WK7 Assignment #6

9. The 0–1 integer programming problem
   a. requires the decision variables to have values between 0 and 1.
   b. requires all the constraints to have coefficients between 0 and 1.
   c. requires the decision variables to have coefficients between 0 and 1.
   d. requires the decision variables to be equal to 0 or 1.
10. Goal programming
   a. requires only that you know whether the goal is direct profit maximization or cost minimization.
   b. allows you to have multiple goals.
   c. is an algorithm with the goal of a quicker solution to the pure integer programming problem.
   d. is an algorithm with the goal of a quicker solution to the mixed-integer programming problem.
11. Nonlinear programming includes
   a. problems in which the objective function is linear but some constraints are not linear.
   b. problems in which the constraints are linear but the objective function is not linear.
   c. problems in which both the objective function and all of the constraints are not linear.
   d. problems that are solvable by quadratic programming.
   e. all of the above.

Discussion Questions and Problems

Discussion Questions

10-1 Compare the similarities and differences of linear and goal programming.

10-2 A linear programming problem was developed, and the feasible region was found. If the additional restriction that all variables must be integers were added to the problem, how would the size of the feasible region change? How would the optimal value of the objective function change?

10-3 List the advantages and disadvantages of solving integer programming problems by (a) rounding off and (b) enumeration.

10-4 How do the three types of integer programming problems differ? Which do you think is most common, and why?

10-5 What is meant by *satisficing*, and why is the term often used in conjunction with goal programming?

10-6 What are deviational variables? How do they differ from decision variables in traditional LP problems?

10-7 If you were the president of the college you are attending and were employing goal programming to assist in decision making, what might your goals be? What kinds of constraints would you include in your model?

10-8 What does it mean to rank goals in goal programming? How does this affect the problem's solution?

10-9 Which of the following are NLP problems, and why?

(a) Maximize profit $= 3X_1 + 5X_2 + 99X_3$
subject to $X_1 \geq 10$
$X_2 \leq 5$
$X_3 \geq 18$

(b) Maximize cost $= 25X_1 + 30X_2 + 8X_1X_2$
subject to $X_1 \geq 8$
$X_1 + X_2 \geq 12$
$0.0005X_1 - X_2 = 11$

(c) Maximize $Z = P_1d_1^- + P_2d_2^+ + P_3^+$
subject to $X_1 + X_2 + d_1^- - d_1^+ = 300$
$X_2 + d_2^- - d_2^+ = 200$
$X_1 + d_3^- - d_3^+ = 100$

(d) Maximize profit $= 3X_1 + 4X_2$
subject to $X_1^2 - 5X_2 \geq 8$
$3X_1 + 4X_2 \geq 12$

(e) Minimize cost $= 18X_1 + 5X_2 + X_2^2$
subject to $4X_1 - 3X_2 \geq 8$
$X_1 + X_2 \geq 18$

Are any of these quadratic programming problems?

Problems

10-10 Elizabeth Bailey is the owner and general manager of Princess Brides, which provides a wedding planning service in southwestern Louisiana. She uses radio advertising to market her business. Two types of ads are available—those during prime-time hours and those at other times. Each prime-time ad costs $390 and reaches 8,200 people, while the off-peak ads each cost $240 and reach 5,100 people. Bailey has budgeted $1,800 per week for advertising. Based on comments from her customers, Bailey wants to have at least two prime-time ads and no more than six off-peak ads.

(a) Formulate this as a linear program.
(b) Find a good or optimal integer solution for part (a) by rounding off or making an educated guess at the answer.

Note: means the problem may be solved with QM for Windows; means the problem may be solved with Excel; and means the problem may be solved with QM for Windows and/or Excel.

n integer programming problem
er.

e students is planning a camping
oming break. The group must hike
igh the woods to get to the camp-
that is needed on this trip must be
ack and carried to the campsite.
ident, Tina Shawl, has identified
e would like to take on the trip, but
ght is too great to take all of them.
o rate the utility of each item on a
vith 100 being the most beneficial.
ht in pounds and utility value are

3	4	5	6	7	8
7	6	3	12	5	14
50	55	50	75	30	70

the hike to the campsite is a long
pounds has been set as the maxi-
of the items to be carried.

s as a 0–1 programming problem
he total utility of the items carried.
psack problem using a computer.
3 is an extra battery pack, which
with several of the other items.
ded that she will take item 5, a CD
she also takes item 3. On the other
akes item 3, she may or may not
Modify the problem to reflect this,
new problem.

