

MATH208 13360 - W22 - Introduction To Linear Algebra

≡ Test: Final Exam

< Question 1 of 25 >

This test: 100 point(s)
This question: 4 point(s)

Determine if the given set is a subspace of $\mathbb{P}_n$. Justify your answer.

The set of all polynomials in $\mathbb{P}_n$ such that $p(0) = 0$

Choose the correct answer below

- ☐ A. The set is a subspace of $\mathbb{P}_n$ because $\mathbb{P}_n$ is a vector space spanned by the given set.
- ☒ B. The set is a subspace of $\mathbb{P}_n$ because the set contains the zero vector of $\mathbb{P}_n$, the set is closed under vector addition, and the set is closed under multiplication by scalars.
- ☐ C. The set is not a subspace of $\mathbb{P}_n$ because the set does not contain the zero vector of $\mathbb{P}_n$.
- ☐ D. The set is not a subspace of $\mathbb{P}_n$ because the set is not closed under multiplication by scalars.
- ☐ E. The set is not a subspace of $\mathbb{P}_n$ because the set is not closed under vector addition.

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Question 2 of 26

This test: 100 point(s) possible
This question: 4 point(s) possible

Is $\lambda = 7$ an eigenvalue of $\begin{bmatrix} 6 & 0 & -3 \\ 3 & 6 & 6 \\ -4 & 2 & 1 \end{bmatrix}$? If so, find one corresponding eigenvector

Select the correct choice below and, if necessary, fill in the answer box within your choice

☒ A. Yes, $\lambda = 7$ is an eigenvalue of $\begin{bmatrix} 6 & 0 & -3 \\ 3 & 6 & 6 \\ -4 & 2 & 1 \end{bmatrix}$. One corresponding eigenvector is $\begin{bmatrix} -3 \\ -3 \\ 1 \end{bmatrix}$.
(Type a vector or list of vectors. Type an integer or simplified fraction for each matrix element.)

☐ B. No, $\lambda = 7$ is not an eigenvalue of $\begin{bmatrix} 6 & 0 & -3 \\ 3 & 6 & 6 \\ -4 & 2 & 1 \end{bmatrix}$.

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< Question 3 of 25

This test: 100 point(s) possible

This question: 4 point(s) possible

Suppose a 7×9 matrix A has four pivot columns. What is $\text{nullity } A$? Is $\text{Col } A = \mathbb{R}^4$? Why or why not?

$\text{nullity } A =$ (Simplify your answer)

Is $\text{Col } A = \mathbb{R}^4$? Why or why not?

- ☒ A. No, because $\text{Col } A$ is a subspace of $\mathbb{R}^7$
- ☐ B. Yes, because the number of pivot positions in A is 4
- ☐ C. Yes, because $\text{rank } A = 4$
- ☐ D. No, because $\text{Col } A$ is a subspace of $\mathbb{R}^9$

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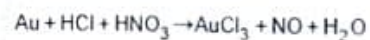
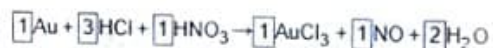
< Question 4 of 25



This test: 100 point(s) possible

This question: 4 point(s) possible

Balance the chemical equation

Assume the coefficient of H_2O is 2. What is the balanced equation?

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Question 5 of 25

This test: 100 point(s) possible
This question: 4 point(s) possible

Let W be the set of all vectors of the form

$$\begin{bmatrix} 5s + 5t \\ 5s \\ 4s \\ 2t \end{bmatrix}$$

Show that W is a subspace of $\mathbb{R}^4$ by finding vectors u and v such that $W = \text{Span}\{u, v\}$.

Write the vectors in W as column vectors

$$\begin{bmatrix} 5s + 5t \\ 5s \\ 4s \\ 2t \end{bmatrix} = s \begin{bmatrix} 5 \\ 5 \\ 4 \\ 0 \end{bmatrix} + t \begin{bmatrix} 5 \\ 0 \\ 0 \\ 2 \end{bmatrix} = su + tv$$

What does this imply about W ?

- ☐ A. $W = s + t$
- ☐ B. $W = u + v$
- ☐ C. $W = \text{Span}\{s, t\}$
- ☒ D. $W = \text{Span}\{u, v\}$

Explain how this result shows that W is a subspace of $\mathbb{R}^4$. Choose the correct answer below.

