The performances of *Zerah Colburn* in London, in 1812, were more remarkable. Colburn,[2] born in 1804, at Cabut, Vermont, U.S.A., was the son of a small farmer. While still less than six years old he showed extraordinary powers of mental calculation, which were displayed in a tour in America. Two years later he was brought to England, where he was repeatedly examined by competent observers. He could instantly give the product of two numbers each of four digits, but hesitated if both numbers exceeded 10,000. Among questions asked him at this time were to raise 8 to the 16th power; in a few seconds he gave the answer 281,474,976,710,-656, which is correct. He was next asked to raise the numbers 2, 3, . . . 9 to the 10th power: and he gave the answers so rapidly that the gentleman who was taking them down was obliged to ask him to repeat them more slowly; but he worked less quickly when asked to raise numbers of two digits like 37 or 59 to high powers. He gave instantaneously the square roots and cube roots (when they were integers) of high numbers, *e.g.*, the square root of 106,929 and the cube root of 268,336,125, such integral roots can, however, be obtained easily by various methods. More remarkable are his answers to questions on the factors of numbers. Asked for the factors of 247,483 he replied 941 and 263; asked for the factors of 171,395 he gave 5, 7, 59, and 83; asked for the factors of 36,083 he said there were none. He, however, found it difficult to answer questions about the factors of numbers higher than 1,000,000. His power of factorizing high numbers was exceptional and depended largely on the method of two-digit terminals described below. Like all these public performers he had to face buffoons who tried to make fun of him, but he was generally equal to them. Asked on one such occasion how many black beans were required to make three white ones, he is said to have at once replied "three, if you skin them"—this, however, has much the appearance of a pre-arranged show.

It was clear to observers that the child operated by certain rules, and

[2] To the authorities mentioned by E. W. Scripture and F. D. Mitchell should be added *The Annual Register*, London, 1812, p. 507 *et seq.*

during his calculations his lips moved as if he was expressing the process in words. Of his honesty there seems to have been no doubt. In a few cases he was able to explain the method of operation. Asked for the square of 4,395 he hesitated, but on the question being repeated he gave the correct answer, namely 19,395,025. Questioned as to the cause of his hesitation, he said he did not like to multiply four figures by four figures, but said he, "I found out another way; I multiplied 293 by 293 and then multiplied this product twice by the number 15." On another occasion when asked for the product of 21,734 by 543 he immediately replied 11,801,562; and on being questioned explained that he had arrived at this by multiplying 65,202 by 181. These remarks suggest that whenever convenient he factorized the numbers with which he was dealing.

In 1814 he was taken to Paris, but amid the political turmoil of the time his exhibitions fell flat. His English and American friends however raised money for his education, and he was sent in succession to the Lycée Napoleon in Paris and Westminster School in London. With education his calculating powers fell off, and he lost the frankness which when a boy had charmed observers. His subsequent career was diversified and not altogether successful. He commenced with the stage, then tried schoolmastering, then became an itinerant preacher in America, and finally a "professor" of languages. He wrote his own biography which contains an account of the methods he used. He died in 1840.

Contemporary with Colburn we find another instance of a self-taught boy, *George Parker Bidder*, who possessed quite exceptional powers of this kind. He is perhaps the most interesting of these prodigies because he subsequently received a liberal education, retained his calculating powers, and in later life analyzed and explained the methods he had invented and used.

Bidder was born in 1806 at Moreton Hampstead, Devonshire, where his father was a stone-mason. At the age of six he was taught to count up to 100, but though sent to the village school learnt little there, and at the beginning of his career was ignorant of the meaning of arithmetical terms and of numerical symbols. Equipped solely with this knowledge of counting he taught himself the results of addition, subtraction, and multiplication of numbers (less than 100) by arranging and rearranging marbles, buttons, and shot in patterns. In after-life he attached great importance to such concrete representations, and believed that his arithmetical powers were strengthened by the fact that at that time he knew nothing about the symbols for numbers. When seven years old he heard a dispute between two of his neighbours about the price of something which was being sold by the pound, and to their astonishment remarked that they were both wrong, mentioning the correct price. After this exhibition the villagers delighted in trying to pose him with arithmetical problems.

