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IC1

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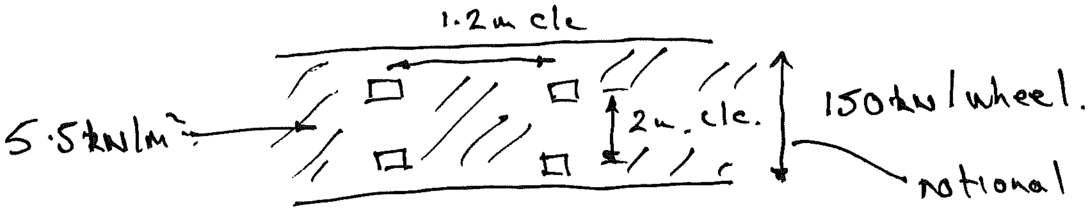
Load Model 1

Bridge Loading - Student Work.

General load applied to all lanes = 5.5 kN/m².

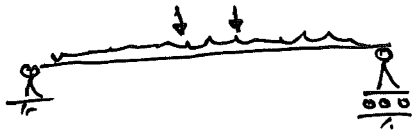
+ a vehicle load of 600kN applied as follows.



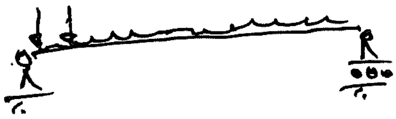
Plan on Lane.

The vehicle load can move to give worst case.

ie



Worst BM vehicle at centre span.

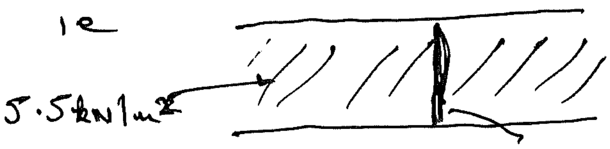


Worst SF vehicle at end.

but what we are allowed to do is

300kN 300kN ↓ ↓ $\xrightarrow{\text{apply as}}$ 600kN ↓

ie



axles combined into one load known as knife edge load (KEL).

IC 2

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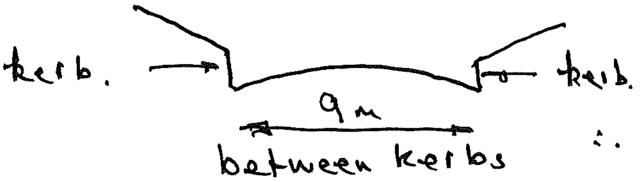
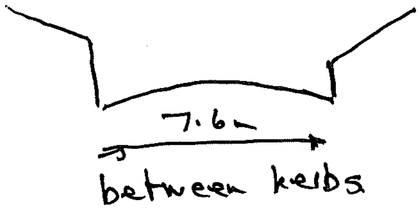
Notional Lanes – not traffic lanes.

These are 3m wide & define where loading occurs



$\therefore \frac{9}{3m} = 3$ notional lanes

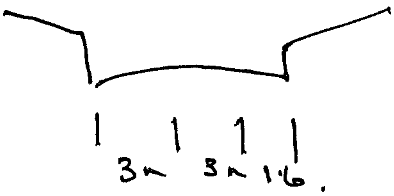


This is typical 2way traffic. ie 3.8m wide lanes.

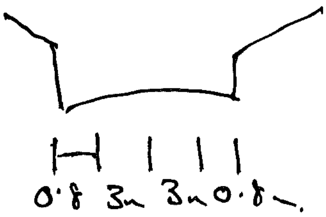
but number of notional lanes

$= \frac{7.6}{3} = 2$ notional lanes + 1.6m spare.

Could give us



or



ie the lanes can move across the width to suit worst case.

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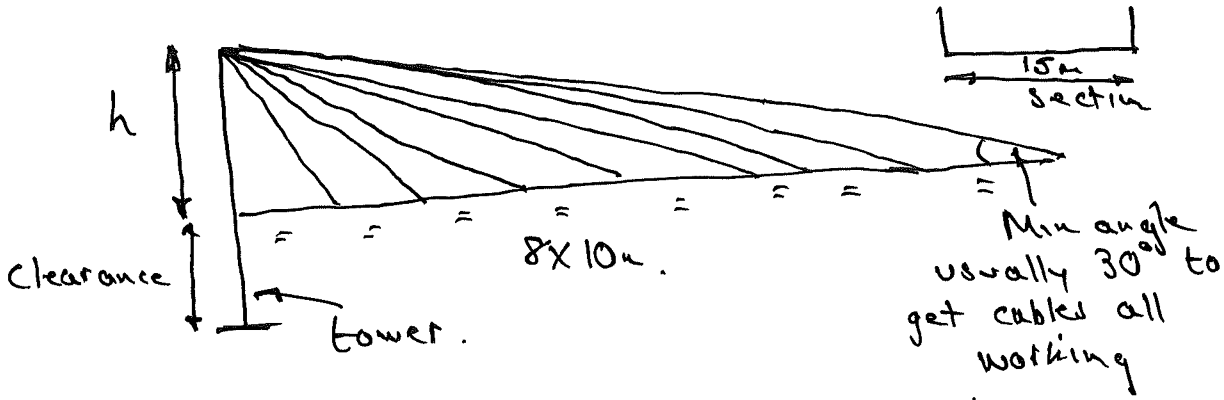
Outline Sizing - Cable Stayed & Arch Solution

Don't need loading, notional lanes at this stage.
Needed for buildability.

Cable stayed



$h = 80 \tan 30° = 46.2m$ say $46m$.

Clearance say ≈ this example is 20m.

Assume initially a single tower & no backstays.

Need to size

Cables - not important here as likely say 50mm up to 150mm diameter.

Deck - depending on what type is chosen.

Tower - will be large.

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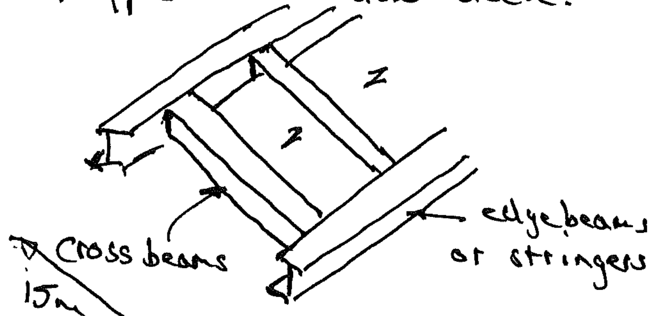
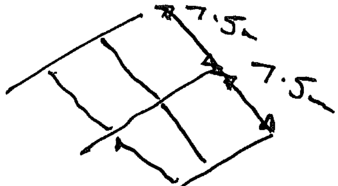
Deck.

Type. – ladder deck.



Spacing of cross beams usually limited to 5m c/c but generally 3.5m c/c to limit buckling of deck.

Single ladder deck. – could try



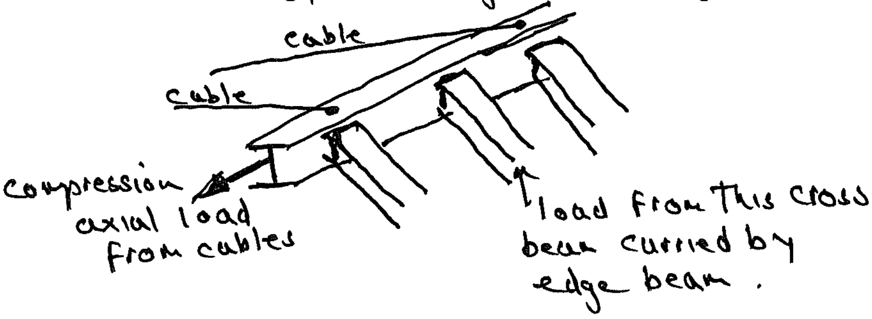
Double ladder deck (Plan).

Sizes

Concrete – usually RC $\frac{span}{20}$ or Prestressed $\frac{span}{20} \times 0.5$ ✓

Cross beams – " $\frac{span}{20}$ + up a size. ✓

Edge " – depends on whether the cables pick up crossbeams & also the axial load generated by the cables.



At this stage for an edge beam assume 1014 deep UB section.

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Tower.

Try to achieve a slenderness of 1 in 100

ie $\frac{Le}{r_{zz}}$ ← effective length / ← weaker radius of gyration

Le usually 2x tower height.
NB This is only indicative depending on tower type ie backstay or no backstay.

Cross beams @ 5m c/c.
∴ Slab Thickness (RC) = $\frac{5000}{20}$ = 250mm

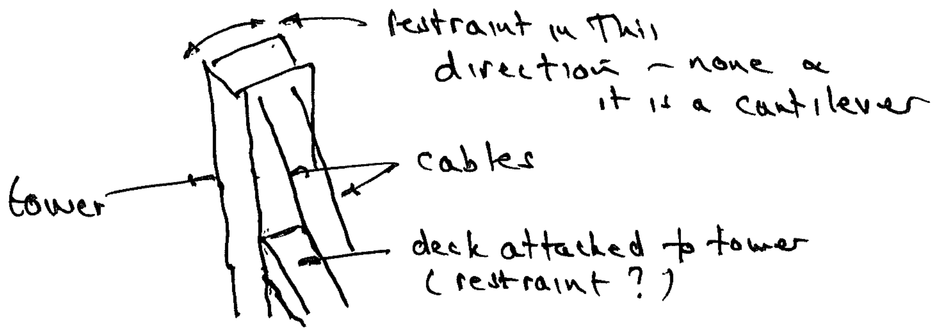
Cross beam depth = $\frac{span}{20}$ + a size
= $\frac{15000}{20}$ + a size = 838 or 914.

Say 914 x 305 x 201 UB as the cross beam forms part of the deck horizontal girder which carries wind load.

Edge beam from earlier = 1016 x 305 x 314 (mid section size)

Tower:

Overall height = 46 + 20 = 66m.
Need tower effective length Le.



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$\therefore$ Effective length = $2 \times h = 2 \times 66 = 132m$. (massive).

$$\frac{L_e}{i_{zz}} = 100 \quad \therefore i_{zz} \text{ requ'd} = \frac{132}{100} = 1{\cdot}32m.$$

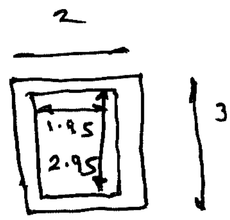
Now guess at a tower size. Make it hollow for access for maintenance & inspection.

Try steel plate $\times$ 25mm thick.

Try $3m \times 2m \times 25m$ thick $\quad i_{zz} = \sqrt{\frac{I_{zz}}{A}}$

I_{zz} = 2nd moment of area

A = cross section area.



$$I_{zz} = \frac{3 \times 2^3}{12} - \frac{2.95 \times 1.95^3}{12} = 0{\cdot}177m^4.$$

$$A = (3 \times 2) - (2{\cdot}95 \times 1{\cdot}95)$$

$\therefore i_{zz} = 0{\cdot}84m. < 1{\cdot}32 \quad \therefore$ too small.

Increase section to say $4{\cdot}2m \times 3{\cdot}2m \times 50mm$ thick

$i_{zz} = 1{\cdot}33m > 1{\cdot}32m$ ✓

At this stage can now:

Look at construction techniques

Talk to others about maintenance etc.

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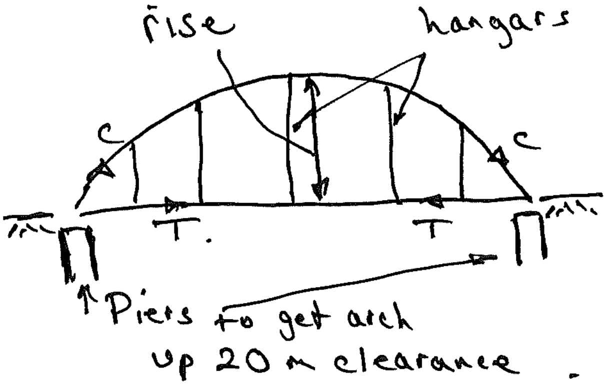
Arch Option.



Assume initially that the bridge deck is at the correct level such that the deck ties the arch.

Assumption is

(1) The hangars are at 10m centres. Therefore the deck ie slab, crossbeams and edge beams are same as cable stayed.

