Math 125 Exam 3 solutions

① a.) $x = 2+3i \quad x = 2-3i$

$x-2-3i=0 \quad x-2+3i=0$

$(x-2-3i)(x-2+3i)=0$

$x^2-2x+3ix-2x+4-6i-3ix+6i-9i^2=0$

$x^2-2x-2x+4-9(-1)=0$

$\boxed{x^2-4x+13=0}$

b.) $x=2+\sqrt{3} \quad x=2-\sqrt{3}$

$x-2-\sqrt{3}=0 \quad x-2+\sqrt{3}=0$

$(x-2-\sqrt{3})(x-2+\sqrt{3})=0$

$x^2-2x+x\sqrt{3}-2x+4-2\sqrt{3}-x\sqrt{3}+2\sqrt{3}-\sqrt{9}=0$

$x^2-2x-2x+4-3=0$

$\boxed{x^2-4x+1=0}$

② $2x^2-4x+5=0$

$2x^2-4x=-5$

$x^2-2x=\frac{-5}{2}$

$-\frac{2}{2}=-1 \rightarrow (-1)^2=1$

$x^2-2x+1=\frac{-5}{2}+1$

$(x-1)^2=\frac{-5}{2}+\frac{2}{2}$

$\sqrt{(x-1)^2}=\sqrt{\frac{-3}{2}}$

$x-1=\pm\frac{i\sqrt{3}\cdot\sqrt{2}}{\sqrt{2}\cdot\sqrt{2}}$

$x=1\pm\frac{i\sqrt{6}}{\sqrt{4}}=\boxed{1\pm\frac{i\sqrt{6}}{2}}$

③ $x(3x+1)=12$

$3x^2+x-12=0$

$x=\frac{-1\pm\sqrt{1^2-4\cdot3\cdot-12}}{2\cdot3}$

$=\frac{-1\pm\sqrt{1+144}}{6}$

$=\boxed{\frac{-1\pm\sqrt{145}}{6}}$

④ $f(x) = 2x^2+4x+1$ $\quad \frac{2}{2}=1 \to 1^2=1$

$f(x)+2 = 2(x^2+2x+1)+1$ (2)

$f(x)+2 = 2(x+1)^2+1$ $\quad V:(-1,-1)$

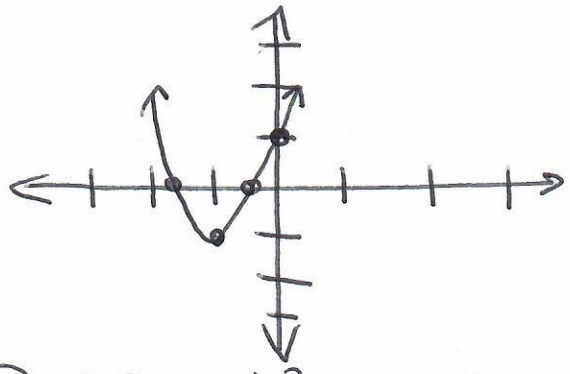
$f(x) = 2(x+1)^2-1$

x-int: $0 = 2x^2+4x+1$

$x = \frac{-4\pm\sqrt{4^2-4\cdot2\cdot1}}{2\cdot2} = \frac{-4\pm\sqrt{8}}{4} = \frac{-4\pm\sqrt{4\cdot2}}{4}$

$= \frac{-4\pm2\sqrt{2}}{4} = -\frac{4}{4}\pm\frac{2\sqrt{2}}{4} = -1\pm\frac{\sqrt{2}}{2}$

$-1+\frac{\sqrt{2}}{2}\approx -.3$ $\quad -1-\frac{\sqrt{2}}{2}\approx -1.7$ $\quad$ y-int: $y=1$



