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# Production and Cost Analysis

## Learning Objectives

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After reading this chapter, you should be able to:

- Understand why the efficiency of inputs in the production process varies according to the ratio of fixed to variable inputs utilized.
- Examine the relationships between production efficiency and average and marginal cost of production.
- Explain the relationships between average and marginal costs and between short-run and long-run costs of production.
- Distinguish between increasing efficiency in production and economies of scale in production.
- Identify increasing, constant, and decreasing economies of scale and distinguish these from economies of scope, purchasing economies, and the learning curve.
- Discuss the meaning and importance of a series of other cost concepts that are used in managerial decision making.

## Introduction

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As first mentioned in Chapter 1, managerial economics is concerned with maximizing profits while taking into consideration nonmonetary issues, such as the firm's risk exposure and its impact upon the social and natural environment. Profits are the surplus of revenues over costs, and the nonmonetary issues can be incorporated into the revenue side or the cost side of the profit calculation. In the first two chapters, we considered the manager's decision making under conditions of risk and uncertainty; in the next pair of chapters we considered the revenue side, and now we turn to the production and costs side of the managerial decision-making problem.

The first and most important distinction to make in production and cost analysis is between *fixed* and *variable* inputs to the production process. **Fixed inputs** do not vary over the time period chosen for the analysis, which we shall call the production period—one day, one week, or one month. Fixed inputs include buildings, factories, machines, vehicles, tools, and furniture (often collectively called “plant and equipment”), and highly skilled employees who cannot be hired or fired at short notice. These fixed inputs are durable assets of the business and continue to be useful in later periods, but generally cannot be varied in the current production period. Since the amount of fixed resources is constant over the current production period it follows that the cost of those resources is a fixed cost over the production period.

**Variable inputs** are those that can be varied at short notice, that is, the input quantity of these can be augmented or reduced during the production period. Examples of variable inputs include raw materials, components, electrical energy, fuel, office supplies, and employees who can be hired, laid-off, or utilized fractionally (paid only for hours worked) during the production period. The costs of these variable inputs will depend on how much of each resource is utilized in the production process during the current production period, and the amounts utilized will vary according to how much output is to be produced. Smaller volumes of output will require lesser amounts of variable inputs and thus lower costs of variable inputs, while larger volumes of output will require greater amounts of variable inputs and thus higher costs of variable inputs.

Economic analysis of the firm goes back hundreds of years to a predominantly agricultural world where there were two main inputs to the production process, namely *land* and *labor*. Tools and other fixed inputs, such as fences, spades, and rakes, were handmade and thus it took time to make additional units of these available to the production process. As the industrial revolution progressed, more and more industrial equipment was made and utilized, such as plows, harvesters, flour mills, steam engines, and so on, and economists chose a collective noun for all these fixed inputs, calling them *capital*. So the word **capital** became used to represent all the fixed inputs that enter the firm's production function. Most people remained relatively unskilled, and in any case, labor was abundant due to high levels of unemployment and was therefore easily augmented or reduced in the production process. Accordingly, the word **labor** was used to represent all the variable inputs to the production process.

This convention, calling the fixed inputs capital and the variable inputs labor, endures today, despite the fact that most human resources are now fixed during the current production period due to the time it takes to recruit and train new hires, the constraints imposed by labor unions and legislation on firms that want to reduce the number of workers, and the difficulty of finding people to hire with the required skills. Particularly, since managerial economists must talk to accountants (about costs) and human resource managers (about wages and salaries), it is better that we adopt the terms fixed and variable inputs to avoid confusion in our communication with these other managers within the firm. So, whenever we hear an economist say “capital and labor” we will know what they mean, but by using the terms “fixed and variable inputs” we will avoid introducing confusion into our communications with other managers.



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Economic analysis can be traced back hundreds of years to a predominantly agricultural world where land and labor were the two main inputs to the production process.

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Economists also make a distinction between the *short run* versus the *long run*. The **short run** refers to the period of time during which the fixed inputs remain fixed. For example, if it would take six months for a firm to change its fixed inputs, then for that firm the short run would be six months. Importantly, the short-run context implies a constraint on the firm’s output level—if the plant and equipment are running at full capacity, it will not be possible to increase output beyond the full capacity output level without increasing the size of the firm’s plant and equipment. The term **plant size** refers to the amount of the fixed inputs in the short run, and plant size can be changed only in the long run.

The **long run** is a hypothetical situation in which all inputs are variable, and the firm (for planning purposes) can contemplate any plant size and, consequently, any output level for production in a future production period. Thus, managers might install any number and combination of machines, vehicles, and other equipment; any number and composition of employees; any size of factory and office buildings; any quantity of raw material, components, energy usage, and so on. So, in any short-run period, managers of the firm will consider whether the current plant size is appropriate for their future sales projections (i.e., planned output levels) and, if not, will begin to make arrangements to augment (if planning expansion) or reduce (if planning contraction) their fixed inputs to enter a new short-run production period with an appropriate range of output levels. For example, suppose a new housing development is planned for the western suburbs of a city. A restaurant that is located near there anticipates that the demand for restaurant meals will increase as a result of the influx of new residents and decides to expand its plant size (seating and serving capacity) to capitalize on the situation. In remodeling and extending

the existing restaurant, the managers have an infinite variety of layouts, seating capacity, kitchen sizes, numbers of permanent employees, and other fixed assets that can be considered, and eventually they would choose one particular configuration of these inputs that would then become fixed for the subsequent short-run periods (until another expansion or reduction in plant size is deemed necessary).

## 5.1 Production and Cost Curves

In the short run, the quantity of output produced depends on both the quantity of the fixed inputs (plant size) and the quantity of the variable inputs. The **production function** shows the exact form of the relationship between the quantity of output and the quantity of the inputs. Because the inputs cost money to purchase, hire, or otherwise utilize, the production function is easily translated into the **cost function**, which relates the cost of production to the various output levels that are possible given the current plant size.

The form of the production and cost functions is an empirical issue—it needs to be investigated from data whether the form of the function is linear or curvilinear. The simplest case is a **linear production function**, where output increases linearly as variable inputs are added to the fixed inputs. With a linear production function the cost function will start with the (lump sum) cost of the fixed inputs and then rise linearly with the output level. **Curvilinear production functions** are more likely, however, because the proportionality between output and the variable inputs is likely to vary.

