The University of Sydney

STUDENT PLAGIARISM: COURSE WORK - POLICY AND PROCEDURE

COMPLIANCE STATEMENT

INDIVIDUAL / COLLABORATIVE WORK

I/We certify that:

(1) I/We have read and understood the *University of Sydney Student Plagiarism: Coursework Policy and Procedure*;

(2) I/We understand that failure to comply with the *Student Plagiarism: Coursework Policy and Procedure* can lead to the University commencing proceedings against me/us for potential student misconduct under Chapter 8 of the *University of Sydney By-Law 1999* (as amended);

(3) this Work is substantially my/our own, and to the extent that any part of this Work is not my/our own I/we have indicated that it is not my/our own by Acknowledging the Source of that part or those parts of the Work.

Name(s): David Mann

Signature(s): David Mann

Date: 9/4/09.

21/5/09.

Sensors and Signals: Assignment 01

David Mann

305169602

dman0888@usyd.edu.au

Problem Definition

General Aim: Track a golf ball through all stages of flight.

Detailed Requirements:

1. The system shall be capable of tracking a golf ball through all stages of its flight, including as it bounces and rolls along the ground.
2. This tracking shall be accurate to within 0.1 m in the ball's position and 0.01 m/s in the ball's velocity.
3. The system shall be man portable and quick and easy to set up.
4. The system shall be automated in its operation.
5. The system shall display the ball's trajectory on a graphical interface.
6. Each shot trajectory shall be able to be analysed.

Target Characteristics:

The characteristics of golf balls are regulated by the governing bodies of golf, and are set out in the official rules of golf. These rules stipulate that golf balls must have the following characteristics:

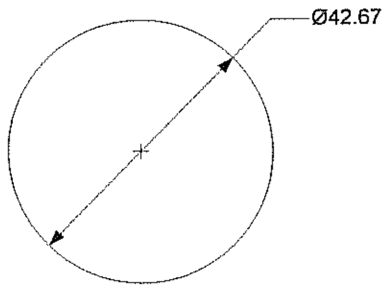


Figure 1. Golf ball dimensions (in mm)

- The ball must be manufactured so that it is spherically symmetric.
- The minimum diameter of the ball may not be less than 42.67 millimetres.
- The maximum weight of the ball may not be more than 45.93 grams.
- The maximum range and velocity of the ball when launched from a test apparatus is restricted.

These rules define the base characteristics of golf balls, but do allow some room variance. The most common variance from a perfect sphere is that many golf balls feature between 240 to 450 dimples on their surface which serve to reduce the aerodynamic drag that the ball feels.

In order to track a golf ball through the air knowledge of both typical and maximal distance trajectories is required. No standard exists for the maximum length of a golf course, but the longest drive on a standard golf course is recorded at 471 metres (575 yards). Longer travelling shots have been recorded but these have been recorded on airfield tarmacs and were able to roll significantly further than they would be able to on a grass surface. The longest golf courses have a six shot par, and are more than 630 metres long[1]. It is known that Tiger Woods typically hits the ball such that it has an initial velocity of approximately 180 miles per hour, or 80.5 metres per second[2]. Using this information, we can assume approximately 100 metres per second as the maximum ball speed.

[1] http://golf.about.com/od/handicaps/f/faq_parlengths.htm
[2] http://sportsillustrated.cnn.com/augusta/cool_stuff/physics/tiger.html

If we assume a maximum launch angle of 45 degrees, this puts the maximum height of the ball following a simple parabolic trajectory with 100m/s initial velocity at:

$$h = \frac{v^2 \sin^2 \theta}{2g}$$
$$h = 255\ metres$$

For this assignment, a maximum height of 300 metres will be used to give a reasonable factor of safety.

For the purpose of this assignment, it will be assumed that the golf ball has the following characteristics (relative to the golfer) during its trajectory:

Characteristic	Value
Maximum Range (m)	500
Minimum Range (m)	0
Maximum Height (m)	300
Maximum Velocity (m/s)	100
Minimum Velocity (m/s)	0

The extreme case is shown below (not to scale):

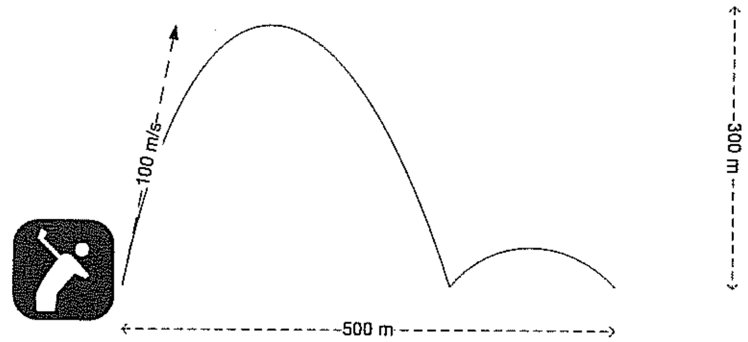


Figure 2. The maximum possible parameters the system will be designed to track.

The next section of this report will evaluate different possible methods for tracking the golf ball over this trajectory.

F = ma a = F/m

Tracking technologies

Several methods could be used to track the golf ball through its trajectory, each with their own benefits and drawbacks. These methods are outlined below. It is anticipated that the final system will be comprised of more than one of these methods.

Radar

Radio detection and ranging (RADAR) is probably the most obvious choice for tracking the range of the ball as it flies through the air. In its most basic form a radar system works by transmitting electromagnetic waves and looking for the reflection of these waves off objects.

Radar Cross Section

The strength of a radar target – how much energy is reflected back to the receiver - is dependant on the target's radar cross section and the transmitted frequency. For a golf ball we assume that the target behaves as a perfect sphere. For a fixed diameter sphere, there are three regions which define the radar cross section as a function of wavelength:

1. Where the radius (a) is much smaller than the wavelength ($\frac{2\pi a}{\lambda} < 1$) the cross section drops off with the wavelength to the power of four. This is the Rayleigh region.
2. Where the radius is comparable to the wavelength ($1 < \frac{2\pi a}{\lambda} < 10$). This is the Mie region where interference effects result in a cyclical variation of RCS as a function of wavelength.
3. Where the radius is much larger than the wave length ($\frac{2\pi a}{\lambda} > 10$). This is the optical region, and in this region the RCS is equal to the projected area of the sphere.

In order to ensure that the system makes use of the maximum RCS, we should ideally choose an operational wavelength that satisfies criterion 3. For the golf ball this means:

$$\frac{42.67 \times \pi}{\lambda} > 10$$
$$\lambda < 13.41\ mm$$

For an electromagnetic wave this corresponds to a frequency of:

$$f > 22.36\ GHz$$

good reasoning

This places it somewhere in the SHF (super high frequency) band.

Distance accuracy

Radars can gauge the distance to a target by using the principle of time of flight. To do this, a pulsed signal can be sent from the antenna and the time measured between transmission and reception. The range resolution of such a system can be estimated by:

$$\delta R = \frac{v\tau}{2}$$

In this case, the worst case range resolution should be 0.1 metres so we have that:

$$\tau < 0.7\ ns$$

We don't need that sort of resolution If we are tracking a point target in space

This requires a matched filter bandwidth of:

$$f = 1.43\ GHz$$

This would be very noisy.

Beam width

The beam width of a single antenna transmitter is inversely proportional to the size of the antenna. A trade off must therefore be made between positional accuracy and the total size of the unit. The requirements state that the system must be man portable and capable to a positional discrimination of 10 cm. In the introduction it was decided that the maximum range of the unit would be 500m, with a factor of safety built in to this value. At 500m range, 10 cm is equal to a beam width of:

$$\theta_{beam} = 0.0057\,°$$

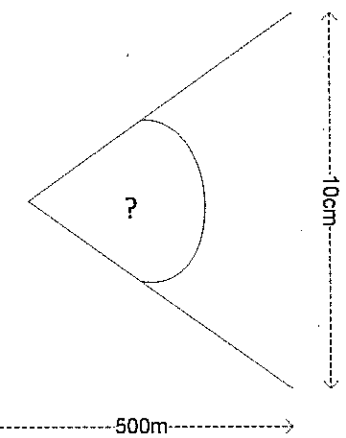


Figure 3. Calculation of required beamwidth.

Now, the beam width of the radar is a function of both the antenna size and the operating frequency. It is given in degrees by:

$$\theta_{3dB} \approx \frac{70\lambda}{d}$$

Again, you have a point target in space, use tracking methods

The wavelength of the system has been constrained by the radar cross section requirements in the previous section. The diameter of the antenna, d, is constrained by the requirement that the system be man portable. A reasonable maximum diameter to use would be 60 cm – using this and the required beamwidth we can determine the allowable operational wavelengths:

$$\lambda < 50\ \mu m$$
$$f > 6\ THz$$

This wavelength places it in far infrared part of the spectrum – far too high a frequency to be useful in a radar application. A time of flight sensor would not be practical for this reason.

Radar phased array

A phase array radar consists of multiple transmitters all separated from each other. In the most common configurations the separation distance between transmitters is constant. Arrays can exist in one, two or three different dimensions. A phased array varies the relative phase of the signal transmitted from each of the sensors to create gain in a particular direction and reduce the gain in other directions. By varying the phase the transmit beam direction can be effectively steered.

A two dimensional phased array would allow for the golf ball to be tracked in three dimensions through its trajectory. The advantage of a phased array is that no mechanical steering of the apparatus is necessary and hence no moving parts are required.

Unfortunately, a phased array would suffer from the same beamwidth problems as mentioned previously.

Doppler

It should be possible to use the Doppler shift to measure the velocity of the ball. Since we know the initial position of the ball, integrating this could give us the position of the ball in space. The kind of Doppler shifts expected can be calculated using:

$$f_d = \frac{v_s}{\lambda}$$

If we assume the radar will operate at a wavelength of 1mm to ensure that we are well within the Rayleigh region as specified above, then this gives the following Doppler shifts:

$$0.01 < v_s < 100 \text{ (m/s)}$$

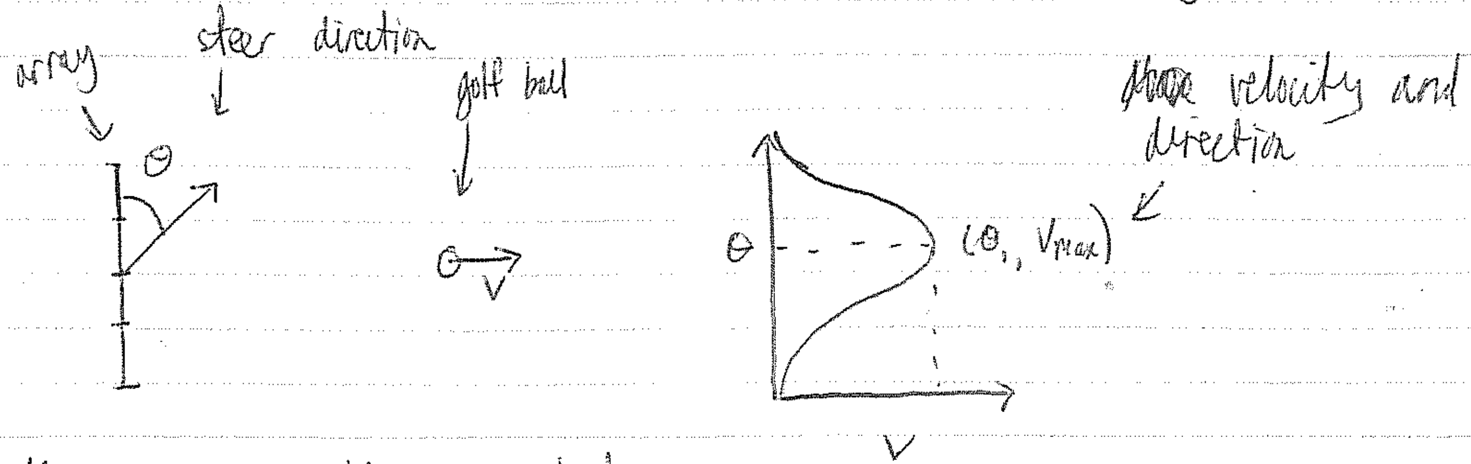
$$10 < f_d < 100{,}000 \text{ (Hz)}$$

By mixing the transmitted signal with the receive signal, and then using a lowpass filter to remove components above 100 kHz, we obtain a frequency measurement that is proportional to the velocity of the ball normal to the sensor. In order to measure the velocity to within 0.01 m/s we must be able to measure the frequency of the mixed signal to within 10 Hz. This is possible if we make frequency measurements on 0.1 seconds of data.

