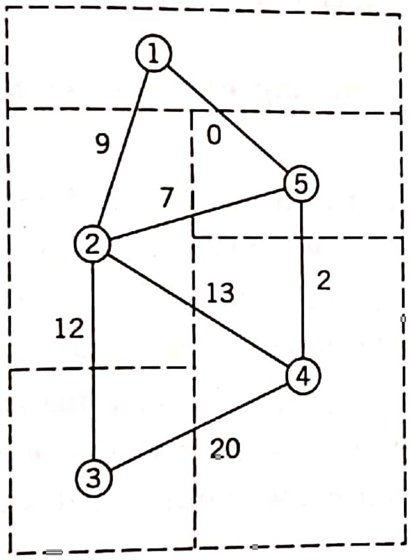
6.4.3 Graph-Based Method

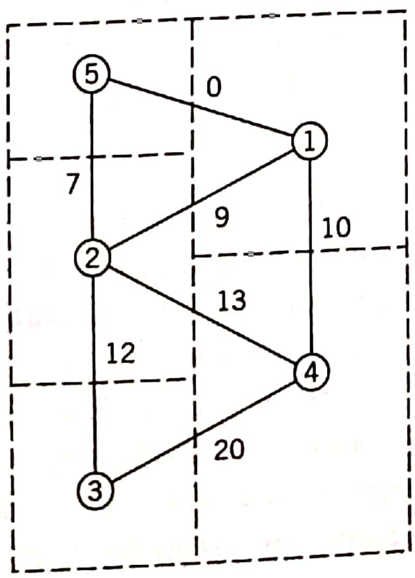
The graph-based method is a construction-type layout algorithm; it has its roots in graph theory. It is often used with an adjacency-based objective. The recognition of the usefulness of graph theory as a mathematical tool in the solution of facilities planning problems dates back to the late 1960s [35] and early 1970s [58]. Graph theory methods have similarities with the SLP method developed by Muther [49].

Consider the block layout shown in Figure 6.11*a*. We first construct an adjacency graph, where each node represents a department, with a connecting arc between two nodes indicating that two departments share a common border. A similar graph is constructed for the alternative block layout shown in Figure 6.11*b*. We observe that the two graphs shown in Figure 6.11 are subgraphs of the graph shown in



Arc	Weight
1-2	9
1-5	0
2-3	12
2-4	13
2-5	7
3-4	20
4-5	2
	63 (total)

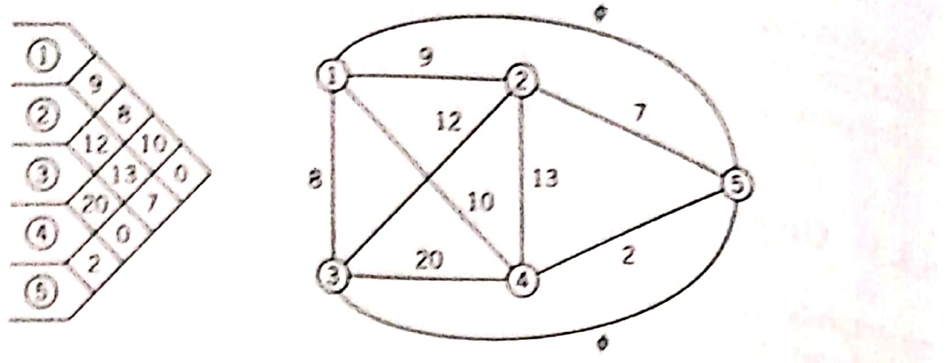
(*a*)



Arc	Weight
1-5	0
2-5	7
1-2	9
1-4	10
2-4	13
2-3	12
3-4	20
	71 (total)

(*b*)

Figure 6.11 Adjacency graphs for alternative block layouts.



(a) Relationship chart (b) Relationship diagram

Figure 6.12 Relationship chart and relationship diagram for graph-based example.

Figure 6.12*b*, which is derived from the relationship chart in Figure 6.12*a*. The relationship chart displays numerical "weights" rather than alphabetic closeness ratings.

Given the adjacency-based objective, block layout (b) is better than block layout (a) with scores of 71 and 63, respectively. Thus, finding a maximally weighted block layout is equivalent to obtaining an adjacency graph with the maximum sum of arc weights.

Before we describe a method for determining adjacency graphs, we first make the following observations:

a. The adjacency score does not account for distance, nor does it account for relationships other than those between adjacent departments.

b. Dimensional specifications of departments are not considered; the length of common boundaries between adjacent departments is also not considered.

c. The arcs do not intersect; this property of graphs is called planarity. We note that the graph obtained from the relationship diagram is usually a nonplanar graph.

d. The score is very sensitive to the assignment of numerical weights in the relationship chart.

There are two strategies we can follow in developing a maximally weighted planar adjacency graph. One way is to start with the graph from the relationship diagram and selectively prune connecting arcs while making sure that the final graph is planar. A second approach is to iteratively construct an adjacency graph via a node insertion algorithm while retaining planarity at all times. A heuristic procedure based on the second approach is described below.