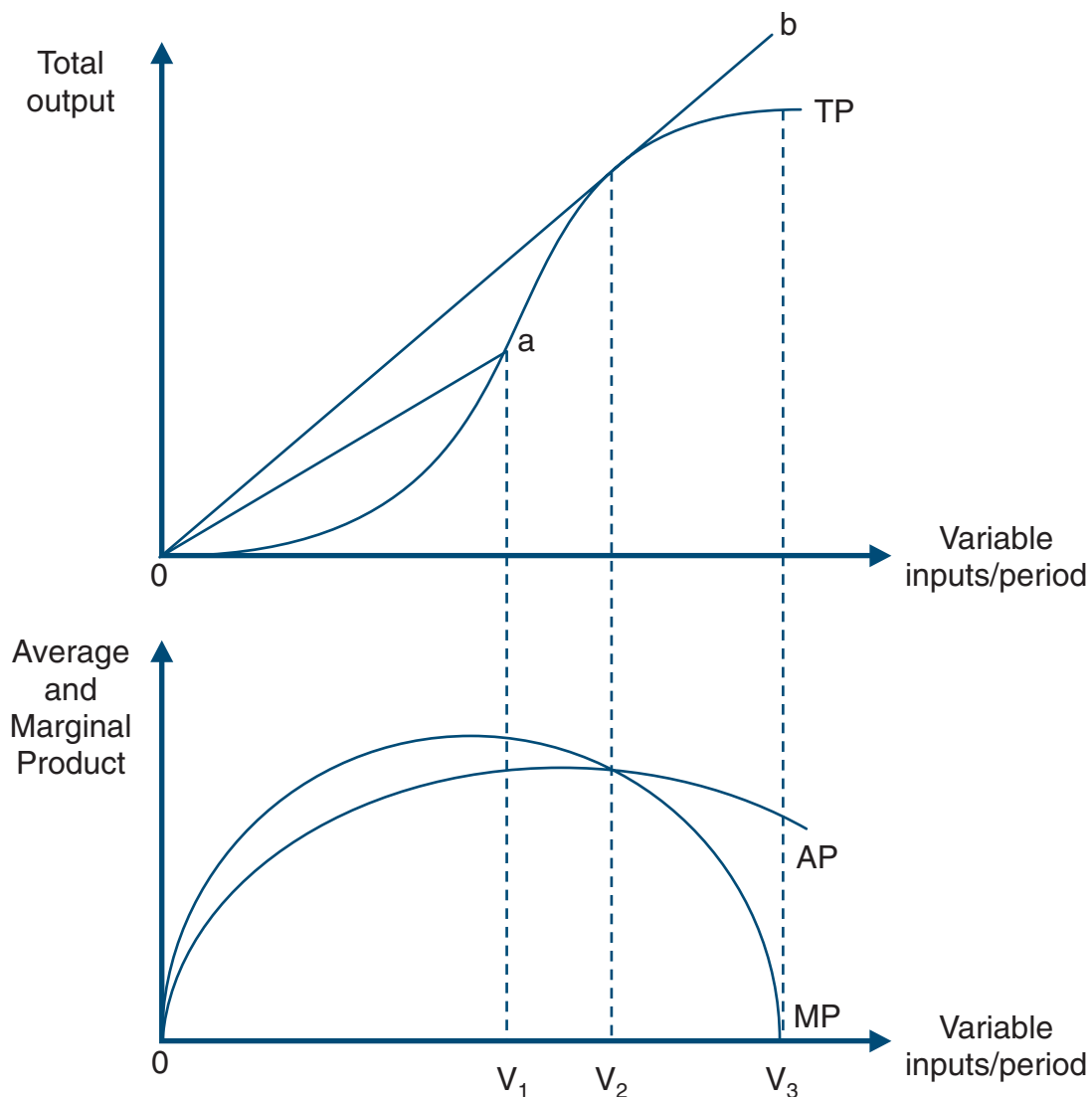
### The Law of Variable Proportions

The **law of variable proportions** states that total output is likely to increase at an increasing rate at first, and then increase at a decreasing rate as we progressively add more and more variable inputs to the fixed inputs. Total output, conventionally called total product (TP), thus rises in a “lazy-S” manner (shown in Figure 5.1). The **total product (TP) curve** shows the total output level that can be produced by a given plant size when augmented by various levels of the variable inputs. For example, in a restaurant with fixed plant size (kitchen and seating capacity) in the short run, as variable inputs (such as food materials, and casual wait staff) are added, output would increase at an increasing rate, at first, as the variable inputs become more efficient in a plant size that is initially “too large” for them. The wait staff would be underutilized and food materials would be wasted—a roast beef would not be completely eaten and the remainder would need to be thrown out at the end of the evening. After the point of inflection (where the TP curve changes from concave from below to convex from below), output increases at a diminishing rate as the variable inputs become progressively less efficient in a plant size that is now “too small” for the amount of variable inputs being applied to the production process. In this region of the production function, the wait staff would be bumping into each other, kitchen staff might be burning food, and the rapid pace of work is likely to induce mistakes in the ordering, cooking, and serving processes.

The increasing and later decreasing efficiency of the variable inputs to the production function can be measured by the average and marginal product values. The **average product (AP) curve** shows the ratio of the output level to the variable input level, at any

particular input level of the variable inputs, or  $TP/V$ . As you can see in the lower part of Figure 5.1, average product (AP) rises at first and then declines as more and more units of the variable inputs are added to the production process. In the upper part of Figure 5.1, a ray from the origin ( $0b$ ) lies just tangent to the TP curve at input level  $V_2$ . Note that the slope of any ray from the origin that hits the TP curve will represent the ratio of total output to the variable inputs, and thus the slope of the ray will indicate the ratio of TP (rise) to  $V$  (run) and thus indicates the AP value. The ray shown in Figure 5.1 indicates the input level ( $V_2$ ) where AP is maximized, since any steeper ray would not touch the TP curve. At lower input levels (such as  $V_1$ ) a ray joining the origin and a point on the TP curve ( $0a$ ) is flatter, and thus AP is lower. At higher input levels ( $V_3$ ), a ray joining the origin and the TP curve (not shown) would also be flatter—thus the AP curve rises to a maximum value at input level  $V_2$  and falls thereafter.

**Figure 5.1: Total, average, and marginal product curves**



The **marginal product (MP) curve** reflects the change in total product for a one-unit change in the variable inputs, that is,  $\Delta TP/\Delta V$  (where  $\Delta V = 1$ ). Since the TP curve is not a straight line, the MP is not constant but varies as additional units of the variable inputs are added. Put another way, the MP curve reflects the *slope* of the TP curve, rising at first, and then falling as the TP curve gets progressively steeper at first (MP rising) and then progressively less steep (MP falling). For example, in a restaurant, adding additional wait staff might cause the total number of meals to increase as displayed in Table 5.1. As you can see, as wait staff are increased from 1–4, MP increases, but as wait staff are increased from 5–10, MP decreases.

**Table 5.1: Total, average, and marginal productivity of wait staff in a restaurant**

Number of wait staff (V) (people)	Total product (TP) (meals served)	Average product (AP) AP = TP/V	Marginal product (MP) MP = $\Delta TP/\Delta V$
1	3	3	3
2	8	4	5
3	15	5	7
4	23	5.75	8
5	30	6	7
6	36	6	6
7	40	5.71	4
8	43	5.38	3
9	44	4.49	1
10	44	4.4	0

Now consider the shape of the marginal product (MP) curve and its relationship with the AP curve. First, since MP equals the slope of the TP curve, it rises from the beginning to a maximum value when the TP curve is steepest, at input level  $V_1$ , and subsequently falls.<sup>1</sup> Note that it must reach zero at input level  $V_3$  because the TP becomes flat at that point. Finally, MP must intersect and be equal to the AP curve when the AP is at its maximum, at input level  $V_2$ , because the MP is equal to the slope of the TP, and at  $V_2$  the AP is equal

1. To reconcile Figure 5.1 (where symbols represent the variable input levels) and Table 5.1 (where numbers represent the input levels) note that  $V_1$  in Figure 5.1 (where MP is maximized) is shown as 4 units of the variable inputs in Table 5.1;  $V_2$  in Figure 5.1 (where AP is maximized and also MP = AP) is shown as 6 units of the variable inputs;  $V_3$  in Figure 5.1 (where MP = 0) is shown as 10 units of the variable inputs.
2. You may note that the TP curve takes the shape of a cubic function,  $TP = a + bV + cV^2 + dV^3$ , where the parameter  $a$  represents the intercept on the vertical axis (zero in this case),  $b$  and  $c$  will take positive values and  $d$  will have a smaller negative value, which ultimately causes the  $V^3$  term to outweigh the  $V$  and  $V^2$  terms and cause the TP to reach a maximum and thereafter decline. Algebraically,  $AP = TP/V = a/V + b + cV + dV^2$  and  $MP = \delta TP/\delta V = b + 2cV + 3dV^2$ . You can see that both AP and MP are quadratic equations and thus have an inverted-U shape.



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Adding additional workers to this production line should be expected to cause average productivity to rise at first and later decrease, due to the law of variable proportions.

to the slope of the ray that is just tangent to (i.e., has the same slope as) the TP curve.<sup>2</sup>

Why do we expect these production curves to bend and intersect like this? Because experience has taught us that the productive efficiency of variable inputs will usually vary in this predictable way, and the common observation of this phenomenon led to it being called the *law of variable proportions*. **Production efficiency** is measured by the average and marginal productivity of the variable inputs in the context of the plant size to which they are added. At low levels of variable inputs the ratio of variable to fixed inputs is very high,

and, in effect, the variable inputs have “too much” plant size to work with. At higher levels of variable inputs, the ratio of variable to fixed inputs is relatively low, and in effect the variable inputs have “too little” plant size to work with.

Another version of the law of variable proportions is the **law of diminishing returns**,<sup>3</sup> which states that as the firm adds variable inputs to its fixed inputs in its production process, after some point the marginal productivity of the variable inputs will begin to decline and will progressively fall, potentially becoming negative if the firm continues to add variable inputs into the production process. Thus, the law of diminishing returns is effectively identical to the law of variable proportions, but refers explicitly to the range of input levels above  $V_1$  in Figure 5.1. We will see that this is the range of input (and output) levels at which the firm will most likely want to be operating.

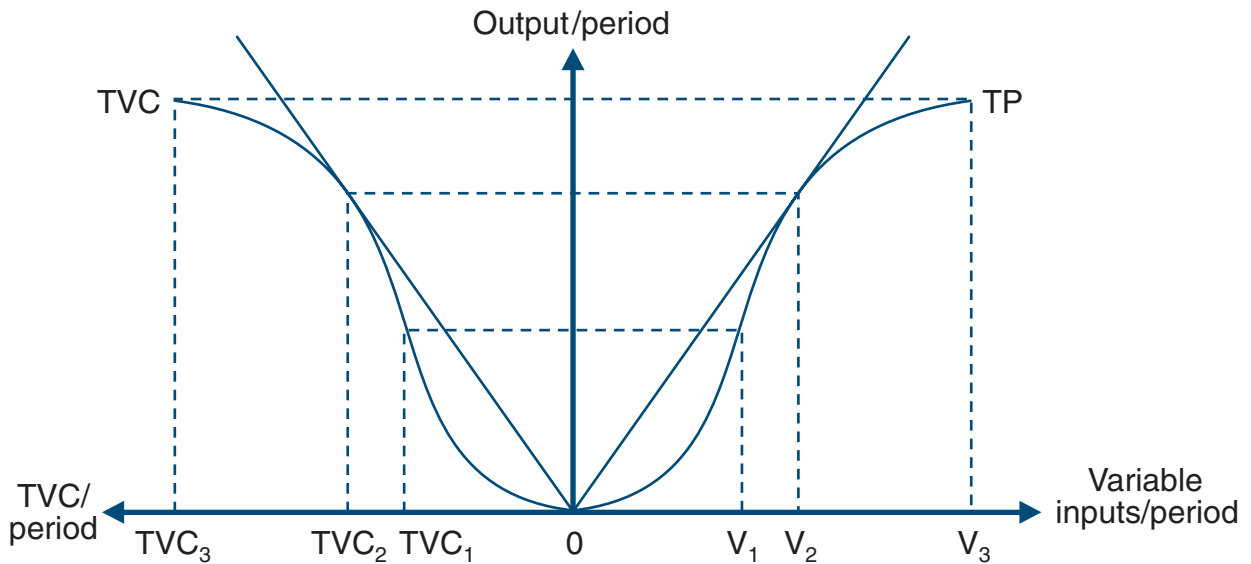
### Total Variable Costs

The **total variable cost (TVC)** is the total cost of the variable inputs to the production process. The total variable cost (TVC) curve will necessarily reflect the shape of the TP curve since increasing, and later, diminishing returns to the variable inputs in production will have a corresponding impact on the TVC curve. In Figure 5.2, we show the TP curve

3. Note that these laws are not legal laws, but instead are empirical laws, that is, they are frequently observed in practice and have been validated by data collection and estimation of the production and cost functions.

on the right-hand side and TVC on the left-hand side, both relating to the vertical axis representing output levels. Note that the left-hand scale (for TVC) is simply the monetary equivalent of the right-hand scale (for TP). Suppose, for example, that units of the variable inputs cost \$100 each, the input quantity levels on the right-hand side are simply multiplied by 100 to find TVC levels on the left-hand side.

