Problem 1. We found that the series solution in powers of x of Airy's equation

$$y'' - xy = 0, \quad -\infty < x < \infty$$

has the form

$$\begin{aligned} y(x) = a_0 & \left[1 + \frac{x^3}{2\cdot 3} + \frac{x^6}{2\cdot 3\cdot 5\cdot 6} + \cdots + \frac{x^{3n}}{2\cdot 3\cdot 5\cdot 6\cdots(3n-1)(3n)} + \dots\right] + \\ a_1 & \left[x + \frac{x^4}{3\cdot 4} + \frac{x^7}{3\cdot 4\cdot 6\cdot 7} + \cdots + \frac{x^{3n+1}}{3\cdot 4\cdot 6\cdot 7\cdots(3n)(3n+1)} + \dots\right]. \end{aligned}$$

Find the intervals of convergence of the two series that you see above.

Problem 2. For the given DEs, seek power series solutions in the form

$$y = \sum_{n=0}^{\infty} a_n x^n$$

and find the recurrence relation for a_n. Then, find the first four nonzero terms of two solutions y_1 and y_2 ($y = a_0 y_1(x) + a_1 y_2(x)$). By evaluating the Wronskian $W(y_1, y_2)(x_0)$, show that y_1 and y_2 form a fundamental set of solutions. Finally, find the general term a_n in each solution.

a) $y'' - xy' - y = 0$,

b) $y'' + xy' + 2y = 0$.

Problem 3. The Hermite polynomial $H_n(x)$ of degree n is defined as the polynomial solution of the Hermite equation

$$y'' - 2xy' + 2ny = 0, \quad -\infty < x < \infty$$

for which the coefficient of x^n (the leading coefficient) is 2^n. Find $H_0(x)$, $H_1(x)$, $H_2(x)$, $H_3(x)$, $H_4(x)$, and $H_5(x)$.