- ☐ A. Since u and v are in $\mathbb{R}^4$ and $W = u + v$, W is a subspace of $\mathbb{R}^4$
- ☒ B. Since u and v are in $\mathbb{R}^4$ and $W = \text{Span}\{u, v\}$, W is a subspace of $\mathbb{R}^4$
- ☐ C. Since s and t are in $\mathbb{R}$ and $W = \text{Span}\{u, v\}$, W is a subspace of $\mathbb{R}^4$
- ☐ D. Since s and t are in $\mathbb{R}$ and $W = u + v$, W is a subspace of $\mathbb{R}^4$

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< Question 6 of 25

This test: 100 point(s) possible
This question: 4 point(s) possible

Assume that A is row equivalent to B. Find bases for Nul A, Col A, and Row A.

$$A = \begin{bmatrix} -2 & 6 & -2 & -6 \\ 2 & -9 & -4 & 3 \\ -3 & 12 & 3 & -6 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 & 7 & 6 \\ 0 & 3 & 6 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

A column vector basis for Nul A is $\left\{ \begin{bmatrix} -7 \\ 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -6 \\ -1 \\ 0 \\ 1 \end{bmatrix} \right\}$
(Use a comma to separate vectors as needed.)

A column vector basis for Col A is $\left\{ \begin{bmatrix} -2 \\ 2 \\ -3 \end{bmatrix}, \begin{bmatrix} 6 \\ 9 \\ -12 \end{bmatrix} \right\}$
(Use a comma to separate vectors as needed.)

A row vector basis for Row A is $\left\{ \begin{bmatrix} 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \end{bmatrix}, \begin{bmatrix} 7 & -2 \end{bmatrix}, \begin{bmatrix} 6 & 1 \end{bmatrix} \right\}$
(Use a comma to separate vectors as needed.)

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Question 7 of 25

This test: 100 point(s) possible
This question: 4 point(s) possible

Determine if $\mathbf{w} = \begin{bmatrix} 4 \\ 3 \\ -1 \end{bmatrix}$ is in Nul A, where $A = \begin{bmatrix} 1 & -3 & 3 \\ 9 & -5 & 1 \\ -7 & 2 & -4 \end{bmatrix}$

Is $\mathbf{w}$ in Nul A? Select the correct choice below and fill in the answer box to complete your choice.

(Simplify your answer.)

☐ A. Yes, because $A\mathbf{w} = \begin{bmatrix} -8 \\ 20 \\ -18 \end{bmatrix}$

☒ B. No, because $A\mathbf{w} = \begin{bmatrix} -8 \\ 20 \\ -18 \end{bmatrix}$

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Question 8 of 25

This test: 100 point(s) possible
This question: 4 point(s) possible

Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 10 \\ 3 \\ 5 \end{bmatrix}$, and $\mathbf{w} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$

- a. Is $\mathbf{w}$ in $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$? How many vectors are in $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$?
b. How many vectors are in $\text{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$?
c. Is $\mathbf{w}$ in the subspace spanned by $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$? Why?

a. Is $\mathbf{w}$ in $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$?

- ☐ A. Vector $\mathbf{w}$ is in $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ because the subspace generated by $\mathbf{v}_1, \mathbf{v}_2$, and $\mathbf{v}_3$ is $\mathbb{R}^3$.
☐ B. Vector $\mathbf{w}$ is not in $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ because it is not a linear combination of $\mathbf{v}_1, \mathbf{v}_2$, and $\mathbf{v}_3$.
☐ C. Vector $\mathbf{w}$ is in $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ because it is a linear combination of $\mathbf{v}_1, \mathbf{v}_2$, and $\mathbf{v}_3$.
☒ D. Vector $\mathbf{w}$ is not in $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ because it is not $\mathbf{v}_1, \mathbf{v}_2$, or $\mathbf{v}_3$.

How many vectors are in $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- ☒ A. The number of vectors in $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is
☐ B. There are infinitely many vectors in $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$.

b. How many vectors are in $\text{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- ☐ A. The number of vectors in $\text{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is
☒ B. There are infinitely many vectors in $\text{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$.

c. Is $\mathbf{w}$ in the subspace spanned by $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$?

