Mathematics Difficulties

CHAPTER 14

I hear and I forget.
I see and I remember.
I do and I understand.

—Chinese Proverb

LEARNING OBJECTIVES

After reading this chapter, you should be able to:

14.1 Explain theories describing mathematics difficulties

14.2 Compare and contrast students with mathematics difficulties and those with mathematics learning disabilities

14.3 Describe characteristics of students with mathematics learning disabilities at the elementary level

14.4 Describe characteristics of students with mathematics learning disabilities at the secondary level

14.5 List the Common Core State Standards for mathematics

14.6 Outline theories for teaching mathematics

14.7 Describe how to assess mathematics achievement

14.8 List teaching strategies to improve mathematics difficulties

14.9 List mathematics strategies to be used in the general education classroom

14.10 Describe the mathematics curriculum

14.11 Describe teaching principles for students with mathematics disabilities

14.12 List activities for teaching mathematics

14.13 Discuss the use of technology for teaching mathematics

STANDARDS Addressed in This Chapter:

CEC

Council for Exceptional Children Initial Level Special Educator Preparation Standards as approved by the National Council for the Accreditation of Teacher Education

CEC Initial Preparation Standard 1: Learner Development and Individual Learning Differences

- 1.0—Beginning special education professionals understand how exceptionalities may interact with development and learning and use this knowledge to provide meaningful and challenging learning experiences for individuals with exceptionalities.
- 1.1—Beginning special education professionals understand how language, culture, and family background influence the learning of individuals with exceptionalities.
- 1.2—Beginning special education professionals use understanding of development and individual differences to respond to the needs of individuals with exceptionalities.

CEC Initial Preparation Standard 3: Curricular Content Knowledge

- 3.0—Beginning special education professionals use knowledge of general and specialized curricula to individualize learning for individuals with exceptionalities.
- 3.1—Beginning special education professionals understand the central concepts, structures of the discipline, and tools of inquiry of the content areas they teach and can organize this knowledge, integrate cross-disciplinary skills, and develop meaningful learning progressions for individuals with exceptionalities.
- 3.2—Beginning special education professionals understand and use general and specialized content knowledge for teaching across curricular content areas to individualize learning for individuals with exceptionalities.
- 3.3—Beginning special education professionals modify general and specialized curricula to make them accessible to individuals with exceptionalities.

CEC Initial Preparation Standard 4: Assessment

- 4.0—Beginning special education professionals use multiple methods of assessment and data-sources in making educational decisions.
- 4.1—Beginning special education professionals select and use technically sound formal and informal assessments that minimize bias.
- 4.2—Beginning special education professionals use knowledge of measurement principles and practices to interpret assessment results and guide educational decisions for individuals with exceptionalities.
- 4.3—Beginning special education professionals in collaboration with colleagues and families use multiple types of assessment information in making decisions about individuals with exceptionalities.
- 4.4—Beginning special education professionals engage individuals with exceptionalities to work toward quality learning and performance and provide feedback to guide them.

CEC Initial Preparation Standard 5: Instructional Planning and Strategies

- 5.0—Beginning special education professionals select, adapt, and use a repertoire of evidence-based instructional strategies to advance learning of individuals with exceptionalities.
- 5.1—Beginning special education professionals consider an individual's abilities, interests, learning environments, and cultural and linguistic factors in the selection, development, and adaptation of learning experiences for individuals with exceptionalities.
- 5.2—Beginning special education professionals use technologies to support instructional assessment, planning, and delivery for individuals with exceptionalities.
- 5.3—Beginning special education professionals are familiar with augmentative and alternative communication systems and a variety of assistive technologies to support the communication and learning of individuals with exceptionalities.
- 5.4—Beginning special education professionals use strategies to enhance language development and communication skills of individuals with exceptionalities.
- 5.6—Beginning special education professionals teach to mastery and promote generalization of learning.
- 5.7—Beginning special education professionals teach cross-disciplinary knowledge and skills such as critical thinking and problem solving to individuals with exceptionalities.

