

Discussion Questions and Problems

Discussion Questions

- 7-1 Discuss the similarities and differences between minimization and maximization problems using the graphical solution approaches of LP.
- 7-2 It is important to understand the assumptions underlying the use of any quantitative analysis model. What are the assumptions and requirements for an LP model to be formulated and used?
- 7-3 It has been said that each LP problem that has a feasible region has an infinite number of solutions. Explain.
- 7-4 You have just formulated a maximization LP problem and are preparing to solve it graphically. What criteria should you consider in deciding whether it would be easier to solve the problem by the corner point method or the isoprofit line approach?
- 7-5 Under what condition is it possible for an LP problem to have more than one optimal solution?
- 7-6 Develop your own set of constraint equations and inequalities, and use them to illustrate graphically each of the following conditions:
- an unbounded problem
 - an infeasible problem
 - a problem containing redundant constraints
- 7-7 The production manager of a large Cincinnati manufacturing firm once made the statement, "I would like to use LP, but it's a technique that operates under conditions of certainty. My plant doesn't have that certainty; it's a world of uncertainty. So LP can't be used here." Do you think this statement has any merit? Explain why the manager may have said it.
- 7-8 The mathematical relationships that follow were formulated by an operations research analyst at the Smith-Lawton Chemical Company. Which ones are invalid for use in an LP problem, and why?

$$\begin{array}{llll}
 \text{Maximize profit} & = & 4X_1 & + 3X_2 + 8X_3 + 5X_4 \\
 \text{subject to} & & 2X_1 & + X_2 + 2X_3 \leq 50 \\
 & & X_1 & - 4X_2 \geq 6 \\
 & & 1.5X_1^2 & + 6X_2 + 3X_3 \geq 21 \\
 & & & 19X_2 - 0.35X_3 = 17 \\
 & & 5X_1 & + 4X_2 + 3\sqrt{X_3} \leq 80 \\
 & & -X_1 & - X_2 + X_3 = 5
 \end{array}$$

- 7-9 Discuss the role of sensitivity analysis in LP. Under what circumstances is it needed, and under what conditions do you think it is not necessary?

- 7-10 A linear program has the objective of maximizing profit = $12X + 8Y$. The maximum profit is \$8,000. Using a computer, we find the upper bound for profit on X is 20 and the lower bound is 9. Discuss the changes to the optimal solution (the values of the variables and the profit) that would occur if the profit on X were increased to \$15. How would the optimal solution change if the profit on X were increased to \$25?
- 7-11 A linear program has a maximum profit of \$600. One constraint in this problem is $4X + 2Y \leq 80$. Using a computer, we find the dual price for this constraint is 3, and there is a lower bound of 75 and an upper bound of 100. Explain what this means.
- 7-12 Develop your own original LP problem with two constraints and two real variables.
- Explain the meaning of the numbers on the right-hand side of each of your constraints.
 - Explain the significance of the technological coefficients.
 - Solve your problem graphically to find the optimal solution.
 - Illustrate graphically the effect of increasing the contribution rate of your first variable (X_1) by 50% over the value you first assigned it. Does this change the optimal solution?
- 7-13 Explain how a change in a technological coefficient can affect a problem's optimal solution. How can a change in resource availability affect a solution?

Problems

- 7-14 The Electrocomp Corporation manufactures two electrical products: air conditioners and large fans. The assembly process for each is similar in that both require a certain amount of wiring and drilling. Each air conditioner takes 3 hours of wiring and 2 hours of drilling. Each fan must go through 2 hours of wiring and 1 hour of drilling. During the next production period, 240 hours of wiring time are available, and up to 140 hours of drilling time may be used. Each air conditioner sold yields a profit of \$25. Each fan assembled may be sold for a \$15 profit. Formulate and solve this LP production mix situation to find the best combination of air conditioners and fans that yields the highest profit. Use the corner point graphical approach.
- 7-15 Electrocomp's management realizes that it forgot to include two critical constraints (see Problem 7-14). In particular, management decides that there should be a minimum number of air conditioners produced.

in order to fulfill a contract. Also, due to an oversupply of fans in the preceding period, a limit should be placed on the total number of fans produced.