**Figure 5.2: Derivation of the TVC curve from the TP curve**

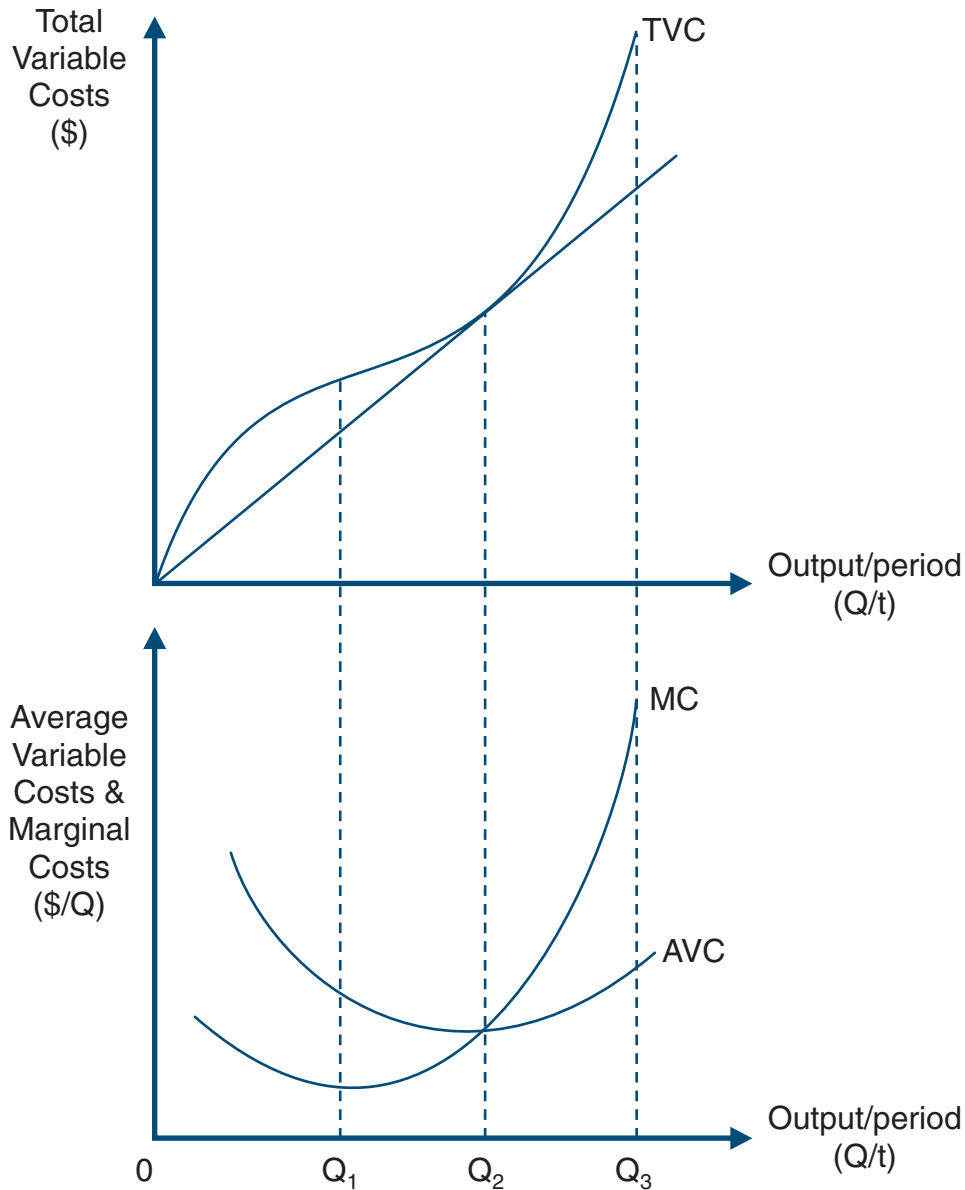


Note that the shape of the TVC curve is absolutely dependent upon the shape of the TP curve—if there are diminishing returns to the variable inputs, there will simultaneously be increasing average and variable costs of production, as shown in Figure 5.3 where we derive the average variable cost (AVC) and marginal cost (MC) curves from the TVC curve.

### **Marginal Costs and Average Variable Costs**

Notice that we have rotated the left-hand side of Figure 5.2 through 90 degrees to show the TVC curve with output ( $Q$ ) on the horizontal axis in Figure 5.3, and thus have our cost data in the same graphical format as the demand and MR curves, as shown in Chapters 3 and 4. From this TVC curve we can now derive average variable cost and marginal cost curves by observing the shape of the TVC curve. **Marginal cost (MC)** is a change in TVC for a one-unit change in the output level (i.e.,  $\Delta\text{TVC}/\Delta Q$ , where  $\Delta Q = 1$ ) and you will note that it is equal to the slope of the TVC curve at any output level. When TVC is at its flattest (at the point of inflection) at output level  $Q_1$ , MC reaches its minimum. When the TVC curve goes vertical (i.e., there is extra variable cost but no extra output, at  $Q_3$ ) then the value of MC becomes infinite.

Figure 5.3: Deriving the AVC and MC curves from the TVC curve



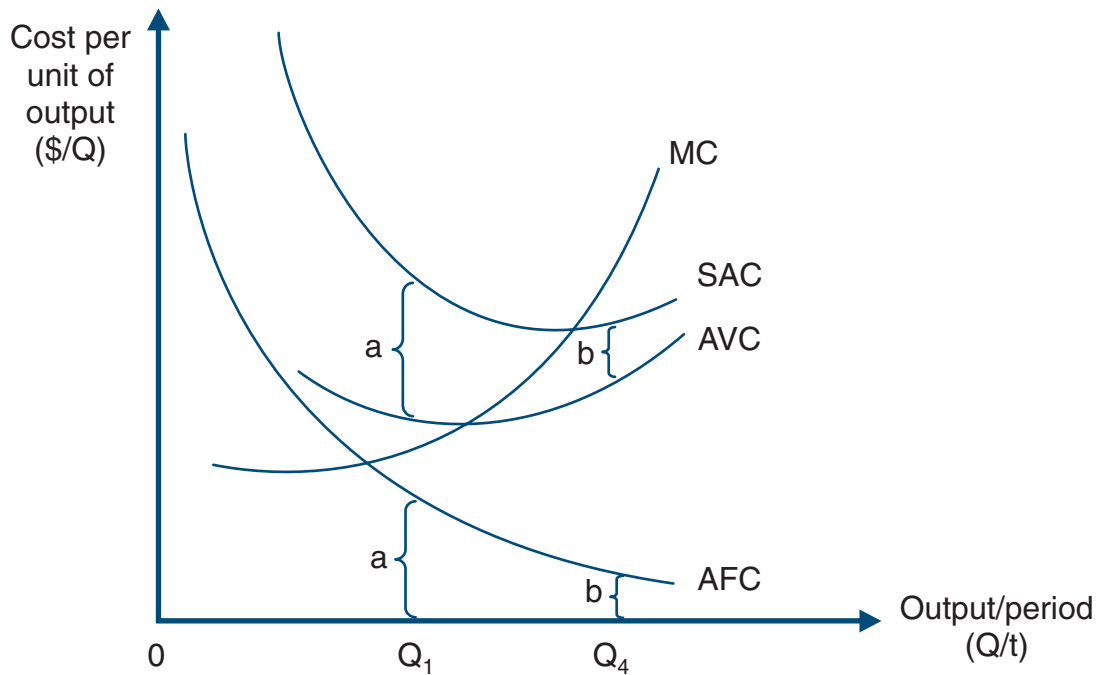
**Average variable cost (AVC)** is the ratio of total variable cost to output level, or  $TVC/Q$ . The slope of a ray from the origin that touches the TVC curve at any output level will give the value of AVC at that output level. You can see that if you were to draw rays from the origin to points on the TVC curve, these rays (not shown) would be progressively flatter at first, reaching a minimum slope (shown) at output level  $Q_2$ , and thereafter the rays would be progressively steeper (not shown) and, thus, AVC must be increasing between output levels  $Q_2$  and  $Q_3$ . It follows that  $MC = AVC$  at the output level ( $Q_2$ ) where AVC is minimized since both are equal to the slope of the TVC curve at that point.<sup>4</sup>

## Total Costs and Short-Run Average Costs

**Total costs (TC)** are the sum of total variable costs and total fixed costs. Since **total fixed costs (TFC)** are constant during the short-run production period, we can simply add a constant vertical amount to the TVC curve to find the TC curve. **Short-run average costs (SAC)** are the total costs divided by the number of units of output, so must be equal to the average variable costs (AVC) plus the average fixed costs. **Average fixed costs (AFC)** are the total fixed costs divided by the number of units of output. Since TFC is a constant, the AFC must decline from a very high number (equal to TFC when  $Q = 1$ ) to a very low number as TFC are spread across larger and larger volumes of output. This type of curve is known as a “rectangular hyperbola.”