- ☐ A. Vector $\mathbf{w}$ is not in the subspace spanned by $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ because $\mathbf{w}$ is not a linear combination of $\mathbf{v}_1, \mathbf{v}_2$, and $\mathbf{v}_3$.
☐ B. Vector $\mathbf{w}$ is not in the subspace spanned by $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ because the rightmost column of the augmented matrix of the system $x_1\mathbf{v}_1 + x_2\mathbf{v}_2 + x_3\mathbf{v}_3 = \mathbf{w}$ is a pivot column.
☒ C. Vector $\mathbf{w}$ is in the subspace spanned by $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ because $\mathbf{w}$ is a linear combination of $\mathbf{v}_1, \mathbf{v}_2$, and $\mathbf{v}_3$ which can be seen because any echelon form of the augmented matrix of the system has no row of zeros.
☐ D. Vector $\mathbf{w}$ is in the subspace spanned by $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ because the subspace generated by $\mathbf{v}_1, \mathbf{v}_2$, and $\mathbf{v}_3$ is $\mathbb{R}^3$.

Time

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This test: 100 point(s) possible
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Previous question

Determine if $w = \begin{bmatrix} 4 \\ 3 \\ -1 \end{bmatrix}$ is in Nul A, where $A = \begin{bmatrix} 1 & -3 & 3 \\ 9 & -5 & 1 \\ -7 & 2 & -4 \end{bmatrix}$

Is w in Nul A? Select the correct choice below and fill in the answer box to complete your choice.
(Simplify your answer.)

☐ A. Yes, because $Aw =$

☒ B. No, because $Aw = \begin{bmatrix} -8 \\ 20 \\ -18 \end{bmatrix}$

Time R

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Question 8 of 25

This test: 100 point(s) possible
 This question: 4 point(s) possible

Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 10 \\ 3 \\ 5 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$

- a Is $\mathbf{w}$ in $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$? How many vectors are in $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$?
 b How many vectors are in $\text{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$?
 c Is $\mathbf{w}$ in the subspace spanned by $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$? Why?

a Is $\mathbf{w}$ in $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$?

- ☐ A. Vector $\mathbf{w}$ is in $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ because the subspace generated by $\mathbf{v}_1, \mathbf{v}_2$, and $\mathbf{v}_3$ is $\mathbb{R}^3$.
☐ B. Vector $\mathbf{w}$ is not in $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ because it is not a linear combination of $\mathbf{v}_1, \mathbf{v}_2$, and $\mathbf{v}_3$.
☐ C. Vector $\mathbf{w}$ is in $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ because it is a linear combination of $\mathbf{v}_1, \mathbf{v}_2$, and $\mathbf{v}_3$.
☒ D. Vector $\mathbf{w}$ is not in $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ because it is not $\mathbf{v}_1, \mathbf{v}_2$, or $\mathbf{v}_3$.

How many vectors are in $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- ☒ A. The number of vectors in $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is
☐ B. There are infinitely many vectors in $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$.

b How many vectors are in $\text{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- ☐ A. The number of vectors in $\text{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is
☒ B. There are infinitely many vectors in $\text{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$.

c Is $\mathbf{w}$ in the subspace spanned by $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$?

- ☐ A. Vector $\mathbf{w}$ is not in the subspace spanned by $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ because $\mathbf{w}$ is not a linear combination of $\mathbf{v}_1, \mathbf{v}_2$, and $\mathbf{v}_3$.
☐ B. Vector $\mathbf{w}$ is not in the subspace spanned by $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ because the rightmost column of the augmented matrix of the system $x_1\mathbf{v}_1 + x_2\mathbf{v}_2 + x_3\mathbf{v}_3 = \mathbf{w}$ is a pivot column.
☒ C. Vector $\mathbf{w}$ is in the subspace spanned by $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ because $\mathbf{w}$ is a linear combination of $\mathbf{v}_1, \mathbf{v}_2$, and $\mathbf{v}_3$ which can be seen because any echelon form of the augmented matrix of the system has no row of the form $[0 \ 0 \ 0 \ | \ c]$ with $c \neq 0$.
☐ D. Vector $\mathbf{w}$ is in the subspace spanned by $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ because the subspace generated by $\mathbf{v}_1, \mathbf{v}_2$, and $\mathbf{v}_3$ is $\mathbb{R}^3$.

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Question 9 of 26

This test: 100 point(s) possible
This question: 4 point(s) possible

For this exercise assume that the matrices are all $n \times n$. The statement in this exercise is an implication of the form "if 'statement 1' then 'statement 2'". Mark an implication as "true" if the truth of "statement 2" always happens to be true. Mark the implication as "false" if "statement 2" is false but "statement 1" is true. Justify your answer.