es sells two sizes of wall posters, a
t poster and a smaller 2- by 3-foot
earned from the sale of each large
smaller poster earns $2. The firm,
le, is not large; it consists of one art
sing, at the University of Kentucky.
lassroom schedule, Jan has the fol-
nstraints: (1) up to three large post-
2) up to five smaller posters can be
to 10 hours can be spent on posters
with each large poster requiring 2
nd each smaller one taking 1 hour.
er almost over, Jan plans on taking
ier vacation to England and doesn't
y unfinished posters behind. Find
on that will maximize her profit.

an aging fleet of Boeing 737 jet air-
sidering a major purchase of up to
nodel 787 and 767 jets. The decision
ccount numerous cost and capabil-
ding the following: (1) the airline
o $1.6 billion in purchases; (2) each
cost $80 million, and each Boeing
10 million; (3) at least one-third of
the planes purchased should be the longer-range 787; (4) the annual maintenance budget is to be no more than $8 million; (5) the annual maintenance cost per 787 is estimated to be $800,000, and it is $500,000 for each 767; and (6) each 787 can carry 125,000 passengers per year, whereas each 767 can fly 81,000 passengers annually. Formulate this as an integer programming problem to maximize the annual passenger-carrying capability. What category of integer programming problem is this? Solve this problem.

: 10-14 Trapeze Investments is a venture capital firm that is currently evaluating six different investment opportunities. There is not sufficient capital to invest in all of these, but more than one will be selected. A 0–1 integer programming model is planned to help determine which of the six opportunities to choose. Variables X_1, X_2, X_3, X_4, X_5, and X_6 represent the six choices. For each of the following situations, write a constraint (or several constraints) that would be used.

(a) At least three of these choices are to be selected.
(b) Either investment 1 or investment 4 must be undertaken but not both.
(c) If investment 4 is selected, then investment 6 must also be selected. However, if investment 4 is not selected, it is still possible to select investment 6.
(d) Investment 5 cannot be selected unless investments 2 and 3 are both also selected.
(e) Investment 5 must be selected if investments 2 and 3 are both also selected.

: 10-15 Horizon Wireless, a cellular telephone company, is expanding into a new era. Relay towers are necessary to provide wireless telephone coverage to the different areas of the city. A grid is superimposed on a map of the city to help determine where the towers should be located. The grid consists of 8 areas labeled A through H. Six possible tower locations (numbered 1–6) have been identified, and each location could serve several areas. The following table indicates the areas served by each of the towers.

TOWER LOCATION	1	2	3	4	5	6
AREAS SERVED	A, B, D	B, C, G	C, D, E, F	E, F, H	E, G, H	A, D, F

Formulate this as a 0–1 programming model to minimize the total number of towers required to cover all the areas. Solve this using a computer.

: 10-16 Innis Construction Company specializes in building moderately priced homes in Cincinnati, Ohio. Tom Innis has identified eight potential locations to construct new single-family dwellings, but he cannot put up homes on all of the sites because he has only $300,000 to invest in all projects. The following table shows the cost of constructing homes in each area and the expected profit to of each home. Note that the ho fer considerably due to lot c and differences in the models that a fraction of a home cann

LOCATION	COST OF BUILDI AT THIS SITE ($
Clifton	60,000
Mt. Auburn	50,000
Mt. Adams	82,000
Amberly	103,000
Norwood	50,000
Covington	41,000
Roselawn	80,000
Eden Park	69,000

(a) Formulate Innis's pro programming.
(b) Solve with QM for Wi

: 10-17 A real estate developer is projects: a small apartmer ping center, and a mini-v requires different funding the net present values of The following table pro ment amounts (in $1,000s (NPV) of each (also expr

	NPV
Apartment complex	18
Shopping center	15
Mini-warehouse	14

The company has $80, $50,000 to invest in yea

(a) Develop an integer l mize the NPV in thi
(b) Solve the problem software. Which of undertaken if NPV money would be us

: 10-18 Refer to the real estate lem 10.17.

(a) Suppose that the s ment complex wo and that the shoppi only if the apartn Formulate the con
(b) Formulate a cons two of the three p

10-16

area and the expected profit to be made from the sale of each home. Note that the home-building costs differ considerably due to lot costs, site preparation, and differences in the models to be built. Note also that a fraction of a home cannot be built.