As previously mentioned, we add AVC and AFC to determine short-run average costs (SAC), as shown in Figure 5.4. Notice that AFC is shown in the lower part of the figure as a monotonically declining line that would progressively approach zero as output levels become very high. We add the AFC curve to the AVC curve by a process of vertical addition at every output level. Since AFC is declining monotonically, and we add this increasing smaller vertical distance to AVC, the vertical distance between AVC and SAC must also become smaller as output levels rise. Accordingly, the SAC continues to fall after the output level where AVC was minimized but, at some point, the rise in AVC exceeds the fall in AFC, and so the summation of these two (i.e., SAC) must also begin to rise. Note that the MC curve must pass through the minimum point of SAC because the TC curve is changing only due to changes in the TVC curve.<sup>5</sup>

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4. There is a general relationship between the average and the marginal value of any statistic. If the marginal observation is below the average, the average must be falling, since the marginal (last) observation will pull the average down, and conversely if the marginal is above the average, the average must be rising, since the marginal observation will push the average up. You already know this in the context of your grade point average (GPA) or your baseball batting percentage, for example. It follows that the marginal value must equal the average value at the minimum value of the average.
  5. We saw earlier that the MC is equal to the slope of the TVC curve. Because the TC curve is simply shifted vertically by the addition of the TFC curve, the TVC and the TC curves must have the same slope at any particular output level.

**Figure 5.4: Finding the SAC curve by adding the AFC curve to the AVC curve**

In Figure 5.4, the vertical addition of the AVC and the AFC curves to find the SAC curve is illustrated at output levels  $Q_1$  and  $Q_4$ . You can see that the value of AFC at output level  $Q_1$  is equal to the vertical distance between the AVC and the SAC curves at that output level, and similarly, the AFC value at output level  $Q_4$  is equal to the vertical distance between the AVC and the SAC curves at that output level.<sup>6</sup>

### ***A Numerical Example: Short-Run Cost Curves for The Robust Coffee Place***

It will help put all these short-run production and cost concepts into focus if we take a look at a practical example. Suppose that entrepreneur Eddie opens a new coffee shop called The Robust Coffee Place and collects data on his variable production inputs and their costs over the first 10 days of operation. His variable costs include the cost of coffee beans, cream, sugar, napkins and coffee stirrers, and casual labor for coffee-making and cleaning the shop. His fixed costs include his rental of the premises, his lease payments on the coffee machine and other equipment, his purchases of coffee cups, utensils, and cleaning materials, and his own salary. During the first 10 days, his output levels (coffees sold) varied up and down according to customer traffic at the time of the day and the day of the week. Eddie arranges his data in ascending order of coffees sold to see how costs of production vary with the output level, as shown in Table 5.2.

6. Because the TVC curve reflects the shape of the TP curve, it is no surprise that the TVC curve is a cubic function of the output level, namely  $TVC = e + fQ + gQ^2 + hQ^3$ , and thus  $AVC = TVC/Q = f + 2gQ + 3hQ^2$  and  $MC = \delta TVC / \delta Q = f + 2gQ + 3hQ^2$ . Thus, AVC and MC are both quadratic expressions and have an inverted-U shape when graphed.

**Table 5.2: Production and variable cost data for The Robust Coffee Place**

Coffees sold (Q) (1)	Coffee bean cost (2)	Cream and sugar cost (3)	Napkins and stirrers cost (4)	Casual labor cost (5)	TVC (6)	AVC = TVC/Q (7)	MC = $\Delta\text{TVC}/\Delta\text{Q}$ (8)
80	40.00	24.00	8.00	80.00	152.00	1.90	
102	45.90	28.56	9.18	85.00	168.64	1.65	0.76
112	48.16	30.24	9.52	90.00	177.92	1.59	0.93
118	53.10	33.04	9.68	95.00	190.82	1.62	2.15
124	62.00	37.20	10.54	98.00	207.74	1.68	2.82
132	72.60	43.56	11.88	106.00	234.04	1.77	3.29
140	81.20	49.00	14.00	120.00	264.20	1.89	3.77
144	86.40	53.28	15.84	124.00	279.52	1.94	3.83
150	97.50	60.00	18.00	140.00	315.50	2.10	6.00

In columns 6, 7, and 8 of Table 5.2, we can calculate the TVC, AVC, and MC values for Eddie's coffee production process. The TVC is simply the sum of the variable cost categories applicable to this business, which are shown in columns 2–5, and it goes up with the numbers of coffees sold, as expected. The average variable costs are equal to TVC divided by the output level (column 1) for each data observation. You can see that AVC falls at first, then rises, as expected. Notice that AVC falls to a minimum value of about \$1.59 per cup somewhere around the output rate of 112 coffees per day, and then rises to more than \$2 per cup at high output rates, being pulled up by the MC value. Marginal costs are estimated over each of the discrete ranges of outputs given by the day-to-day variations in the number of coffees sold. We calculate MC as the change in TVC ( $\Delta\text{TVC}$ ) divided by the change in quantity of number of coffees produced ( $\Delta\text{Q}$ ). As you can see, the MC value rises and continues to rise as output levels rise, indicating diminishing marginal productivity of the variable inputs as they are applied to the fixed inputs of the coffee shop.

You might be thinking that this data indicates that Eddie should try to keep his AVC down near its minimum level by restricting coffee sales to around 110–120 coffees per day. But, before we conclude anything like that, we need to consider Eddie's fixed costs as well and add them to the variable costs already considered. In Table 5.3, we show TFC, AFC, and SAC data in addition to the cost data repeated from Table 5.2.

**Table 5.3: Short-run total costs and average costs for The Robust Coffee Place**

Output (Q) (1)	TVC (\$) (2)	TFC (\$) (3)	TC (\$) (4)	AVC (\$) (5)	AFC (\$) (6)	SAC (\$) (7)	MC (\$) (8)
80	152.00	200.00	352.00	1.90	2.50	4.40	
102	168.64	200.00	368.64	1.65	1.96	3.61	0.76
112	177.92	200.00	377.92	1.59	1.79	3.37	0.93
118	190.82	200.00	390.82	1.62	1.69	3.31	2.15
124	207.74	200.00	407.74	1.68	1.61	3.29	2.82
132	234.04	200.00	434.04	1.77	1.52	3.29	3.29
140	264.20	200.00	464.20	1.89	1.43	3.32	3.77
144	279.52	200.00	479.52	1.94	1.39	3.33	3.83
150	315.50	200.00	515.50	2.10	1.33	3.44	6.00

As shown in Table 5.3, we assume that Eddie's fixed costs are \$200 per day. Total costs (TC) in column 4 are the summation of the TVC and the TFC values in columns 2 and 3. The AFC value (column 6) is the TFC divided by the output level (column 1). Notice that the AFC values decline quickly as output levels increase because the constant level of TFC is spread over more and more units of output. The SAC values (column 7) may be found either by dividing the TC values by the output levels, or by adding together the AVC and AFC values at each output level. Now, notice what is happening to the SAC values; although SAC is relatively high at low output levels due to the heavy burden of AFC, it falls relatively quickly to a minimum of \$3.29 in the output range of about 124–132 coffees a day, after which it starts to rise slowly. Note that the SAC value is relatively stable in the range \$3.29 to \$3.33 over a quite wide range of output levels from about 115 to 144 coffees per day. Also note that the SAC keeps falling after the AVC has started to rise, because the rise in AVC is outweighed by the fall in AFC as output levels continue to rise. Even though the MC is above the AVC, it is below the SAC value until it intersects the SAC curve at \$3.29 per cup.



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So, is Eddie making any money in his new coffee business? That depends, of course, on what he earns per coffee sold, that is, what price he is charging. Although we will see in Chapter 7 exactly what price he should charge to maximize profits, for now let's assume that he is

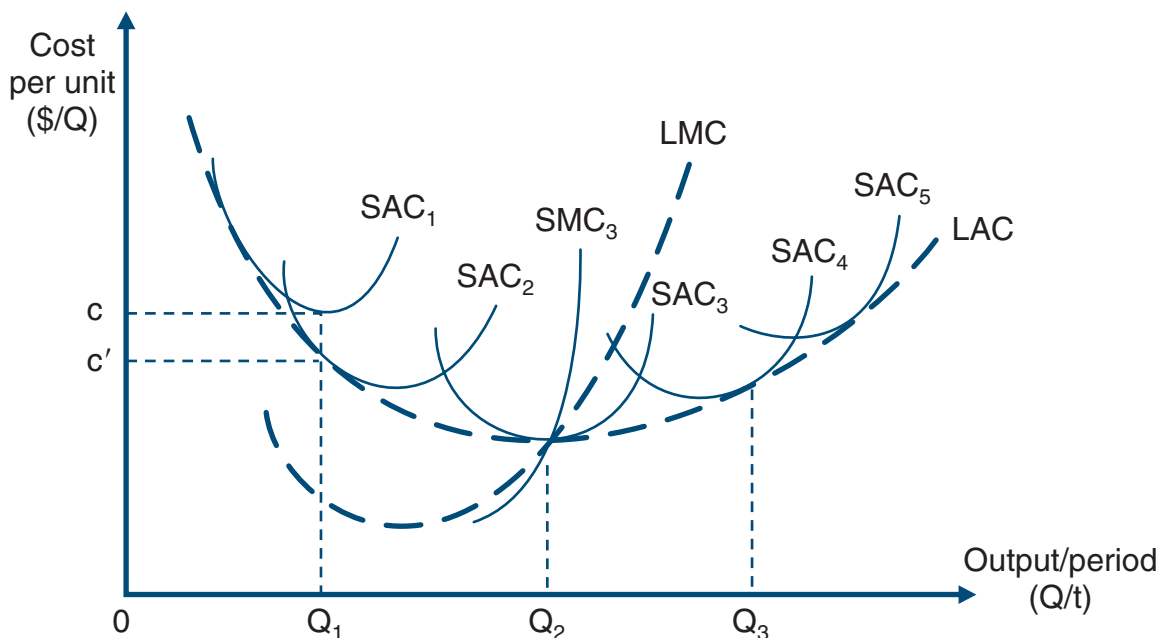
Variable costs for a coffee shop include the cost of coffee beans, cream, sugar, napkins, mugs, and employees.