If the linear transformation $x \mapsto Ax$ maps $\mathbb{R}^n$ into $\mathbb{R}^n$, then A has n pivot positions.

Choose the correct answer below.

- ☒ A. The statement is false. The linear transformation $x \mapsto Ax$ will always map $\mathbb{R}^n$ into $\mathbb{R}^n$ for any $n \times n$ matrix. According to the Invertible Matrix Theorem, A has n pivot positions only if $x \mapsto Ax$ maps $\mathbb{R}^n$ onto $\mathbb{R}^n$.
- ☐ B. The statement is true. The linear transformation $x \mapsto Ax$ will always map $\mathbb{R}^n$ into $\mathbb{R}^n$ for any $n \times n$ matrix. Therefore, according to the Invertible Matrix Theorem, A has n pivot positions.
- ☐ C. The statement is false. According to the Invertible Matrix Theorem, $x \mapsto Ax$ maps $\mathbb{R}^n$ into $\mathbb{R}^n$ if and only if A has n pivot positions.
- ☐ D. The statement is true. According to the Invertible Matrix Theorem, if $x \mapsto Ax$ maps $\mathbb{R}^n$ into $\mathbb{R}^n$, then A is invertible, and if a matrix is invertible it has n pivot positions.

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Question 10 of 25

This test: 100 point(s) possible
This question: 4 point(s) possible

Solve the system

$$x_1 - 3x_3 = 10$$

$$2x_1 + 2x_2 + 9x_3 = 2$$

$$x_2 + 5x_3 = -4$$

Select the correct choice below and, if necessary, fill in the answer boxes to complete your choice

- ☒ A. The unique solution of the system is $\begin{pmatrix} 4 & 5 & -2 \end{pmatrix}$
(Type integers or simplified fractions.)
- ☐ B. The system has infinitely many solutions
- ☐ C. The system has no solution

Find the coordinate vector x_B of x relative to the given basis $B = \{b_1, b_2\}$.

$$b_1 = \begin{bmatrix} 1 \\ -3 \end{bmatrix} \quad b_2 = \begin{bmatrix} 2 \\ -7 \end{bmatrix} \quad x = \begin{bmatrix} 3 \\ -12 \end{bmatrix}$$

$$[\mathbf{x}]_1 = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$$

(Sin.p², out answer)

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Jose

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This test: 100 point(s) possible

This question: 4 point(s) possible

Let $B = \{b_1, b_2\}$ and $C = \{c_1, c_2\}$ be bases for a vector space V , and suppose $b_1 = -4c_1 + 5c_2$ and $b_2 = 6c_1 - 8c_2$

- Find the change-of-coordinates matrix from B to C .
- Find $[x]_C$ for $x = -9b_1 + 4b_2$. Use part (a).

a $P_{C \leftarrow B} = \begin{bmatrix} -4 & 6 \\ 5 & -8 \end{bmatrix}$

b $[x]_C = \begin{bmatrix} 60 \\ -77 \end{bmatrix}$

(Simplify your answer.)

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Question 13 of 25

This test: 100 point(s) possible
This question: 4 point(s) possible

Use the factorization $A = PDP^{-1}$ to compute A^k , where k represents an arbitrary positive integer

$$\begin{bmatrix} 11 & -40 \\ 2 & -7 \end{bmatrix} = \begin{bmatrix} 4 & 5 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} -1 & 5 \\ 1 & -4 \end{bmatrix}$$

$$A^k = \begin{bmatrix} -4 + 5 \cdot 3^k & 20 - 20 \cdot 3^k \\ -1 + 1 \cdot 3^k & 5 - 4 \cdot 3^k \end{bmatrix}$$

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Question 14 of 25

This test: 100 point(s) possible

This question: 4 point(s) possible

Use the factorization $A = PDP^{-1}$ to compute A^k , where k represents an arbitrary positive integer

$$\begin{bmatrix} 11 & -40 \\ 2 & -7 \end{bmatrix} = \begin{bmatrix} 4 & 5 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} -1 & 5 \\ 1 & -4 \end{bmatrix}$$

$$A^k = \begin{bmatrix} -4 + 5 \cdot 3^k & 20 - 20 \cdot 3^k \\ -1 + 1 \cdot 3^k & 5 - 4 \cdot 3^k \end{bmatrix}$$

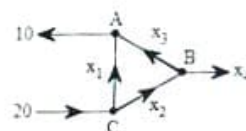
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This test: 100 point(s) possible
This question: 4 point(s) possible

Next question

Find the general flow pattern of the network shown in the figure. Assuming that the flows are all nonnegative, what is the largest possible value for x_3 ?