This technique could be practically implemented to determine both the trajectory and velocity of the ball.

Phased array Doppler

A phased array that functions using the doppler principle could be used to track the balls position in 3 dimensions. A beam could be steered using a 2D phased array. The direction the ball is travelling in could be inferenced by finding the steer direction that gives the highest velocity. good idea



More on this later

Active Acoustic

Acoustic sensors rely on transmitting and receiving sound waves from the target. They primarily work using the time of flight principle for range and the Doppler principle for velocity. Most of these sensors use sound above the audible frequency range – typically 40 KHz or higher. For the golf ball tracker this would be a requirement as we wouldn't want the crowd to hear the sensor being used.

The specifications of a typical ultrasonic sensor are shown below:

Model	Sick UM30-215118
Max Range	8m
Frequency	80 KHz
Resolution	0.18 mm

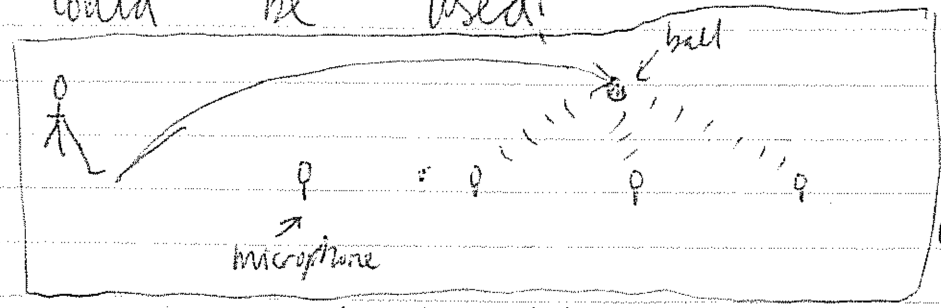
For this sensor, the range is a limiting factor when compared against the requirements of the golf ball tracker.

Acoustic sensors would be impractical for use in this situation for the following reasons:

- The maximum range of an ultrasonic sensor is severely limited by the attenuation of sound through the air. For example, in a best case scenario an ultrasonic sensor operating at 40 KHz has an attenuation of about 1dB/m. Over a range of 500 metres to a target, this translates to 1000dB of round trip attenuation, not including the effect of spherical spreading or target characteristics.
- The angular spreading resolution of acoustic sensors is typically poor (greater than 20 degrees). This could be improved through the use of an ultrasonic phased array, but the range limitation is still present.

Passive Acoustic

Passive acoustic sensors rely on the target emitting enough noise to be detected by a receiver (microphone). If the ball did emit sound as it travels through the air a system such as the following could be used:



By measuring the difference in arrival time to each sensor of the sound, the position in 1 dimension would be determined. To track in 3D, a ~~[illegible]~~ 3D array would be required. Such a system would require deployment over the entire golf field, making it impractical to set up. interesting idea, though

Laser

Laser based sensors operate using one of two main principles – either time of flight or triangulation. Time of flight laser ranging devices measure the time it takes for a transmitted light pulse to reflect back to a receiver. Triangulation sensors monitor the position of a laser dot from an axis at a non-parallel angle to the laser and in doing so can use trigonometric principles to determine the range to the dot.

Time of flight sensors

Time of flight laser distance sensors are available from a number of manufacturers. They tend to have an analogue or digital output. The characteristics of an example TOF sensor are shown below:

Sensor	Sick DMT10-2-2211
Max range	155m
Accuracy	+- 10 mm
Beam size	20mm + (5mm * distance in metres)

As can be seen from this, the maximum range is relatively low – only 155m. Furthermore, the system loses angular resolution quite severely as the target moves away from the sensor. At the maximum range, the beam size is approximately 0.8 metres in diameter.

This sensor is typical of a laser based sensing system – longer ranges than this require that a reflector be attached to the target which the rules of golf will not allow for. The system does not meet our target range or the target angular resolution as calculated in the radar section.

Triangulation sensors

The principle of operation of a triangulation sensor is shown below:

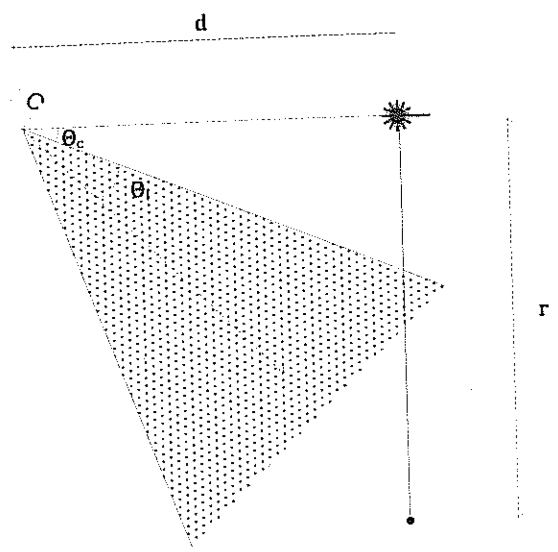


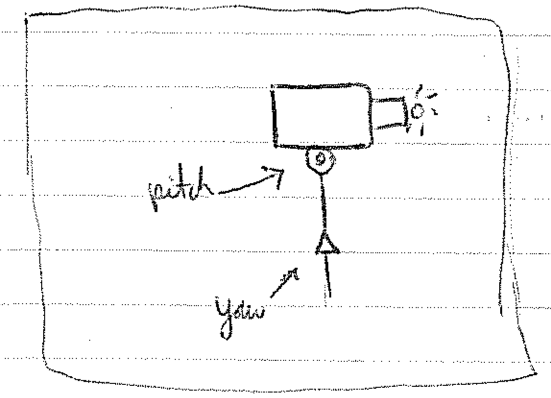
Figure 4. Triangulation sensor principle of operation

A CCD camera monitors the position of a laser dot in its field of view, shown as the shaded area. The range can be determined by monitoring the position of this dot using the camera. Such a sensor would be impractical for tracking a golf ball for the following reasons:

- Extremely sensitive to ambient light conditions.
- Poor range discrimination over the range requirement – a typical CCD resolution of 640 pixels would give this many bins over a range of 500m, giving just under one meter of range resolution. This is further degraded due to the spread of the beam with distance.

<u>Mechanically Steered Time of Flight</u>

For a laser based system to work it would require mechanical steering as shown below:



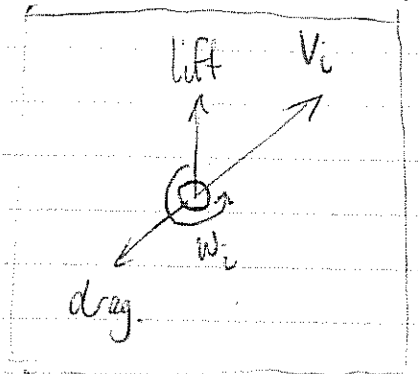
One servo would pitch the laser and the other would control it in yaw.

Mechanically steering a sensor in this way will ~~require be heavy~~ result in the system being heavy and cumbersome. Moving mechanical parts also require maintenance and are more susceptable to breakage than non-moving systems.

Track Based On Known Initial Conditions?

If the initial velocity and spin of the ball can be measured it may be possible to calculate the balls trajectory.

Given the initial position of the ball it should be possible to integrate the velocity over the entire time of flight to determine the trajectory.



How does the velocity of the ball V, evolve as a function of time?

The ball will experience several forces as it moves through the air:

* Lift (a spinning object generates lift)
* drag (oppose the direction of motion)
* wind (would vary unpredictably).

The lift on a spinning golf ball is due to the ball turning a flow of air as it spins[1] The lift of ~~such a cylinder~~ a sphere is given by:

$$L = 4/3\,(4\pi^2 r^3 w \rho v)$$

where r is the radius, w is the rotation, ρ is the density of the air and v is the linear velocity of the ball.

1. http://www.grc.nasa.gov/WWW/K-12/airplane/beach.html

The drag on a golf ball is reduced by the dimpling of it's surface. For a perfect sphere, the drag is given by:

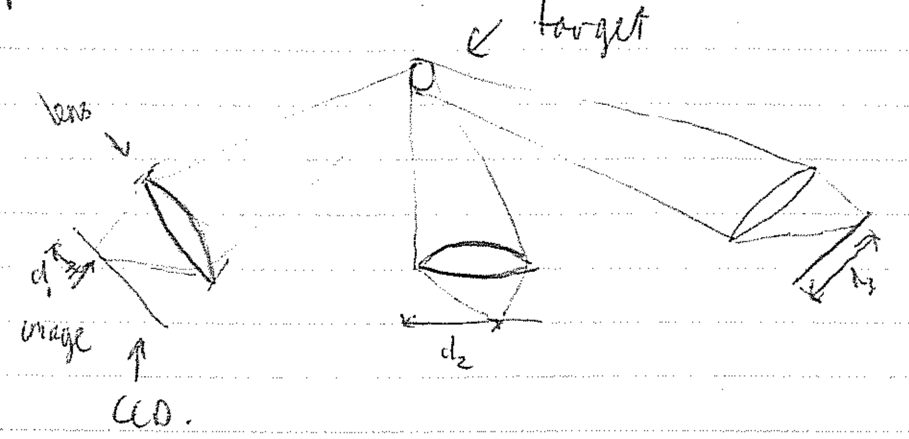
$$F_d = 1/2\,\rho v^2 \times 0.1 \times \pi r^2$$

The wind will produce another force on the ball. This force will be highly variable as a function of time, especially in the case where the wind is gusting.

As stated previously, in order to determine the trajectory we must be able to describe the balls velocity as a function of time. Whilst we can calculate the lift and drag felt by the ball, we cannot reasonably determine the wind ~~velocity~~ at every point on its trajectory. For this reason it is impractical to use a sensor that only measures the initial velocity and rotation rate of the ball.

Camera Triangulation

Multiple cameras would be used to track the position of the golf ball through space. A 2 dimensional simplification of this technique is shown below:



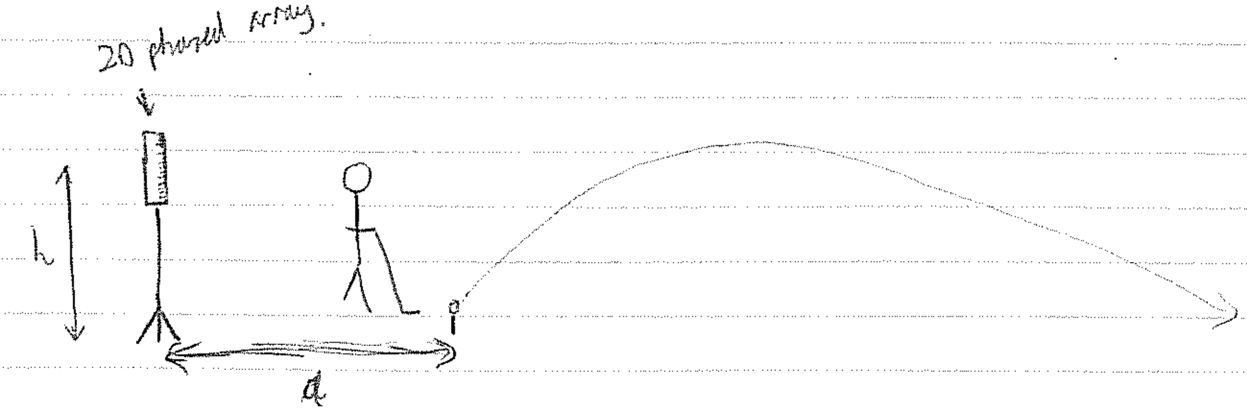
By comparing the location of the target in each image and with the positions of the cameras relative to each other known, the ball can be tracked in 2 Dimensions with 3 cameras. More cameras would be required to locate the ball in 3Dimensions.