LOCATION	COST OF BUILDING AT THIS SITE ($)	EXPECTED PROFIT ($)
Clifton	60,000	5,000
Mt. Auburn	50,000	6,000
Mt. Adams	82,000	10,000
Amberly	103,000	12,000
Norwood	50,000	8,000
Covington	41,000	3,000
Roselawn	80,000	9,000
Eden Park	69,000	10,000

(a) Formulate Innis's problem using 0–1 integer programming.
(b) Solve with QM for Windows or Excel.

10-17 A real estate developer is considering three possible projects: a small apartment complex, a small shopping center, and a mini-warehouse. Each of these requires different funding over the next 2 years, and the net present values of the investments also vary. The following table provides the required investment amounts (in $1,000s) and the net present value (NPV) of each (also expressed in $1,000s):

		INVESTMENT	
	NPV	YEAR 1	YEAR 2
Apartment complex	18	40	30
Shopping center	15	30	20
Mini-warehouse	14	20	20

The company has $80,000 to invest in year 1 and $50,000 to invest in year 2.

(a) Develop an integer programming model to maximize the NPV in this situation.
(b) Solve the problem in part (a) using computer software. Which of the three projects would be undertaken if NPV is maximized? How much money would be used each year?

10-18 Refer to the real estate investment situation in Problem 10.17.

(a) Suppose that the shopping center and the apartment complex would be on adjacent properties and that the shopping center would be considered only if the apartment complex were also built. Formulate the constraint that would stipulate this.
(b) Formulate a constraint that would force exactly two of the three projects to be undertaken.

10-19 Triangle Utilities provides electricity for three cities. The company has four electric generators that are used to provide electricity. The main generator operates 24 hours per day, with an occasional shutdown for routine maintenance. Three other generators (1, 2, and 3) are available to provide additional power when needed. A start-up cost is incurred each time one of these generators is started. The start-up costs are $6,000 for 1, $5,000 for 2, and $4,000 for 3. These generators are used in one of the following ways: a generator may be started at 6:00 A.M. and run for either 8 hours or 16 hours, or it may be started at 2:00 P.M. and run for 8 hours (until 10:00 P.M.). All generators except the main generator are shut down at 10:00 P.M. Forecasts indicate the need for 3,200 megawatts more than provided by the main generator before 2:00 P.M., and this need goes up to 5,700 megawatts between 2:00 and 10:00 P.M. Generator 1 may provide up to 2,400 megawatts, generator 2 may provide up to 2,100 megawatts, and generator 3 may provide up to 3,300 megawatts. The cost per megawatt used per 8-hour period is $8 for 1, $9 for 2, and $7 for 3.

(a) Formulate this problem as an integer programming problem to determine the least-cost way to meet the needs of the area.
(b) Solve using computer software.

10-20 The campaign manager for a politician who is running for reelection to a political office is planning the campaign. Four ways to advertise have been selected: TV ads, radio ads, billboards, and social media advertising buys. The costs of these are $900 for each TV ad, $500 for each radio ad, $600 for a billboard for 1 month, and $180 for each buy on social media (approximately 40,000 unique impressions). The audience reached by each type of advertising has been estimated to be 40,000 for each TV ad, 32,000 for each radio ad, 34,000 for each billboard, and 17,000 for each social media buy. The total monthly advertising budget is $16,000. The following goals have been established and ranked:

1. The number of people reached should be at least 1,500,000.
2. The total monthly advertising budget should not be exceeded.
3. Together, the number of ads on either TV or radio should be at least 6.
4. No more than 10 ads/buys of any one type should be used.

(a) Formulate this as a goal programming problem.
(b) Solve this using computer software.
(c) Which goals are exactly met and which are not?

10-21 Geraldine Shawhan is president of Shawhan File Works, a firm that manufactures two types of metal file cabinets. The demand for her two-drawer model is up to 600 cabinets per week; the demand for

needed per week
(eating, personal
ry, and so on)
and studying
time off base
its, and so on)

udy 30 hours a week
al. This is his most
at students need at
average and that this
iat goal number 3 is
week of social time.

gramming problem.
nputer software.

cial planner (CFP),
vest $250,000. This
, bonds, or a mutual
return on investment
and 10% for real es-
a very high expected
ith a 10% expected
considerations, sev-
o keep the risk at an
put at least 30% of
l is that the amount
t exceed 50% of the
ds combined. In ad-
absolute restriction.
more than $150,000

gramming problem.
e equally important.
to solve this prob-
ld be put in each of
t is the total return?
et?

luces four- and six-
ngines. The firm's
ngine sold during
is $1,800 − 50X_1$,
d. Hinkel makes
larger engines sold,
ix-cylinder engines
oduction time avail-
le. A four-cylinder
production time,
130 hours to man-
ion planning prob-

ces two models of
XJ8. In any given
rcross has 40 hours
Each XJ6 requires
hours. The revenue

(in $1,000s) for the firm is nonlinear and is stated as (Number of XJ6s)(4 − 0.1 number of XJ6s) + (Number of XJ8s)(5 − 0.2 number of XJ8s).