⑤ $(x^2+2x)^2-11(x^2+2x)+24=0$

$u = x^2+2x$

$u^2 = (x^2+2x)^2$

$u^2-11u+24=0$

$-8\ -3$

$(u-8)(u-3)=0$

$u=8 \quad u=3$

$8 = x^2+2x$

$0 = x^2+2x-8$

$0 = (x+4)(x-2)$

$x=-4 \quad x=2$

$3 = x^2+2x$

$0 = x^2+2x-3$

$0 = (x+3)(x-2)$

$x=-3 \quad x=2$

6. $u = x^{1/6}$

$u^2 = (x^{1/6})^2 = x^{2/6} = x^{1/3}$

$u^2 + 3u - 40 = 0$

$(u+8)(u-5) = 40$

$u = -8 \quad u = 5$

$-8 \neq x^{1/6}$

$5 = x^{1/6}$

$(5)^6 = (\sqrt[6]{x})^6$

$15625 = x$

7. $u = \frac{x}{x-1}$

$u^2 = \left(\frac{x}{x-1}\right)^2$

$u^2 + u - 2 = 0$

$(u+2)(u-1) = 0$

$u = -2 \quad u = 1$

$-2 = \frac{x}{x-1}$

$-2x + 2 = x$

$2 = 3x$

$2/3 = x$

$1 = \frac{x}{x-1}$

$x - 1 = x$

$-1 = 0$

$\emptyset$

8. $u = \sqrt{r}$

$u^2 = (\sqrt{r})^2 = r$

$u^2 - 6u - 27 = 0$

$(u-9)(u+3) = 0$

$u = 9 \quad u = -3$

$(9)^2 = (\sqrt{r})^2$

$81 = r$

$-3 = \sqrt{r}$

$\emptyset$

9. $u = x^2$

$u^2 = (x^2)^2 = x^4$

$3u^2 + 5u - 2 = 0$

$3u^2 + 6u - 1u - 2 = 0$

$3u(u+2) - 1(u+2) = 0$

$(u+2)(3u-1) = 0$

$u = -2 \quad 3u = 1$

$u = \frac{1}{3}$

$3 \cdot -2 = -6$

$6 \quad -1$

$\sqrt{-2} = \sqrt{x^2}$

$\pm i\sqrt{2} = x$

$\sqrt{\frac{1}{3}} = \sqrt{x^2}$

$\pm \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = x$

$\pm \frac{\sqrt{3}}{\sqrt{9}} = x$

$\pm \frac{\sqrt{3}}{3} = x$

(10.) $\sqrt{6x+7} - \sqrt{3x+3} = 1$

$(\sqrt{6x+7})^2 = (1+\sqrt{3x+3})^2$

$6x+7 = (1+\sqrt{3x+3})(1+\sqrt{3x+3})$

$6x+7 = 1+\sqrt{3x+3}+\sqrt{3x+3}+(\sqrt{3x+3})^2$

$6x+7 = 1+2\sqrt{3x+3}+3x+3$

$6x+7 = 4+2\sqrt{3x+3}+3x$

$-3x \quad -4 \quad -4 \quad -3x$

$(3x+3)^2 = (2\sqrt{3x+3})^2$

$(3x+3)(3x+3) = 4(3x+3)$

$9x^2+9x+9x+9 = 12x+12$

$9x^2+18x+9 = 12x+12$

$-12x \quad -12 \quad -12x \quad -12$

$9x^2+6x-3=0 \qquad 9\cdot -3 = -27$

$9 \quad -3$

$(9x^2+9x)(-3x-3)=0$

$9x(x+1)-3(x+1)=0$

$(9x-3)(x+1)=0$

$9x=3 \qquad x=-1$

$x=\frac{1}{3}$

Check:

$\sqrt{6\cdot\frac{1}{3}+7} - \sqrt{3\cdot\frac{1}{3}+3} = 1$

$\sqrt{2+7} - \sqrt{1+3} = 1$

$\sqrt{9} - \sqrt{4} = 1$

$3-2=1$ ✓

$\sqrt{6\cdot -1+7} - \sqrt{3\cdot -1+3} = 1$

$\sqrt{-6+7} - \sqrt{-3+3} = 1$

$\sqrt{1} - \sqrt{0} = 1$

$1-0=1$ ✓

(11) $\sqrt{6x^2y^3}\sqrt{3x^4y^6} = \sqrt{18x^6y^9} = \sqrt{9\cdot 2x^6y^8\cdot y}$
$= 3x^3y^4\sqrt{2y}$

(12) $\dfrac{\sqrt{48a^4b^7}}{\sqrt{6ab^5}} = \sqrt{\dfrac{48a^4b^7}{6ab^5}} = \sqrt{8a^3b^2} = \sqrt{4\cdot 2a^2\cdot ab^2}$
$= 2ab\sqrt{2a}$

(13) $\sqrt{x^3+3x^2} - \sqrt{4x+12} = \sqrt{x^2(x+3)} - \sqrt{4(x+3)}$
$= x\sqrt{x+3} - 2\sqrt{x+3} = \sqrt{x+3}\,(x-2)$