charging a competitive price of \$4 per cup. Since this price exceeds average cost for all output levels, he has made a profit every day so far, but notice that when the output levels are at or near 150 cups a day, his marginal cost exceeds \$4 so he is losing money on the last few coffees! Although he should not turn away the last few customers (since they might turn into repeat and regular customers), he should start thinking about increasing his plant size (e.g., install a larger or faster coffee machine) to produce higher output levels at lower average and marginal cost levels.<sup>7</sup>

## 5.2 The Long-Run Cost Curves

As mentioned earlier, the long run is a hypothetical situation in which all inputs and costs are variable, so managers can choose whichever plant size they think is appropriate for their demand situation. Thus, managers can visualize many different SAC curves (and their underlying AVC, AFC, and MC curves) all at the same time, and will then select one set of short-run curves, build the chosen plant size, and proceed ahead into a new short-run production period. The **long-run average cost (LAC)** curve shows the least cost of production for each output level when all inputs are variable. It is composed of a small segment of many different SAC curves. In Figure 5.5, we show the long-run average costs (LAC) curve as the *envelope curve* of a series of SAC curves, where each of the SAC curves sits on the LAC and is tangent to the LAC for a small distance.<sup>8</sup>

Figure 5.5: The LAC curve is an envelope curve for the SAC curves



7. This example assumes that all costs of operation are included. If Eddie was staying open until midnight on some days and only sells a few coffees after 9 p.m., and was not accounting for the opportunity cost of his own time, he should revise his decision.

8. An envelope curve forms the outer boundary of a set of observations. The LAC curve shows the minimum SAC for every output level, presuming that the size of plant can be varied infinitesimally to minimize the SAC for each output level.

Note that this does *not* mean the LAC joins the minimum point of each SAC curve. On the downward sloping (left-hand side) section of the LAC curve, the part of the SAC that contributes to the LAC is found *to the left* of the minimum point on the SAC curve. You can see in Figure 5.5 that at output level  $Q_1$  it is  $SAC_2$ , rather than the minimum point on  $SAC_1$ , that contributes to the LAC curve since the average cost of  $Q_1$  is minimized at  $c'$  dollars rather than at  $c$  dollars. Oppositely, on the upward-sloping right-hand side of the LAC curve, the part of the SAC that contributes to the LAC is *to the right* of the minimum point on each SAC curve.

In Figure 5.5, we also show the **long-run marginal cost (LMC)** curve, which is the locus of least-cost short-run marginal cost levels when all inputs to the production process are variable. The LMC curve has the expected relationship with LAC, lying below LAC when the latter is falling, lying above LAC when the latter is rising, and intersecting LAC when the latter is at its minimum level. At the minimum point on the LAC curve, note that  $LAC = LMC$  and also that  $SAC = SMC$  so that all four values are equal at output level  $Q_2$ . (Note that we are using SMC to refer to short-run marginal cost to distinguish it from LMC.)

Because the fixed inputs are infinitely variable in the long run, there would be an infinite number of SAC curves, which allows the LAC curve to be a smooth curve composed of infinitely small sections of each of the possible SAC curves. We have shown only five SAC curves in Figure 5.5, but those five curves are enough to demonstrate that we expect the LAC curve to be U-shaped. Said another way, we expect the production efficiency of the firm to improve at first and later reduce as we progressively increase the size of the firm's plant.

### Economies and Diseconomies of Scale

The increase in production efficiency as plant size is increased is known as **economies of scale** and is characterized by successive SAC curves lying below and to the right of the preceding SAC curve (such that the LAC curve is downward sloping). Economies of scale are also known as economies of plant size. In Figure 5.5, economies of plant size are evident up until output level  $Q_2$ . **Diseconomies of plant size** (or scale) are evident when successively larger plant sizes cause the SAC curves to lie above and to the right of the preceding SAC curve, and thus the LAC curve is upward sloping and LAC values are rising.

What causes economies of plant size? They occur because the ratio of fixed to variable inputs is becoming progressively more efficient. This increased efficiency occurs because some of the fixed resources with unused capacity (such as management time, or factory, office, and storage space) become more fully utilized without costing any more money, and because larger plant sizes allow more and more workers to specialize on those parts of the work where they are most productive, rather than be "jacks of all trades, masters of none." Economies of scale may also arise due to purchasing economies (e.g., buying in bulk), as we shall see later. Diseconomies of plant size may occur when plants become very large and there are many people working in the same workplace. Communication among coworkers, and between bosses and other employees, becomes less and less efficient, and employee morale might break down leading to reduced personal efficiency of individual workers.



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When communication among managers and other employees becomes less efficient in large plants, diseconomies of plant size may occur leading to reduced efficiency.

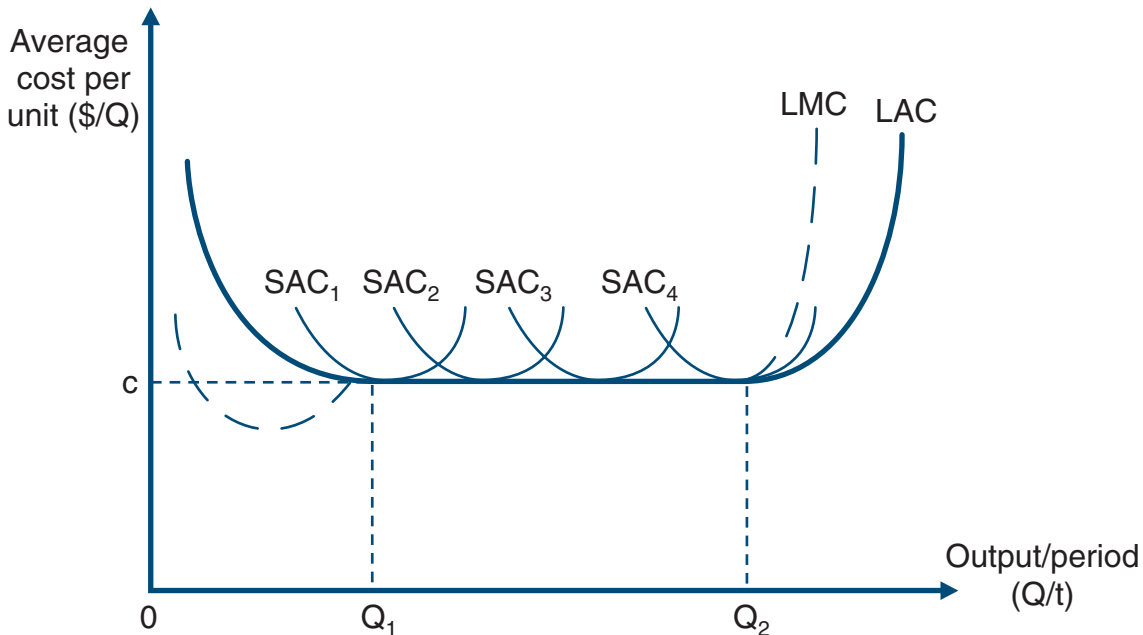
The SAC curve that nestles in the bottom of the U-shaped LAC curve is known as the **optimum plant size**, or the **optimal scale of plant**. The optimum scale of plant is the plant size that allows the product to be produced at the least cost per unit when all inputs to the production process can be varied.

### Constant Returns to Scale

It is possible that the firm might experience **constant returns to plant size**, where the LAC of production remains constant at a particular level as the scale of plant is increased, as shown in Figure 5.6. In such cases, **constant returns to scale** are usually

preceded by economies of scale and are later followed by diseconomies of scale, but there is a range of plant sizes over which LAC neither falls nor rises. In Figure 5.6, a selection of SAC curves lie on a horizontal section of the LAC curve between output levels  $Q_1$  and  $Q_2$ . Note the LMC curve is the dashed line that joins the left-hand section of the LAC when constant returns to plant size begin, and is then equal to LAC while the latter is constant, and finally LMC rises above LAC, pulling that curve upwards after diseconomies of scale set in.

**Figure 5.6: Constant returns to plant size**



In this case, there are multiple optimal plant sizes, as each SAC curve that is tangent to the flat bottom of the LAC curve can produce the product at the minimum average cost of production. Constant returns to plant size are preferable to diseconomies of plant size, since the firm can keep its costs per unit of output at a stable level in the long run and, thus, avoid price increases and the loss of market share that might otherwise have been necessary. Constant returns might occur when the forces that deliver economies of scale are just balanced by the forces that deliver diseconomies of scale over a range of output levels. They might occur because managerial decisions have been taken to improve the productivity of resources that were seen to be approaching the point of diminishing marginal productivity, such as decisions to supply workers with more efficient computers, tools, machines, vehicles, and so on. In the case of The Robust Coffee Place, if demand was sufficient, Eddie could potentially gain constant returns to plant size by doubling his plant size by leasing the shop next door and installing similar coffee-making equipment and personnel there to replicate his existing operation.