His reputation increased and, before he was nine years old, his father found it profitable to take him about the country to exhibit his powers. A couple of distinguished Cambridge graduates (Thomas Jephson, then tutor of St. John's, and John Herschel) saw him in 1817, and were so impressed by his general intelligence that they raised a fund for his education, and induced his father to give up the rôle of showman; but after a few months Bidder senior repented of his abandonment of money so easily earned, insisted on his son's return, and began again to make an exhibition of the boy's powers. In 1818, in the course of a tour young Bidder was pitted against Colburn and on the whole proved the abler calculator. Finally the father and son came to Edinburgh, where some members of that University intervened and persuaded his father to leave the lad in their care to be educated. Bidder remained with them, and in due course graduated at Edinburgh, shortly afterwards entering the profession of civil engineering in which he rose to high distinction. He died in 1878.

With practice Bidder's powers steadily developed. His earlier performances seem to have been of the same type as those of Buxton and Colburn which have been already described. In addition to answering questions on products of numbers and the number of specified units in given quantities, he was, after 1819, ready in finding square roots, cube roots, &c. of high numbers, it being assumed that the root is an integer, and later explained his method which is easy of application: this method is the same as that used by Colburn. By this time he was able to give immediate solutions of easy problems on compound interest and annuities which seemed to his contemporaries the most astonishing of all his feats. In factorizing numbers he was less successful than Colburn and was generally unable to deal at sight with numbers higher than 10,000. As in the case of Colburn, attempts to be witty at his expense were often made, but he could hold his own. Asked at one of his performances in London in 1818, how many bulls' tails were wanted to reach to the moon, he immediately answered one, if it is long enough.

Here are some typical questions put to and answered by him in his exhibitions during the years 1815 to 1819—they are taken from authenticated lists which comprise some hundreds of such problems: few, if any, are inherently difficult. His rapidity of work was remarkable, but the time limits given were taken by unskilled observers and can be regarded as only approximately correct. Of course all the calculations were mental without the aid of books, pencil, or paper. In 1815, being then nine years old, he was asked:—If the moon be distant from the earth 123,256 miles, and sound travels at the rate of 4 miles a minute,* how long would it be before the inhabitants of the moon could hear of the battle of Waterloo: answer, 21 days, 9 hours, 34 minutes, given in less than one minute. In 1816, being then ten years old, just learning to write, but unable to form figures, he answered questions such as the following:—What is the interest on £11,111 for 11,111 days at 5 per cent. a year: answer, £16,911. 11*s*., given in one minute. How many hogsheads of cider can be made from a million of apples, if 30 apples make one quart: answer, 132 hogsheads, 17 gallons, 1 quart, and 10 apples over, given in 35 seconds. If a coach-wheel is 5 feet 10 inches in circumference, how many times will it revolve in running 800,000,000 miles: answer, 724,114,285,704 times and 20 inches remaining, given in 50 seconds. What is the square root of 119,-550,669,121: answer 345,761, given in 30 seconds. In 1817, being then eleven years old, he was asked:—How long would it take to fill a reservoir whose volume is one cubic mile if there flowed into it from a river 120 gallons of water a minute: answered in 2 minutes. Assuming that light travels from the sun to the earth in 8 minutes, and that the sun is 98,000,-000 miles off, if light takes 6 years 4 months travelling from the nearest fixed star to the earth, what is the distance of that star, reckoning 365 days 6 hours to each year and 28 days to each month—asked by Sir William Herschel: answer, 40,633,740,000,000 miles. In 1818, at one of his performances, he was asked:—If the pendulum of a clock vibrates the distance of 9¾ inches in a second of time, how many inches will it vibrate in 7 years 14 days 2 hours 1 minute 56 seconds, each year containing 365 days 5 hours 48 minutes 55 seconds: answer, 2,165,625,744¾ inches, given in less than a minute. If I have 42 watches for sale and I sell the first for a farthing, and double the price for every succeeding watch I sell, what will be the price of the last watch: answer, £2,290,649,224. 10*s*. 8*d*. If the diameter of a penny piece is 1⅜ inches, and if the world is girdled with a ring of pence put side by side, what is their value sterling, supposing the distance to be 360 degrees, and a degree to contain 69.5 miles: answer, £4,803,340, given in one minute. Find two numbers, whose difference is 12, and whose product, multiplied by their sum, is equal to 14,560: answer, 14 and 26. In 1819, when fourteen years old, he was asked:—Find a number whose cube less 19 multiplied by its cube shall be equal to the cube of 6: answer, 3, given instantly. What will it cost to make a road for 21 miles 5 furlongs 37 poles 4 yards, at the rate of £123. 14*s*. 6*d*. a mile: answer, £2688. 13*s*. 9¾*d*., given in 2 minutes. If you are now 14 years old and you live 50 years longer and spend half-a-crown a day, how many farthings will you spend in your life: answer, 2,805,120, given in 15 seconds. Mr. Moor contracted to illuminate the city of London with 22,965,321 lamps, the expense of trimming and lighting was 7 farthings a lamp, the oil consumed was ⅜ths of a pint for every three lamps, and the oil cost 3*s*. 7½*d*. a gallon; he gained 16½ per cent. on his outlay: how many gallons of oil were consumed, what was the cost to him, and what was the amount of the contract: answer, he used 212,641 gallons of oil, the cost was £205,996. 16*s*. 1¾*d*., and the amount of the contract was £239,986. 13*s*. 2*d*. If the distance of the earth from the moon be 29,531,531¼ yards, what is the weight of a thread which will extend that distance, supposing 7¹¹⁄₁₆ yards of it weigh ⅟₁₆th part of a drachm: answer, 8 cwt. 1 qr. 13 lbs. 9 oz. 1 dr. and ¹³⁄₁₆ths of a drachm.