For students, the Common Core Standards for Math can be found at: *www.corestandards.org*

Some individuals with learning disabilities and related disabilities do well in language and reading, but their nemesis is with mathematics and quantitative learning. Two mathematics problem areas for students with learning disabilities are identified in the law through the Individuals With Disabilities Education Improvement Act of 2004 (IDEA-2004) are: (1) mathematics calculation and (2) mathematics reasoning. Difficulties in either of these areas of mathematics can interfere with mathematics achievement in school and with success in later life.

In the "Theories" section of this chapter, we examine (1) mathematics as a universal language, (2) mathematics difficulties, (3) early number concepts and number sense, (4) characteristics of mathematics disabilities, (5) mathematics disabilities at the secondary level, (6) mathematics standards, (7) learning theories for mathematics instruction, and (8) assessing mathematics skills.

In the "Teaching Strategies" section of this chapter, we discuss strategies and methods for teaching mathematics.

14.1 Theories Describing Difficulties With Mathematics

We live in a mathematical world. Every culture and language group uses concepts involved in quantity and math. Mathematics is a symbolic language, which enables human beings to think about, record, and communicate ideas about the elements and relationships of quantity. Mathematics is also a universal language because it has meaning for all cultures and civilizations. In every culture, social class, language, and ethnic group, children live in a natural environment that is rich in quantitative information and events. Human beings in all cultures, languages, social classes, and ethnic groups think about, record, and communicate ideas through quantity. Children in some cultures count blocks; children in other cultures count stones. Students rely on mathematical concepts when they think about the scores of their favorite baseball team, compare player standings, plan to purchase a CD, or pay for a movie ticket. When adolescents and adults plan their budget, balance a checkbook, or use a spreadsheet, they are using mathematics. The level of mathematical thinking and problem solving needed in the workplace and in day-to-day living has increased dramatically (National Council of Teachers of Mathematics, 2000, 2006).

problem solving
The kind of thinking needed to work out mathematics word problems.

Did You Get It?

Given that mathematics is universal in its scope, meaning, and use across all geographical areas and cultures, the statement that mathematics is ______ is inaccurate.

a. used for problem solving
b. a language based on symbols
c. considered the language of the intelligent but typically too difficult for a majority of the masses
d. used to explain and quantify

14.2 Students With Mathematics Difficulties and Students With Mathematics Learning Disabilities

Many students have difficulty in acquiring and using mathematics skills. Researchers differentiate 2 different groups: (1) students with math difficulties and (2) students with mathematics learning disabilities. In this book, we offer strategies for teaching both groups. Students with math difficulties perform poorly in mathematics achievement tests. Over 30% of eighth-grade students score below basic math performance on the National Assessment of Educational Progress (NAEP) (Maccini, et al., 2008; Mazzocco, 2007).

mathematics learning disabilities
Students whose learning disability is in the area of mathematics.

In contrast, students with mathematics learning disabilities comprise about 6% of the general population (Mazzocco, 2007). Mathematics learning disabilities is a biologically based disorder and is related to difficulties in cognitive processing and brain functioning. Research with functional Magnetic Resonance

Imaging (fMRI) shows these cognitive processing dysfunctions (Mazzoco, 2007; Gersten, Clarke, & Mazzocco, 2007).

Approximately 26% of students with learning disabilities exhibit problems in the area of mathematics. Over 50% of students with disabilities have mathematics goals written into their individualized education programs (IEPs) (Kunsch, Jitendra, & Sood, 2007; Miller & Hudson, 2007; Cass et al., 2003).

dyscalculia
A medical term indicating lack of ability to perform mathematical functions. The condition is associated with neurological dysfunction.

The term dyscalculia is a medically oriented term that describes a severe disability in mathematics with medical connotations. When an adult suffers a brain injury and loses abilities in arithmetic, medical professionals identify the loss of math skills related to the neurological impairment as dyscalculia. An analogous term in reading is *dyslexia*, the loss of reading skills that has medical and cognitive connotations.