Such a system would be impractical because:

* The ball must be located in the image of each camera and its position determined.
* The processing required to do this is quite intensive and may require multiple computers
* The accuracy of the measurements depends on the spacing of the cameras and will drop quickly with range.

Chosen Method

The method that will be used to track the ball in this project is a phased array doppler radar. The system would be placed behind the golfer as they tee off in the manner shown below:



The radar would be elevated using a telescopic stand to a height, h, that provides it with a view of the field. It would be placed directly behind the golfer, and be pointed down the line that the golfer is aiming to hit the ball.

The reasons that this system has been chosen are as follows:

* Both time of flight and doppler methods can be used to determine the range and velocity of the ball.
* Doppler measurements of the balls velocity should be resistant to surrounding clutter whilst the ball is in the air. On the ground this should also be the case in short grass only.

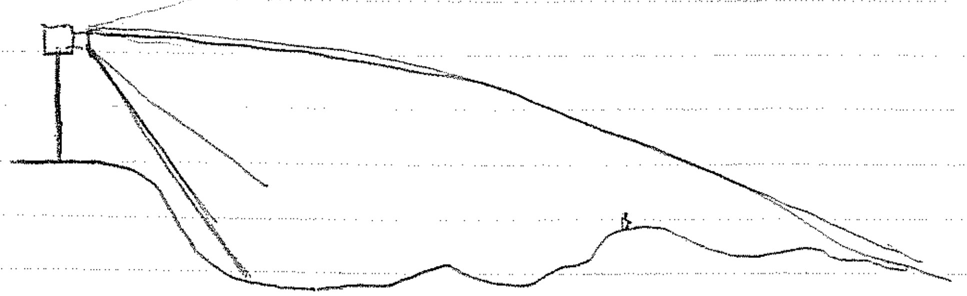
* The system will require no moving mechanised parts and therefore should be able to be made compact and portable. It will also be easier to set up and maintain.
* The accuracy of the system is the best out of the systems considered in this report.

Still outstanding in the implementation of a doppler radar system are the following unknowns that will need to be solved in the second phase:

* How can the ~~range~~ angular resolution of the system be improved?
* How high would the system need to be placed above the golf course to track the ball as it rolls?

If the ball cannot be accurately tracked using this method whilst it is on the ground the following could be added to the system.

A camera could be mounted to the top of the doppler radar stand such that it looks down on the field as shown:



If a plan of the golf course is known (ie a height map) then the position of the ball as it rolls along the ground can be determined.

Some interesting ideas, pity you have not read the section on tracking in the book, it would have made solving your pulsewidth and beamwidth problems a lot easier.

8

Phase 2: Preliminary Design

~~Traj~~ Trajectory prediction

Given the initial conditions of the ball, what model will be able to predict the trajectory?

We need to know the balls initial velocity and the other forces on the ball.

Initial Velocity: We require a 3 vector to represent the balls initial velocity

$$\hat{V} = [v_x \ v_y \ v_z]$$

Where we take the z axis as the vertical, and the x axis joining the tee to the hole. The y axis is then perpendicular to the plane formed by the x and z axes.

Drag: The drag on a sphere is a force that is proportional to the velocity but acting in the opposite direction[1]

$$\underline{D} = C_d \rho V^2 A/2$$

Where ~~...~~ $0.07 < C_d < 0.5$ for a sphere, ρ is the density, v is the velocity, A is the cross sectional area and D is the drag force.

For a golf ball, one source gives C_d as $C_d \approx 0.4$[2]

Lift: A spinning ball generates lift, given by the following equation:

$$\underline{L} = 4/3 \left(4\pi^2 b^3 s \rho V\right)$$

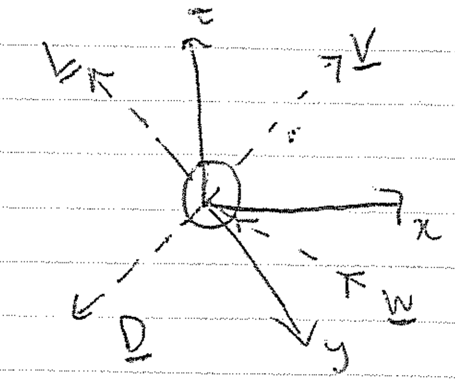
Where s is the rotation rate in ~~rad~~ revs/sec, b is the balls radius, and ρ is the density of the air and V is the velocity. The lift force always acts perpendicular to the velocity.

Wind: Wind can be modelled as another form of drag, where the ball is forced in the direction of the wind velocity, ie.

$$\underline{W} = C_d \rho V_w^2 A/2 \ \hat{V}_w$$

where V_w is the wind velocity, and $\hat{V}_w$ is the unit vector in the direction of the wind.

Given the various forces on the ball, how can we model it's trajectory?



1. http://www.grc.nasa.gov/WWW/K-12/airplane/shaped.html.

2. http://www.acoustoscan.com.au/ISSS/ Free Flight Aerodynamics of Sports Balls II.pdf.

If we denote the initial ball position

$$P = [x, y, z] = [0, 0, 0].$$

Then $P(T+dt) = P(T) + V \times dt$

But as time progresses the velocity changes too, so

$$V(T+dt) = V(T) + \left(\frac{D}{m} + \frac{L}{m} + \overset{+g}{\frac{W}{m}}\right) \times dt$$

combining these expressions:

$$P(T+dt) = P(T) + V(T) \times dt + \left(\frac{D}{m} + \frac{L}{m} + \overset{+g}{\frac{W}{m}}\right) \times dt^2$$

We can then choose a sufficiently small dt such that we can model the trajectory accurately.

~~To do this, the matlab function shown was used.~~

When plotting the trajectory using the lift equation shown before, a large discontinuity was seen in the trajectory. This was because the lift force from the equation was too large. Instead, the following model will be used:

✓

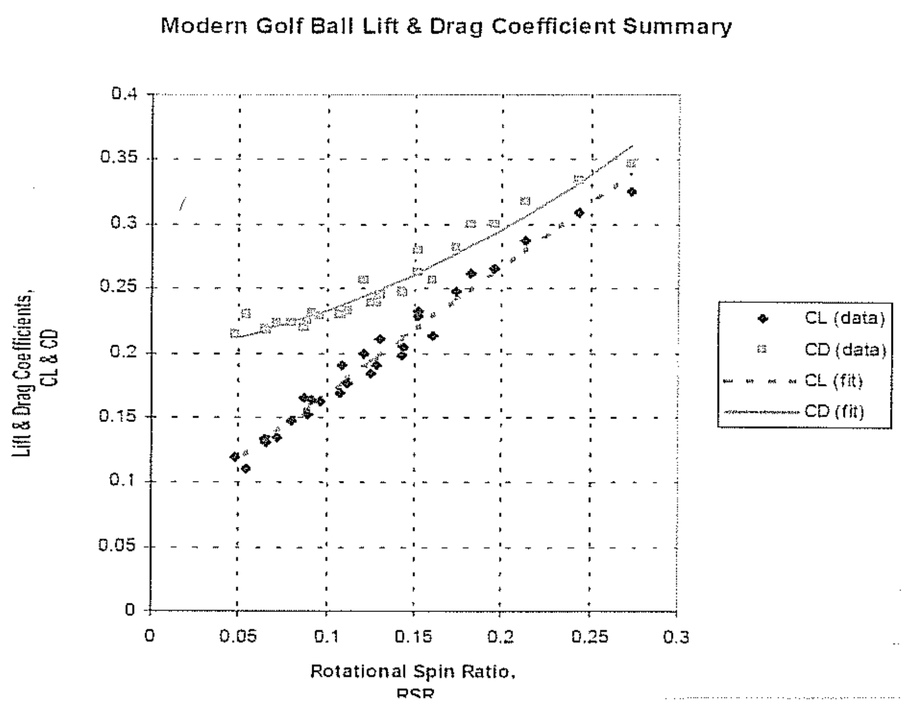
$$L = \tfrac{1}{2} \rho V^2 A C_L$$

Where C_L is the lift ~~ratio~~ coefficient. This coefficient ~~ratio~~ is a function of both the rotation rate and linear velocity of the ball.

We define the RSR as:

$$RSR = \frac{2\pi \omega r}{V}$$

Then the following graph gives a C_L for a specific RSR: (3)



good to see that you have a pair of finger nail scissors

There is thus a linear relationship between C_{L} ~~spin~~ and RSR:

$$C_L = 1.027[RSR] + 0.063.$$

✓

This source also provides an average C_D of ~ 0.3, which will be used from now on.

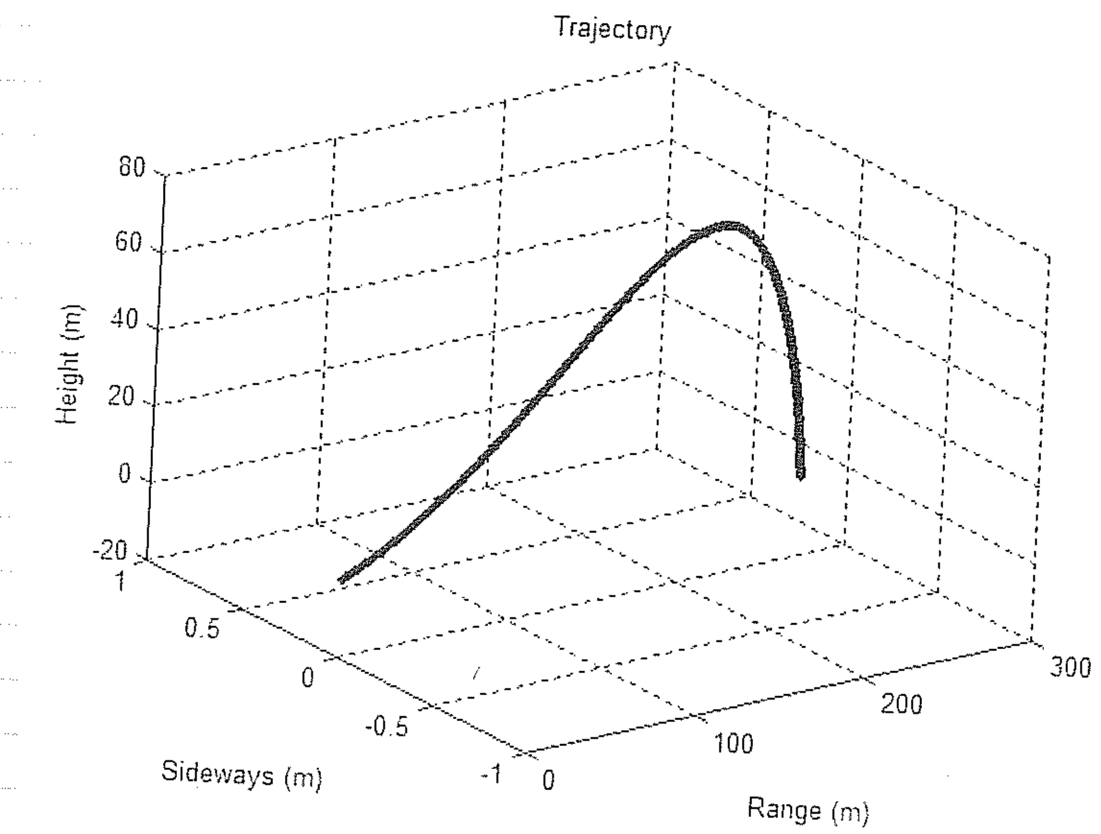
3. http://sections.asme.org/canaveral/special/golf/asmegolf.pdf.