- If Electrocomp decides that at least 20 air conditioners should be produced but no more than 80 fans should be produced, what would be the optimal solution? How much slack or surplus is there for each of the four constraints?
- If Electrocomp decides that at least 30 air conditioners should be produced but no more than 50 fans should be produced, what would be the optimal solution? How much slack or surplus is there for each of the four constraints at the optimal solution?

Q: 7-16 A candidate for mayor in a small town has allocated \$40,000 for last-minute advertising in the days preceding the election. Two types of ads will be used: radio and television. Each radio ad costs \$200 and reaches an estimated 3,000 people. Each television ad costs \$500 and reaches an estimated 7,000 people. In planning the advertising campaign, the campaign manager would like to reach as many people as possible, but she has stipulated that at least 10 ads of each type must be used. Also, the number of radio ads must be at least as great as the number of television ads. How many ads of each type should be used? How many people will this reach?

Q: 7-17 The Outdoor Furniture Corporation manufactures two products, benches and picnic tables, for use in yards and parks. The firm has two main resources: its carpenters (labor force) and a supply of redwood for use in the furniture. During the next production cycle, 1,200 hours of labor are available under a union agreement. The firm also has a stock of 3,500 board feet of good-quality redwood. Each bench that Outdoor Furniture produces requires 4 labor hours and 10 board feet of redwood; each picnic table takes 6 labor hours and 35 board feet of redwood. Completed benches will yield a profit of \$9 each, and tables will result in a profit of \$20 each. How many benches and tables should Outdoor Furniture produce to obtain the largest possible profit? Use the graphical LP approach.

Q: 7-18 The dean of the Western College of Business must plan the school's course offerings for the fall semester. Student demands make it necessary to offer at least 30 undergraduate and 20 graduate courses in the term. Faculty contracts also dictate that at least 60 courses be offered in total. Each undergraduate course taught costs the college an average of \$2,500 in faculty wages, and each graduate course costs \$3,000. How many undergraduate and graduate courses should be taught in the fall so that total faculty salaries are kept to a minimum?

Q: 7-19 MSA Computer Corporation manufactures two models of smartphones, the Alpha 4 and the

Beta 5. The firm employs five technicians, working 160 hours each per month, on its assembly line. Management insists that full employment (i.e., all 160 hours of time) be maintained for each worker during next month's operations. It requires 20 labor hours to assemble each Alpha 4 computer and 25 labor hours to assemble each Beta 5 model. MSA wants to see at least 10 Alpha 4s and at least 15 Beta 5s produced during the production period. Alpha 4s generate \$1,200 profit per unit, and Beta 5s yield \$1,800 each. Determine the most profitable number of each model of smartphone to produce during the coming month.

Q: 7-20 A winner of the Texas Lotto has decided to invest \$50,000 per year in the stock market. Under consideration are stocks for a petrochemical firm and a public utility. Although a long-range goal is to get the highest possible return, some consideration is given to the risk involved with the stocks. A risk index on a scale of 1–10 (with 10 being the most risky) is assigned to each of the two stocks. The total risk of the portfolio is found by multiplying the risk of each stock by the dollars invested in that stock.

The following table provides a summary of the return and risk:

STOCK	ESTIMATED RETURN	RISK INDEX
Petrochemical	12%	9
Utility	6%	4

The investor would like to maximize the return on the investment, but the average risk index of the investment should not be higher than 6. How much should be invested in each stock? What is the average risk for this investment? What is the estimated return for this investment?

Q: 7-21 Referring to the Texas Lotto situation in Problem 7-20, suppose the investor has changed his attitude about the investment and wishes to give greater emphasis to the risk of the investment. Now the investor wishes to minimize the risk of the investment as long as a return of at least 8% is generated. Formulate this as an LP problem, and find the optimal solution. How much should be invested in each stock? What is the average risk for this investment? What is the estimated return for this investment?

Q: 7-22 Solve the following LP problem using the corner point graphical method. At the optimal solution, calculate the slack for each constraint:

$$\begin{aligned}
 &\text{Maximize profit} = 4X + 4Y \\
 &\text{subject to} \quad 3X + 5Y \leq 150 \\
 &\quad \quad \quad X - 2Y \leq 10 \\
 &\quad \quad \quad 5X + 3Y \leq 150 \\
 &\quad \quad \quad X, Y \geq 0
 \end{aligned}$$