Both mathematics difficulties and mathematics learning disabilities that emerge in elementary school often continue through the secondary school years. Not only is a mathematics disability a debilitating problem for individuals during school years, but it also continues to impair them as adults in their daily lives (Maccini, Mulcahy, & Wilson, 2007; Adelman & Vogel, 2003; Cass et al., 2003). Almost one-half of the children who are identified with severe mathematics difficulties in the fourth grade are still classified as having serious mathematics difficulties three years later (Gersten & Jordan, 2005; National Center for Learning Disabilities, 2006; Swanson, 2007).

It should be emphasized that not all students with learning disabilities or related disabilities encounter difficulty with number concepts. In fact, some individuals with severe reading disabilities do well in mathematics and exhibit a strong aptitude in quantitative thinking.

The identification and treatment of mathematics disabilities have received much less attention than problems associated with reading disabilities (Fuchs, Fuchs, & Hollenbeck, 2007; Gersten, Clarke, & Mazzocco, 2007). For students with mathematics difficulties, the mathematics curriculum in most general education classrooms does not pay sufficient attention to learning differences in mathematics among students. Moreover, the general education mathematics curriculum does not allot sufficient time for instruction, for guided practice, or for practical applications. Further, mathematical concepts are introduced at too rapid a rate for students who have difficulty with math. If students do not have sufficient time to fully grasp a mathematical concept and to practice it before another mathematical concept is introduced, they feel overwhelmed and become confused (Cawley & Foley, 2001; Butler et al., 2003).

14.2a Early Number Concepts and Number Sense

one-to-one correspondence
A relationship in which one element of a set is paired with one and only one element of a second set.

Number sense refers to the facility to think about quantity. Examples of number sense for young children include the ability to count, match and sort objects, and understand one-to-one correspondence. For some children, difficulties with number sense begin at an early age. Number sense hinges on the child's experience in manipulating objects. A child with unstable perceptual skills, attention problems, or difficulties in motor development may have insufficient experiences with the activities of manipulation that serve to pave the way for understanding quantity, space, order, time, or distance (Berch & Mazzocco, 2007; Kephart, 1971).

When children are expected to perform mathematics assignments, they may not have yet acquired the early skills needed for mathematics learning.

If these children are introduced to a number concept before they have the necessary prerequisite experiences, they will not understand, and they will be confused. Learning mathematics is a sequential process, and children must acquire skills at an earlier stage before going on to the next stage (Jordan et al., 2007).

Early number learning and number sense include skills in (1) one-to-one correspondence, (2) counting, (3) spatial relationships, (4) visual-motor and visual-perception skills, and (5) concepts of time and direction.

early number learning
The young child's early learning of quantitative concepts.

One-to-One Correspondence This refers to the ability to pair one element of a set to another element of a second set. For example, the child is able to place one cookie on a table for each child in a group.

Counting Counting entails the ability to count objects by numbers. Children may first have to point to objects in a set and then count each object verbally. Some children are unable to see objects in groups (or sets)—an ability needed to identify the number of objects quickly. Two developmental counting procedures are called *counting-all* and *counting-on*. With counting-all, when children count the objects in 2 groups, they count each object starting with number 1. With counting-on, children start with the number in the larger group and the count-on each number in the smaller group. Even when adding a group of 3 with a group of 4, some young children with mathematics difficulties persist in counting the objects starting with the number 1, instead of adding onto the number of the larger group (Bley & Thornton, 2001; Van de Walle, Karp, & Bay-Williams, 2010).

Spatial Relationships Typically, young children learn by playing with objects such as pots and pans, boxes that fit into each other, and objects that can be put into containers. These play activities help develop a sense of space, sequence, and order. Parents of children with mathematics difficulties often report that their child did not enjoy or play with blocks, puzzles, models, or construction-type toys as preschoolers. These children may have missed these early number-learning experiences.

Many concepts of spatial relationships are normally acquired at the preschool age. Children destined to have mathematics disabilities are baffled by such concepts as up-down, over-under, top-bottom, high-low, near-far, front-back, beginning-end, and across. The child may be unable to perceive distances between numbers on number lines or rulers and may not know whether the number 3 is closer to 4 or to 6.

spatial relationships
Concepts such as up-down, over-under, top-bottom, high-low, near-far, beginning-end, and across. A disturbance in spatial relationship can interfere with the visualization of the entire number system.

number lines
A sequence of numbers forming a straight line that allows the student to manipulate computation directly. Number lines help students develop an understanding of number symbols and their relationship to each other.