Therefore only size the main arch.

Don't need the rise of the arch at this stage

Use the same approach as the cable stayed tower. as no BM in parabolic arch.

Limit $\frac{Le}{i_{zz}} = 100$.

So if assuming whole length of arch $\simeq$ 80m is unbraced then $Le \simeq 80$m.

$\therefore$ i_{zz} required $= \frac{\cancel{800}\ 80}{100} = 0.8$m.

Trial & error time

Try section 2.5×3m $\times$ 50mm thick

$$i_{zz} = \sqrt{\frac{\frac{3 \times 2.5^3}{12} - \frac{2.9 \times 2.4^3}{12}}{(2.5 \times 3) - (2.4 \times 2.9)}} = 1.02\text{m} > 0.8\text{m} \checkmark$$

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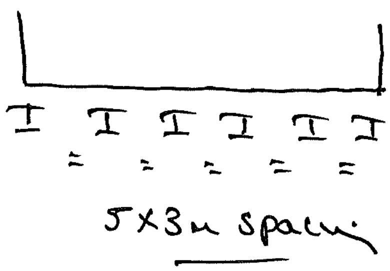
Plate girder option – unlikely for 80m.

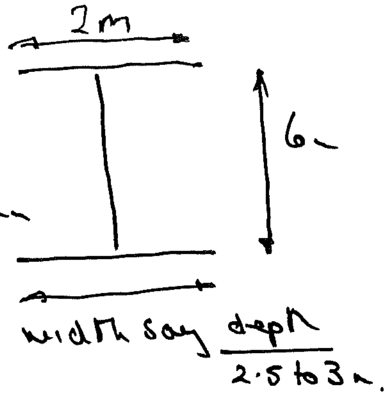


Rule of Thumb for depth $= \dfrac{\text{span}}{15 \text{ to } 20}$.

Try $\dfrac{80}{15} = 5.3\text{m}$ say 6m.

Section could be say

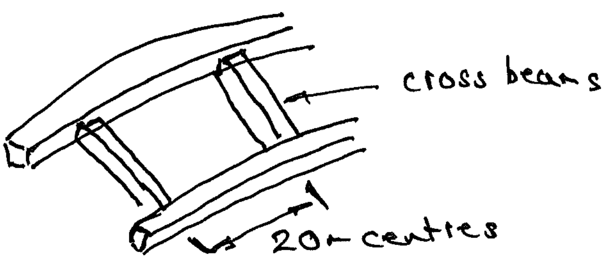
thickness guess = 50mm



Weight of one girder = 392 tonnes!

Try restraining the arches with cross beams

ie



new $i_{zz} = \dfrac{20}{100} = 0.2\text{m}$.

smaller section

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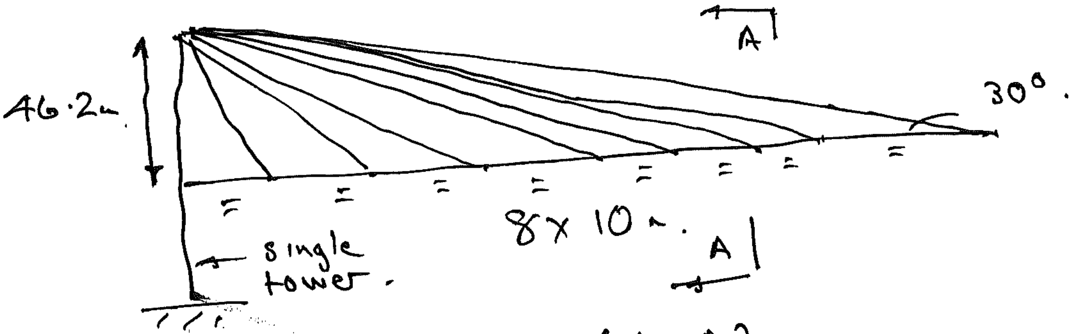
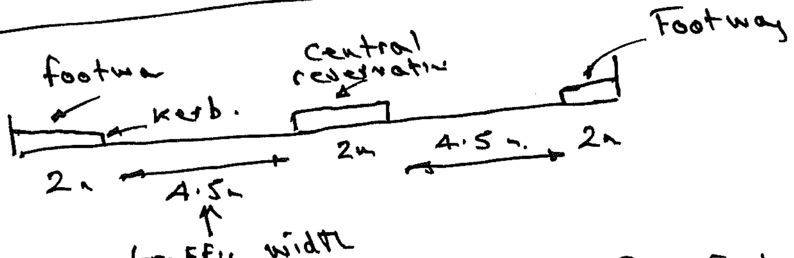
Full Design.

Cable Stayed Structure.



Traffic X-section (A-A).



Notional traffic lane is 3m wide

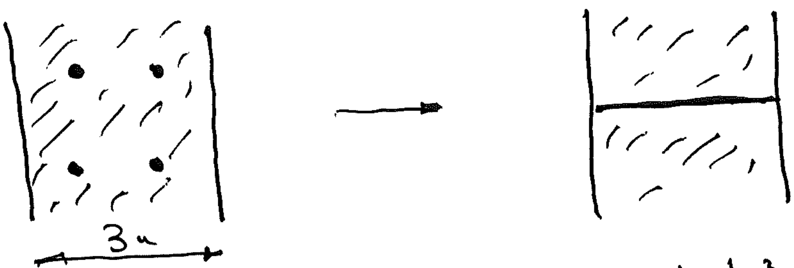
Notional traffic lane every carriageway

∴ only 1 notional taffic lane every carriageway

ie

We are using Load Model 1

ie



/// = 5.5 kN/m²

• = wheel of 150kN.

/// = 5.5 kN/m²

— = 600kN knife edge load across whole lane.

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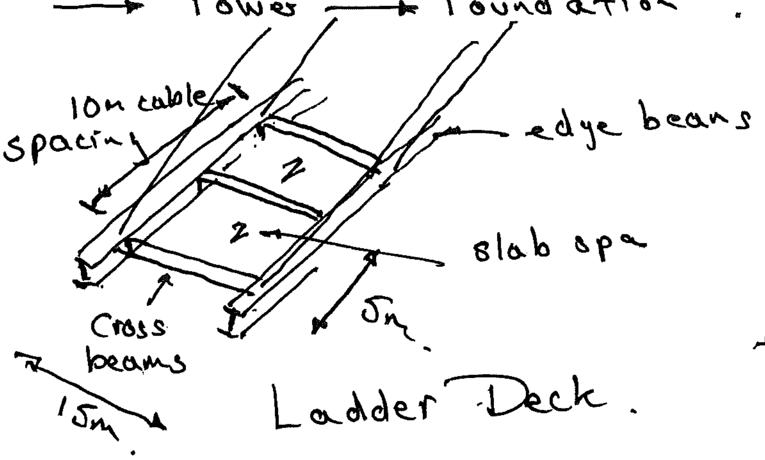
Assumption.

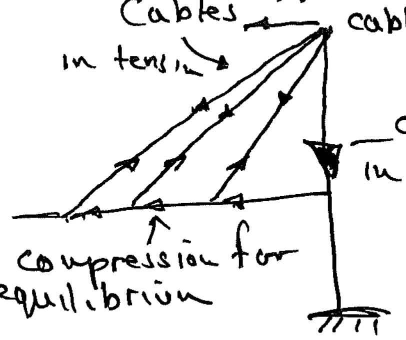
Within the 4·5m carriageway there is a 3m notional lane + 1·5m wide cyle-track (load = 3kN/m²).

What are we designing?

Slab → Cross beams → Edge beams → Cables.
→ Tower → Foundation.

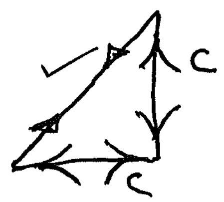


Ladder Deck.



H creates a BM.

simplify



▶ know - tension in cables

← resulting forces for equilibrium

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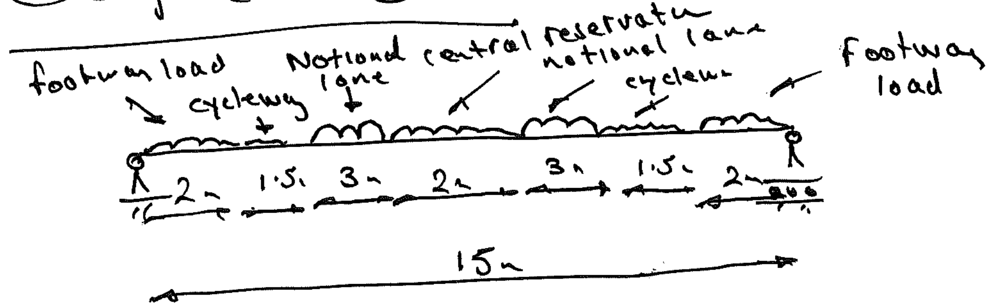
Design Cross Beam.



Simplify by ignoring s/w of footway & central reservation and all the ~~surf~~ surfacing.
Also make the footway & central reservation variable loads same as cycleway ie $3 kN/m^2$.
Now need to factor the loads.

Permanent Design Action

Assume 250mm Thick concrete deck. ie $\left(\frac{5m\ span}{20}\right)$.

$= 0.25 \times 25 \times 1.35 = 8.44 kN/m^2$

(Student design guide gives concrete density = $24 kN/m^3$).

Variable Design actions

Notional Lane full load = $5.5 kN/m^2 \times 1.35 = 7.43 kN/m^2$

Wheel loads now as a knife edge load (KEL)

$= 600 \times 1.35 = 810 kN$ across whole lane.

Footway & cycle lane loads = $3 \times 1.35 = 4.05 kN/m^2$.

Now chase the loads back to the cross beams, which are spaced at 5m centres.

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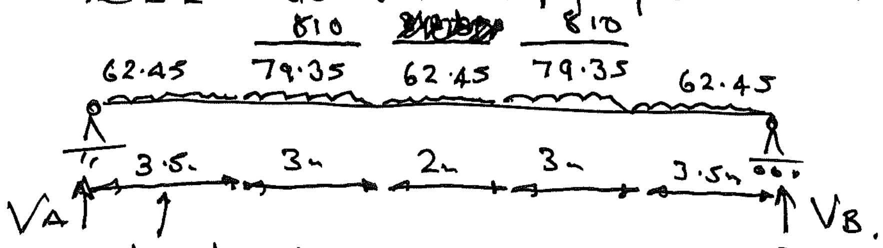
∴ Total footway & cycleway load in (kN/m)

$= (8.44 + 4.05) \times 5 = 62.45$ kN/m.

Total notional lane udl

$= (7.43 + 8.44) \times 5 = 79.35$ kN/m.

KEL – don't multiply by 5 it is not a udl.



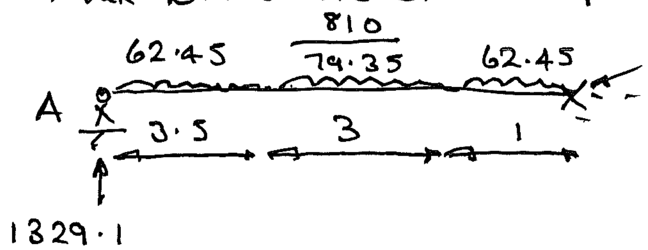
combined cycleway & footway.

Need BM for design

$V_A = V_B = (62.45 \times 3.5) + (79.35 \times 3) + 810 + (62.45 \times 1)$

$= 1329.1$ kN.