(14) $(2\sqrt{3}-3\sqrt{6})^2 = (2\sqrt{3}-3\sqrt{6})(2\sqrt{3}-3\sqrt{6})$
$= 4\sqrt{9} - 6\sqrt{18} - 6\sqrt{18} + 9\sqrt{36} = 4\cdot 3 - 12\sqrt{18} + 9\cdot 6$
$= 12 - 12\sqrt{9\cdot 2} + 54 = 66 - 12\cdot 3\sqrt{2} = 66 - 36\sqrt{2}$

(15) $(4+5i)^2 = (4+5i)(4+5i) = 16+20i+20i+25i^2$
$= 16+40i+25(-1) = -9+40i$

(16) $4\overline{)67}$ R 3 (16, 4, 27, 24, 3)

$i^{67} = i^3 = -i$

(17) $\dfrac{2+6i}{2-6i}\cdot\dfrac{(2+6i)}{(2+6i)} = \dfrac{4+12i+12i+36i^2}{4+12i-12i-36i^2} = \dfrac{4+24i+36(-1)}{4-36(-1)}$
$= \dfrac{-32+24i}{40} = \dfrac{-32}{40} + \dfrac{24i}{40} = -\dfrac{4}{5} + \dfrac{3i}{5}$

(18) $\dfrac{3}{\sqrt[3]{4x}}\cdot\dfrac{\sqrt[3]{2x^2}}{\sqrt[3]{2x^2}} = \dfrac{3\sqrt[3]{2x^2}}{\sqrt[3]{8x^3}} = \dfrac{3\sqrt[3]{2x^2}}{2x}$

(19) $\dfrac{2-\sqrt{5}}{2+\sqrt{5}}\cdot\dfrac{(2-\sqrt{5})}{(2-\sqrt{5})} = \dfrac{4-2\sqrt{5}-2\sqrt{5}+\sqrt{25}}{4-2\sqrt{5}+2\sqrt{5}-\sqrt{25}}$
$= \dfrac{4-4\sqrt{5}+5}{4-5} = \dfrac{9-4\sqrt{5}}{-1} = \dfrac{9}{-1} - \dfrac{4\sqrt{5}}{-1} = -9+4\sqrt{5}$

(20) $D = 1^2 - 4 \cdot 3 \cdot 6 = 1 - 72 = -71 < 0$

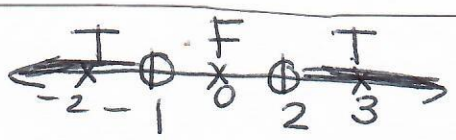
2 imaginary (complex)

(21) $x^2 - x > 2$

$x^2 - x - 2 > 0$

$(x-2)(x+1) > 0$

$x = 2 \quad x = -1$



$(-2-2)(-2+1) > 0$

$(-4)(-1) > 0$ T

$(0-2)(0+1) > 0$

$(-2)(1) > 0$ F

$(3-2)(3+1) > 0$

$1(4) > 0$ T

$(-\infty, -1) \cup (2, \infty)$

(22) $\frac{x}{x+1} \geq 2$

$\frac{x}{x+1} - \frac{2(x+1)}{1(x+1)} \geq 0$

$\frac{x}{x+1} - \frac{2x+2}{x+1} \geq 0$

$\frac{x - 2x - 2}{x+1} \geq 0$

$\frac{-x-2}{x+1} \geq 0$

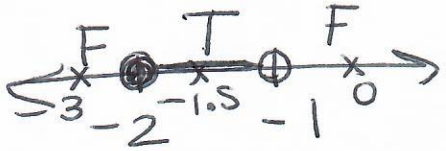
$-x - 2 = 0$

$-x = 2$

$x = -2$

$x + 1 = 0$

$x = -1$



$\frac{-(-3)-2}{-3+1} \geq 0$

$\frac{3-2}{-2} \geq 0$

$\frac{1}{-2} \geq 0$ F

$\frac{-(-1.5)-2}{-1.5+1} \geq 0$

$\frac{1.5-2}{-0.5} \geq 0$

$\frac{-0.5}{-0.5} \geq 0$

$1 \geq 0$ T

$\frac{-0-2}{0+1} \geq 0$

$\frac{-2}{1} \geq 0$ F

$[-2, -1)$