Visual-Motor and Visual-Perception Abilities Children with mathematics difficulties may have trouble with activities requiring visual-motor abilities and visual-perception abilities. Visual-motor abilities combine motor movement with what one sees, such as copying a figure or shape. Visual perception refers to the ability to interpret what one sees, for example, the ability visually to perceive a geometric shape as a complete and integrated entity. For a child with difficulty with visual perception, a square may not appear as a square shape, but rather as 4 unrelated lines. Some children may be unable to count objects in a series by pointing to each of them and saying, "1, 2, 3, 4, 5." These children must first learn to count by physically grasping and manipulating objects. Some children have difficulty in learning to perceive number symbols visually. They might confuse the vertical strokes of the number 1 and the number 4, or they may confuse the upper half of the number 2 with portions of the number 3.

Other children are unable to see objects in groups (or sets)—an ability needed to identify the number of objects quickly. Even when adding a group of 3 with a group of 4, some young children with mathematics difficulties persist in counting the objects starting with the number 1 to determine the total number in the groups instead of using the counting-on strategy. With the counting-on strategy, children learn to add onto the number of the larger group (Bley & Thornton, 2001; Van de Walle, Karp, & Bay-Williams, 2010).

Children with inadequate mathematics abilities often do poorly in visual-motor tasks. Because of their difficulties in perceiving shapes, recognizing spatial relationships, and making spatial judgments, they are unable to copy geometric forms, shapes, numbers, or letters. These children are likely to struggle with handwriting, as well as in arithmetic. When children cannot write numbers easily, they cannot properly align the numbers that they write, which leads to computation errors (Bley & Thornton, 2001).

Concepts of Time and Direction Basic concepts of time are typically acquired during the preschool years. For example, a 4-year-old counted the time until his grandmother would come to visit in terms of "sleeps" (e.g., Grandma will be here after three sleeps). Expressions such as "10 minutes ago," "in a half hour," and "later" are usually part of the preschooler's understanding and speaking vocabulary. By the end of first grade, children are expected to tell time to the half hour, and by the middle grades to the nearest minute.

Many students with mathematics difficulties have a poor sense of time and direction. They become lost easily and cannot find their way to a friend's house or to their own home from school. They sometimes forget whether it is morning or afternoon and may even go home during the recess period, thinking the school day has ended. Because they have difficulty estimating the time span of an hour, a minute, several hours, or a week, they cannot estimate how long a task will take. They may not be able to judge and allocate the time needed to complete an assignment.

Did You Get It?

While math disability affects approximately 6% of the general population, math difficulty affects approximately __________ out of 10 students.

a. 1
b. 3
c. 5
d. 7

14.3 Characteristics of Students With Mathematics Disabilities

A number of characteristics of students with learning disabilities and related disabilities affect quantitative learning. However, each student who encounters difficulties in mathematics is unique; not all exhibit the same traits. In this section, we discuss the following characteristics of students who have

TABLE 14.1

Information Processing and Problems in Mathematics

Information-Processing Problems	Effects on Mathematics Functioning
Motor problems	• Problems in writing numbers, illegible, slow, and inaccurate • Difficulty writing numbers in small spaces
Attention problems	• Poor attention doing the steps of mathematics in calculation • Poor attention during mathematics instruction
Problems in memory and retrieval	• Cannot remember math facts • Forgets the sequence of steps • Forgets the multiple steps in word problems
Problems in visual-spatial processing	• Difficulty in visual • Problems aligning numbers
Problems with auditory processing	• Difficulty with remembering auditory arithmetic facts • Difficulty with "counting-on"

mathematics difficulties or mathematics learning disabilities: (1) information-processing difficulties, (2) language and reading abilities, and (3) math anxiety.