Max BM occurs at mid span due to symmetry.



mid span.

work from A → X

$BM = (1329.1 \times 7.5) - (62.45 \times 3.5 \times 5.75) - (810 \times 2.5)$
$- (79.35 \times 2.5 \times 3) - (62.45 \times 1 \times 0.5)$

$= \underline{6060.1 \text{ kNm}}$

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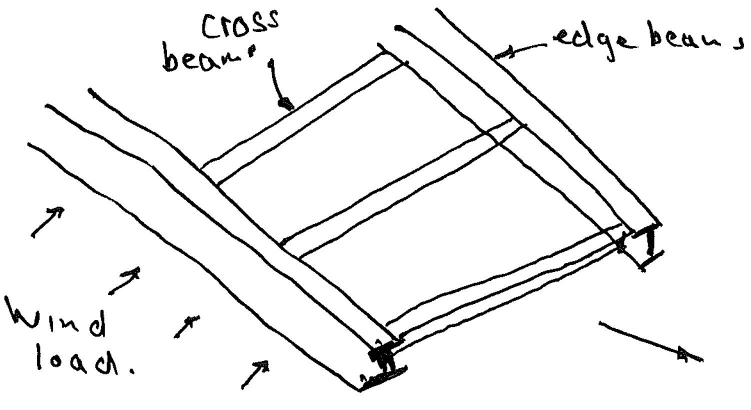
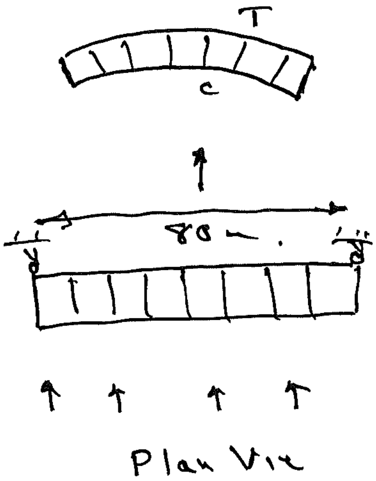
Now need to consider bridge wind loading.

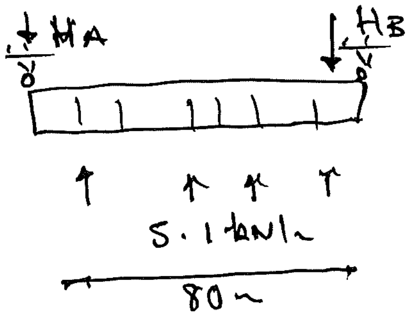




Assume wind load = $2\,kN/m^2$.

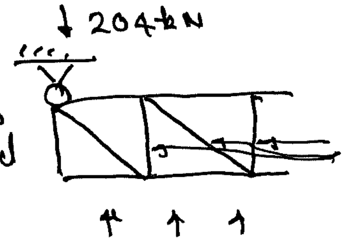
Also " depth of deck + parapet is say 1.5m. (might be low but take it for now).

Need factored wind load = $2 \times 1.5 \times 1.7 = 5.1\,kN/m$. (1.7 = Factor)



$$H_A = H_B = \frac{80 \times 5.1}{2} = \underline{204\,kN}$$

Should do analysis of this to find worst loaded internal member



Look for worst loaded internal member.

Because I am aware that 204 kN axial load is small compared to 6060 kNm bending moment, for speed use 204 kN as worst case axial load in the cross beam. Could argue ignore it.

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∴ Cross beam subject to axial load & bending.

From page 51 of student guide

$$\frac{N_{Ed}}{N_{b,Rd}} + \frac{k\,M_{Ed}}{M_{b,Rd}} \leq 1.$$

$$\therefore \frac{204}{N_{b,Rd}} + \frac{6060 \cdot 1}{M_{b,Rd}} \leq 1$$

Could argue that the cross beam has full lateral restraint from the slab over ie use full strength not buckling resistance

$$\therefore \frac{204}{N_{c,Rd}} + \frac{6060 \cdot 1}{M_{c,Rd}} \leq 1.$$

N_c = cross section area x yield stress

M_c = plastic modulus x yield stress.

Try 838 x 292 x 176 UB section

$\therefore N_{cRd} = 224\,cm^2 \times 10^2 \times 350 \times 10^{-3} = 7840\,kN$

$M_{c,Rd} = 6808 \times 350 \times 10^{-3} = 2383\,kNm$ ← too small

(NB should have used yield stress of $335\,N/mm^2$ not $350\,N/mm^2$ as sections this size have thickness ~~[illegible]~~ $\geq 40mm$)

Try 1016 x 305 x 487 UB

$M_{c,Rd} = 23208 \times 350 \times 10^{-3} = 8122\,kNm$

now okay.

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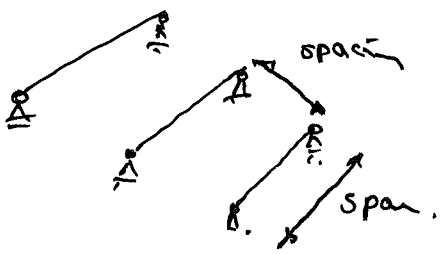
Now try design it compositely (likely to start with this).

From previous notes use exactly the same approach.

Find capacity of the concrete section & then the steel section and depending on outcome use one of the three formulae.

Force capacity of concrete slab $R_c = 0.567 \times B_e \times D \times f_{ck}$. (effective flange width; depth of slab.)

" " " steel beam R_s = section area × yield stress.



For equal spacing.

$B_e = \frac{span}{4}$ or spacing (lesser value).

in this case $= \frac{15}{4} = 3.75m$.

C40/50 concrete.

$\therefore R_c = 0.567 \times 3750 \times 250 \times 40 \times 10^{-3} = 21263 kN$.

$R_s = 256 \times 10^2 \times 350 \times 10^{-3} = 8960 kN$ (assume steel beam 914×305×201 UB).

$R_c > R_s$ $\therefore$ Neutral axis is in the slab.

$\therefore$ Moment capacity $M_c = R_s\left[\frac{D}{2} + D_s - \left(\frac{R_s}{R_c} \times \frac{D_s}{2}\right)\right]$

$= 8960\left[\frac{903}{2} + 250 - \left(\frac{8960}{21263} \times \frac{250}{2}\right)\right] \times 10^{-3}$

$= 5813 < 6060.1 kNm$ $\therefore$ fails.

(D = depth of steel section
Ds = depth of slab)

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Increase beam to 1016 x 305 x 222 UB

gives $M_c = 6705 kNm > 6060 kNm$ ✓

Look at difference

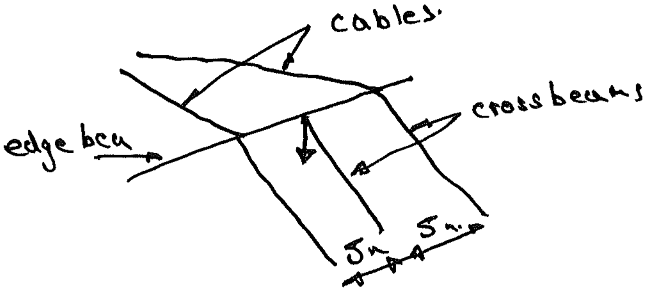
Non-compositely - 487 kg/m

Compositely - 222 kg/m

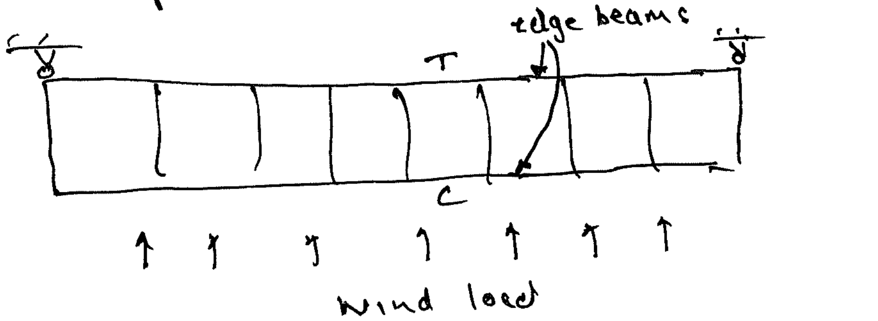
Edge Beam

Subjected to The following 3 load types

1. Point load from cross beam.

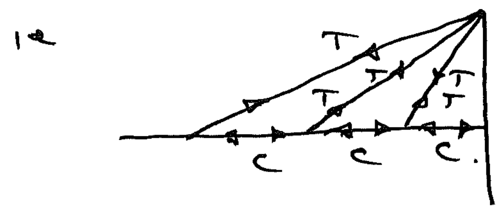


2. Axial compression / tension force from wind load



(Plan view)

3. Axial compression from the cable forces



1. Point load outcome.

End reaction from cross beam from earlier calc.

$= 1329.1$ kN.

$\therefore$ $BM = \dfrac{WL}{4} = \dfrac{1329.1 \times 10}{4} = 3323$ kNm

(ignoring beam s/w which is heavy I know)

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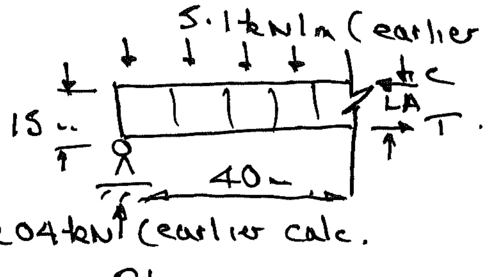
2. Axial load from wind (both C & T)



use method of sections

Max BM in truss from wind load occurs at mid point

$= (204 \times 40) - 5.1 \times 40 \times \frac{40}{2}$

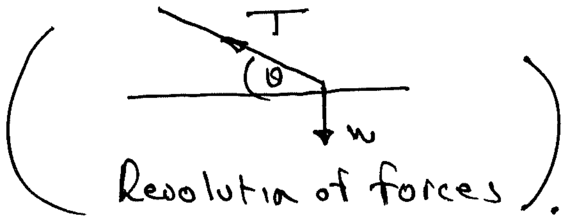
$= 4080 \text{ kNm}.$

but BM = C x lever arm or T x lever arm (C = T).

$\therefore 4080 = C \times 15 \quad \therefore \underline{C = 272 \text{kN}}$

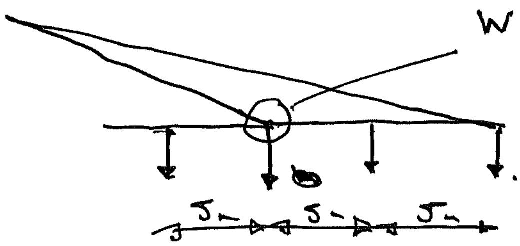
3. Axial compression from cables

Load per cable $= \dfrac{W}{\sin\theta}.$



$T \sin\theta = W.$

Need W for each cable.

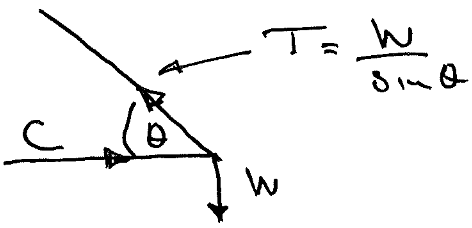


W = 1 edge beam at cable + 2x (½ edge beam either side)

↓ = edge beam reactions

$\therefore W = 1329.1 + \left(2 \times \frac{1329.1}{2}\right) = \underline{2658 \text{ kN}}$

Need axial force in edge beam in terms of W & θ.



For horiz equilibrium

$C = T\cos\theta = \dfrac{W\cos\theta}{\sin\theta} = \underline{\dfrac{W}{\tan\theta}}.$

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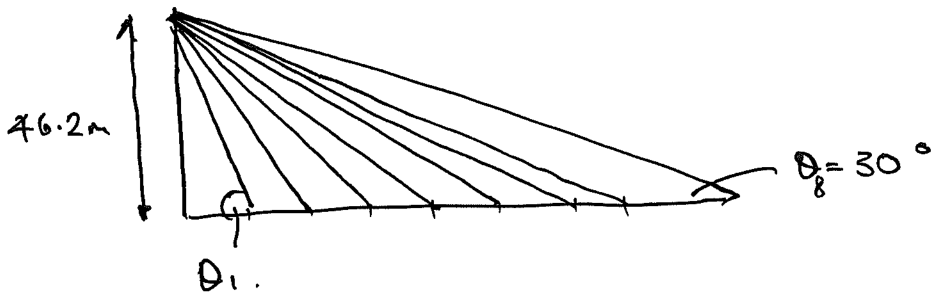

$\theta_1 = \tan^{-1} \frac{46.2}{10} = 77.8°$

$\theta_2 = \tan^{-1} \frac{46.2}{20} = 66.6°$

$\theta_3 = 57°$

$\theta_4 = 49.2°$

$\theta_5 = 42.7°$

$\theta_6 \; 37.6°$

$\theta_7 = 33.4°$

$\theta_8 = 30°$.

