Problem 4

Describe the hypotheses behind the concept of boundary layer in immersed bodies. Under which conditions can we separate the inviscid region far from the wall from the viscous region close to the wall?

Problem 3

Air at $T = 25\,\mathrm{C}^\circ$ and $p = 1\,\mathrm{atm}$ flows normal to a circular cylinder with $D = 1\,\mathrm{cm}$ at velocity $U = 10\,\mathrm{m/s}$. According to the inviscid theory, the external velocity on the surface of the cylinder profile is given by Eq. 8.39 in the textbook with $K \equiv 0$,

$$v_\vartheta(\vartheta) = -2U\sin\vartheta \quad \rightarrow \quad u_s(x_s) = 2U\sin\left(2\frac{x_s}{D}\right) \tag{1}$$

where $u_s(x_s)$ is the same velocity as $v_\vartheta(\vartheta)$ but expressed in curvilinear coordinates originating at the stagnation point ($x_s = 0$ or $\vartheta = \pi$) as shown in the figure below. In the vicinity of the

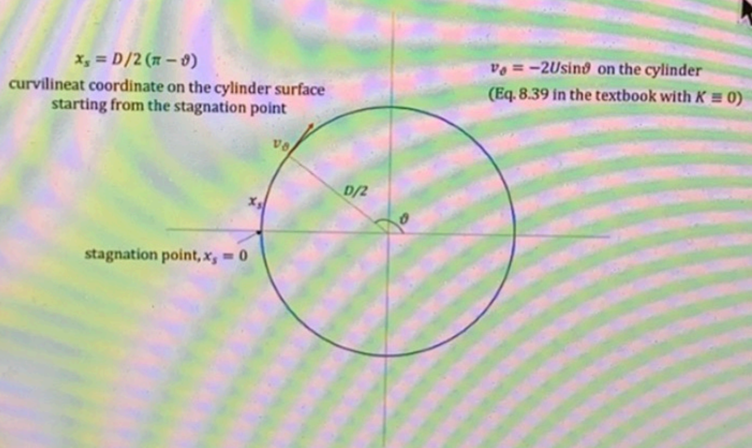

Figure 3: Circular cylinder with no rotation.

stagnation point the above external velocity can be approximated as

$$u_s(x_s) = \frac{4Ux_s}{D} \tag{2}$$

(a) Derive Eq. 2 starting from Eq. 1; (b) use Thwaite's method and Eq. 2 to calculate the wall shear stress τ_w, the displacement thickness δ^* and momentum thickness θ in the vicinity of the stagnation point. Hint: use Karman Eq. 7.51 in the textbook to obtain δ^*.

BONUS. Repeat the calculation in (b) but using the exact expression of u_s in Eq. 1 (you will need to use numerical integration).

Problem 1

Water from a large tank discharges into the atmosphere through the pipeline shown in the figure. Calculate (a) the discharge velocity V at the exit of the pipeline (station 2); (b) the head loss h_f; and (c) the volume flow rate Q. Assume constant diameter of the pipeline, $d = 15\,\text{cm}$, and roughness $\epsilon/d = 0.0017$. Use Moody's formula (Eq. 6.48 in the textbook).

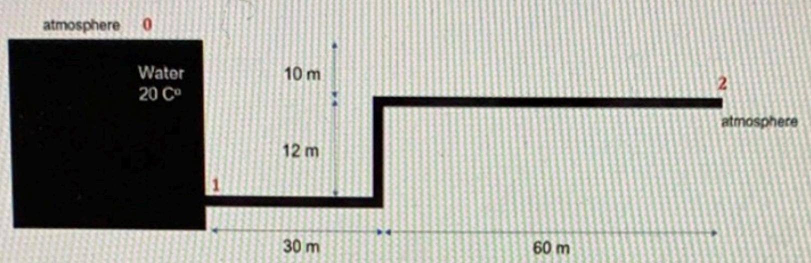

Figure 1: Pipeline.

Problem 2

Air at $T = 25\,\text{C}^\circ$ and $p = 1\,\text{atm}$ flows past a long flat plate, at the end of which is placed a narrow scoop, as shown in the figure below. (a) Determine if the boundary layer is laminar or turbulent at the position of the scoop assuming $\text{Re}_{x,trans} = 5 \times 10^5$; (b) estimate the height h of the scoop if it is to extract 4 kg/s per meter of width b into the paper. (c) Find the drag on the plate per meter of width, D/b up to the inlet of the scoop.

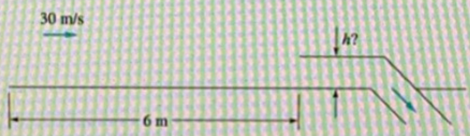

Figure 2: Flat plate with scoop.

Problem 3

Air at $T = 25\,\text{C}^\circ$ and $p = 1\,\text{atm}$ flows normal to a circular cylinder with $D = 1\,\text{cm}$ at velocity $U = 10\,\text{m/s}$. According to the inviscid theory, the external velocity on the surface of the cylinder profile is given by Eq. 8.39 in the textbook with $K \equiv 0$,

$$v_\vartheta(\vartheta) = -2U \sin\vartheta \quad \rightarrow \quad u_s(x_s) = 2U \sin\left(2\frac{x_s}{D}\right) \tag{1}$$

where $u_s(x_s)$ is the same velocity as $v_\vartheta(\vartheta)$ but expressed in curvilinear coordinates originating at the stagnation point ($x_s = 0$ or $\vartheta = \pi$) as shown in the figure below. In the vicinity of the