14.3a Information-Processing Difficulties

The information-processing model of learning is discussed in Chapter 5. Briefly, information processing traces the flow of information during learning. Many of the elements of information processing are linked to mathematics learning, such as paying attention, visual-spatial processing, auditory processing, long-term memory and retrieval, working memory, and motor skills (Geary et al., 2007; Cirino et al., 2007; Wilson & Swanson, 2001). Table 14.1 shows how problems with elements of information processing affect mathematics.

14.3b Language and Mathematics Abilities

Early concepts of quantity are evidenced by the child's use of language, such as *all gone, that's all, more, big,* and *little.* Although some children with mathematics disabilities have superior verbal language skills and may even be excellent readers, for many children the mathematics disability is compounded by oral language and reading deficiencies. Their language problems may cause them to confuse mathematics terms such as *plus, take away, minus, carrying, borrowing,* and place value. Mathematics word problems are particularly difficult for students with reading disabilities. If they are unable to read or do not understand the underlying language structure of the mathematics problem, they cannot plan and perform the tasks required to solve the problem (Bley & Thornton, 2001).

place value
The aspect of the number system that assigns specific significance to the position a digit holds in a numeral.

14.3c Math Anxiety

Math anxiety is an emotion-based reaction to mathematics that causes individuals to freeze up when they confront math problems or when they take

math anxiety
Refers to a debilitating emotional reaction to mathematics situations.

math tests. The anxiety may stem from the fear of school failure and the loss of self-esteem. Brain research using brain imaging (functional Magnetic Resonance Imaging—fMRI) shows that triggers for stress and anxiety are actually located in specific areas of the brain (Lytle & Todd, 2009). Anxiety has many repercussions. It can block the school performance of students with mathematics disabilities by making it difficult for them to initially learn the mathematics, it impedes their ability to use or transfer the mathematics knowledge they do have, and it becomes an obstacle when they try to demonstrate their knowledge on tests (Ashcraft, Krause, & Hopko, 2007; Barkley, 2005; Slavin, 2009).

Many students and adults with learning disabilities and related mild disabilities report that anxiety is a constant companion. One individual said that she sprinkled anxiety wherever she went, making calm people nervous and nervous people fall apart. She described how she couldn't get out the right words and how she trembled. Teaching Tips 14.1, "Guidelines for Dealing With Math Anxiety," gives suggestions for dealing with math anxiety.

Did You Get It?

According to the information-processing model, a 6-year-old student who has difficulty writing numbers legibly and coherently, probably has a deficit in

- **a.** attention
- **b.** visual-spatial ability
- **c.** motor control
- **d.** memory

TEACHING TIPS 14.1

Guidelines for Dealing With Math Anxiety

- Use competition carefully. Have students compete with themselves rather than with others in the class or school. In a competitive situation, make sure that students have a good *chance* of succeeding.
- Provide abundant practice with similar tests. Students become familiar with the test procedure.
- Use clear instructions. Make sure that students understand what they are to *do* in math assignments. Ask students to work sample problems and be sure that they understand the assignment. When doing a new math procedure, give students plenty of practice and examples or models to show how the work is done.
- Avoid unnecessary time pressures. Give students ample time to complete math assignments in the class period. Give occasional take-home tests. If necessary, reduce the number of problems to be completed.
- Try to remove pressure from test-taking situations. Teach students test-taking strategies. Give practice tests. Make sure that the test format is clear and that students are familiar with the format. For example, a student may be familiar with the problem in the following format:

$$\begin{array}{r} 7 \\ +\ 3 \end{array}$$

The same child may be unfamiliar with a test format that presents the same problem in a different form:

$$7 + 3$$

Professional Resource Download

14.4 Students With Mathematics Disabilities at the Secondary Level

The mathematics problems of students with learning disabilities and related mild disabilities in middle school and high school differ from those at the elementary level. The secondary mathematics curriculum becomes increasingly more sophisticated and abstract and it is based on the presumption that the basic skills have been learned. The increased mathematics requirements at the high school level and the pressure of more testing are likely to adversely affect students with mathematics difficulties (Maccini & Gagnon, 2006; Deshler et al., 2001).