### 5.3 Other Cost Concepts

It is important to understand a variety of other cost concepts that are important for managerial decision making. As indicated in Chapter 1, economic and accounting concepts of costs and profits differ due to the use of different cost and revenue conventions. Thus, it is important here to clarify several cost concepts and to make it perfectly clear what we mean by costs of production in managerial economics.

#### Economic Costs Versus Historical Costs

In business situations, there are times when the *actual cost* of an input is not the *economic cost* of the input. The *economic cost* of an item is defined as its value in the best alternative utilization of that item. The best alternative use of the item is also known as the best alternative opportunity, and so economic costs are also known as *opportunity costs*. For example, suppose a machine had been purchased last month for \$200,000 but, due to currency exchange rate fluctuations, it would now cost \$250,000 to buy and import the same machine. Rather than use the machine in its production process, the firm could alternatively sell it to another firm for \$250,000, assuming it was still in as-new condition. This means that the economic cost (opportunity cost) of the machine is \$250,000 not the initial purchase cost. The initial purchase cost is known as the **historical cost**, or what it actually cost in the past period.

Another example is the inventory of copper tubing that a plumbing firm holds, which cost, let's say, \$100,000 to buy last year. Now, let's suppose that the price of copper had recently risen significantly and would now cost \$150,000 to buy the same amount of copper tubing. Again, the historical cost would be \$100,000 whereas the economic cost would be \$150,000 since it would cost that much to replace the copper tubing, or, alternatively, it could be sold to another plumber for that amount. Thus, the value of the inventory of copper tubing held by the firm would need to be revised upwards to reflect the change in copper prices. Oppositely, if the market value of another inventory item, say, finished goods, were to fall significantly due to the design becoming outmoded, the

value of those inventories would need to be revised downwards to reflect their economic value to the firm.

It is important to understand that the cost concept underlying the cost curves in our analysis is economic (or opportunity) costs. To rely on historical costs would potentially undervalue (or overvalue) the true economic costs of producing any particular output level.

### Sunk Costs, Unavoidable Costs, and Incremental Costs

Costs that have been incurred previously and cannot be retrieved are called **sunk costs**. Most fixed costs will be sunk costs unless the asset can be sold to someone else who wants to buy it, and usually the price that someone else will pay for used equipment or other asset is less than the historical cost of those items. The price the firm can obtain by selling these used items is known as the *salvage value*, and, as mentioned, unless the value is appreciating (e.g., a collector's item), the salvage value is usually less than the historical cost. To account for the declining value of a fixed asset, accountants apply a depreciation charge that reduces the depreciated value of the asset to reflect its reduced salvage value.

Similar to sunk costs are **unavoidable costs**—these are costs that the firm is contractually committed to pay regardless of output levels. Lease costs of a fixed asset, for example, are set by contractual agreement and must be paid monthly in most cases. Salaries of top management are also unavoidable costs for the firm in most cases, although in dire circumstances even these might be avoidable or at least postponed.

**Incremental costs** are those costs that will be incurred in the future because of a decision to be made. Thus, variable costs are incremental costs that follow the decision to produce output at a particular level, whereas fixed costs are not incremental. In making forward-looking decisions about output levels in the current or future periods, we will see that only incremental costs are relevant and that sunk costs and unavoidable costs are not relevant since they will occur anyway. And, continuing the discussion above, the incremental costs that we do consider must be valued at their economic (or opportunity) cost rather than at their historical costs.

### Economies of Scope

**Economies of scope** are reductions in average costs that occur because the cost of an input can be spread across more than one product line. Our analysis has been concerned with the production and cost curves for a single product line, but, in most firms, there will be multiple product lines. Economies of scope arise when fixed or variable inputs to the production process have underutilized capacity and this underutilized capacity may be used in another production process. For fixed inputs, there might be unused or underutilized space in buildings, time of managers, or time of machines and equipment that could be utilized in the production process for another product line. Thus, Toyota, which has at least 45 different vehicle types and models, gains economies of scope when it introduces a new model because the extra product line does not require an entirely new set of buildings, managers, and machines but can instead utilize the spare space and time of existing fixed inputs. Accordingly, some fraction of the fixed costs will be allocated to the new model and thereby reduce the fixed cost allocation to the previously existing models.

Economies of scope are also possible for variable inputs that are incompletely utilized in the production of a particular product. Consider a metal-working firm that has a production process in its factory (fixed inputs) that processes mild steel (variable input) into steel gates and fences for sale to firms in the construction industry. The off-cuts of steel that are generally too small to be used in fences and gates might be called waste products and simply disposed of; alternatively, these can be used to produce one or more other product lines, such as small brackets. Because products of saleable value (brackets) can be made from the waste product of the metal-working operation, accountants will want to assign some of the cost of the steel to the bracket product line and thus, reduce the cost of the steel to the gates and fences product line.

Increasing the scope of the firm's operation to utilize the underutilized space and time of fixed input, or the unused time or capability of variable inputs, serves to reduce the fixed or variable costs associated with the other product lines produced by the firm. Economies of scope, therefore, cause the SAC and the LAC curves to shift vertically downwards at every output level due to a downward shift of the AFC or the AVC curves.

### Purchasing Economies

**Purchasing economies** are the reduction in average costs that are due to purchasing inputs in larger volumes, where the firm receives discounts for buying in bulk. For example, a glass manufacturing company might pay \$30 per ton of sand for orders of 10 tons or less, but the price of sand might fall to \$27 per ton (i.e., 10% discount) for order quantities between 10 and 50 tons, and fall again to \$24 per ton (i.e., 20% discount) for order quantities greater than 50 tons. Because larger glass manufacturing firms would use more sand per period in their production process, they will gain a cost advantage over smaller firms whose lesser rate of production makes it economically inefficient to take delivery of huge amounts of sand and have their factory space (and their funds) tied up in a large inventory of sand. Along with the reduced cost per ton of sand, the delivery cost per ton will generally be smaller for larger delivery volumes, so this is another purchasing economy accruing to larger firms.

As indicated earlier, purchasing economies are one of the main causes of economies of scale—firms with larger plant sizes gain purchasing economies on all kinds of fixed and variable inputs that cause the LAC curve to slope downward as plant size is increased, up to the point of the optimum scale of plant. While very large firms might still be gaining purchasing economies as they expand to larger and larger plant sizes, the rise of other inefficiencies might offset the gains from



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Purchasing economies are the reduction in average costs due to purchasing inputs in larger volumes, where the firm receives discounts for buying in bulk.

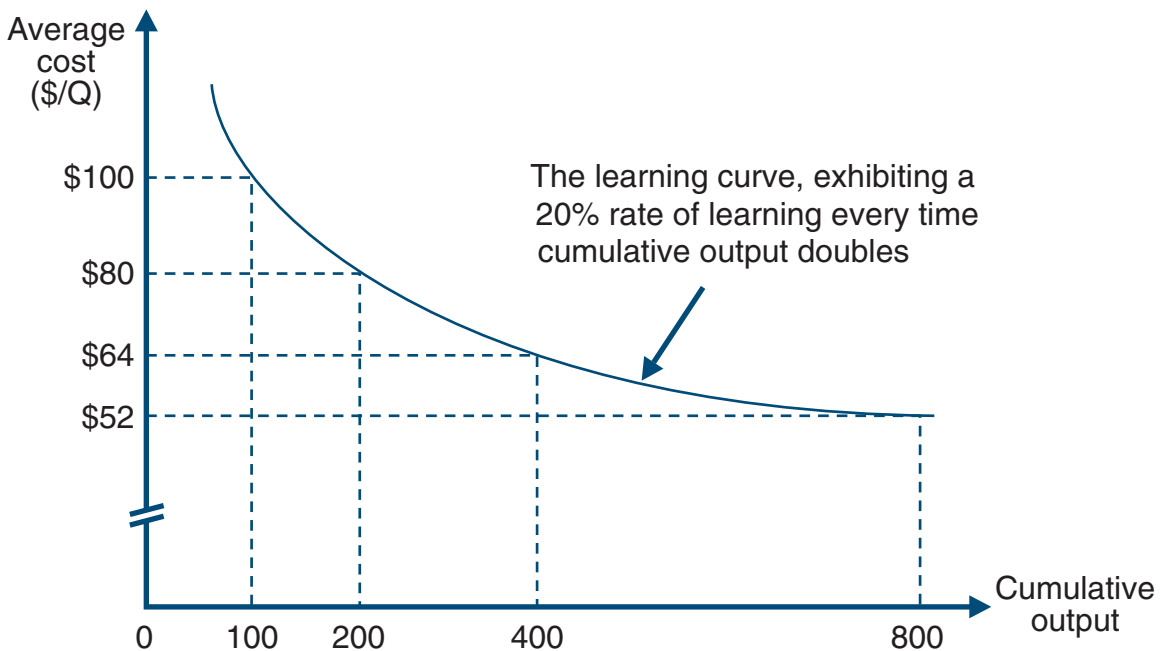
While very large firms might still be gaining purchasing economies as they expand to larger and larger plant sizes, the rise of other inefficiencies might offset the gains from

purchasing economies such that overall the firm eventually experiences diseconomies of plant size.

### Learning Curves

**Learning curves**, also known as experience curves, show the decline in average cost per unit of output as the firm's experience producing that product accumulates. As the firm's experience in producing a particular product increases, the managers and other employees discover ways to cut time and materials cost in the production of that product. The famous economist Kenneth Arrow called this "learning by doing," but it might equally be called learning by making mistakes (and not repeating the mistakes!) (Arrow, 1962, 1970). Empirical studies indicate that average costs tend to decline by a relatively stable percentage each time that cumulative output doubles. Figure 5.7 displays a learning curve where the average cost declines by 20% each time cumulative output doubles. This can be verified using simple arithmetic: The average cost falls to 80% of the preceding level each time cumulative output doubles.