$\therefore$ Total compression in edge beam $= \sum \frac{W}{\tan\theta}$

$$= 2658 \left\{ \frac{1}{\tan 77.8} + \frac{1}{\tan 66.6} + \frac{1}{\tan 57°} + \frac{1}{\tan 49.2} + \frac{1}{\tan 42.7} + \frac{1}{\tan 37.6} + \frac{1}{\tan 33.4} + \frac{1}{\tan 30°} \right\}$$

$= \underline{18409 \text{ kN}}$.

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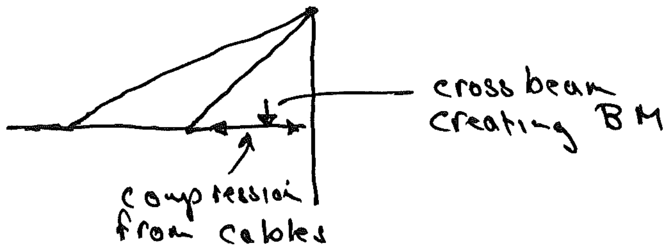
Worst case is the edge beam adjacent to the tower which is subjected to axial load 18409 kN and BM = 3323 kNm.



Now need to combine these as previously in cross beam design

$$\frac{N_{Ed}}{N_{b,Rd}} + \frac{k\,M_{ED}}{M_{b,Rd}} \leq 1.$$

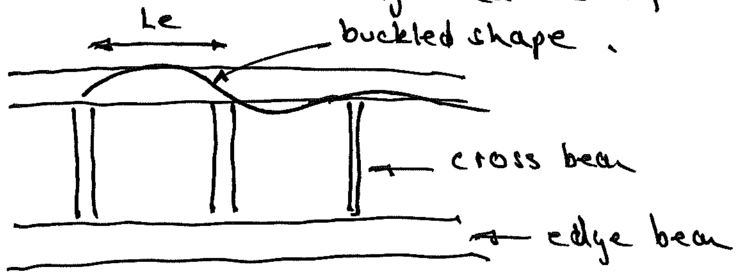
Is edge beam laterally restrained. If not what is the effective length ie where is it restrained.

Not too sure of detail of slab on edge beam. Slab is designed to sit on cross beams

∴ Assume here (academic exercise) that slab does not restrain edge beams ∴ Le = ?

Cross beams attached to edge beam every 5m.

ie



Plan on deck.

∴ Le = 5m.

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Assume that we use a 1016 x 305 x 487 (UB section)

$$\frac{18409}{N_{b,Rd}} + \frac{3323}{M_{b,Rd}} \leq 1.$$

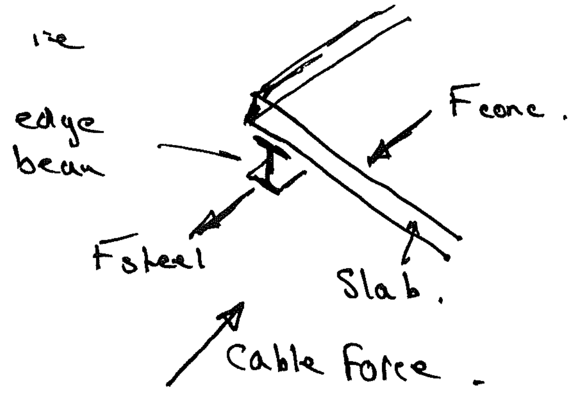
$N_{b,Rd}$ for $L_e = 5m$ = 11600kN < 18409 fails.

$M_{b,Rd}$ " " = 5m = 5650kN

Could use a plate girder and design still as $L_e = 5m$.

but try composite edge beam with deck slab.

ie



By tying the steel beam to the slab, they act together to resist the axial load.

But how much is carried by the slab & how much by the steel beam.

Use the idea that the strain in both the steel beam and concrete slab is the same ie they must both "move" the same distance under compression force.

$$\text{strain in edge beam} = \frac{F_{steel}}{\text{steel sect. area}} \times \frac{1}{E_{steel}} \quad \left(\text{ie } \frac{\text{stress}}{\text{strain}} = E. \right)$$

$$= \frac{F_{steel}}{620 \times 10^2} \times \frac{1}{205} = 7.868 \times 10^{-8} F_{steel}.$$

$$\text{Strain in slab} = \frac{F_{conc}}{\text{sect. area}} \times \frac{1}{E_{conc}}.$$

half width of slab deck.

$$= \frac{F_{conc}}{(7.5 \times 0.25) \times 10^6} \times \frac{1}{15.} = 3.555 \times 10^{-8} F_{conc}.$$

but these strains are the same.

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$\therefore$ 7.868 Fsteel = 3.555 Fconc.

$F_{conc} = \frac{7.868}{3.555} F_{steel}$

$(\therefore = 2.21\ F_{steel})$

but also. Fsteel + Fconc = 18409 kN.

sub between these 2 equations gives

Fsteel = 5729 kN & Fconc = 12680 kN

At this stage, before going on to check edge beam compositely, quick buckling check of slab.

Euler buckling check. $P_E = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 \times 15 \times \left(\frac{7500 \times 250^3}{12}\right)}{5000^2}$

$= 57844\ kN > 12680\ kN$

$\therefore$ Likely okay, but need a more detailed check

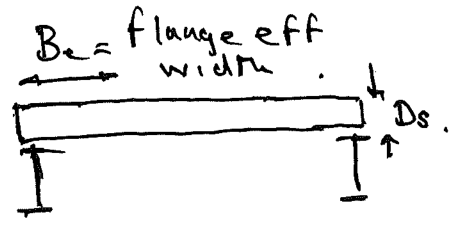
Now design the edge beam compositely exactly as previous composite design.

{ Find force in concrete slab Rc and force in steel beam Rs and then use 1 of 3 formulae }.

$R_c = 0.567\ B_E D_s f_{ck}$

$= 0.567 \times 2500 \times 250 \times 40 \times 10^{-3}$

$= 14175\ kN$.



$B_E = \frac{\text{span of beam}}{4}$ or $\frac{\text{spacing of beams}}{2}$ (lesser)

NB only slab on one side.

$= \frac{10}{4}$ or $\frac{15}{2}$ ie 2.5m

R_s = section area x steel yield stress

$= 620 \times 10^2 \times 350 \times 10^{-3}$

as previous should be 335 N/mm²?

$= 21700\ kN$ (assume 1016x305x487 UB)

Since $R_s > R_c$ the neutral axis is now in the steel section.

{ Not met before, but all it means is that we use formula 2 or 3 off composite handout. }

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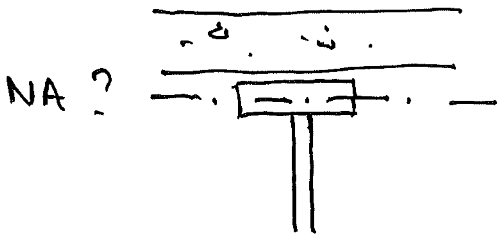
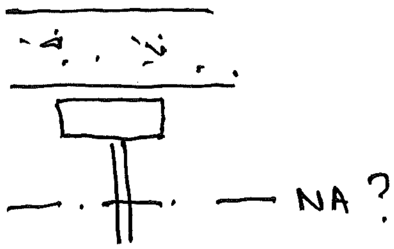
So where is the NA. In the flange or web of the steel beam

ie





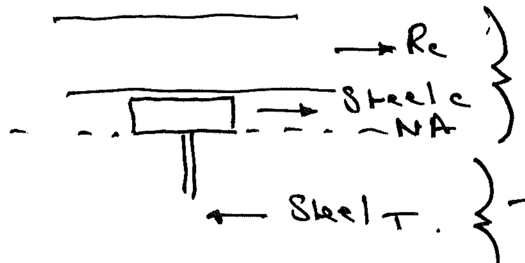
In order to know which it is we only need to calculate the capacity of the flange shown.

ie area of flange x yield stress as before.

$= (54.1 \times 308.5) \times 350 \times 10^{-3} = 5841$ kN.

thickness (54.1) width (308.5)

So if NA moves to bottom of flange



$\therefore$ Total comp = 14175 + 5841
= 20016 kN

Total tension = 21700 − 5841
= 15859 kN

ie if NA moves to bottom of flange C now > T.

$\therefore$ as C now > T then NA is in the flange.

(if C were still < T then NA is in the web)

As NA is in the flange, use formula 2.

ie
$$M_c = R_s \frac{D}{2} + R_c \frac{D_s}{2} - \frac{(R_s - R_c)^2}{R_f} \times \frac{T}{4}.$$

where D = depth of steel beam
D_s = " " slab.
R_f = resistance of flange
T = thickness " "

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$$= \left(21700 \times \frac{1036.1}{2} \times 10^{-3}\right) + \left(14175 \times \frac{250}{2} \times 10^{-3}\right)$$

$$- \left[\frac{(21700 - 14175)^2}{5841} \times \frac{54.1}{4}\right] \times 10^{-3}$$

$$= 12882 \text{ kNm.}$$

Now rework.

$$\frac{N_{ED}}{N_{b,RD}} + \frac{M_{ED}}{M_{b,Rd}} = \frac{5729}{11600} + \frac{3323}{12882} = 0.75 < 1 \checkmark$$

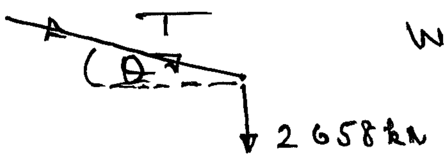
safe load tables

Cable Sizing.

Vert load per cable from deck = 2658kN (from earlier calcs).

Worst case cable load is where the angle is least (θ)

ie



Worst θ = 33.4° as this cable takes twice the load of the 30° cable.

$$T \sin\theta = 2658 \text{ kN}$$

$$\therefore T = \frac{2658}{\sin 33.4} = 4829 \text{ kN}$$

Assume cable breaking strength = 1600N/mm² and that a spiral wound cable is used.

$$F_{RD} = \frac{F_{uk}}{1.5}$$ where $F_{uk} = k d^2 R_R$ (Page 50 design guide bridges)

take k as 0.5

R_R = breaking strength

d = cable diameter

$\therefore F_{RD}$ ~~required~~ = 2 × 4829

allows for cable replacement or corruption of structure.

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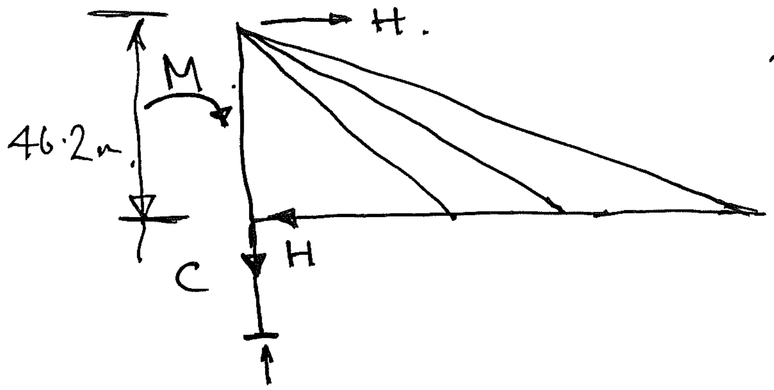
Rearranging gives $\dfrac{2 \times 4829 \times 1.5}{0.5 \times 1600 \times 10^{-3}} = d^2$.

$\therefore$ $d = \underline{135mm}$.

Tower Design.

We are working with a single tower initially, but unlikely to work for road bridge. Pedestrian more likely



Tower subjected to BM from pull of cables plus axial load from deck.

H for one edge beam = 18409 kN.

As we only have assumed one tower then both edge beams must produce H ie H for the tower = 2 x 18409
= 36818 kN.

$\therefore$ BM in tower = $H_{\text{tower}} \times 46.2 = 36818 \times 46.2$
= 1700992 kNm !!!.

Axial load C in tower = $(\underset{\text{vert cable load}}{2658} \times 8) \times \underset{\text{edge beams}}{2}$
= 42528 kN

At this stage undertake a quick check, not proper design, to get a feel for the numbers.

{ Assume outline size = 4m x 3m x 25mm thick. It was just a bit larger }.