In the United States, high school mathematics requirements for graduation are becoming more rigorous. In most states, high school graduation is contingent upon passing mathematics courses, such as algebra, that previously were required only of students in a college preparatory curriculum. Many states now include algebra as a graduation requirement for all students (National Council of Teachers of Mathematics, 2000, 2006; Witzel, Mercer, & Miller, 2003).

Many secondary students with mathematics difficulties are able to be successful in advanced mathematics courses, but others shy away from geometry, statistics, and calculus. In the past, students with learning disabilities who faced mathematics disabilities were advised to continue remedial or basic mathematics courses. However, because algebra is now required for a high school diploma in most states, we must consider how to teach algebra to students with learning disabilities and related mild disabilities.

Common mathematics problems at the secondary level include basic operations (including fractions), decimals and percentages, fraction terminology, multiplication of whole numbers, place value, measurement skills, and division (Cass et al., 2003). Adolescents with learning disabilities and related mild disabilities continue to have memory deficits that interfere with the automatic learning of computation facts. These adolescents appreciate techniques that will help them learn and remember calculation facts. Students with severe problems in mathematics need direct instruction, with emphasis on learning basic skills to help them acquire functional abilities for successful living.

direct instruction
A method associated with behavioral theories of instruction. The focus is directly on the curriculum or task to be taught and the steps needed to learn that task.

Many students with learning disabilities and related disabilities can succeed in advanced mathematics courses. Many of these students will be going on to postsecondary education and college, and many will enter professions such as engineering or computer science that require competencies in advanced mathematics.

Effective instructional strategies in mathematics for secondary students include the following (Maccini & Gagnon, 2006; Cass et al., 2003; Witzel et al., 2003):

- **Provide many examples.** Students need to have many examples that illustrate the concept being taught. Teachers often provide too few examples.
- **Provide practice in discriminating various problem types.** Secondary students with mathematics disabilities have problems with discrimination. They ignore the operation sign and add instead of subtract. Once a skill is learned, the mathematics problem should be placed with different problems so that the student will learn to discriminate and generalize.
- **Provide explicit instruction.** Students with mathematics disabilities need direct instruction that is organized with step-by-step presentations.

Did You Get It?

The sophistication and __________ nature of the secondary mathematics curriculum increases.

a. concrete
b. anxiety-producing
c. abstract
d. hypothetical

14.5 Mathematics Standards

14.5a High Standards and Annual Testing

Federal and state governments now require the establishment of high mathematics standards and annual testing that uses those standards as a measure of achievement. Under the No Child Left Behind Act of 2002, schools are accountable for results, and schools are punished or rewarded on the basis of students' test results. The scores that students receive on these mathematics tests affect high-stakes decisions, such as whether the student will be promoted to the next grade or will receive a high school diploma. Garrison Keillor, the satirist, describes Lake Wobegon as where "all the women are strong, all the men are good looking, and all the children are above average." Schools in high socioeconomic areas tend to have students who do well under high-stakes mathematics assessment, while schools in impoverished areas struggle to have their students perform at the expected levels. In general, students with mathematics disabilities do not fare well under the high-stakes assessment approach to mathematics education without special considerations and accommodations (Witzel et al., 2003; Ysseldyke et al., 2001). (See more information on high-stakes testing in Chapter 2, "Assessment and the IEP Process.")

The Obama administration's Department of Education proposes to continue promoting high-stakes testing. Federal incentive grants were awarded to states and districts that have set high standards for the students they serve. The majority of states have now adopted the Common Core Standards for math, and students will be tested based on the Common Core Standards, which are described further in this chapter.

14.5b Common Core State Standards for Mathematics

COMMON CORE STATE STANDARDS

The Common Core State Standards for math define what students should understand and what students should be able to do in their study of math. There are standards for practice and for content.

In math there are eight standards for mathematical practice.

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.