6.4.3.1 Procedure

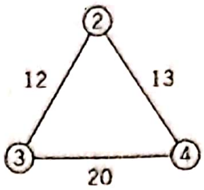
Step 1. From the relationship chart in Figure 6.12*a*, select the department pair with the largest weight. Ties, if any, are broken arbitrarily. Thus, departments 3 and 4 are selected to enter the graph.

Step 2. Next, select the third department to enter. The third department is selected based on the sum of the weights with respect to departments 3 and 4. From Figure 6.13*a*, department 2 is chosen with a value of 25. The columns in this figure correspond to the departments already in the adjacency graph, and the rows correspond to departments not yet selected. The last column gives the sum of the weights for each unassigned department.

a. Step 2

	3	4	Total
1	8	10	18
2	12	13	25 (best)
5	0	2	2

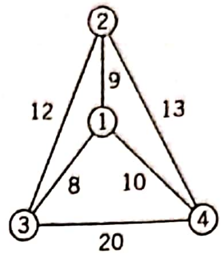
→



b. Step 3

	2	3	4	Total
1	9	8	10	27 (best)
5	7	0	9	9

→



c. Step 4

	1	2	3	4
5	0	7	0	2

Faces	Total
1-2-3	7
1-2-4	9 (best)
1-3-4	2
2-3-4	9 (best)

→

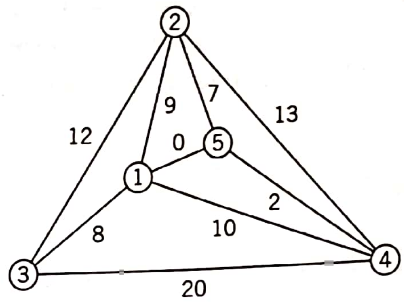
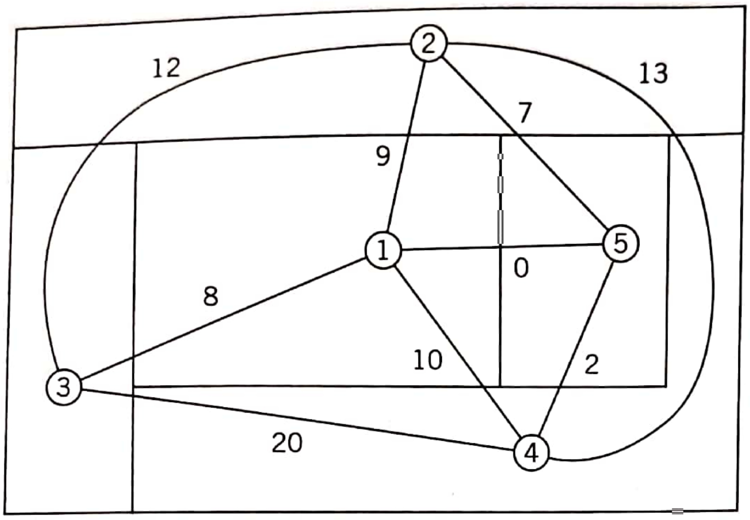


Figure 6.13 Steps of the graph-based procedure.

Step 3. We then pick the fourth department to enter by evaluating the value of adding one of the unassigned departments represented by a node on a face of the graph. A face of a graph is a bounded region of a graph. For instance, a triangular face is the region bounded by arcs 2-3, 3-4, and 4-2 in Figure 6.13*a*. We will denote this face as 2-3-4. The outside region is referred to as the external face. For our example, the value of adding departments 1 and 5 is 27 and 9, respectively. Department 1 is selected and placed inside the region 2-3-4, as shown in Figure 6.13*b*.

Step 4. The remaining task is to determine on which face to insert department 5. For this step, department 5 can be inserted on faces 1-2-3, 1-2-4, 1-3-4, and 2-3-4. Inserting 5 on faces 1-2-4 and 2-3-4 yields identical values of 9. We arbitrarily select 1-2-4. The final adjacency graph is given in Figure 6.13*c*. This solution is optimal, with a total sum of arc weights equal to 81.

Step 5. Having determined an adjacency graph, the final step is to construct a corresponding block layout. A block layout based on the final adjacency graph is shown in Figure 6.14. The manner by which we constructed the block layout is analogous to the SLP method. We should note that in constructing the block layout, the original department shapes had to be altered significantly in order to satisfy the requirements of the adjacency graph. In practice, we may not have as much latitude in making such alterations since department shapes are generally derived from the geometry of the individual machines within the department and the internal layout



Arc	Weight
1-2	9
1-3	8
1-4	10
1-5	0
2-3	12
2-4	13
2-5	7
3-4	20
4-5	2
	81 (total)

Figure 6.14 Block layout from the final adjacency graph.

configuration. We will discuss department shapes and their control later in this chapter (see Section 6.5). Finally, we should point out that there are algorithmic methods for performing this step as demonstrated by Giffin et al. [20] and Hassan and Hogg [23].