So we have a second order ODE, which is solved using ~~forward~~ finite differencing in the function below:

```
function [ traj, t] = golftraj( v, spin, wind, dt )
%GOLFTRAJ Calculate the trajectory of a golf ball given it's initial
%conditions.

%% Initialisation
dia = 0.04267; %ball diameter
mass = 45.93/1e3; %ball mass kg
A = pi*(dia/2)^2; %cross sectional area
C_d = 0.2; %coefficient of drag
rho = 1.2041; % air density kg/m^3 at 20 degs
g = [0 0 -9.81]; % gravitational
P(1, :) = [0, 0, 0]; %initial position
t = 0; %initial time
i = 2; %iterator
P = zeros(100000, 3); %preallocate for speed.

%% Forward differencing.
while 1
    % NORMAL VECTORS
    v_hat = v/norm(v); %unit vector for velocity
    v_xy = [v(1) v(2) 0]; %velocity in xy plane
    wind_hat = wind/norm(wind); %unit vector in wind direction
    v_norm_xy = cross(v_xy, v)/ norm(cross(v_xy, v)); %unit vector normal to velocity of ball in xy plane (ie,
spin axis)
    v_norm = cross(v_norm_xy, v)/norm(cross(v_norm_xy, v)); %unit vector normal to ball velocity and up.

    % FORCES
    D = -C_d*rho*dot(v,v)*(A/2)*v_hat;
    %D = 0;
    if spin > 0
        RSR = 2*pi*spin*(dia/2)/norm(v);
            if RSR > 0.3
                fprintf('   ',i, RSR);
            end
        CL = 1.027*RSR + 0.063;
        L = rho*dot(v,v)*A*CL*v_norm;
        % L = (1/3)*(4*pi^2*(dia/2)^3)*spin*rho*sqrt(dot(v,v))*v_norm;
        % lift method.
    else
        L = 0;
    end
    if norm(wind) > 0
        W = C_d*rho*dot(wind,wind)*(A/2)*wind_hat;
    else
        W = 0;
    end

    % VELOCITY CALCULATION

    % TRAJECTORY CALCULATION
    P(i,:) = P((i-1),:) + v*dt;
    v = v + (D/mass + L/mass + W/mass + g)*dt;
    t(i) = t(i-1) + dt;
    %if the ball hits the ground, stop!
    if P(i,3) < 0
        break
    end
    i = i+1;
end
    traj = P;
```

~~Shown examples~~ An example of the output of this function is shown on the next page.

Nice



It is apparent that it will not be sufficient to track the ball based on its initial conditions alone because:

* The initial conditions can never be precisely known
* The wind will gust in real life, making it's effect unpredictable
* In any numerical integration method there will be some error present

Thus we need to use an active tracking method.

yes!

Analysis of the chosen method

As stated in the first phase, a phased array radar will be used in this system. It is anticipated that this system will be pulsed. It is not known as yet whether doppler measurements will be needed to obtain the required ~~a~~ velocity accuracy.

We need to look at:

- required array elements
- operating frequency
- tracking
- signal processing
- transmit power
- antenna size
- mounting etc.
- electronics
- User interface.

✓

Operating Frequency

As ~~data~~ calculated in phase 1, to operate in the optical region we require that the frequency be:

$$f_{op} > 22.36 \text{ GHz}$$

Such a frequency gives the maximum effective RCS. Using a frequency much higher than this will not provide much more benefit.

✓

Thus we choose an ~~oppe~~ operating frequency of 23GHz, ensuring good returns from the target.

Tracking

A phased array is capable of steering multiple beams using the relative phase shift between multiple transmitters.

We can use a phased array to track the golf ball in both range and in bearing.

Phased Array Considerations

A phased array provides a convenient alternative to a mechanically steered array for this system. The reasons for using a phased array are:

* No need for moving parts
* This will eliminate mechanical wear on the system
* It means we dont need a high powered servo to steer the system, reducing power usage.

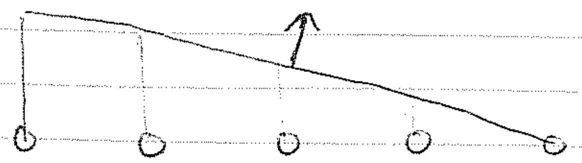
When designing a radar phased array we ~~a~~ need to take into account the ✓

following parameters:

* Array dimensionality
* Number of elements
* Size of the elements
* Interelement spacing.

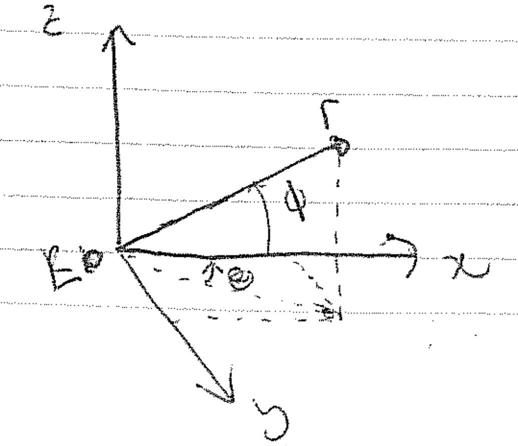
The operational principle of a phased array is as follows:

A set of transmitters are spaced apart from each other as shown

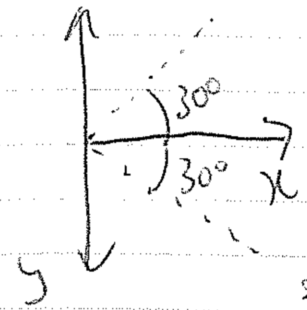


$\phi = 4\psi$ $\phi = 3\psi$ $\phi = 2\psi$ $\phi = \psi$ $\phi = 0$

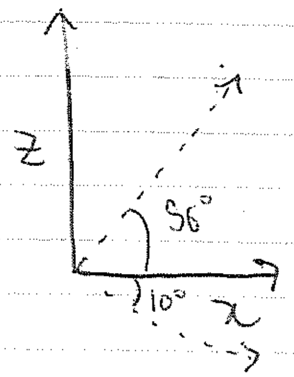
The transmitters operate at the same frequency, f, with the phase of the transmit signal increased in increments along the array. This phase is adjusted ~~so that~~ such that the ~~beam~~ signals transmitted combine constructively in a particular direction - the steer direction. The example shown is the 1D linear array case. For our tracker, we need to be able to track the golf ball in 3 dimensions, which we can denote r, θ, ϕ as shown below



There are however some constraints to this:



* We can assume that the golfer hits the ball in the "right" direction - ie down the +ve x axis thus we only have to consider a 180° range of θ
* 180° is probably excessive - it should be enough to look at a region approximately around $-30° < \theta < 30°$.
* We previously made an assumption that the ball will only travel to a height of around 300m. The maximum height is typically acheived 2/3rds of the way along the trajectory, say 200m. So the maximum positive angle we need to look up to is approximately



$$\phi < 56°$$

and we will assume that we need not look down more than 10°, so that

$$-10° < \phi < 56°$$

Since again, the negative x direction can be ignored.

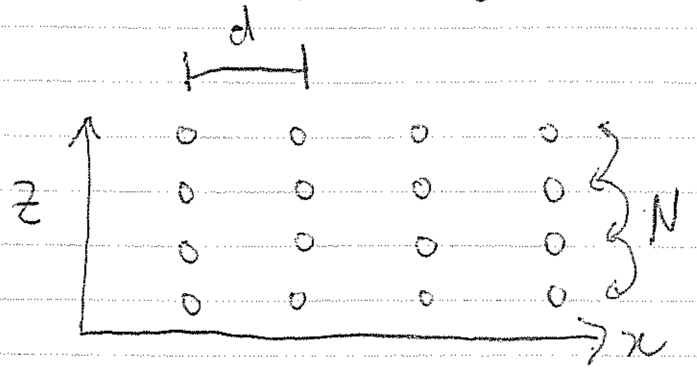
We now need to do 3 things:

* Generate a beam to point at the golf ball.
* Avoid the appearance of grating lobes
* Make the beam as narrow as possible

We can affect these ~~beam width~~ parameters by adjusting the following parameters:

* operating frequency, f
* number of array elements
* Array dimensionality
* element spacing

In this case we should use a 2 dimensional ~~array~~ array, as shown below:



Using such an array we can steer a beam into one half of the area either side of a plane in the zy axis.

The inter element spacing, d, and the frequency will affect when grating lobes appear. These lobes are large gains in the beam pattern that exist in directions other than the desired steer direction.

They occur because the array spacing can be such that certain steer directions can "require" a phase shift of more than $\lambda/2$ between elements. We can prevent grating lobes by spacing the elements according to the field of view required.

$$d < \frac{\lambda}{1+\sin\theta} \quad (1)$$

where θ is half the required field of view. Thus with our operating frequency of $f = 23$ GHz

$$\Rightarrow \lambda = 0.013 \text{ m}$$
$$= 1.3 \text{ cm.}$$

Interelement spacing must then be:

$$d < \frac{1.3}{1+\sin(33°)}$$

$$d < 0.8416 \text{ cm.}$$

in ~~the~~ the vertical. To keep things simple, we will also use this value for the horizontal spacing. This interelement spacing also specifies the maximum size of each element.

Making the operating frequency higher will decrease d, so it is probably in our best interests to leave f as 23 GHz here.

Now we need to decide on the number of elements to use in this array. The number of elements will affect the gain and beamwidth of the transmitted pulses. At broadside (perpendicular to the array) the beamwidth is narrowest. As the steer direction moves away from broadside, the effective aperture size decreases and so the beamwidth increases.

(1) http://www.microwaves101.com/encyclopedia/phasedarrays.cfm.

The half power points of the beam can be expressed as:

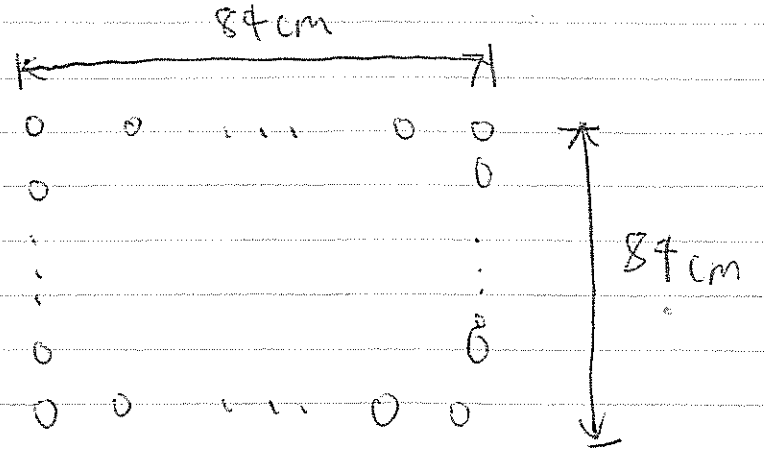
$$\theta_{3dB} = 0.886\lambda / N d \cos\theta_0$$

We want the beam to be symmetric, so N should be the same in both ~~the~~ z and y directions. The graphs on the opposite page show the beamwidth vs. number of array elements. As can be seen, the beam width varies from ~~0.78~~ 0.79 to 0.94 degrees as the steer angle is moved from broadside ($\theta = 0$) to the max steer angle of θ 33°, for 100 elements.