It should be noted that Bidder did not visualize a number like 984 in symbols, but thought of it in a concrete way as so many units which could be arranged in 24 groups of 41 each. It should also be observed that he, like Inaudi whom I mention later, relied largely on the auditory sense to enable him to recollect numbers. "For my own part," he wrote, in later life, "though much accustomed to see sums and quantities expressed by the usual symbols, yet if I endeavour to get any number of figures that are represented on paper fixed in my memory, it takes me a much longer time and a very great deal more exertion than when they are expressed or enumerated verbally." For instance suppose a question put to find the product of two numbers each of nine digits, if they were "read to me, I should not require this to be done more than once; but if they were represented in the usual way, and put into my hands, it would probably take me four times to peruse them before it would be in my power to repeat them, and after all they would not be impressed so vividly on my imagination."

Bidder retained his power of rapid mental calculation to the end of his life, and as a constant parliamentary witness in matters connected with engineering it proved a valuable accomplishment. Just before his death an illustration of his powers was given to a friend who talking of then recent discoveries remarked that if 36,918 waves of red light which only occupy one inch are required to give the impression of red, and if light travels at 190,000 miles a second, how immense must be the number of waves which must strike the eye in one second to give the impression of red. "You need not work it out," said Bidder, "the number will be 444,433,-651,200,000."

Other members of the Bidder family have also shown exceptional powers of a similar kind as well as extraordinary memories. Of Bidder's elder brothers, one became an actuary, and on his books being burnt in a fire he rewrote them in six months from memory but, it is said, died of consequent brain fever; another was a Plymouth Brother and knew the whole Bible by heart, being able to give chapter and verse for any text quoted. Bidder's eldest son, a lawyer of eminence, was able to multiply together two numbers each of fifteen digits. Neither in accuracy nor rapidity was he equal to his father, but then he never steadily and continuously devoted himself to developing his abilities in this direction. He remarked that in his mental arithmetic he worked with pictures of the figures, and said "If I perform a sum mentally it always proceeds in a visible form in my mind; indeed I can conceive no other way possible of doing mental arithmetic": this it will be noticed is opposed to his father's method. Two of his children, one son and one daughter representing a third generation, have inherited analogous powers.