Find the general flow pattern of the network shown in the figure. Choose the correct answer below and fill in the answer boxes to complete your choice.

☒ A.
$$\begin{cases} x_1 = 10 - x_3 \\ x_2 = 10 + x_3 \\ x_3 \text{ is free} \\ x_4 = 10 \end{cases}$$

☐ B.
$$\begin{cases} x_1 = \square \\ x_2 \text{ is free} \\ x_3 \text{ is free} \\ x_4 \text{ is free} \end{cases}$$

☐ C.
$$\begin{cases} x_1 = \square \\ x_2 = \square \\ x_3 = \square \\ x_4 = \square \end{cases}$$

☐ D.
$$\begin{cases} x_1 \text{ is free} \\ x_2 = \square \\ x_3 \text{ is free} \\ x_4 = \square \end{cases}$$

Assuming that the flows are all nonnegative, what is the largest possible value for x_3 ?

The largest possible value for x_3 is .

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Joseph Malo

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Question 15 of 25

This test: 100 point(s) possible
This question: 4 point(s) possible

Matrix A is factored in the form PDP^{-1} . Use the Diagonalization Theorem to find the eigenvalues of A and a basis for each eigenspace

$$A = \begin{bmatrix} 5 & 0 & -5 \\ 2 & 6 & 10 \\ 0 & 0 & 6 \end{bmatrix} = \begin{bmatrix} -5 & 0 & -1 \\ 0 & 1 & 2 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 2 & 1 & 10 \\ -1 & 0 & -5 \end{bmatrix}$$

Select the correct choice below and fill in the answer boxes to complete your choice.
(Use a comma to separate vectors as needed.)

☐ A. There is one distinct eigenvalue, $\lambda =$. A basis for the corresponding eigenspace is .

☒ B. In ascending order, the two distinct eigenvalues are $\lambda_1 =$ and $\lambda_2 =$. Bases for the corresponding eigenspaces are $\left\{ \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix} \right\}$ and $\left\{ \begin{bmatrix} -5 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$, respectively.

☐ C. In ascending order, the three distinct eigenvalues are $\lambda_1 =$, $\lambda_2 =$, and $\lambda_3 =$. Bases for the corresponding eigenspaces are , , and , respectively.

Let A be a 4×6 matrix. What must a and b be in order to define $T: \mathbb{R}^3 \rightarrow \mathbb{R}^6$ by $T(\mathbf{x}) = A\mathbf{x}$?

...

$$a = 6$$

(Simplify your answer.)

$$b = 4$$

(Simplify your answer.)



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Joseph M

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This test: 100 point(s) possible

This question: 4 point(s) possible

For this exercise assume that the matrices are all $n \times n$. The statement in this exercise is an implication of the form "If 'statement 1', then 'statement 2'". Mark an implication as True if the truth of "statement 2" always happens to be true. Mark the implication as False if "statement 2" is false but "statement 1" is true. Justify your answer.

If the columns of A span $\mathbb{R}^n$, then the columns are linearly independent.

Choose the correct answer below

- ☐ A. The statement is true. The Invertible Matrix Theorem states that if the linear transformation $\mathbf{x} \mapsto A\mathbf{x}$ does not map $\mathbb{R}^n$ into $\mathbb{R}^n$, then A is invertible. Therefore, the columns are linearly independent.
- ☐ B. The statement is false. The Invertible Matrix Theorem states that if the columns of A span $\mathbb{R}^n$, then matrix A is not invertible. Therefore, the columns are linearly dependent.
- ☒ C. The statement is true. The Invertible Matrix Theorem states that if the columns of A span $\mathbb{R}^n$, then matrix A is invertible. Therefore, the columns are linearly independent.
- ☐ D. The statement is false. The Invertible Matrix Theorem states that if the linear transformation $\mathbf{x} \mapsto A\mathbf{x}$ maps $\mathbb{R}^n$ into $\mathbb{R}^n$, then A is not invertible. Therefore, the columns are linearly dependent.