~~We now have the array parameterised.~~

100 elements sets the total array size to ~ 84 cm, in both dimensions. It does not appear to be very beneficial in terms of narrower beamwidth to use more elements than this.

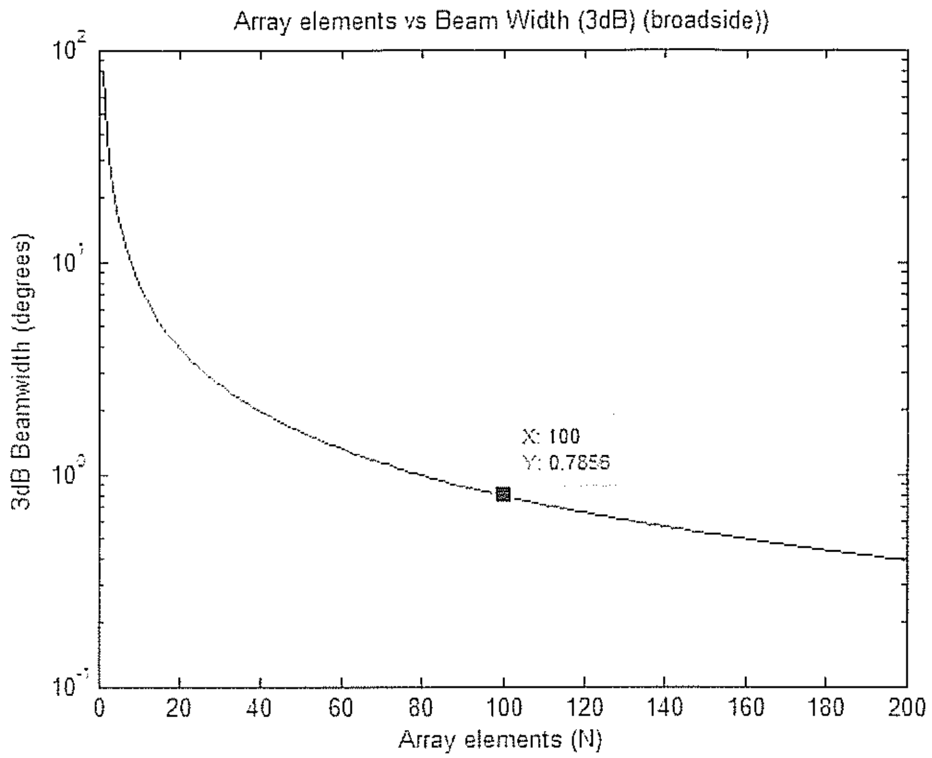
We now have our phased array fully parameterised:

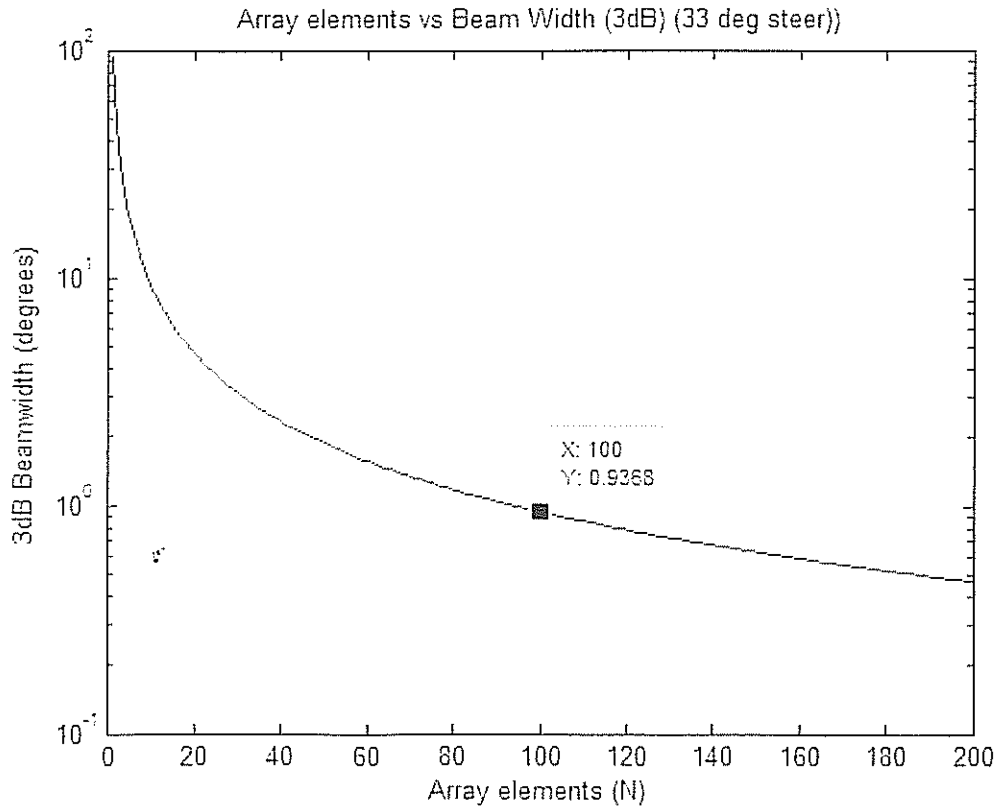


Total elements = 100 × 100 = 10,000 Wow

Element size: 0.84 cm.
Beamwidth (max): 0.94° (3dB @ 33°)
Beam width (min): 0.79° (3dB @ 0°).

~~Target~~



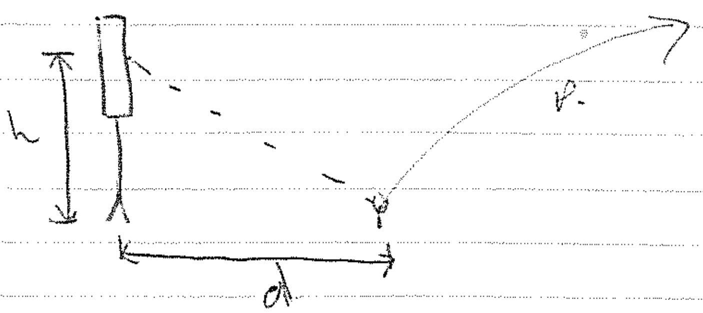


Target Tracking And Range Resolution.

In order to track the golf ball we need to measure the three parameters described previously: θ, ϕ and r. We may also need to ~~track~~ measure the ball velocity directly, v.

~~Tracking in range.~~

To track the ball in range we need to have some sort of knowledge about it's initial position and possibly velocity. We can constrain the balls initial position by ensuring that the radar is always mounted a fixed distance and height behind the tee.



For the moment, let us ignore the fact that there will be 2 ~~m~~ moving targets - the club and the golf ball and idealise the problem by assuming that the golf ball ~~starts~~ just starts moving of its own accord from the tee.

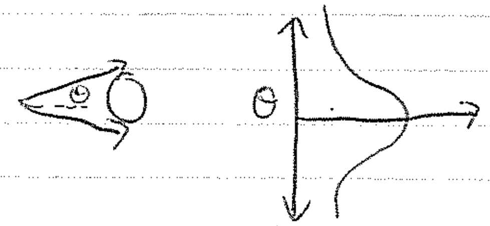
✓

~~Range gating~~

→ Tracking in angle.

→ Scanning across the target

Since we have a spherical target, we can assume that its return levels as a function of beam position is symetrical, ie:



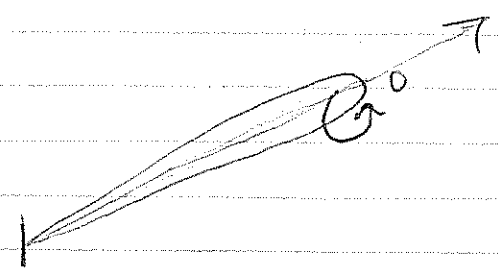
Thus, if we scan the beam about a region such that $p(\theta_1) = p(\theta_2)$, then we can assume that the target really lies at $\theta = (\theta_1 + \theta_2)/2$.

A method such as this is required because with a beamwidth of 0.94 degrees (as calculated before), the beam diameter at 300m is 2.5m, much larger than our required accuracy of 0.1m in position.

→ Conical Scan (conscan).

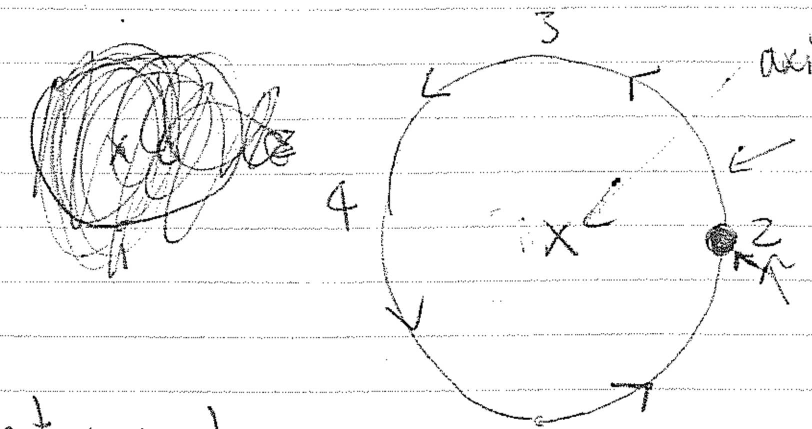
The conical scan tracking method is one such way of tracking across the target

dr



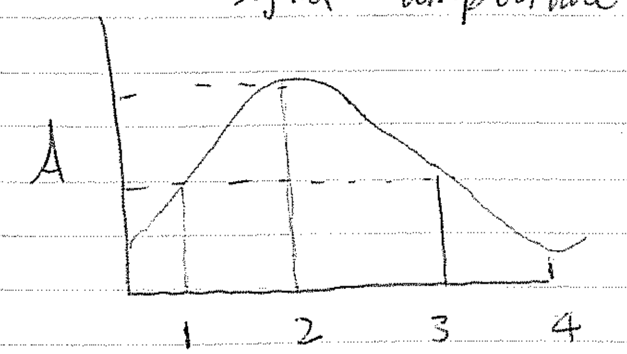
The beam is rotated conically about the estimate of where the target is. If the target lies directly in the middle of the line, then all the returns about a single revolution will give the same amplitude. If however, the target does not sit on the axis of rotation then the signal will be amplitude modulated

ie:

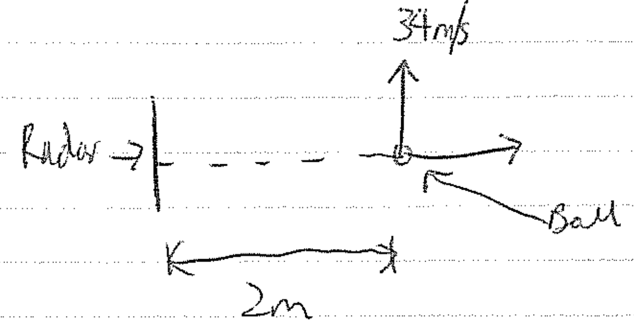


This is not a great method if you have a phased array - monopulse is superior as it can be formed by the array

Received signal amplitude vs Beam position



For our system, the maximum bearing rate, defined as $d\phi/dt$, will be on when the ball is closest to us. Given a max ball velocity of 100 m/s and ~~angle of~~ loft angle of $\theta = 20°$ this gives a linear velocity of 34 m/s ~~at~~ vertically. If we assume that the radar is to be 2 metres behind the golfer at tee off, we have the following situation:

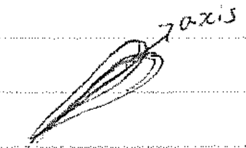


Which gives an angular velocity

$$d\phi/dt = 34/2 = 17 \text{ rad/sec.}$$

In order to use conscan under these circumstances we must:

* Have 5-25 pulses per revolution.
* The target cannot move "too far" in the total scan time. Define "too far" as half a beamwidth in angle.
* The beams must overlap a little, ie



Half a beamwidth in angle is ~0.4° or 0.007rad. This will occur ~~at~~ in $(0.007/17)$ sec or 0.41 ms. In this amount of time we need between 5 and 25 pulses, giving our pulse repetition frequency:

$$17\,\mu s \leq f_p \leq 82\,\mu s.$$

So with a PRF of 17 μs, we can scan the

ball 25 times in a rotation without it having moved "too far", at it's maximum bearing rate

~~Tracking~~ range

Using linscan we can determine the parameters θ and ϕ, but how do we determine range? To do this, we must use a technique that maximises our range resolution.