I mention next the names of *Henri Mondeux*, and *Vito Mangiamele*. Both were born in 1826 in humble circumstances, were sheep-herds, and became when children, noticeable for feats in calculation which deservedly procured for them local fame. In 1839 and 1840 respectively they were brought to Paris where their powers were displayed in public, and tested by Arago, Cauchy, and others. Mondeux's performances were the more striking. One question put to him was to solve the equation $x^3 + 84 = 37x$: to this he at once gave the answer 3 and 4, but did not detect the third root, namely, -7. Another question asked was to find solutions of the indeterminate equation $x^2 - y^2 = 133$: to this he replied immediately 66 and 67; asked for a simpler solution he said after an instant 6 and 13. I do not however propose to discuss their feats in detail, for there was at least a suspicion that these lads were not frank, and that those who were exploiting them had taught them rules which enabled them to simulate powers they did not really possess. Finally both returned to farm work, and ceased to interest the scientific world. If Mondeux was self-taught we must credit him with a discovery of some algebraic theorems which would entitle him to rank as a mathematical genius, but in that case it is inconceivable that he never did anything more, and that his powers appeared to be limited to the particular problems solved by him.

Johann Martin Zacharias Dase, whom I next mention, is a far more interesting example of these calculating prodigies. Dase was born in 1824 at Hamburg. He had a fair education, and was afforded every opportunity to develop his powers, but save in matters connected with reckoning and numbers he made little progress and struck all observers as dull. Of geometry and any language but German he remained ignorant to the end of his days. He was trustworthy and filled various small official posts in Germany. He gave exhibitions of his calculating powers in Germany, Austria, and England. He died in 1861.

When exhibiting in Vienna in 1840, he made the acquaintance of Strasznicky who urged him to apply his powers to scientific purposes. This Dase gladly agreed to do, and so became acquainted with Gauss, Schumacher, Petersen, and Encke. To his contributions to science I allude later. In mental arithmetic the only problems to which I find allusions are straightforward examples like the following:—Multiply 79,532,853 by 93,758,479: asked by Schumacher, answered in 54 seconds. In answer to a similar request to find the product of two numbers each of twenty digits he took 6 minutes; to find the product of two numbers each of forty digits he took 40 minutes; to find the product of two numbers each of a hundred digits he took 8 hours 45 minutes. Gauss thought that perhaps on paper the last of these problems could be solved in half this time by a skilled computator. Dase once extracted the square root of a number of a hundred digits in 52 minutes. These feats far surpass all other records of the kind, the only calculations comparable to them being Buxton's squaring of a number of thirty-nine digits, and Wallis' extraction of the square root of a number of fifty-three digits. Dase's mental work however was not always accurate, and once (in 1845) he gave incorrect answers to every question put to him, but on that occasion he had a headache, and there is nothing astonishing in his failure.

Like all these calculating prodigies he had a wonderful memory, and an hour or two after a performance could repeat all the numbers mentioned in it. He had also the peculiar gift of being able after a single glance to state the number (up to about 30) of sheep in a flock, of books in a case, and so on; and of visualizing and recollecting a large number of objects. For instance, after a second's look at some dominoes he gave the sum (117) of their points; asked how many letters were in a certain line of print chosen at random in a quarto page he instantly gave the correct number (63); shown twelve digits he had in half a second memorized them and their positions so as to be able to name instantly the particular digit occupying any assigned place. It is to be regretted that we do not know more of these performances. Those who are acquainted with the delightful autobiography of Robert-Houdin will recollect how he cultivated a similar power, and how valuable he found it in the exercise of his art.

Dase's calculations, when also allowed the use of paper and pencil, were almost incredibly rapid, and invariably accurate. When he was sixteen years old Strasznicky taught him the use of the familiar formula $\pi/4 = \tan^{-1}(1/2) + \tan^{-1}(1/5) + \tan^{-1}(1/8)$, and asked him thence to calculate π. In two months he carried the approximation to 205 places of decimals, of which 200 are correct.[3] Dase's next achievement was to calculate the natural logarithms of the first 1,005,000 numbers to 7 places of decimals; he did this in his off-time from 1844 to 1847, when occupied by the Prussian survey. During the next two years he compiled in his spare time a hyperbolic table which was published by the Austrian Government in 1857. Later he offered to make tables of the factors of all numbers from 7,000,000 to 10,000,000 and, on the recommendation of Gauss, the Hamburg Academy of Sciences agreed to assist him so that he might have leisure for the purpose, but he lived only long enough to finish about half the work.

[3] The result was published in *Crelle's Journal*, 1844, vol. XXVII, p. 198.