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Combine the axial load & bending as per 1st year work.

ie $\sigma = \dfrac{\text{axial load}}{\text{area}} \pm \dfrac{\text{BM}}{\text{section modulus}}$.

Area = $(4 \times 3) - (3.95 \times 2.95) = 0.3475\,m^2$

$I = \left(\dfrac{3 \times 4^3}{12} - \dfrac{2.95 \times 3.95^3}{12}\right) = 0.849\,m^4$ (bending about major axis)

$Z = \dfrac{I}{y} = \dfrac{0.849}{2} = 0.425\,m^3$.

$\therefore \sigma = \dfrac{42528}{0.3475} \pm \dfrac{1700992}{0.425} = 4124717\ kN/m^2\ (c)$ & 3879951 " (T)

122 N/mm². ✓

$= 4125\,N/mm^2\,(c)$ & $3880\,N/mm^2\,(T)$.

Spectacular failure.

Rework with a larger section

Try 10m x 12m x 40mm thick tower. It gets better. Bending stress reduces to 258 N/mm².

So it might work but would need to undertake full design of the section (see later).

Options.

(1) Use 2 towers - likely from the start, which would halve the BM.

IC 27.

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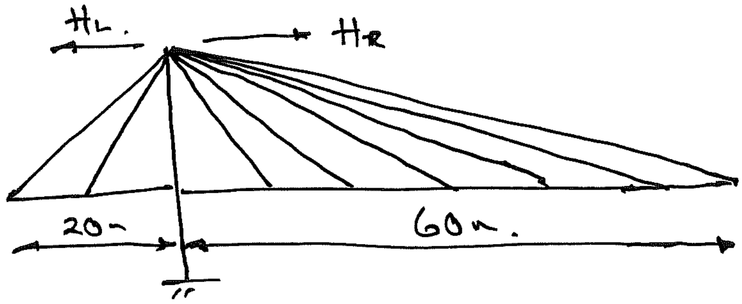
Use backstays ie move the tower.



Assume same height tower

$$H_R = 2658\left(\frac{1}{\tan 77.8}+\frac{1}{\tan 66.6}+\frac{1}{\tan 57}+\frac{1}{\tan 49.2}+\frac{1}{\tan 42.7}+\frac{1}{\tan 37.6}\right)$$

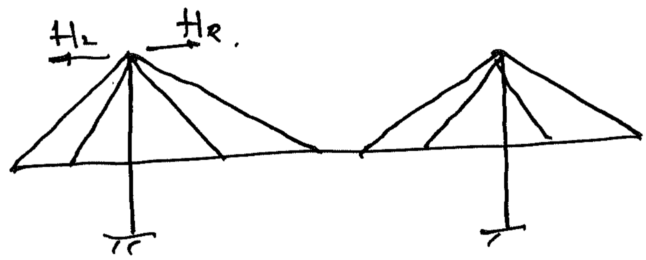
$= 12076$ kN

$$H_L = 2658\left(\frac{1}{\tan 77.8}+\frac{1}{\tan 66.6}\right) = 1725 \text{ kN}$$

$\therefore$ BM $= (12076 - 1725)\times 46.2 = 478216$ kNm

ie large reduction.

or.



Use towers either end of span plus backstays.

BM virtually eliminated or minimised.

$H_L = H_R$

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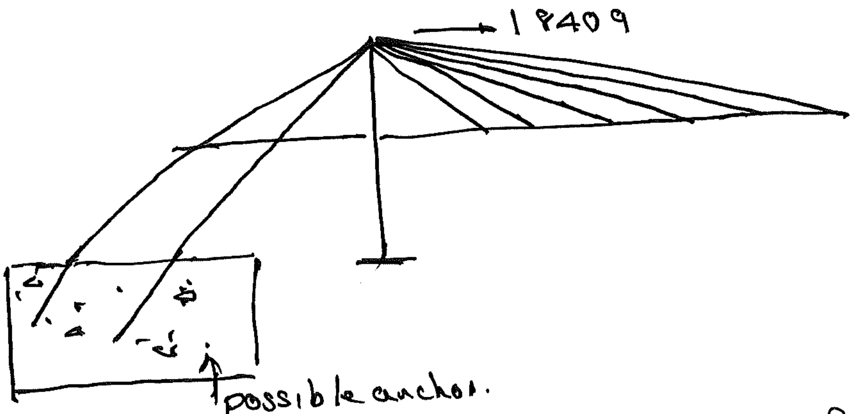
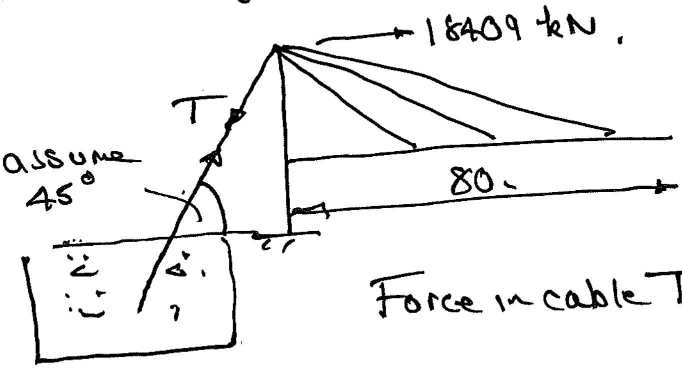
Use Anchored Backstays.



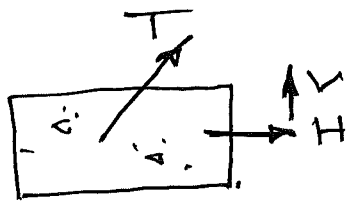
Assume single backstay cable for our example



$$\text{Force in cable } T = \frac{18409}{\cos 45^\circ} = 26034 \text{ kN.}$$

The cable can be designed as previously but will be large.

What about the anchor block.



$H = T\cos\theta$

$V = T\sin\theta$.

Block must be sized to resist sliding (H) and uplift (V).

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$V = 26034 \sin 45° = 18409$ kN

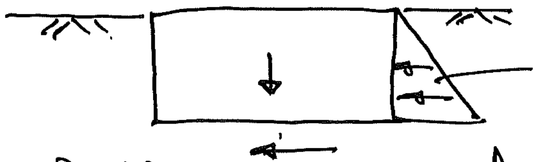
Try a block 20m x 10m x ~~5~~5m

$= 20 \times 10 \times 5 \times 25 = 25000$ kN.

FoS against uplift $= \frac{25000}{18409} = 1.36$. (might ask for FoS of 2).

$H = 26034 \cos 45° = 18409$ kN.

Resistance to sliding is provided by friction against the soil below the block and passive resistance of soil against the block ie



Assume $k_p' = 3$ $\omega = 20$ kN/m³

$\mu = 0.4$ (coeff of friction)

Resistance $= \overset{\mu W}{0.4(20 \times 10 \times 5 \times 25)} + \overset{k_p' \frac{\omega h^2}{2}}{\left(3 \times 20 \times \frac{5^2}{2}\right)}$

$= 10750$ kN < 18409 Fails

Try 20 x 5 plan x 10m deep. still Fails

Make bigger.

IC 30

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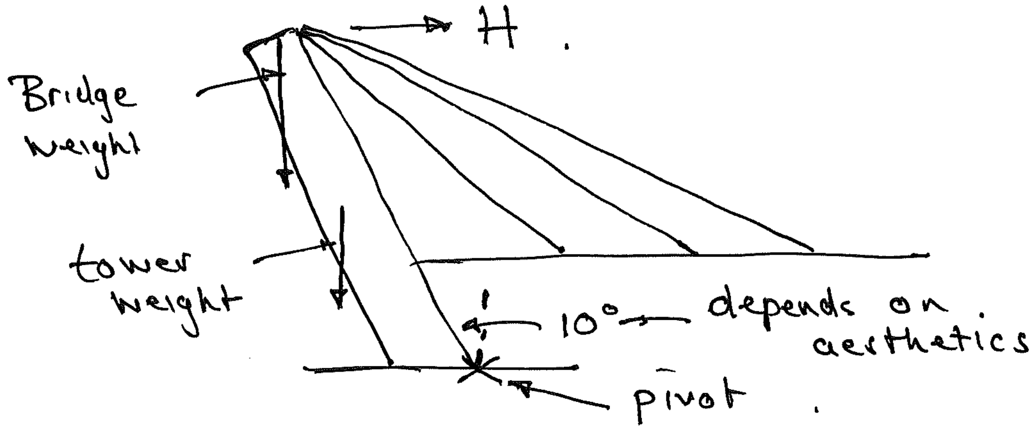
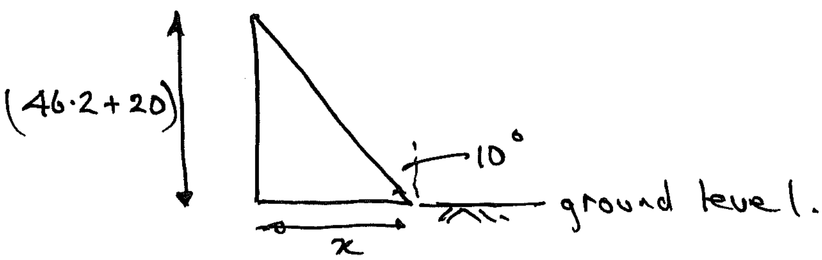
Leaning the Tower Back.



By leaning the bridge back (away from the cables) the bridge weight is offset from the centroid of the tower and creates a countering BM.



$x = 11.67\,m.$

Total weight of bridge = 42528 kN.

$\therefore$ Restoring moment = 42528 × 11.67 = 496302 kNm.

OM (for single tower) = 1700992 kNm.

$\therefore$ Reduced OM = 1700992 − 496302 = 1204690

ie 30° approx.

(tower weight won't add much to this)

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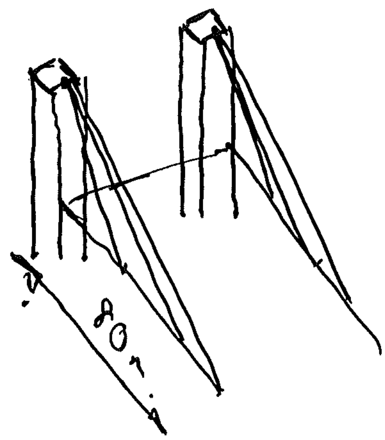
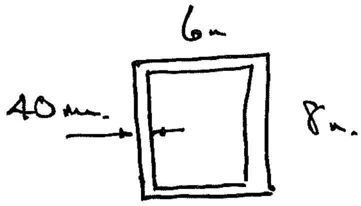
Tower Design.

Assume That 2 towers are to be used with no backstays, no leaning etc, no anchorages.
Guess at a section size 6m x 8m x 40mm Thick based on earlier spectacular failure.

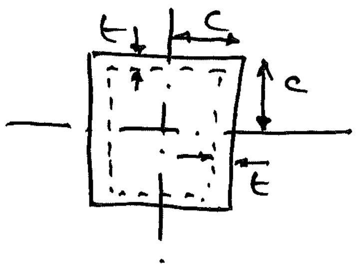
ie



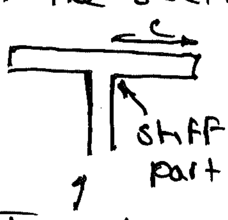


Need section classification ie class 1, class 2, class 3 or class 4 ← don't want this

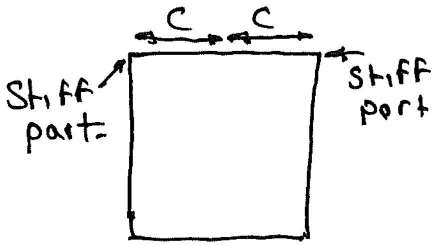
Need $\frac{c}{t}$ for classification



C is the outstand from the "stiff" part of the section



I section



for our section $\frac{c}{t} = \frac{3000}{40}$ & $\frac{4000}{40}$ ($\frac{6m}{2}$, $\frac{8m}{2}$)

$= 75$ & 100.

from table on page 28 of Bridge Guide for Class 3 section $c/t \leq 11.3$!!!

