9i^2}{n^2} - 4 - \frac{12i}{n} + 2) = \lim_{n\to\infty} \frac{3}{n} \sum_{i=1}^{n} (\frac{9i^2}{n^2} - \frac{6i}{n} - 1)$$

$$= \lim_{n\to\infty} \frac{3}{n} (\frac{9}{n^2} \sum_{i=1}^{n} i^2 - \frac{6}{n} \sum_{i=1}^{n} i - \sum_{i=1}^{n} 1) = \lim_{n\to\infty} [\frac{27}{n^3} \frac{n(n+1)(2n+1)}{6} - \frac{18}{n^2} \frac{n(n+1)}{2} - \frac{3}{n} \cdot n \cdot 1]$$

$$= \lim_{n\to\infty} [\frac{9}{2} \frac{(n+1)(2n+1)}{n^2} - 9\frac{n+1}{n} - 3] = \lim_{n\to\infty} [\frac{9}{2} \frac{n+1}{n} \frac{2n+1}{n} - 9(1+\frac{1}{n}) - 3]$$

$$= \lim_{n\to\infty} [\frac{9}{2}(1+\frac{1}{n})(2+\frac{1}{n}) - 9(1+\frac{1}{n}) - 3] = \frac{9}{2} \cdot 1 \cdot 2 - 9 \cdot 1 - 3 = -3$$

5.3

9. $f(t) = (t-t^2)^8$ $g(s) \int_5^s (t-t^2)^8 dt$

by FTC1 $g'(s) = f(s) = (s-s^2)^8$

14. Let $u = \sqrt{x}$ $\frac{du}{dx} = \frac{1}{2\sqrt{x}}$ $\frac{dh}{dx} = \frac{dh}{du}\frac{du}{dx}$

$$h(x) = \frac{d}{dx}\int_1^{\sqrt{x}} \frac{z^2}{z^4+1} dz = \frac{d}{du}\int_1^u \frac{z^2}{z^4+1} dz \cdot \frac{du}{dx} = \frac{u^2}{u^4+1} \cdot \frac{du}{dx} = \frac{x}{x^2+1} \frac{1}{2\sqrt{x}} = \frac{\sqrt{x}}{2(x^2+1)}$$

20. $\int_{-1}^{1} x^{100} dx = [\frac{1}{101} x^{101}]_{-1}^{1} = \frac{1}{101} - (-\frac{1}{101}) = \frac{2}{101}$

23. $\int_1^9 \sqrt{x} dx = \int_1^9 x^{\frac{1}{2}} dx = [\frac{x^{\frac{3}{2}}}{\frac{3}{2}}]_1^9 = \frac{2}{3}[x^{\frac{3}{2}}]_1^9 = \frac{2}{3}(9^{\frac{3}{2}} - 1^{\frac{3}{2}}) = \frac{2}{3}(27-1) = \frac{52}{3}$

24. $\int_1^8 x^{-\frac{2}{3}} dx = [\frac{x^{\frac{1}{3}}}{\frac{1}{3}}]_1^8 = 3[x^{\frac{1}{3}}]_1^8 = 3(8^{\frac{1}{3}} - 1^{\frac{1}{3}}) = 3(2-1) = 3$

25. $\int_{\frac{\pi}{6}}^{\pi} \sin\theta d\theta = [-\cos\theta]_{\frac{\pi}{6}}^{\pi} = -\cos\pi - (-\cos\frac{\pi}{6}) = -(-1) - (-\frac{\sqrt{3}}{2}) = 1 + \frac{\sqrt{3}}{2}$

28. $\int_0^4 (4-t)\sqrt{t} dt = \int_0^4 (4-t)t^{\frac{1}{2}} dt = \int_0^4 (4t^{\frac{1}{2}} - t^{\frac{3}{2}}) dt = [\frac{8}{3}t^{\frac{3}{2}} - \frac{2}{5}t^{\frac{5}{2}}]_0^4$

$$= \frac{8}{3} \cdot 8 - \frac{2}{5} \cdot 32 = \frac{320-192}{15} = \frac{128}{15}$$

34. $\int_0^3 (2\sin x - e^x)dx = [-2\cos x - e^x]_0^3 = (-2\cos 3 - e^3) - (-2-1) = 3 - 2\cos 3 - e^3$

36. $\int_1^{18} \sqrt{\frac{3}{z}}\,dz = \int_1^{18} \sqrt{3} z^{-\frac{1}{2}} dz = \sqrt{3}[2z^{\frac{1}{2}}]_1^{18} = 2\sqrt{3}(18^{\frac{1}{2}} - 1^{\frac{1}{2}}) = 2\sqrt{3}(3\sqrt{2} - 1)$

39. $\int_{\frac{1}{\sqrt{3}}}^{\sqrt{3}} \frac{8}{1+x^2} dx = [8\arctan x]_{\frac{1}{\sqrt{3}}}^{\sqrt{3}} = 8(\frac{\pi}{3} - \frac{\pi}{6}) = 8\cdot\frac{\pi}{6} = \frac{4\pi}{3}$

43. $\int_{\frac{1}{2}}^{\frac{1}{\sqrt{2}}} \frac{4}{\sqrt{1-x^2}} dx = [4\arcsin x]_{\frac{1}{2}}^{\frac{1}{\sqrt{2}}} = 4(\frac{\pi}{4} - \frac{\pi}{6}) = 4\cdot\frac{\pi}{12} = \frac{\pi}{3}$