At grade 5, the following math content standard areas are:

Operations and Algebraic Thinking
Number and Operations in Base Ten
Number and Operations-Fractions
Measurement and Data
Geometry (Common Core Standards, 2010)

FIGURE **14.1**
Fifth Grade Areas/Domains for Common Core State Standards for Mathematics

5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning. (Common Core State Standards Initiative, 2010)

These eight practices describe the ways in which students should engage with the subject matter as they grow in mathematical maturity. The content standards are a combination of procedure and understanding. Students who do not understand have difficulty engaging in the mathematical practices. The content standards then at each grade level set an expectation of understanding (Common Core State Standards for Mathematics, 2010). There are standards for kindergarten through 8. Domains are the broad areas. Then there are clusters of standards that are related and there are the individual standards.

The areas for the Common Core Content Standards (domains) at Grade 5 are listed in Figure 14.1.

At each grade level, within each domain there are specific clusters and individual standards. A complete set of the standards is available at *http://www.corestandards.org*. A free app is available to download for your iPhone or iPad and is known as Common Core.

Did You Get It?

Annual mathematics assessments, comprehensive standards, and a system of reward and punishment for educational institutions dependent on student grades are hallmarks of which piece of educational legislation?

a. No Child Left Behind Act
b. IDEA
c. Section 504 of the Rehabilitation Act
d. Elementary and Secondary Education Act

14.6 Learning Theories for Mathematics Instruction

14.6a Active Involvement

Learning mathematics should be an active process that involves doing. Use of hands-on learning materials allows students to explore ideas for themselves. Manipulative materials enable students to see, to touch, and to move objects. As students become actively involved in mathematics, they should be encouraged to use mathematics for solving real-life problems. This active view of

STUDENT STORIES 14.1

Active Involvement in Mathematics

The following examples illustrate how a young child uses estimation skills.

- Four-year-old Lee had just had his first experience sleeping overnight in a tent. Lee, his brother and his grandparents put up their tent, in which they place 4 sleeping bags for their overnight campout. After Lee excitedly described the experience to his parents the next day, they asked if they could come along next time. Lee did not answer immediately but spent some time considering the question. After estimating the space, he responded to his parents, "No you cannot come with us because the tent is not big enough to hold 2 more sleeping bags."
- The following problems show how young children construct solutions to subtraction problems.

  Problem A: Jane had 8 trucks. She gave 3 to Ben. How many trucks does she have left?

  Problem B: Jane has 8 trucks. Ben has 6 trucks. How many more trucks does Jane have than Ben?

In problem A, a young child counts out 8 trucks and gives 3 away. Then the child counts the trucks that are left. In Problem B, the child counts out 8 trucks for Jane and a set of 6 trucks for Ben. The child then matches Jane's trucks to Ben's. Finally the child counts to see how many more trucks Jane has than Ben. The child has constructed meaning and does not need to ask, "Should I add or subtract?"

REFLECTIVE QUESTION

1. What process did Lee use to estimate how many sleeping bags would fit into the tent? Draw a picture to show the solution of problem A or problem B.

mathematics learning is epitomized in the following Chinese proverb: "I hear and I forget. I see and I remember. I do and I understand."

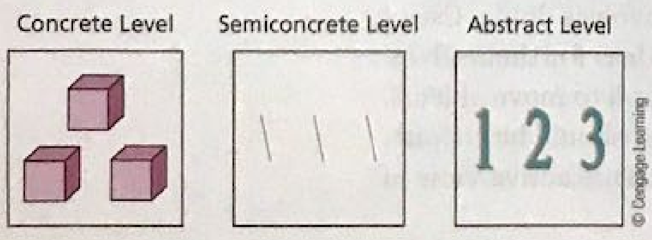
Student Stories 14.1, "Active Involvement in Mathematics," illustrates the active process approach to mathematics.

14.6b Progression From Concrete Learning to Abstract Learning

The learning of mathematics is a gradual process. It is not a matter of either knowing it or not knowing it. Instead, the learning of mathematics follows a continuum that gradually increases in strength. The Common Core Standards reflect this progression. As mathematics learning progresses, knowledge slowly builds from concrete to abstract learning, from incomplete to complete knowledge, and from unsystematic to systematic thinking. To help students progress from concrete to abstract learning, three sequential levels of mathematics instruction are suggested (Mercer & Pullen, 2009; Miller & Hudson, 2007; Cass et al., 2003).