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This test: 100 point(s) possible
This question: 4 point(s) possible

Determine if the columns of the matrix form a linearly independent set

$$\begin{bmatrix} 1 & 3 & -3 & 2 \\ 3 & 10 & -6 & 0 \\ 2 & 9 & 6 & -20 \end{bmatrix}$$

Select the correct choice below and, if necessary, fill in the answer box(es) within your choice.

- ☐ A. The columns of the matrix do not form a linearly independent set because there are more entries in each vector, , than there are vectors in the set, .
(Type whole numbers.)
- ☒ B. The columns of the matrix do not form a linearly independent set because the set contains more vectors, , than there are entries in each vector, .
(Type whole numbers.)
- ☐ C. Let A be the given matrix. Then the columns of the matrix form a linearly independent set since the vector equation, $A\mathbf{x} = \mathbf{0}$, has only the trivial solution.
- ☐ D. The columns of the matrix form a linearly independent set because at least one vector in the set is a constant multiple of another.

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This test: 100 point(s) possible
This question: 4 point(s) possibleDescribe all solutions of $A\mathbf{x} = \mathbf{0}$ in parametric vector form, where A is row equivalent to the given matrix.

$$\begin{bmatrix} 1 & -2 & -9 & 4 \\ 0 & 1 & 4 & -4 \end{bmatrix}$$

$$\mathbf{x} = x_3 \begin{bmatrix} 1 \\ -4 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 4 \\ 4 \\ 0 \\ 1 \end{bmatrix}$$

(Type an integer or fraction for each matrix element.)

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This test: 100 point(s) possible
This question: 4 point(s) possibleDescribe all solutions of $Ax = 0$ in parametric vector form, where A is row equivalent to the given matrix

$$\begin{bmatrix} 1 & -2 & -9 & 4 \\ 0 & 1 & 4 & -4 \end{bmatrix}$$

$$x = x_3 \begin{bmatrix} 1 \\ -4 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 4 \\ 4 \\ 0 \\ 1 \end{bmatrix}$$

(Type an integer or fraction for each matrix element.)

Compute each matrix sum or product if it is defined. If an expression is undefined, explain why. Let $A = \begin{bmatrix} 1 & 0 & -1 \\ 4 & -6 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 7 & -4 & 2 \\ 1 & -4 & -3 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$, and $D = \begin{bmatrix} 3 & 5 \\ -2 & 3 \end{bmatrix}$.

$-2A$, $B - 2A$, AC , CD

Compute the matrix product $-2A$. Select the correct choice below and, if necessary, fill in the answer box within your choice.

☒ A. $-2A = \begin{bmatrix} -2 & 0 & 2 \\ -8 & 12 & -4 \end{bmatrix}$

(Simplify your answer.)

- ☐ B. The expression $-2A$ is undefined because matrices cannot have negative coefficients.
- ☐ C. The expression $-2A$ is undefined because matrices cannot be multiplied by numbers.
- ☐ D. The expression $-2A$ is undefined because A is not a square matrix.

Compute the matrix sum $B - 2A$. Select the correct choice below and, if necessary, fill in the answer box within your choice.

☒ A. $B - 2A = \begin{bmatrix} 5 & -4 & 4 \\ -7 & 8 & -1 \end{bmatrix}$

(Simplify your answer.)

- ☐ B. The expression $B - 2A$ is undefined because A is not a square matrix.
- ☐ C. The expression $B - 2A$ is undefined because B and A have different sizes.
- ☐ D. The expression $B - 2A$ is undefined because B and $-2A$ have different sizes.

Compute the matrix product AC . Select the correct choice below and, if necessary, fill in the answer box within your choice.

☐ A. $AC = \begin{bmatrix} & \\ & \end{bmatrix}$
(Simplify your answer.)

- ☐ B. The expression AC is undefined because the number of rows in A is not equal to the number of columns in C .
- ☒ C. The expression AC is undefined because the number of columns in A is not equal to the number of rows in C .
- ☐ D. The expression AC is undefined because the number of rows in A is not equal to the number of rows in C .

Compute the matrix product CD . Select the correct choice below and, if necessary, fill in the answer box within your choice.

MATH208 13360 - W22 - Introduction To Linear Algebra

Joseph

≡ **Test: Final Exam**

Question 20 of 25

This test: 100 point(s) possible
This question: 4 point(s) possible

Compute each matrix sum or product if it is defined. If an expression is undefined, explain why. Let $A = \begin{bmatrix} 1 & 0 & -1 \\ 4 & -6 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 7 & -4 & 2 \\ 1 & -4 & -3 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$ and $D = \begin{bmatrix} 3 & 5 \\ -2 & 3 \end{bmatrix}$.