**Figure 5.7: The learning curve**



Note that the learning curve relates the average cost of production to the *cumulative volume* of production and therefore refers to SAC values across several different production periods, potentially involving several different plant sizes, differing input prices, and so on. Thus, a learning curve shows the decline in average costs where nothing (necessarily) stays the same. Indeed, the learning curve accounts for all of the cost reductions that take place over time, including those due to economies of scale, economies of scope, and purchasing economies, as well as those that are due to changes in technology causing the inputs to be more productive than before.

Question: How does the learning curve impact the SAC and LAC curves? Answer: It causes them to sink downward gradually from one period to the next. Early in the life of a firm, for a new business venture for example, the downward shift of the LAC and SAC curves would be quite significant as that firm doubles then doubles again its cumulative output level. But as the firm gets older and accumulates greater and greater cumulative output (i.e., greater production experience) the relatively large cumulative output figures need to double again to see the same percentage decline in average costs. So, as the firm matures and has behind it a relatively long history of production, the period-to-period downward shift of the SAC curve will become negligible. Note that the rate of learning varies across production processes, being relatively low for simplistic production methods and being higher for more complex production methods, but it usually falls within the range of 5–20%.

## Summary

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In this chapter, we have laid the groundwork for our analysis of production and costs and have encountered a variety of cost concepts that are useful for decision making by managerial economists. We first introduced the concepts of fixed and variable inputs in the production process, noting that while there are fixed inputs in the short run, in the long run all inputs are theoretically variable as the firm is free to choose any size plant that it wants. In the short run, the productivity of the variable inputs will increase at first and later decrease in accordance with the law of variable proportions, the latter part of which is also known as the law of diminishing marginal productivity. Essentially, the marginal product of the variable inputs (MP) might increase at first but after some point begins to decline and continues to decline as more and more variable inputs are added to the fixed inputs.

The total variable cost (TVC) curve mirrors the shape of the total product (TP) curve because it is essentially the monetary version of the TP curve. At first, when average product (AP) is rising, average variable cost (AVC) is falling, and later, when AP is falling, AVC is rising. Similarly, when marginal product (MP) is initially rising, marginal cost (MC) is falling, and later when MP is falling, MC is rising. The MC curve lies below the AVC and SAC curves when they are falling, and lies above those two curves when they are rising. Inevitably, the MC curve cuts the minimum point on both the AVC and the SAC curves.

Total costs (TC) are the sum of total variable costs (TVC) and total fixed costs (TFC). TFC are constant at a particular level relating to the plant size throughout the current production period. To find TC we simply add the constant TFC amount to the varying TVC amount at each output level. Short-run average costs (SAC) are the sum of AVC and average fixed costs (AFC) at each output level, or alternatively are equal to  $TC/Q$  at each output level.

In the long run the firm may choose any size of plant and thus can regard the long-run average cost (LAC) curve as a smorgasbord of available plant sizes and choose the plant size that best suits its output plans. The long-run marginal cost (LMC) curve cuts the LAC curve from below at its minimum point when the LAC curve is U-shaped, that is, exhibiting both economies and diseconomies of plant size (or scale). In some cases the LAC might be found to have a flat bottom, and thus exhibit more than one optimal size of plant,

in which case the LMC curve is coextensive with the LAC curve for the duration of plant sizes that exhibit such constant returns to scale.

Finally, we considered a series of cost concepts that are important for managerial decision making. In economic analysis we always consider the economic costs of resources or assets, which is equal to the opportunity costs of those items. Historical costs might overstate or understate the economic costs of an input to the production process or an asset held by the firm if the opportunity cost has changed since the time the item was purchased. Sunk costs are costs that have been paid and cannot be retrieved, while unavoidable costs may not yet have been paid but must be paid regardless of the output level chosen. The fixed costs of production are usually either sunk costs or unavoidable costs. Incremental costs are those that are incurred because of a decision, so variable costs are always incremental costs.

The concept of economies of scope was introduced to illustrate that adding additional product lines can cause the short-run cost curves to shift vertically downwards as the cost of underutilized fixed or variable inputs is transferred to the new product lines. Purchasing economies refer to the decline in average costs that is due to the firm's ability to gain discounts by buying in bulk as the firm becomes larger. This led to a discussion of the learning curve, whereby the average cost of production declines over time as the firm (and its employees) learns from cumulative production experience and also benefits from economies of scale, scope, and purchasing.

### Questions for Review and Discussion

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1. Suppose your production process has three inputs—machinery, highly skilled labor, and raw materials. If you wanted a new (larger or smaller) machine, it would take six months to be fabricated, delivered, and installed. Your highly skilled workers are all under contract for another eight months. New skilled workers take three months to acquire, because of the lengthy process of advertising, interviewing, signing them up, and familiarizing them with your operations. Raw materials must be ordered four weeks in advance.
  - a. How long is your short run?
  - b. When can you make your long run decision to change plant size?
2. How does the law of diminishing returns differ from the law of variable proportions?
3. Why is the point of inflection on the total product curve the point where diminishing returns begin?
4. Why is marginal product maximized at the same variable input level at which marginal cost is minimized?
5. Why does the marginal cost (MC) curve cut the minimum point of the average variable cost (AVC) curve? Why does it also cut the minimum point of the short-run average cost (SAC) curve?
6. Why is the long-run average cost (LAC) curve *not* a line joining the minimum points on the possible SAC curves?
7. Define economies of scale, diseconomies of scale, and constant returns to scale.
8. Explain the difference between economic costs and historical costs.

9. Distinguish between sunk costs and unavoidable costs.
10. Explain the learning curve in terms of economies of plant size, economies of scope, purchasing economies and changes in technology.

### Decision Problems

1. Donald's Oyster & Pearl Company operates a pearl-diving business in the North Pacific Ocean. Donald owns a large trawler boat and hires local divers from the nearby islands and pays them on the basis of the weight of oysters recovered. He sells the pearls and the oyster meat separately. Over the past month he has been out pearling eight times in the same general area, each time taking all the divers who showed up looking for work. The details of the number of divers and the weight of oysters recovered are listed in the following table:

Trip number	Divers employed	Oysters recovered (kilograms)
1	6	38
2	17	76
3	9	56
4	5	32
5	12	74
6	3	15
7	14	80
8	15	78

- a. Over what ranges do there appear to be increasing, constant and/or diminishing returns to the variable factor?
  - b. What number of divers appears to be most efficient in terms of output per diver?
  - c. What number of divers appears to minimize the marginal cost of oysters and pearls?
2. The Peaches and Plums Cosmetics Company (PPCC) produces face cream products mostly for women. Management has established the following relationships between units of the variable inputs and units of output. One variable input unit comprises one (unskilled) person working 40 hours per week, the electric power to run the mixing machines, the ingredients in the required proportions, and the small glass jars and other packaging materials needed for a week's production. The variable costs are \$1,200 per unit of the variable inputs. Overhead (i.e., fixed) costs are \$120,000 per week.

Variable input (units)	Output (jars)
100	6,500
200	14,300
300	20,200
400	24,400
500	27,800
600	30,000

- a. Derive a table showing the firm's AVC and SAC values for the output levels shown.

- b. Graph the AVC and SAC curves and sketch in your best estimate of the marginal cost curve.
  - c. At what output do you think diminishing returns first start? Explain with reference to the graph and reconcile this with the input–output data.
3. Suppose that one of the very expensive ingredients in Peaches and Plums cosmetic cream (refer to problem 2 for details) was suddenly found in abundance, such that cost per unit of the variable inputs fell to \$500 each.
  - a. What is the impact of this discovery on the firm’s SAC, AVC, and MC curves?
  - b. Does it change the point where diminishing returns set in? Why or why not?
  - c. Now suppose that after the cost reduction for the expensive ingredient, worker morale improves such that worker productivity goes up by 10% at each input level. What is the impact of this on the firm’s AVC, SAC, and MC curves?
4. Panache Shirts Limited is a small manufacturing business that makes men’s shirts in distinctive designs that appeal to a limited segment of the market. Sales have been growing steadily and since beginning the business with one production center, Mr. Panache has first doubled, then tripled and finally quadrupled the number of production centers by leasing larger spaces within a warehouse building. Each production center consists of a cutting machine, three sewing machines, and three skilled operators. Variable inputs include cotton and silk cloth, sewing threads, buttons, casual labor, and packaging materials. Throughout the expansion, Mr. Panache has personally supervised all of the operators and has handled all other aspects of the business, including the marketing and financial aspects. He has kept records of the daily production of shirts from each production center, as follows:

Production center	Shirts per day (average)
1	61.8
2	127.2
3	182.4
4	228.9

Each production center costs \$12,000 per month in fixed and variable costs. Mr. Panache draws \$9,000 a month from the business for his salary, and the remaining fixed costs are \$3,000 a month. Assume there are 20 working days in each month.