The range resolution is defined as the minimum resolvable range difference between 2 targets. To meet our system specification, we need it to be of the order of 0.1m or less. only if you have more than one target in space

For a traditional pulsed radar, the range resolution is given by:

$$\partial R = c/2\Delta f$$

Where Δf is the effective bandwidth of the signal. If we are using a perfect correlator, then the 3dB points are at $\Delta f \doteq 1/\tau$ where τ is the pulse width.

Making the substitutions with our 0.1m range resolution:

$$0.1 = c/2\Delta f = \frac{c\tau}{2}$$

$$0.2 = c\tau$$

$$\tau = 0.667 ns.$$ Too small

Which is very short, and as seen in phase 1, requires a very 'high' matched filter bandwidth, which will be prone to noise.

Other techniques can be used to gain range resolution. These techniques work on the following principle: We increase the bandwidth of the transmitted pulse, and use a matched filter to compress it to a duration that is the inverse of the bandwidth. This allows us to have a longer pulse duration and/or better range resolution.

Linear Chirp

Using linear chirp is perhaps the easiest way to improve the range resolution. To do this, the frequency is ~~[illegible]~~ increased linearly over the pulse duration. ~~[illegible]~~

The range resolution in this case is given by:

$$\partial R = c/2\Delta f$$

We need $0.1 = c/2\Delta f$

$$\Delta f = c/0.2$$

$$\Delta f = 1.5 \times 10^9 Hz$$
$$= 1.5 GHz.$$

So we could for example chirp from our 23GHz up to 24.5GHz.

Is this practical?

Given our pulse repetition frequency:

$$17\mu s \le PRF \le 82\mu s$$

As constrained by the conscan requirements, what are the equivalent maximum ranges?

$$Max(R) = \frac{c T_{PRT}}{2}$$

~~5100 ≤ R ≤~~

~~5100 ≤ Max(R) ≤~~

$$2550 \le Max(R) \le 12300 \text{ metres.}$$

So our maximum range, as constrained by the necessary PRF, is well above our 500m requirement.

How long is the pulse length?

We know it must be smaller than the PRT.

$$\tau < PRT$$

We also know that it must include "many" cycles of the transmitted pulse frequency. So that $\tau \times f$ is very large. If we set τ to 1/10 th of the PRT, then

so $$1.7\mu s \le \tau \le 8.2\mu s$$

Which equates to a number of cycles per pulse at 23GHz of:

$$3.91 \times 10^4 \le N \le 1.89 \times 10^5$$

Which seems sufficient.

If we choose a round values of

PRT = 50 μs
τ = 5 μs

we get a chirp rate of:

$$\frac{\partial f}{\partial t} = \frac{1.5GHz}{5\mu s} = 3.0 \times 10^{14} \text{ Hz/s.}$$ can be done

Which will need to be validated to see if such a rate is possible.

Tracking

We have seen how we can use methods to obtain range and bearing measurements that satisfy the system specifications. But how do we track the ball?

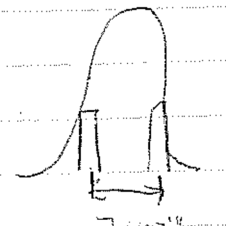
As mentioned before, conscan will be used to track in bearing (θ, ϕ) but what about range?

We should be able to use a split gate tracker for this purpose. Such a tracker requires us to sample and hold the output of our matched filter.

ok

Since we are using a chirped pulse, the output of the matched filter will have a 3dB duration of:

$$T = 1/\Delta f = 1/1.5\,\mathrm{GHz} = 7\times10^{-10}\,s.$$



So this wont work, since very accurate timing circuitry would be needed thus we will have to rely ~~on~~ on range gating of a 5μs pulse.

The equations that give these errors for conscan and range gating are:

$$\sigma_r = \frac{\tau}{2.5\sqrt{SNR\times2}} \quad \text{(range gating)}$$

$$\sigma_\theta = \frac{\theta_{3dB}}{k\sqrt{2\times SNR}} \quad \left(\begin{array}{c}\text{conscan}\\ k=1.4\end{array}\right)^{(1)}$$

So ~~we~~ we need to know the worst case SNR! ~~To do this we will use the radar range equation and in the process determine our required transmit power.~~

pp 559 textbook.

~~Determine SNR~~ worst

Using the previous two equations, we can determine the required SNR!

For the linear range case, we know that

$$\Delta r = c\sigma_r \le 0.1$$

Lets try to do a bit better, so

$$c\sigma_r = 0.05$$

$$\frac{c\tau}{2.5\sqrt{SNR\times2}} = 0.05$$

shorter pulse makes this smaller! How short can we go?

$$\frac{c\tau}{2.5\times0.05} = \sqrt{SNR\times2}$$

$$SNR = \frac{c^2\tau^2}{(2.5\times0.05)^2\times2}$$

$$= 78.6\,dB \leftarrow \text{wow!}$$

Interesting result

~~For angular range, we want~~

Maybe we need a shorter pulse width! and not try to be so optimistic ($c\sigma_r = 0.1$).

Note also that the pulse width will set our minimum range! So ~~maybe pulsed radar wont work; so since we are looking at such short ranges and our antenna is monostatic.~~

Say we want a minimum range of 5m, so that the radar is placed 5m behind the golfer, then the pulse width can be at a maximum

of $\tau_{max} = 10/c = 33\,ns.$

At this ~~frequency~~ width, we require an SNR of

of:

Better SNR = 29 dB.

How about the radial case?

At 300m we want less than 0.1m in equivalent error so:

$$300\sigma_t = \frac{300\,\theta_{3dB}}{k\sqrt{2SNR}} \leq 0.1 \qquad (k = 1.4 \text{ for conscan}).$$

SNR = 27dB.

So the restricting case is the linear ~~radial~~ case, so lets determine our transmit power!

Past 300m we lose accuracy in our measurement (ie $\sigma_r > 0.1$), but we can still track it.

Range Equation

$$S_{dB} = P_{dB} + 2G_{dB} + 10\log_{10}\left(\frac{\lambda^2}{(4\pi)^3}\right) + \sigma_{dB} - L_{dB} - 40R_{dB}.$$

We need to determine the required S_{dB} for a return off a target at 300m.

Parameters we have:

$R = 300m$ $\quad K_{dB} = 24dB.$

$\sigma = 0.0114\,m^2 \Rightarrow \sigma_{dB} = -19.4\,dB$

$\lambda = 1.3\,cm.$ @ 23GHz.

→ Antenna gain

$$\left[\begin{array}{l} G_o \doteq 4\pi A/\lambda^2 \\ \doteq 47\,dB \end{array}\right] \nearrow \text{Non scanned gain}$$

$$\left[\begin{array}{l} G_s \doteq (4\pi A/\lambda^2)\cos\theta \\ \doteq 46\,dB \end{array}\right] \nearrow \text{scanned gain at } 33°.$$

→ use this figure as worst case gain.

$G = 46dB$

→ System losses

Since we have not designed the system yet we have to give a rough estimate of losses. Assume 3dB of losses in the receive circuitry and a 3dB fudge factor → total of 6dB losses

$L = 6dB.$

So S_{dB} has to be:

$$S_{dB} = P_{dB} + 2\times 46 + 10\times\log_{10}\left(\frac{0.013^2}{(4\pi)^3}\right) - 19.4 - 6 - 4\times 24$$

And also it has to be

$$S_{dB} > NF + 29\,dB.$$

Where NF is the noise floor.

What's the noise floor? matched B/W $= 1/\tau = \frac{1}{33ns}$.

$$NF = 10\times\log_{10}(kTB) + 6\,dB.$$

290K room temp.

$= -129.16 + 6\,dB$

dn $= -123\,dBW$

So we need at 300m a return S_{dB} of

$$S_{dB} = -123 + 29$$
$$= -94\,dBW$$

So transmit power is:

$P_{dB} = 5.94\,dBW$

Looks good

$= 3.9\,W$

$= 4\,W$

Which seems reasonable.

Set to $P_{dB} = 5W$ to beat losses and extend range a little.

The story so far

* Phased array, $f_{op} = 23GHz$.
* 5 Watt transmit power
* 0.86×0.86 m in size (max)
* $0.79 < \Theta_{3dB} < 0.94$ for angle
* Monoscan gives 0.1m error in position at 300m
* linear range gates give 0.1m across the full range.

What else to consider?

* Signal processing
* Circuitry
* Power
* Operation → user interface → procedure
* Thinning of phased array
* Tracking in more detail.
* aesthetics.

Thinning the array

~~The phased array directivity sensitivity to various directions is~~

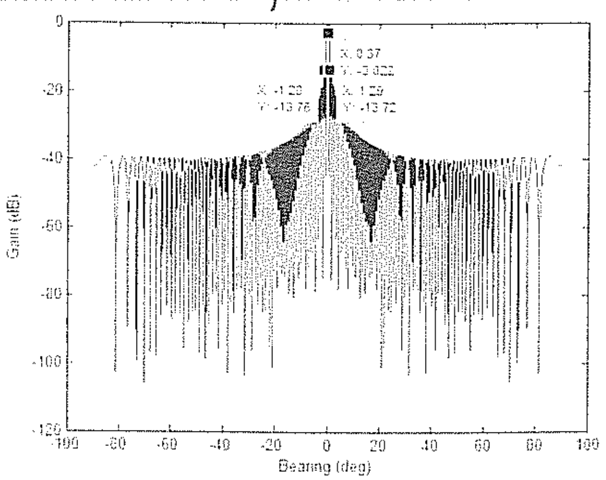
At the moment, the phased array has 10,000 elements. This may prove to be overly expensive. We can thin the array, such that array stays the same size, but the number of elements is reduced.

We can see how much we can thin the array by looking at it in one dimension. The script shown is then used to simulate sending a signal to the array from a ~~[illegible]~~ bearings between -90° to 90° whilst it is steered towards broadside.