PROJECT Arch Bridge

ELEMENT Outline Sizing

SHEET NO ABOS1

DESIGNED BY dje

DATE

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University of Plymouth
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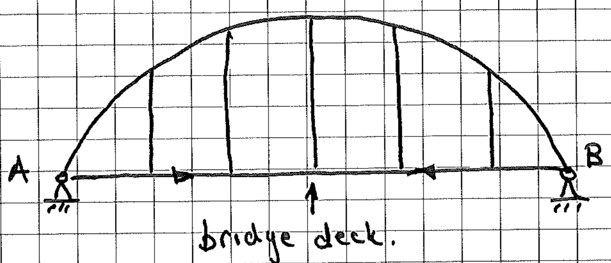
CALCULATIONS | OUTPUT



Assume initially that the bridge deck is at the correct level to "tie" the arch together ie it carries the horizontal thrust at A & B, which is zero as the thrust is in the deck in tension.

The hangars support the deck and transfer the vertical load to the arch.

Therefore the deck and hangars behave similarly to the cable stayed bridge, the calcs for which can be referred to for the design.

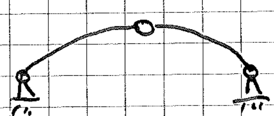
Size of Main Arch.

This will depend upon the rise of the arch for any given span. The higher the rise, the lower the forces involved. However a "high rise" arch can look ugly. There is therefore a "trade off" between arch size and arch rise. It is usual to draw several arch profiles before selecting one to design, based usually on the aesthetics.

Basis for Sizing

Initially choose a parabolic arch and for simplicity make it a 3 hinged arch.

ie



This makes the analysis easy to start with.

PROJECT Arch Bridge

ELEMENT Outline Sizing

SHEET NO AB0S2

DESIGNED BY dje.

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CALCULATIONS | OUTPUT

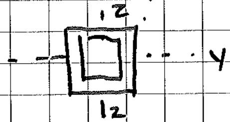
A parabolic arch subjected to a udl along it's entire length has no bending moment.

Initial sizing

This can be achieved in a "rough" manner if the slenderness of the arch is limited initially to 120 ie $\frac{L_e}{r_{zz}} = 120$.

ie if $L_e = 20m$ then r_{zz} required $= 0.167m$.

Using a hollow plate girder section ie

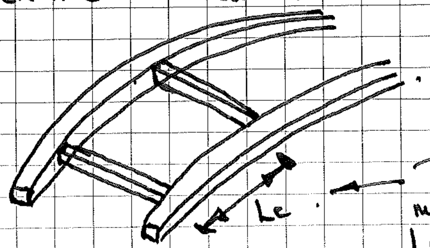


$$r = \sqrt{\frac{I}{A}}$$

Now use a trial & error approach to guessing a size of section ie breadth & depth and also a thickness of say 25mm or 40mm.

Look at the "Newport Bridge" Construction Notes on the portal which indicates how the use of cross members between two parallel arch members can limit the effective length.

ie



This length will need to be multiplied by a number between 1 and 2 depending on the relevant stiffnesses of the cross members and the main arch. At outline, assume a factor of 1.5.

More accurate approach.

Undertake an analysis of a 3 hinged arch & then design for axial load only.

PROJECT Bridge Design

ELEMENT Outline Sizing

SHEET NO ABOS 3

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CALCULATIONS | OUTPUT

Arch.

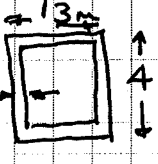
(a) Very "rough & ready" arch size. Take the span of the arch and assume that the maximum allowable slenderness ie $\frac{Le}{r} = 120$

This will provide a value for r. eg if 20m span $r = 0.167m$.

Modern steel arches comprise of "hollow" sections but are unlikely to be standard RHS sections but rather "made up" plate girders.

Using $r = \sqrt{\frac{I}{A}}$ determine from trial and error a depth & width of "hollow section" required based upon a 25mm thick section. The r value varies little if the section is made thicker than this.

For an arch span of 200m this method produces a section which is approx

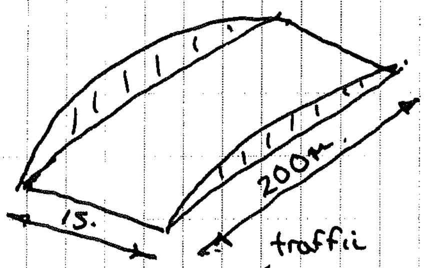


This provides the correct r for the stronger axis buckling as cross members around the arch prevent weaker axis buckling.

(b). More accurate approach.

Assume a bridge deck 15m wide supported by 2 arches

ie



should be 1.25 but this is outline

Deck load = $(5.5 \times 1.35) + (0.25 \times 1.35 \times 25) = 15.86\,kN/m^2$

Knife edge load = 100 kN/m across the lanes. (There are possibly 5 lanes across the 15m and the KEL is only applied to 3, but for speed assume applied to them all). The worst position will be at the centre of the 200m span

PROJECT Bridge Design

ELEMENT Outline Sizing

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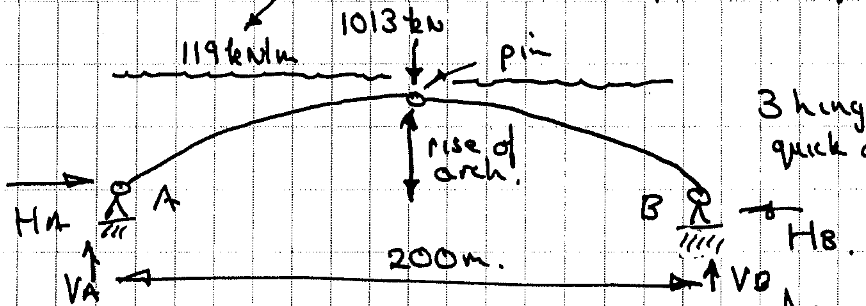
CALCULATIONS | OUTPUT

∴ Load per arch (assuming the load is taken as a udl although applied through the hangers)

$= (15.86 \times \frac{15}{2})$ kN/m & a point load of $100 \times \frac{15}{2} = 750$kN.
$\times 1.35$
1013 kN



3 hinged arch for quick analysis

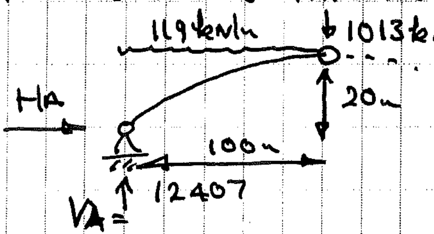
It is up to the designer to choose the rise of the arch. However, the shallower the arch (smaller rise) the bigger the axial thrust & therefore the larger the section. There is likely to be trade off between aesthetics & size. Size probably won't affect aesthetics over such a scale but is likely to be a construction problem due to weight problems.

In this instance assume that the rise = 20m.

The arch can be either circular or parabolic form. This arch is assumed to be parabolic.

$\therefore V_A = V_B = (119 \times \frac{200}{2}) + \frac{1013}{2} = 12407$ kN.

Now determine $H_A = H_B$ by taking moments about the pin.



$$H_A = \frac{(12407 \times 100) - 119 \times \frac{100^2}{2}}{20} = 32285 \text{ kN}$$

(NB The thrust 3228 tonnes)

V_A & H_A can now be combined at the support to provide an axial thrust in the arch. (A parabolic arch with symmetrical udl & single load at the crown has no BM.)

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PROJECT: Bridge Design

ELEMENT: Outline Sizing

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CALCULATIONS | OUTPUT

In order to combine the loads V_A & H_A, need to determine the tangent of the arch at A.

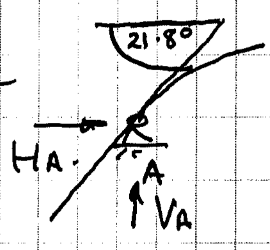
The equation of a parabola is : $y = ax^2$

for this arch when $y = 20m$ $x = 100m$ $\therefore a = 0.002$

$\therefore \frac{dy}{dx} = 2ax$. $\therefore$ When $x = 100m$ ie at A.

$\frac{dy}{dx} = 0.4$ $\therefore$ tangent angle $= \tan^{-1} 0.4 = 21.8°$

ie



$\therefore$ Combining V_A & H_A tangential to A

gives $H_A \cos\theta + V_A \sin\theta$

$= 32285 \cos 21.8° + 12407 \sin 21.8°$

$= 34584 kN.$

This is the axial compression force in the arch.

Therefore need to design a compression member 200m long which will support this force.

Section 15 of the Student Simple Guide should be used.

Resistance Axial Load $N_{b,Rd} = \frac{\chi A f_y}{\gamma_{M1}}$

χ = slenderness reduction factor from Fig 8.2.

$\bar{\lambda} = \frac{L_{cr}}{i} \times \frac{1}{\lambda_1}$ (where L_{cr} = eff length of arch. i = radius of gyration $\lambda_1 = 76$ for S355 steel)

Try using a section 2.5m w x 3.5m deep x 40mm thick (trial & error.)

$I = 0.865 m^4$ $A = 0.474 m^2$ $\therefore i = \sqrt{\frac{I}{A}} = 1.35m.$

$\therefore \bar{\lambda} = \frac{200}{1.35} \times \frac{1}{76} = 1.95.$ $\therefore \chi \approx 0.185$ from Fig 8.2.

$\therefore N_{b,Rd} = \frac{0.185 \times 0.474 \times 10^6 \times 355 \times 10^{-3}}{1.0} = \underline{31130 kN}.$

ie fails. $\therefore$ Need to increase section size

PROJECT Bridge Design

ELEMENT Outline Sizing

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CALCULATIONS

OUTPUT

At this stage wind loading has not been considered which will cause bending in the weaker axis depending on the bracing used between the two arches.

This is sufficient to allow construction considerations to decide on the type of bridge to be used for the location.

PROJECT Arch Bridge

ELEMENT Full Design

SHEET NO ABFD 1

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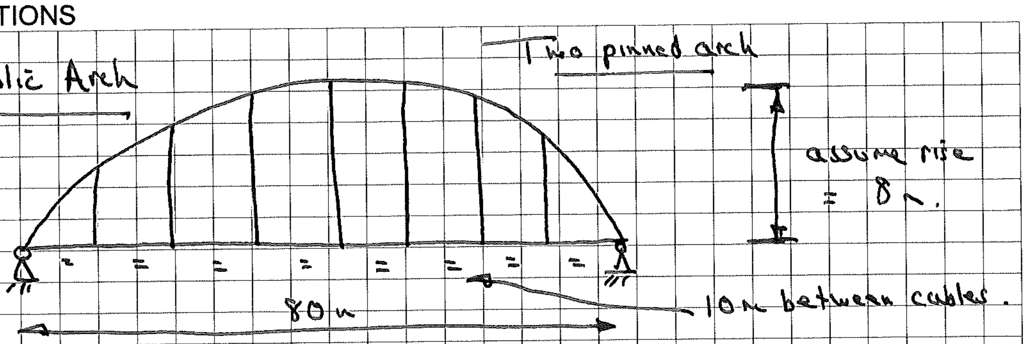
CALCULATIONS | OUTPUT

Parabolic Arch

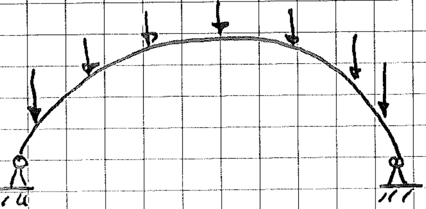


The deck design is exactly the same as for the cable stayed option, but there is no axial cable force to consider in the edge beams. However if the arch is tied through the deck then it will be a tension force which must be considered alongside bending.

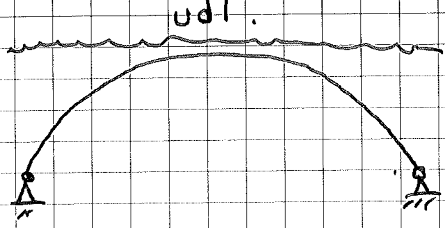
Arch analysis.

The arch can be analysed in two ways. The outline design was undertaken as a 3 hinged arch which is statically determinate. The 2 hinged arch is indeterminate & needs to be analysed differently.

In reality



Simplified



As a series of point loads
See fully worked analysis at the end of the calcs.