- a. Has Panache Shirts Limited increased the rate of output or the scale of output? Explain.
  - b. Are there economies or diseconomies of scale apparent in the data? Explain.
  - c. Indulge in some speculation as to the causes of the economies and diseconomies, if any.
5. Greenfield Farms Bakery (GFB) is currently producing below full capacity with a relatively stable demand of 15,000 loaves of bread per week for its regular wholesale customers. At this time, the business is reasonably profitable—average cost per loaf

is \$1.48 and the delivered price is \$1.70 per loaf to the various food stores that retail the bread to the end-user customers. The CEO of GFB, Ms. Brianna Puddington, has been negotiating to supply bread to another large food chain that currently buys its bread from a rival bakery. This would involve a minimum fixed order of 10,000 loaves per week with the requirement that GFB must also supply any additional demand by this food chain up to another 20,000 loaves per week (i.e., 30,000 loaves total). Ms. Puddington considers the probability distribution of the extra demand from this wholesaler to be as follows:

Quantity demanded	10,000	15,000	20,000	25,000	30,000
Probability	0.4	0.3	0.15	0.1	0.05

While GFB's present production facilities could supply the additional 30,000 loaves per week, it would prove to be very expensive to run the plant at such a high production rate. Alternatively, Ms. Puddington is considering the purchases of a new continuous-process mixing and baking machine that would cost \$416,000 (installed and ready to start). This machine could always be sold at its market value, which is expected to decline linearly at one sixth of its value per year, and would have no scrap value at the end of its 6-year life. Alternatively, the funds could be invested at 18% per annum at similar risk. The new plant would not require any other changes in overhead costs, which are currently \$7,200 per week including management and skilled labor salaries. This current level of fixed cost includes no allowance for depreciation since the present plant has been completely depreciated. Variable costs for the present and the proposed plant are as follows (the missing data for each plant indicates that these output levels are not possible in that plant):

Output per week	Present plant TVC per week	Proposed plant TVC per week
10,000	12,600	—
15,000	15,000	—
20,000	18,400	—
25,000	25,500	30,000
30,000	37,200	30,600
35,000	53,900	30,800
40,000	80,800	31,200
45,000	153,000	45,900
50,000	—	71,000

- Calculate the AVC and SAC values, plot the curves for each plant size, and comment on the differences.
- Calculate the expected value of weekly profits under three alternative scenarios:
  - Present plant with no additional sales
  - Present plant with new sales contract
  - Proposed plant with new sales contract
- Advise Ms. Puddington whether to install the proposed new plant, and explain your decision.

## Key Terms

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**average fixed costs (AFC)** Total fixed cost (TFC) divided by the number of output units ( $Q$ ). Since TFC is a constant, AFC is a constant divided by an increasing number (as  $Q$  increases) and so the AFC curve takes the shape of a rectangular hyperbola.

**average product (AP) curve** Shows the ratio of the output level (or Total Product, TP) to the variable input level, ( $V$ ), or  $TP/V$ , at any particular input level of the variable inputs.

**average variable cost (AVC)** The value of total variable cost (TVC) divided by the quantity of output units ( $Q$ ).

**capital** The economic value of the firm's investment in fixed inputs (such as land, buildings, equipment, vehicles, and skilled human resources) that enter the firm's production function, referred to as *fixed inputs* in this book, since *capital* has other meanings, often associated with financial issues.

**constant returns to plant size** A situation where the long-run average cost (LAC) of production remains constant at a particular level as the scale of the plant is progressively increased.

**constant returns to scale** Another expression that means the same as constant returns to plant size, since plant *size* is often referred to as *scale*.

**cost function** A mathematical expression showing the firm's total costs as a function of the fixed and variable costs necessary to produce a range of output levels.

**curvilinear production functions** Reflect a state where the proportionality between output and the variable inputs varies, such that there is either increasing returns to the variable inputs, or diminishing returns to the variable inputs, or both in sequence.

**diseconomies of plant size** Occur when successively larger plant sizes cause the next short-run average cost (SAC) curve to lie above and to the right of the preceding SAC curve, and thus the long-run average cost (LAC) curve is upward sloping.

**economies of scale** Occurs when the short-run average cost (SAC) curves associated with successively larger plant sizes lie on a downward-sloping section of the long-run average cost (LAC) curve, and thus allow reduced average costs per unit of output as production volume increases in the long run.

**economies of scope** Reductions in the average cost of production per unit of output (SAC or LAC) that are due to (a) the spreading of overhead (fixed) costs over a broader product line, thus reducing the average fixed costs (AFC) of an existing product; and/or (b) utilizing material off-cuts, idle time of variable labor, and other underutilized variable inputs to produce another product thus reducing the average variable cost (AVC) of existing products offered.

**fixed inputs** Inputs to the production process that do not vary over the time period chosen for the analysis (the production period). These include land, buildings, equipment, and skilled personnel that take time to build, acquire, assemble, or recruit.

**historical cost** The actual monetary cost that was incurred to purchase a fixed input or an item in inventory in a past period. The present period cost of that asset or item (its opportunity cost) might be quite different from its historical cost due to inflation, current shortage or abundance of supply, obsolescence, or collectors' value recognized.

**incremental costs** Costs that will be incurred in the current or future production period because of a decision to be made. In making forward-looking decisions about output levels in the current or future periods, only incremental costs are relevant.

**labor** Historically used by economists to refer to the variable inputs to the production function, but these days only *unskilled* human resources can be varied from day to day according to production requirements. Conversely, *skilled* labor takes time to recruit and train and is thus treated as a fixed input to the production process.

**law of diminishing returns** A phenomenon in production situations reflecting decreasing productivity of the variable inputs that causes the marginal product (MP) of the variable inputs to decline progressively, after some point, as more and more variable inputs are added to the fixed inputs in a production process.

**law of variable proportions** A rule that states that the firm's total output is likely to increase at an increasing rate at first, and then increase at a decreasing rate, as we progressively add more and more variable inputs to the fixed inputs. The law of diminishing returns refers to the latter part of this law.

**learning curves** The downward-sloping curvilinear trajectory of the firm's average costs of output as production experience increases over multiple production periods (they are also known as *experience curves*). Learning curves incorporate all the reasons for changes in average cost per unit of output, including economies of scale, economies of scope, purchasing economies, changes in technology and changes in employee efficiency.

**linear production function** A situation where output increases linearly as variable inputs are added to the fixed inputs. With a linear production function the total cost (TC) curve would start with the (lump sum) cost of the fixed inputs and then rise linearly with the output level.

**long run** A hypothetical situation in which a firm can contemplate the cost implications of various plant sizes, that is, the long-run average (LAC) curve. The firm can choose the plant size (a specific SAC curve) that best serves its production plans. After this plant-size decision is made, it takes time to purchase the fixed inputs and set up the new plant, after which point the firm will transition from the old short-run situation to the new short-run situation.

**long-run average cost (LAC) curve** The curve that shows the least cost of production for each output level assuming that all inputs are variable (in the long run). It is composed of a small segment of many different SAC curves.

**long-run marginal cost (LMC)** The curve that is marginal to the LAC curve. It lies below LAC when the latter is falling, lies above LAC when the latter is rising, and intersects LAC when the latter is at its minimum level. It is the locus of short-run marginal cost (SMC) levels at the output levels where the SAC curves relating to each SMC are tangent to the LAC curve, and necessarily requires that all inputs to the production process are variable.

**marginal cost (MC)** The change in total cost for a one-unit change in the output level, or  $\Delta TC / \Delta Q$ . Since  $\Delta TC$  only happens (in the short run) due to  $\Delta TVC$ , short-run marginal cost (SMC) is also equal to  $\Delta TVC / \Delta Q$ .

**marginal product (MP) curve** A curve that reflects the change in total product for a one-unit change in the variable input level. Due to the law of variable proportions, the MP is typically not constant but increases at first and later diminishes as additional units of the variable inputs are added to the fixed inputs.

**optimal scale of plant** See *optimum plant size*.

**optimum plant size** The SAC curve that sits in the bottom of the U-shaped LAC curve. The optimum size (or scale) of plant is the plant size that allows the product to be produced at the least cost per unit when all inputs to the production process can be varied. With constant returns to scale (horizontal section of LAC curve) there will be several optimum plant sizes.

**plant size** A term that refers to the amount of the fixed inputs involved in the production process in the short run—plant size can be changed only in the long run.

**production efficiency** A condition of business operations when output can only be increased by also increasing the costs of production, that is, there is no slack in the system.

**production function** A formula that shows the dependence of total product (TP) or output (Q) on the quantities of fixed and variable inputs to the production process.

**purchasing economies** The reduction in average costs per unit of a variable input that is due to purchasing that input in larger volumes, where the firm receives discounts for buying in bulk.

**short run** The period of time in which the firm is unable to change the size of its plant (and thus its maximum output capacity) due to the time it takes to purchase and assemble additional fixed inputs to the production process.

**short-run average costs (SAC)** The (short run) total costs divided by the number of units of output. SAC is equal to the average variable costs (AVC) plus the average fixed costs (AFC).

**sunk costs** Costs that have been incurred previously and cannot now be retrieved, such as the historical costs of assets purchased. In most cases only the salvage value of these assets (if any) can be retrieved.

**total costs (TC)** The sum of all indirect (fixed) and direct (variable) costs of production that are involved in a business firm.

**total fixed costs (TFC)** The cost of all fixed inputs to the production process. Since fixed inputs cannot be varied in the short run, TFC will remain constant in the short run unless the prices of any of the fixed inputs changes—such as managers' salaries and lease costs per month.

**total product (TP) curve** A curve that shows the total output level that can be produced by a given plant size when augmented by various levels of the variable inputs.

**total variable cost (TVC)** A value that increases as more units are produced because higher levels of output require higher levels of the variable inputs to be added to the firm's fixed inputs.

**unavoidable costs** Costs that the firm is contractually committed to pay regardless of output levels, such as management salaries, rental of factory and office space, lease payments for equipment, and so on.

**variable inputs** Inputs to the production process that can be varied at short notice, such as raw materials, components, and unskilled labor, because the firm can readily purchase these inputs in the markets for these resources.