- 2A, B - 2A, AC, CD

- ☐ D. The expression $-2A$ is undefined because A is not a square matrix

Compute the matrix sum $B - 2A$. Select the correct choice below and, if necessary, fill in the answer box within your choice.

- Ⓐ. $B - 2A = \begin{bmatrix} 5 & -4 & 4 \\ -7 & 8 & -1 \end{bmatrix}$
(Simplify your answer.)

- ☐ B. The expression $B - 2A$ is undefined because A is not a square matrix.
☐ C. The expression $B - 2A$ is undefined because B and A have different sizes.
☐ D. The expression $B - 2A$ is undefined because B and $-2A$ have different sizes.

Compute the matrix product AC . Select the correct choice below and, if necessary, fill in the answer box within your choice.

- ☐ A. $AC = \square$
(Simplify your answer.)
- ☐ B. The expression AC is undefined because the number of rows in A is not equal to the number of columns in C .
- ☒ C. The expression AC is undefined because the number of columns in A is not equal to the number of rows in C .
- ☐ D. The expression AC is undefined because the number of rows in A is not equal to the number of rows in C .

Compute the matrix product CD . Select the correct choice below and, if necessary, fill in the answer box within your choice.

- ☒ A. $CD = \begin{bmatrix} -1 & 11 \\ -8 & -7 \end{bmatrix}$
(Simplify your answer.)

- ☐ B. The expression CD is undefined because square matrices cannot be multiplied
☐ C. The expression CD is undefined because the corresponding entries in C and D are not equal
☐ D. The expression CD is undefined because matrices with negative entries cannot be multiplied

If T is defined by $T(\mathbf{x}) = A\mathbf{x}$, find a vector $\mathbf{x}$ whose image under T is $\mathbf{b}$, and determine whether $\mathbf{x}$ is unique. Let $A = \begin{bmatrix} 1 & 4 & 5 \\ 0 & 1 & -2 \\ 4 & -17 & 20 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} -4 \\ -12 \\ -2 \end{bmatrix}$.

Find a single vector $\mathbf{x}$ whose image under T is $\mathbf{b}$.

$$\mathbf{x} = \begin{bmatrix} -55 \\ -14 \\ -1 \end{bmatrix}$$

Is the vector $\mathbf{x}$ found in the previous step unique?

- ☐ A. No, because there are no free variables in the system of equations.
- ☐ B. Yes, because there is a free variable in the system of equations.
- ☐ C. No, because there is a free variable in the system of equations.
- ☒ D. Yes, because there are no free variables in the system of equations.

≡ Test: Final Exam

< Question 22 of 25



This test: 100 point(s) possible
This question: 4 point(s) possible

Compute the determinant by cofactor expansion. At each step, choose a row or column that involves the least amount of computation.

$$\begin{vmatrix} 3 & 0 & 0 & 4 \\ 2 & 8 & 3 & -8 \\ 2 & 0 & 0 & 0 \\ 9 & 2 & 1 & 8 \end{vmatrix} = \boxed{16} \text{ (Simplify your answer)}$$

$T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$, rotates points (about the origin) through $-\frac{\pi}{6}$ radians

$$A = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

(Type an integer or simplified fraction for each matrix element. Type exact answers, using radicals as needed.)

Find the inverse of the matrix

$$\begin{bmatrix} 8 & 6 \\ 9 & 7 \end{bmatrix}$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

☒ A.

The inverse matrix is

$$\begin{bmatrix} \frac{7}{2} & -3 \\ -\frac{9}{2} & 4 \end{bmatrix}$$

(Type an integer or simplified fraction for each matrix element.)

☐ B. The matrix is not invertible



Find the change-of-coordinates matrix from B to the standard basis in $\mathbb{R}^3$.

$$B = \left\{ \begin{bmatrix} 2 \\ 8 \\ 5 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ -6 \end{bmatrix}, \begin{bmatrix} 3 \\ -3 \\ 2 \end{bmatrix} \right\}$$

$$P_B = \begin{bmatrix} 2 & -3 & 3 \\ 8 & 0 & -3 \\ 5 & -6 & 2 \end{bmatrix}$$