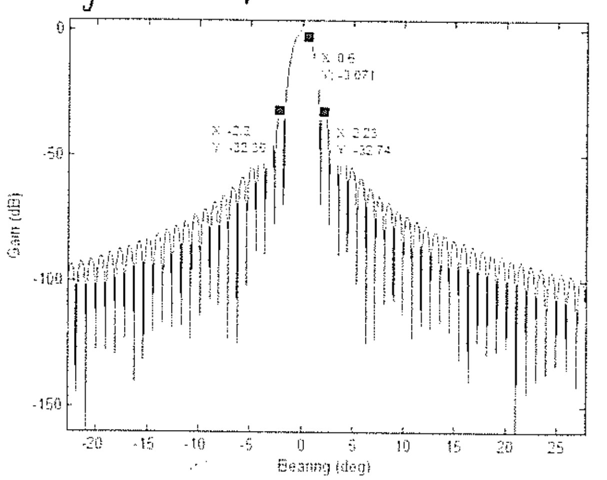
```
clear all
array = [1, 40:60,100]; %elements in the array!
spacing = 0.6474; %n*lambda spacing
p = 1;
for i = -90:0.01:90;
    %calculate interelement phase delay.
    phi = spacing*cosd(90-i);
    t = 0:0.1:10;
    sig = zeros(size(t));
    for j = array;
        k = 0.5*(1-cos(2*pi*(j-1)/(N-1))); %hamming window
      % k=1; %rectangular window
        sig = sig + k*cos(t - 2*pi*j*phi);
    end
    rms(p, 1) = i;
    rms(p, 2) = sqrt(sum(sig.^2)/max(size(t)));
    p = p + 1;
end
rms(:,2) = rms(:,2)/(max(rms(:,2)));
rms(:,2) = 20*log10(rms(:,2));
plot(rms(:,1), rms(:,2))
```

~~Specifics how many array illustrated~~
The ~~array~~ ~~[illegible]~~ is parameterised in terms of the elements present. The example shown is a array of 100 elements, with elements 2→39 and 61→99 removed. It generates the second beam pattern seen

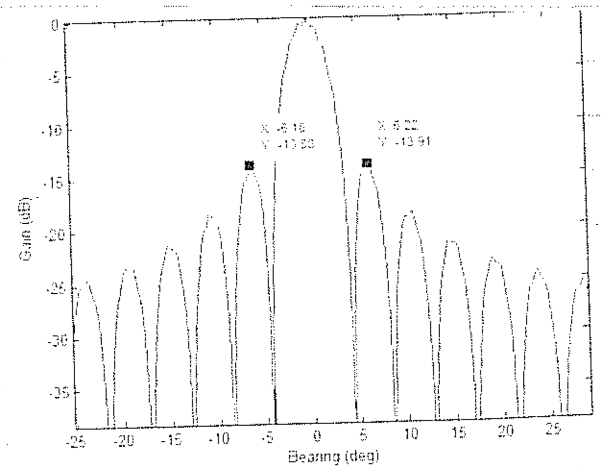
With rect window; 100 elements:



Hamming window, 100 elements



Thinned [1, 40:60, 100] Hamming ~~(42% thinned)~~ (22%) thinned



→ using a hamming window reduces sidelobe level, but increases 3dB width
→ sidelobes rise up as array is thinned
→ Windowing can suppress this effect

→ The actual level of thinning and window used will need to be determined in detailed design phase. For now, lets assume a non-thinned, rectangular window array.

good

Tracking and Signal Processing

We know most of the parameters of the system now, and have looked at how we will track the golf ball. ~~[illegible]~~ This section will look at the tracking in a bit more detail.

How will we know when the target has begun it's trajectory? The easiest way to test for this is to ~~threshold~~

* point the ~~[illegible]~~ beam at the tee to start with
* monitor the doppler shift of the returns from the target
* if the returns are consistent with a velocity greater than ~~an~~ threshold velocity then use the measured velocity as an initial estimate of range ~~[illegible]~~ and rate start tracking

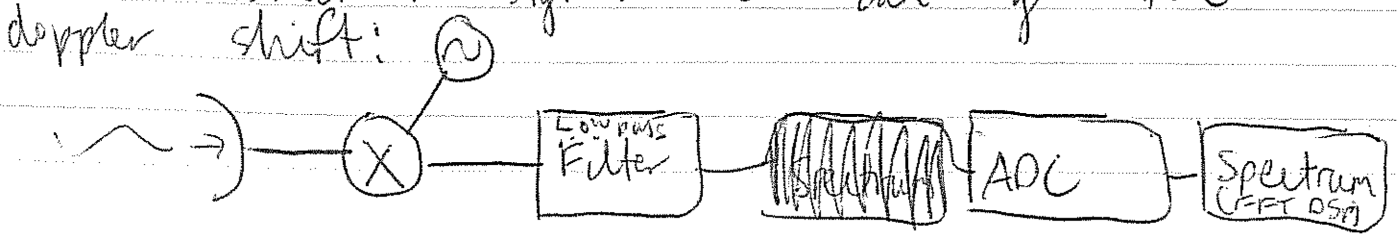
As calculated in phase 1, the doppler shifts expected will be

$$0.01 < V < 100 \quad (m/s)$$

$$10 < f_d < 100{,}000 \quad (Hz).$$

✓

By mixing the received signal with ~~a~~ the transmitted signal we can get the doppler shift:



To do this, we use an ADC to sample the mixed ~~RX signal~~ and filtered RX signal at a rate of 200 kHz, giving us up to 100 m/s of speed measurement.

The FFT should be ~~short~~ on a time period that is short enough to give reasonable time discrimination around the hit and also long enough to give good enough speed discrimination.

A 1024 point FFT at this ~~rate~~ sample rate (200 kHz) corresponds to a 5 ms time period, which should be ~~long enough for~~ short enough to resolve the swing in time. It also gives us a velocity ~~read~~ resolution of ~ 0.1 m/s (100 m/s ÷ 1024) which is more than adequate for our purposes.

This FFT would be done on a PC → laptop that has sampled the doppler signal using an ADC. When the detected speed is above a swing threshold tracking using range gating and conscan will begin. The threshold speed will be determined during testing.

Bearing Tracking

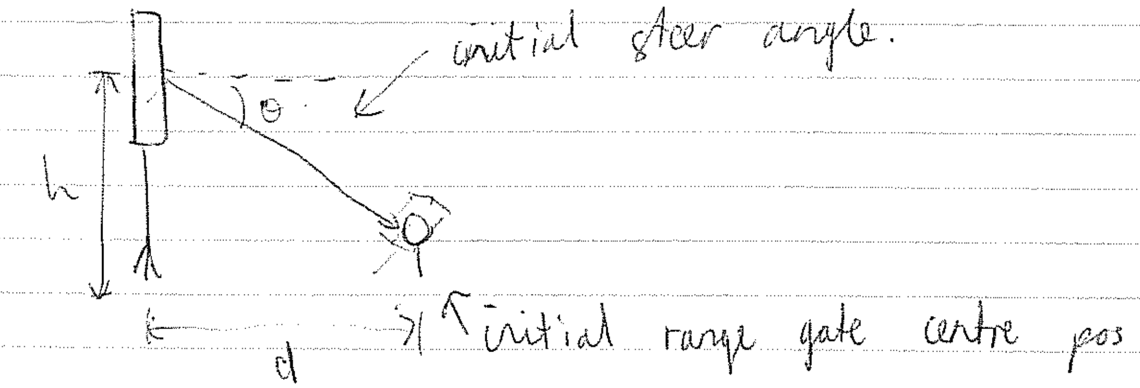
As mentioned previously, conscan will be used to ~~~~ track the ball in azimuth and elevation.

✓

Range Tracking

The first stage of the active tracking process will be to track the target in range. To do this, a range ~~~~ gated system will be used.

The initial velocity estimate will be used to as an estimate of the range rate. The initial position of the ball will be known and this can be used to track the target.



~~~~

Bearing Tracking

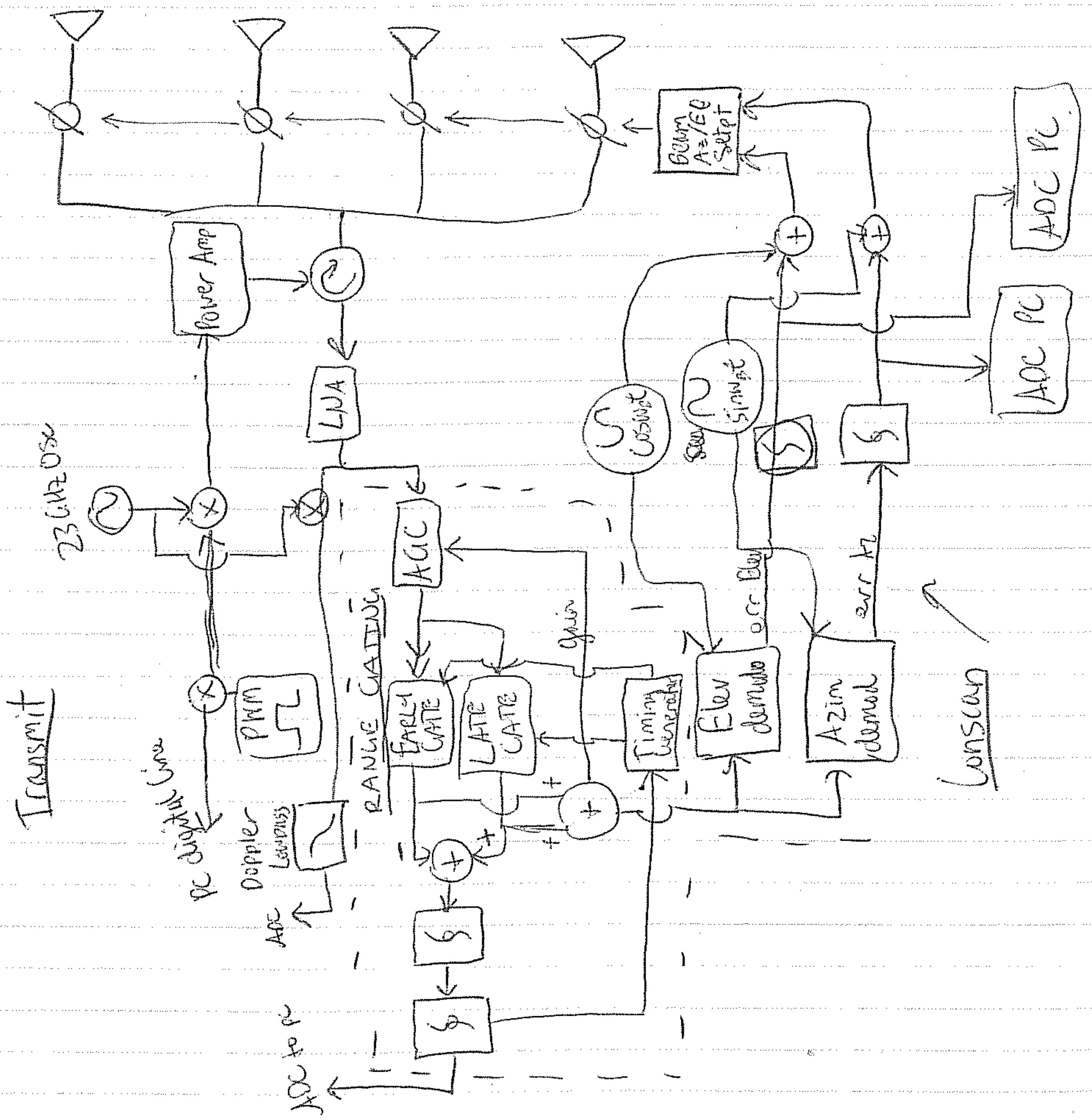
The second stage will involve using the predicted range to select range bins from each pulse for the conscan.

~~An system and~~

~~Mixed array tracking~~

✓
~~~~

A diagram of the ~~are~~ functional elements required is shown.