This is the analysis that follows

It can be seen from the results for both of these analyses that H (in reality) = 26,233 kN & H (simplified) = 26,580 kN

ie simplified errs on the high side by approx 1%.

and to make life even simpler the value of H for a 3 hinged arch gives the same value for a parabolic arch with a udl & 2 hinged.

PROJECT Arch Bridge

ELEMENT Full Design

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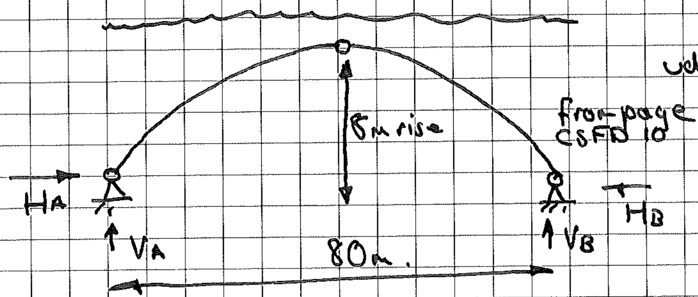
CALCULATIONS | OUTPUT

$\therefore$ Arch Analysis (only $\div$ this case with a symmetrical load).



$$udl = \frac{\text{total load on arch}}{80}$$

from page CSFD 10

$$= \frac{2658 \times 8}{80}$$

$$= 265 \cdot 8 \text{ kN/m}$$

$\therefore V_A = V_B = 265 \cdot 8 \times \frac{80}{2} = 10632 \text{ kN}$ (ignoring arch s/w).

Moments about pin

$$H_A = \frac{(10632 \times 40) - \frac{265 \cdot 8 \times 40^2}{2}}{8} = 26{,}580 \text{ kN}$$

Now combine these to calculate the worst force in the arch.

Since it is an arch which has a udl on it and it is parabolic there is no BM on the arch.

$\therefore$ Combine H_A & V_A at A to determine max axial compression in the arch,

Axial Force $= H_A \cos\theta + V_A \sin\theta$. where $\theta = \tan^{-1}\frac{dy}{dx}$.

$= 24679 + 3949 = 28628 \text{ kN}$. $= \tan^{-1} 2ax$

$= \tan^{-1} 2 \times 0 \cdot 005 \times 40$

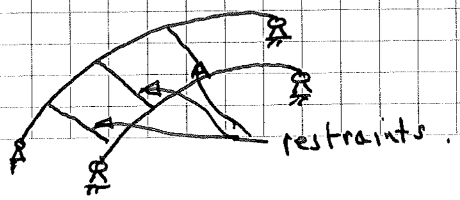
$= 21 \cdot 8^\circ$.

Now design the arch member itself.

Guess size reqd is such that $L_e/_{r_{zz}} = 100$.

L_e = length of arch between restraints.

ie



PROJECT Arch Bridge

ELEMENT Full Design

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CALCULATIONS

OUTPUT

In this case we will assume that no cross members are provided (for aesthetic reasons).

$\therefore$ $L_e \simeq 80m$ $\therefore$ i_{zz} required $= \frac{80}{100} = 0.8m$.

Try section 2m wide x 3m deep x 40mm thick.

$I_{zz} = 0.278\,m^4$ $A = 0.394\,m^2$ $\therefore i_{zz} = \sqrt{\frac{I}{A}} \simeq 0.84m$

Using section 15 of the "Student Style Guide"

Axial load capacity $N_{b,Rd} = \frac{\chi A f_y}{\gamma_{m1}}$

χ = slenderness reduction factor from fig 8.2

$\bar{\lambda} = \frac{L_{cr}}{i} \times \frac{1}{\lambda_1}$ (where L_{cr} = eff length of arch, i = radius of gyration, $\lambda_1 = 76$ for S355 steel)

$\therefore \bar{\lambda} = \frac{80}{0.84} \times \frac{1}{76} = 1.25$ $\therefore \chi = 0.35$ from fig 8.2.

$\therefore N_{b,Rd} = \frac{0.35 \times 0.394 \times 10^6 \times 355 \times 10^{-3}}{1.0} = 48954\,kN$

ie well ~ (28628 kN required).

$\therefore$ Could rework to obtain a smaller section.

PROJECT Arch Bridge

ELEMENT Full Design

SHEET NO ABFD 4

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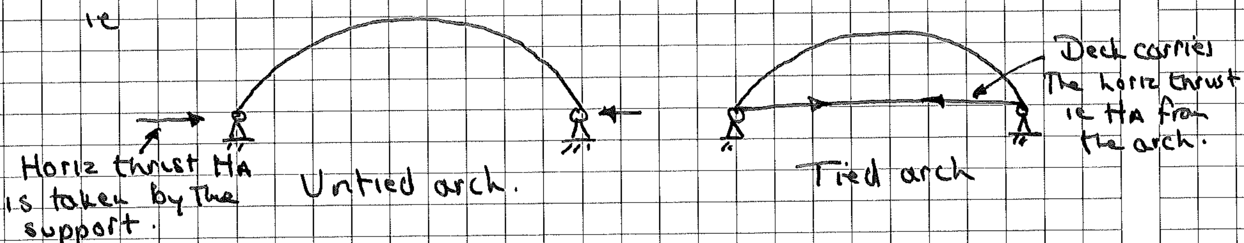
CALCULATIONS

OUTPUT

Bridge Deck.

This is the same as for the cable stayed example, except that there is no compression in the deck (from the cables) but there will be tension if the deck is used to tie the arch.

ie



From page CSFD 9 BM in edge beam = 3323 kNm.

Now need to combine with horizontal reaction HA = 26580 kN

$$\frac{N_{Ed}}{N_{t,Rd}} + \frac{M_{Ed}}{M_{b,Rd}} \leq 1$$

Try same section as for the cable stayed bridge (page CSFD 9).

$M_{b,Rd} = 5650$ kNm

$$N_{t,Rd} = \frac{A \times f_y}{\gamma_m} = \frac{620 \times 10^2 \times 355 \times 10^{-3}}{1 \cdot 0} = 22010 \text{ kN}.$$

$$\therefore \frac{26580}{22010} + \frac{3323}{5650} \geq 1 \quad \therefore \text{ Fails}.$$

However if the slab is composite then could argue that beam carries 31% of axial load (page CSFD 10 – $\frac{5729}{18409}$).

which gives $\frac{26580 \times 0 \cdot 31}{22010} + \frac{3323}{5650} = 0 \cdot 96$ ✓

Would need to design the slab such that there was sufficient reinforcement to carry the tensile force.

PROJECT Arch Bridge

ELEMENT Full Design

SHEET NO ABFD 5

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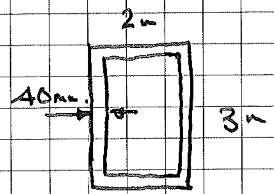
CALCULATIONS | OUTPUT

Look at the section classification of the main arch.



$\frac{c}{t} = \frac{1500}{40} = 37.5$

$\frac{d_{tw}}{t} = \frac{3000 \text{ (whole section in compression)}}{40} = 75.$

both limits mean this is a Class 4 section (page 28 of Simplified Guide)

Simplified method for Class 4 sections is to reduce the thickness of the section which reduces I and hence the allowable stress.

The class 3 limit for the web = 45.

Multiply this by t_w^2 to give $45 \times 40^2 = 72000 \text{ mm}^2$.

Actual depth of web = 3m $\therefore$ Revised thickness $= \frac{72000}{3000} = 24\text{mm}$.

Now recalculate I_{zz} & A & i_{zz} etc

$\therefore I_{zz} = 0.17 \text{ m}^4 \quad A = 0.237 \text{ m}^2 \quad \therefore i_{zz} = \sqrt{\frac{I}{A}} = 0.847\text{m}.$

$\therefore \lambda = \frac{L_{cr}}{i} \times \frac{1}{\lambda_1} = \frac{80}{0.847} \times \frac{1}{76} = 1.24 \quad \therefore \chi = 0.35$ from fig 8.2

$\therefore N_{b,Rd} = \frac{0.35 \times 0.237 \times 10^6 \times 355 \times 10^{-3}}{1.0} = 29447 \text{ kN}.$

Still just ✓ ie > 28628 kN

PROJECT Arch Bridge

ELEMENT Full Design

SHEET NO ABFD - Appendix 1.1

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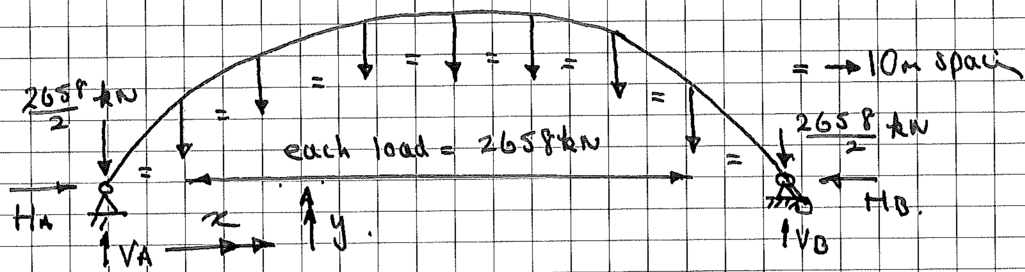
CALCULATIONS

OUTPUT

Full Worked Analysis of 2 hinged arch. - (for future reference only).

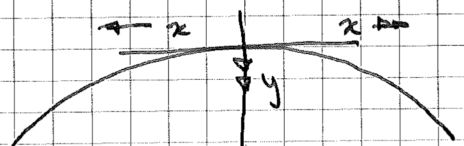


$$H = \frac{\int_0^L My\,dx}{\int_0^L y^2\,dx}.$$

(for a parabolic arch $\int_0^L y^2\,dx$ can be taken as $\frac{8h^2L}{15}$.
where h = rise of arch
L = span of arch.)

Where M is the bending moment calculated in terms of x, with H_A removed and y is a generic value for y anywhere on the arch, in terms of x.
Where there are a number of point loads, then M must be determined and integrated for the value between each hangar as shown below.

The parabola



The directions of x & y are different than that for the arch analysis above.
The parabola values of x & y are used to find the equation of the arch.

$$y = ax^2$$

for our arch - at $x = 40$m $y = 8$m

$$\therefore a = \frac{8}{40^2} = 0.005.$$

Using the directions of x & y in the top diagram

$$y = 8 - a(x-40)^2 \quad \text{(for any value of } x \text{ from } 0 \rightarrow 80\text{)}.$$

$$= 8 - 0.005(x-40)^2 = \cancel{8} - 0.005x^2 + 0.4x \cancel{-8}$$

$$= -0.005x^2 + 0.4x.$$

PROJECT Arch Bridge

ELEMENT Full Design

SHEET NO ABFD – Appendix 1.2

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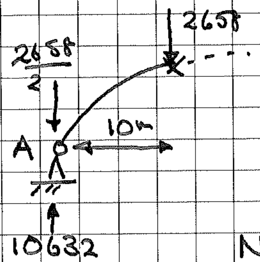
CALCULATIONS | OUTPUT

Now calculate the value of BM anywhere from A to the First point load ie over 10m. (ignoring H_A).