(a) Formulate this problem.
(b) Solve using Excel.

10-29 During the busiest season of the year, Green-Gro Fertilizer produces two types of fertilizers. The standard type (X) is just fertilizer, and the other type (Y) is a special fertilizer and weed-killer combination. The following model has been developed to determine how much of each type should be produced to maximize profit subject to a labor constraint:

$$\text{Maximize profit} = 12X - 0.04X^2 + 15Y - 0.06Y^2$$
$$\text{subject to} \quad 2X + 4Y \leq 160 \text{ hours}$$
$$X, Y \geq 0$$

Find the optimal solution to this problem.

10-30 Pat McCormack, a financial advisor for Investors R Us, is evaluating two stocks in a particular industry. He wants to minimize the variance of a portfolio consisting of these two stocks, but he wants to have an expected return of at least 9%. After obtaining historical data on the variance and returns, he develops the following nonlinear program:

$$\text{Minimize portfolio variance} = 0.16X^2 + 0.2XY + 0.09Y^2$$
$$\text{subject to} \quad X + Y = 1 \quad \text{(all funds must be invested)}$$
$$0.11X + 0.08Y \geq 0.09 \quad \text{(return on the investment)}$$
$$X, Y \geq 0$$

where

X = proportion of money invested in stock 1
Y = proportion of money invested in stock 2

Solve this using Excel and determine how much to invest in each of the two stocks. What is the return for this portfolio? What is the variance of this portfolio?

10-31 Summertime Tees sells two very popular styles of embroidered shirts in southern Florida: a tank top and a regular T-shirt. The cost of the tank top is $6, and the cost of the T-shirt is $8. The demand for these is sensitive to the price, and historic dicate that the weekly demands are given b

$$X_1 = 500 - 12P_1$$
$$X_2 = 400 - 15P_2$$

where

X_1 = demand for tank top
P_1 = price for tank top
X_2 = demand for regular T-shirt
P_2 = price for regular T-shirt

(a) Develop an equation for the total profi
(b) Use Excel to find the optimal solution lowing nonlinear program. Use the p tion developed in part (a).

Maximize profit
subject to
$$X_1 = 500 - 12P_1$$
$$X_2 = 400 - 15P_2$$
$$P_1 \leq 20$$
$$P_2 \leq 25$$
$$X_1, P_1, X_2, P_2 \geq 0$$

10-32 The integer programming problem belov developed to help First National Bank dec out of 10 possible sites, to locate four new fices: X_i represents Winter Park, Maitlan Downtown, South Orlando, Airport, Wint Apopka, Lake Mary, and Cocoa Beach, f 1 to 10, respectively.

(a) Where should the four new sites be l what will be the expected return?
(b) If at least one new branch *must* in Maitland or Osceola, will this answers? Add the new constraint and
(c) The expected return at Apopka wa mated. The correct value is $160,0 (i.e., 160). Using the original assump is, ignoring part (b)—does your ans (a) change?

IP for Problem 10.32

Maximize expected returns = $120X_1 + 100X_2 + 110X_3 + 140X_4 + 155X_5 + 128X_6 + 145X_7 + 190X_8 + 170X_9 +$

subject to

$$\begin{aligned}
20X_1 + 30X_2 + 20X_3 + 25X_4 + 30X_5 + 30X_6 + 25X_7 + 20X_8 + 25X_9 + 30X_{10} &\leq 110 \\
15X_1 + 5X_2 + 20X_3 + 20X_4 + 5X_5 + 5X_6 + 10X_7 + 20X_8 + 5X_9 + 20X_{10} &\leq 50 \\
X_2 + X_6 + X_7 + X_9 + X_{10} &\leq 3 \\
X_2 + X_3 + X_5 + X_8 + X_9 &\geq 2 \\
X_1 + X_3 + X_{10} &\geq 1 \\
X_1 + X_2 + X_3 + X_4 + X_5 + X_6 + X_7 + X_8 + X_9 + X_{10} &\leq 4 \\
\text{All } X_i = 0 \text{ or } 1
\end{aligned}$$