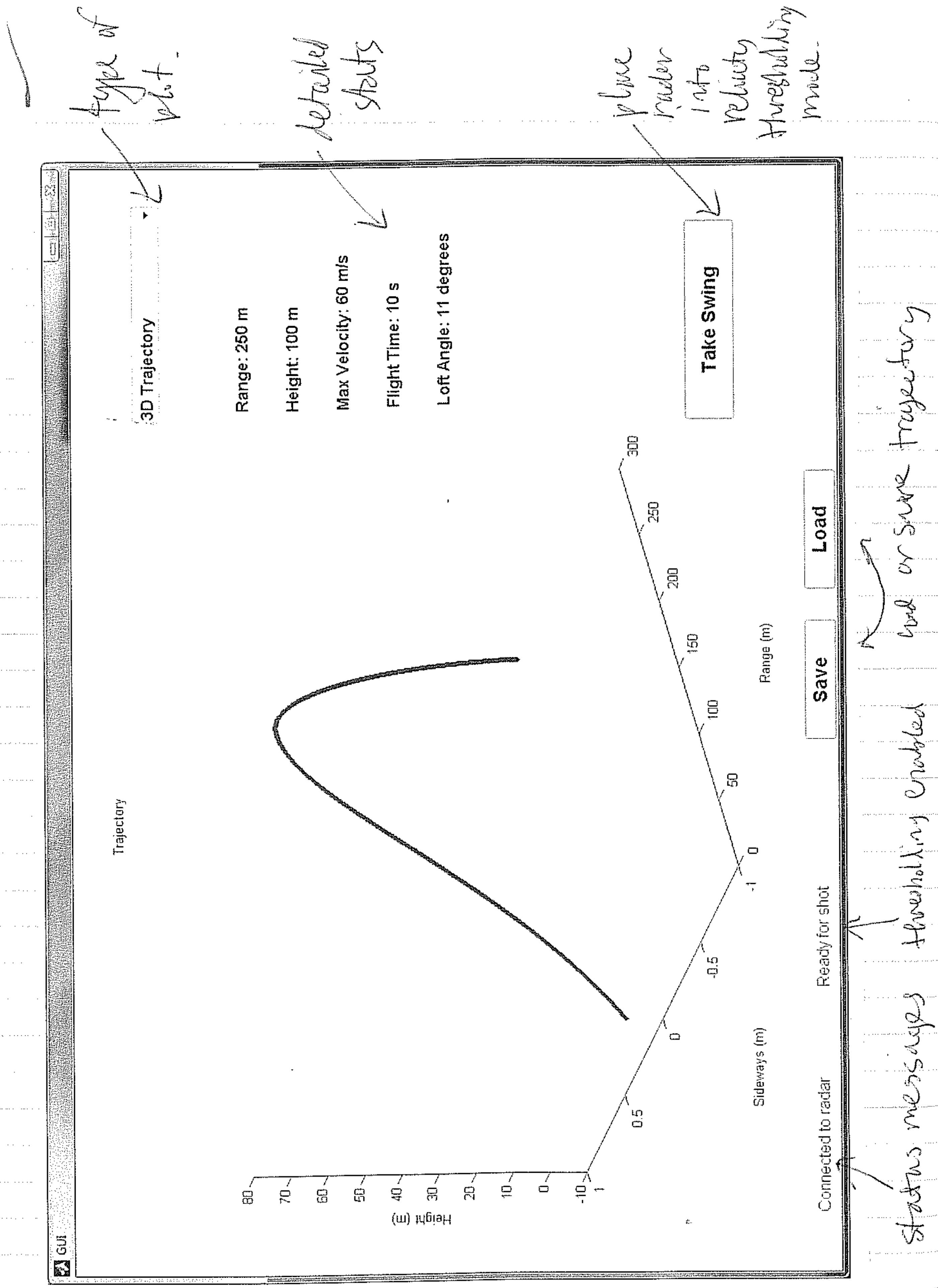
User interface

The user interface will run on a laptop computer that has a built in analog to digital convertor. It should be kept as simple as possible since the user will most likely be unaware of any ~~complex radar~~ or the actual radar technology.

In line with the tracking method described before, the user should position the radar 5m behind the tee and a metre off the ground. When it is positioned, they will click a button that forces the radar to form a beam in the direction of the tee. The previous velocity thresholding algorithm ~~than~~ is then used to initiate tracking when the swing is detected.

~~Apart from this~~ Once the ball is in motion, the software will display the path of the ball in 3 Dimensions, and once the ball hits the ground it will display detailed statistics. It also allows the trajectory to be saved so that it can be viewed at a later time.

The design is shown overleaf.



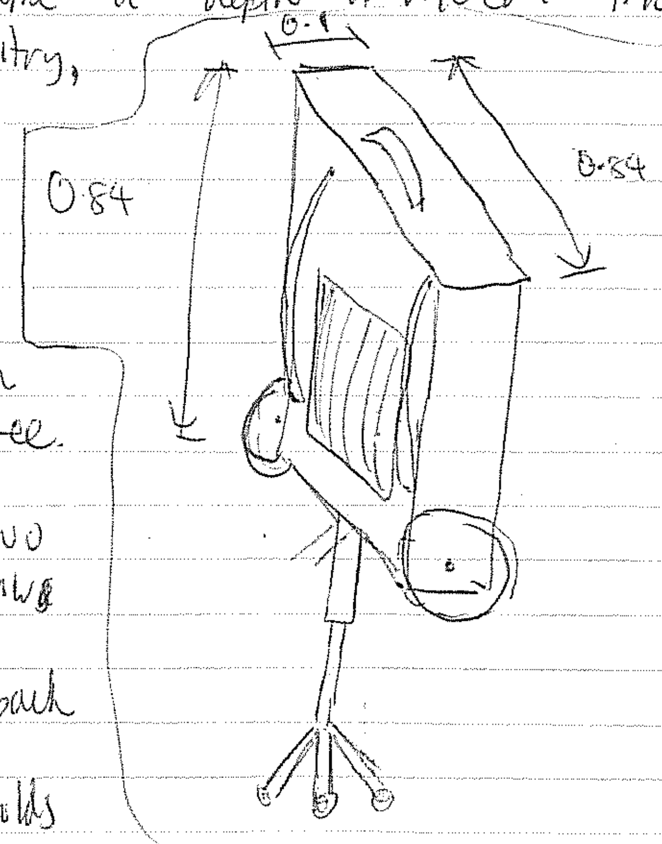
Electrical Requirements

- Laptop is battery powered
- Transmit peak power is 5W, assuming 50% efficiency say 10W to drive the array
- Recieve circuitry will require power too, assume 2W
- ADC module powered via USB from Laptop.

→ The system would be powered via a battery capable of 7W instantaneous output power. Many standard 12V batteries are easily capable of this.

System Size

As stated, the array is 0.84 x 0.84 m in size. Assume a depth of ~10cm including the receive circuitry, and battery



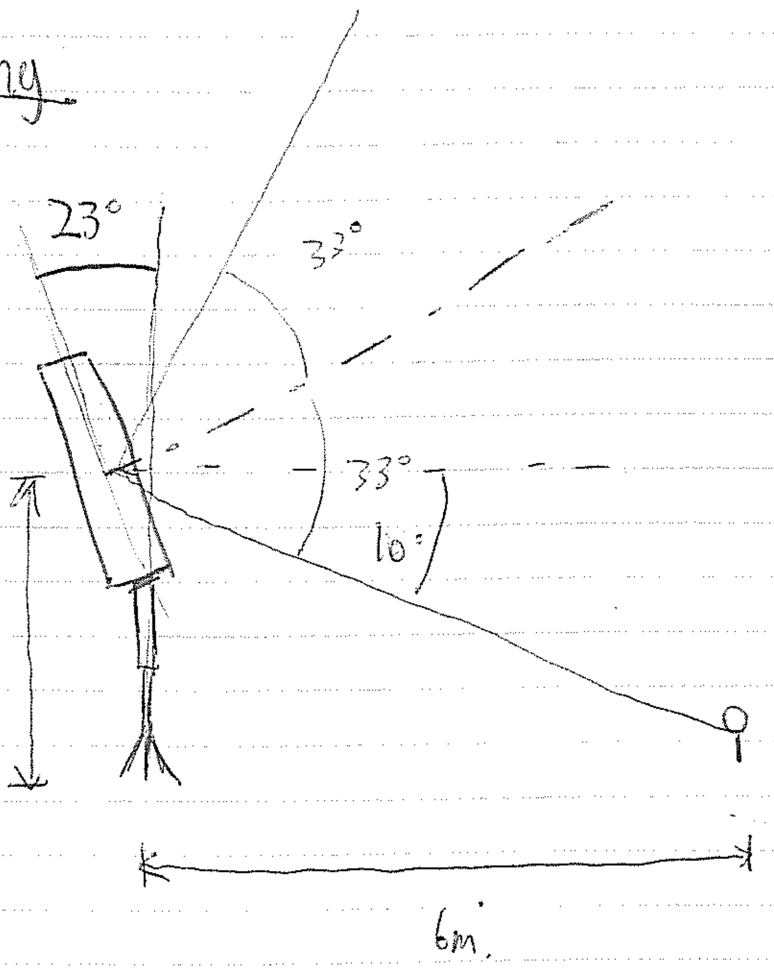
A telescopic stand which extends from inside the array is used to mount and orient it with respect to the tee.

For portability, two shoulder straps allow the unit to be worn on the back of the user, and a pouch holds the laptop with ADC.

Ball spin

Two wheels allow the system to be rolled around the course if the backpack method turns out to be too uncomfortable.

Mounting



As discussed before, the radar has a 33° maximum steer angle in the vertical plane. This means that it must be mounted at an angle in order to see both the tee and the maximum height of the trajectory.

To acheive this the phased array will be able to pivot on the telescopic

✓

stand, with it locking when the angle is 23°; as shown.

To see the tee, the radar must then be placed at a minimum of 6m behind the ball and 1m above it.

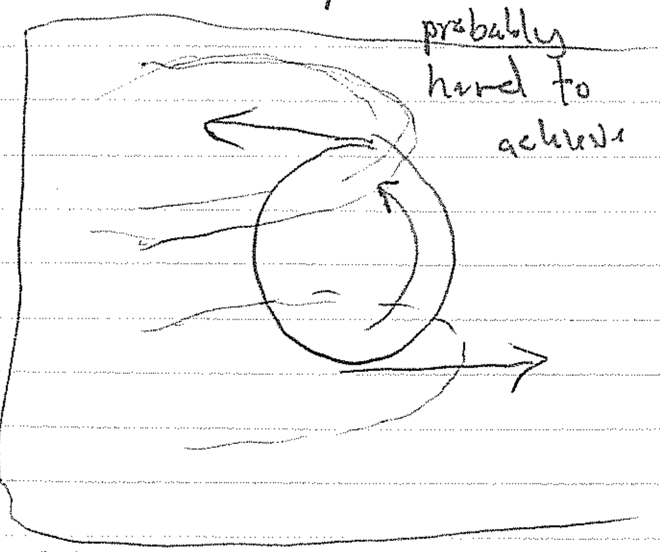
Ball Spin

Can we measure the ball spin rate directly with this system?

Potentially but it will have to be verified. The following method could be trialled.

~~If the number of dimples on the ball is known, then the received doppler signal could be analysed to determine the spin rate~~

If the beam is pointed at the top of the ball, and the doppler return is measured, and then pointed at the bottom of the ball and the return measured again the difference between these two signals will be proportional to the spin rate of the ball.



This method could be incorporated into the lonscan pulses and would likely only work well at close range, where the angular divergence of the beam is low.

✓

Nice piece of work you have a good understanding of the concepts involved

92

Summary

f_{op} = 23GHz [Reasoning: Optical region, good RCS, reasonable attenuation through both atmosphere and rain]

T_x Power = 5W [Reasoning: Required to get SNR at maximum range for 0.1m discrimination in conscan and range gating]

Max elements: 100x100 [Gives good beamwidth and appropriate steer angle maximums. Can be thinned later in the design process.]

Tracking: Conscan, Range Gating and Doppler [Reasoning: Doppler gives velocity information and initiates tracking. Conscan and range gating give required position to the accuracy]

Pulse width τ = 33ns [Gives required minimum range, still allows for many cycles of the signal per pulse]

PRT = 17μs [Sets maximum range at 2550m, allows for 25 scans around ball (conscan) at balls maximum angular rate]

ϕ θ