BM anywhere between 0m & 10m at a distance of xm from A

$$= \left(10632x - \frac{2658x}{2}\right).$$

Now find $\int_0^{10} My\,dx$

$$= \int_0^{10} \left(10632x - \frac{2658x}{2}\right)\left(-0{\cdot}005x^2 + 0{\cdot}4x\right)$$

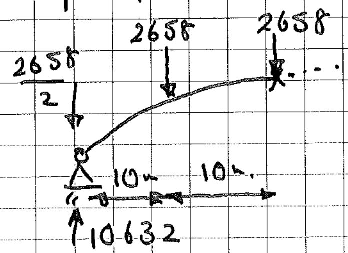
$$= \int_0^{10} -53{\cdot}16x^3 + 4252{\cdot}8x^2 + 6{\cdot}645x^3 - 531{\cdot}6x^2$$

$$= \int_0^{10} -46{\cdot}515x^3 + 3721{\cdot}2x^2.$$

$$= \left[-\frac{46{\cdot}515x^4}{4} + \frac{3721{\cdot}2x^3}{3}\right]_0^{10} = 1{,}124{,}113\text{ kN}.$$

Now repeat the exercise for the BM value between 10m & 20m.

ie



BM anywhere between 10m & 20m at a distance xm from A

$$= \left[10632x - 2658\left(\frac{x}{2} + (x-10)\right)\right]$$

Now find $\int_{10}^{20} My\,dx.$

$$= \int_{10}^{20} (10632x - 3987x + 26580)(-0{\cdot}005x^2 + 0{\cdot}4x)$$

$$= \int_{10}^{20} (6645x + 26580)(-0{\cdot}005x^2 + 0{\cdot}4x)$$

$$= \int_{10}^{20} -33{\cdot}225x^3 + 2658x^2 - 132{\cdot}9x^2 + 10632x$$

$$= \left[-\frac{33{\cdot}225x^4}{4} + \frac{2658x^3}{3} - \frac{132{\cdot}9x^3}{3} + \frac{10632x^2}{2}\right]_{10}^{20}$$

PROJECT Arch Bridge

ELEMENT Full Design

SHEET NO ABFD – Appendix 1.3

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CALCULATIONS | OUTPUT

$= 7531000 - 1290238 = 6,240,762$ kN.

Repeat for BM from 20m – 30m

$$= \int_{20}^{30} \left[\left(10632x - 2658\left(\frac{x}{2} + (x-10) + (x-20)\right)\right)\right] \times \left(-0.005x^2 + 0.4x\right)$$

$= 12\,309\,863$ kN

Repeat for BM from 30m – 40m

$$= \int_{30}^{40} \left[\left(10632x - 2658\left(\frac{x}{2} + (x-10) + (x-20) + (x-30)\right)\right)\right] \times \left(-0.005x^2 + 0.4x\right)$$

$= 16141813$ kN.

Due to symmetry there is no need to continue, but add up all the values and multiply by 2.

$$\int_0^L y^2 dx = \frac{8h^2L}{15} = \frac{8 \times 8^2 \times 80}{15} = 2730.67$$

$$\therefore H = \frac{\Sigma \int_0^L M y \, dx}{\int_0^L y^2 dx} = \frac{(1124113 + 6240762 + 12309863 + 16141813) \times 2}{2730.67}$$

$= 26233$ kN.

This can now be used in conjunction with VA etc to determine BM & axial forces in the arch in order to design the section.

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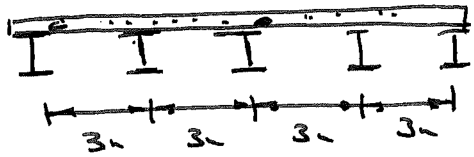
Plate Girder Design Example. – Simply Supported Span

10 kN/m² unfactored load (given).



Span of beam = 34.65m.

Outline Design – Depth only

Anticipated Depth (to check if UB section would be adequate)

Approx depth = $\frac{\text{span}}{20} = \frac{34.65}{20} = \underline{1.7m}$. ie larger than UB

"Student guide to steel bridges" by SCI suggests depth anywhere from $\frac{\text{span}}{20}$ to $\frac{\text{span}}{30}$.

In this case try a size slightly lower than 1.7m ie 1.6m.

Width of flange is somewhere between $\frac{\text{o/a depth}}{2.5 \text{ to } 3} = 640$ to $533mm$.

In this case say 500 mm.

In order to work out the flange & web thickness need the loading and the BM.

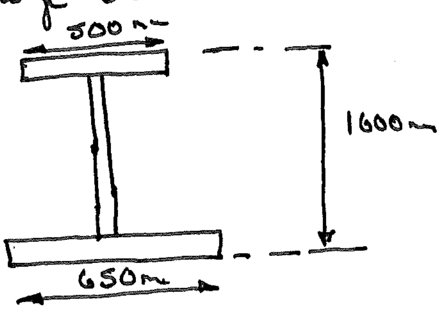
The loading is given as $10 kN/m^2$ ∴ $udl = (10 \times 3) \times 1.35 = 40.5 kN/m$

∴ $BM = \frac{wl^2}{8} = \frac{40.5 \times 34.65^2}{8} = 6078 kNm$.

If designing this for exam or test, make the plate girder symmetrical. However in this example, with a concrete slab over the top which can be used compositely, make the section unsymmetrical with a smaller top flange

∴ Make top flange 500mm wide & bottom flange 650mm wide

ie



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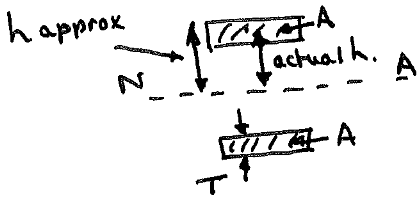
Try the beam on its own.

$W_{el,yy}$ requ'd $= \dfrac{BM}{p_y}$ (assume grade 355 steelwork)

$= \dfrac{6078\times10^6}{355} = 17.1\times10^6\ mm^3$.

For speed assume NA is at mid height (just for sizing flanges)

Ignoring the web $I = 2Ah^2$ giving $W_{el,yy} \simeq 2Ah$.



$\therefore \dfrac{1600}{2}\times(2\times500\times T) = 17.1\times10^6$

$\therefore T = 21.4$ mm.

Could round up to 25mm but try 40mm. (see below for why)

Web takes the SF $= 40.5\times\dfrac{34.65}{2} = 702$ kN

Assume allowable shear stress (is full) $= 355\ N/mm^2$.

$\therefore$ Area requ'd $= \dfrac{702\times10^3}{355} = 1978\ mm^2$.

Web depth $= 1600 - (2\times40) = 1520$ mm.

$\therefore$ Web width $= \dfrac{1978}{1520} = 1.3$mm.

This might fit a strength - theory, but think of the "slenderness"

ie $\dfrac{1520}{1.3} = 1170$ ie way, way too high, it will buckle.

$\therefore$ Try a 12mm thick web. (over the first metre of span - av shear $= \dfrac{702 + (702-40.5)}{2} = 681.75$ kN.

Slight difference so use max shear & assume over 1m.

radius of gyration $= 0.289d = 0.289\times12$.

$\therefore$ Slenderness $= \dfrac{1520}{0.289\times12} = 438$ which is still likely to buckle.

If it is 25mm then Slenderness $= \dfrac{1520}{0.289\times25} = 210.3$

likely to be on the limit for local buckling.

— This completes the outline design. —

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This is the start of the full design.

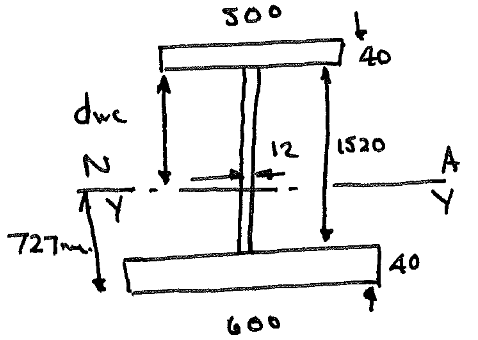
Try section classification - see fig 5.2 of Simplified Guide

Flanges $\frac{c}{t} = \frac{250}{40}$ (top flange) $= 6.25$. or $\frac{325}{40}$ (bottom flange) $= 8.2$

For grade 355 steel this is $> 8.1 < 11.3$ ∴ Class 3 ie elastic.

{ if we had chosen the 25mm which was indicated earlier then the flanges would have been Class 4 and subject to local buckling }

Web. $\frac{d_{wc}}{t} = \frac{d_{wc}}{12}$. where d_{wc} = the depth of the web in compression.



Using parallel axis Theorem

$I_{yy} = 31164 \times 10^6 \text{ mm}^4$.

$\bar{y} = 727\text{mm}$ (depth to neutral axis).

∴ $d_{wc} = 1520 - (727 - 40) = 833\text{mm}$.

∴ $\frac{d_{wc}}{12} = \frac{833}{12} = 69.4$. ∴ from fig 5.2. for a class 3 section $\frac{d_{wc}}{t} < 45$. which it isn't.

∴ Section is Class 4 & will buckle locally.

[if doing this calc under exam / test conditions I would make the web 20mm thick to give $\frac{d_{wc}}{t} = 41.7$ ie now Class 3
The section is however now heavier if there is a lifting limit]

Design for bending.

As the beam is simply supported with a slab on it, the compression flange is laterally restrained.

In the first instance, forget composite construction.

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∴ Stress in outer edges of flanges $= \frac{M}{W_{el,yy}}$ $\left[W_{el,yy} = \frac{I_{yy}}{y} \right]$.
(as unsymmetric $W_{el,yy}$ is different for top & bottom flanges)

∴ Top flange stress $= \dfrac{6078 \times 10^6}{\frac{31164 \times 10^6}{(1600-727)}} = 170.3\ N/mm^2 < 355$ ✓.

Bottom " " $= \dfrac{6078 \times 10^6}{\frac{31164 \times 10^6}{727}} = 141.8\ N/mm^2 < 355$ ✓.

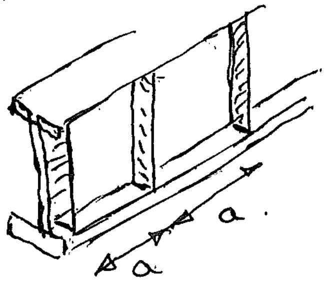
It can be seen that the 40mm thickness of flange was OTT and the 25mm might have worked but the calculation would be based on local buckling as the web shown below.

Design for Shear.

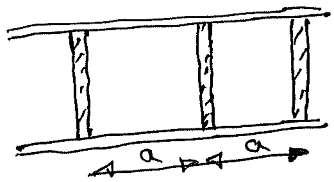
Ignore the flanges as contribution is small (remember analysis lectures on shear)

Need to decide if and where transverse stiffeners are placed.

ie

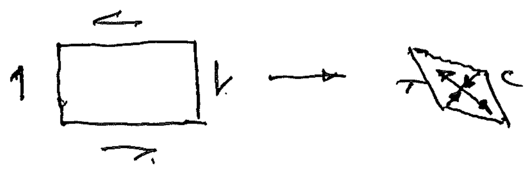


a is the spacing of the stiffeners, which are plates welded to the web for the depth between the flanges.



The effect of providing stiffeners is to reduce the length over which buckling occurs (similar to reducing the effective length).

ie



The shear forces produce a squashed element with compression from bottom left to top right.

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Consider the case where no stiffeners are provided in the length of the beam and only at the supports. $\therefore a = 34.65m$.

Depth between flanges = 1520mm. $\therefore \dfrac{a}{h_w} = \dfrac{34.65}{1.52} = 22.8 > 1$.

Use equation 7.5 of Simplified Guide

where $k_\tau = 5.34 + 4\left(\dfrac{h_w}{a}\right)^2 = 5.34 + 4\left(\dfrac{1.52}{34.65}\right)^2 = 5.35$.

$$\therefore \bar{\lambda}_w = \frac{h_w}{30.3\, t_w \sqrt{k_\tau}} \quad \text{(grade 355 steel)}$$

$$= \frac{1520}{30.3 \times 12 \sqrt{5.35}} = 1.81.$$

$\therefore$ Reduction factor $\chi = 0.47$ Fig 7.2 Simplified Guide

(or use one of the equations in 7.6.)

$\therefore$ Web shear capacity $V_{bw,Rd} = \dfrac{\chi f_{yw} h_w t_w}{\sqrt{3}\, \gamma_{M1}}$ eqn 7.7 Simplified Guide

$$= \frac{0.47 \times 355 \times 1520 \times 12}{\sqrt{3} \times 1.1} \times 10^{-3} = \underline{1597} \text{ kN}$$

Actual = 702 kN. $\therefore$ okay.

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if it wasn't satisfactory then need to provide stiffeners at closer centres.

eg if stiffeners are spaced at 1500mm c/c. $\dfrac{a}{h_w} = \dfrac{1500}{1520} < 1$.

$$\therefore k_\tau = 4 + 5.34\left(\frac{h_w}{a}\right)^2 = 9.5$$

$\therefore \bar{\lambda}_w = 1.36 \qquad \therefore \chi = 0.61.$

$\therefore$ Capacity = $\underline{2073 \text{ kN}}$.