The three levels from concrete thinking to abstract thinking are shown in Figure 14.2.

FIGURE **14.2**
Concrete to Abstract Learning



1. **The concrete level.** At this level, students manipulate actual materials such as blocks, cubes, marbles, plastic pieces, poker chips, or place-value sticks. Students can physically touch, move, and manipulate these objects as they work out solutions to number problems.
2. **The semiconcrete level (or the representational level).** Once the students master the skill on the concrete level, instruction progresses to the semiconcrete

or representational level. Students use pictures or tallies (i.e., marks on the paper) to represent the concrete objects as they work on mathematics problems.

3. **The abstract level.** At this level, students use only the numbers to solve mathematics problems without the help of semiconcrete pictures or tallies.

14.6c Direct Instruction of Mathematics

Direct instruction is a method of mathematics teaching that helps students achieve mastery of mathematics skills through instruction that is explicit, carefully structured, and planned. It is a comprehensive system that integrates curriculum design with teaching techniques to produce instructional programs in mathematics (Miller & Hudson, 2007; Marchand-Marella, Slocum, & Martella 2004; Kroesbergan & Van Luit, 2003; Swanson & Hoskyn, 2001). The sequential nature of mathematics makes the direct instruction approach particularly adaptable to the content of mathematics.

Mathematics programs that are based on direct instruction are highly organized and carefully sequenced. Instruction follows an ordered plan. Teachers determine the objectives of the teaching, plan the teaching through task analysis, provide explicit instruction, and plan for continuous testing. Direct instruction has been shown to be very effective for students with learning disabilities and related mild disabilities (Miller & Hudson, 2007; Marchand-Marella et al., 2004; Jones & Southern, 2003). To use direct instruction, teachers do the following:

1. Break tasks into small steps
2. Administer probes to determine whether the students are learning
3. Supply immediate feedback
4. Provide diagrams and pictures to enhance student understanding
5. Give ample independent practice

14.6d Learning Strategies Instruction

Learning strategies instruction helps students with mathematics disabilities acquire specific procedures for meeting the challenges of mathematics in their curriculum and to take control of their own mathematics learning (Deshler, 2003). Intervention practices that use learning strategy instruction are effective in increasing achievement. Teachers who implement a learning strategies instruction model perform the following (Deshler, 2003; Mainzer et al., 2003):

1. Provide elaborate explanations to model learning processes
2. Provide prompts to use strategies
3. Engage in teacher–student dialogues
4. Ask processing questions

See Chapter 9, "Adolescents and Adults With Learning Disabilities and Related Disabilities," for more information about learning strategies instruction.

14.6e Problem Solving

Problem solving was identified as the top priority for the mathematics curriculum by the National Council of Teachers of Mathematics (NCTM, 2000) and is reflected in the Common Core Standards. Moreover, problem solving is rapidly assuming a larger part of the curriculum in both general education and special education (Cawley & Foley, 2001; NCTM, 2000; Van de Walle, Karp, & Bay-Williams, 2010). Mathematics problem solving involves the kind of thinking needed to work out mathematics word problems. In addition, a current view of mathematics expands the perspective of problem solving to the processes by which a student resolves unfamiliar situations. Implicit in the teaching of problem solving are the following underlying beliefs about mathematics: (1) there is no single way to do mathematics, (2) there is no single way to organize mathematics for instructional purposes, and (3) important mathematical concepts are actually learned through problem solving (Van de Walle, Karp, & Bay-Williams 2010).

An example of a problem-solving task is to ask students to think about the number 8 and to draw a picture of how the number 8 can be broken in two different amounts. Then ask the students to tell a story to go with their pictures (Van de Walle, Karp, & Bay-Williams, 2010).

Problem solving is the most difficult area of mathematics for many students with mathematics difficulties. Students with math difficulties need extensive guidance and practice to learn to combine thinking and language with the calculation skills and concepts required to solve mathematics problems. To solve mathematics problems, students must analyze and interpret information so that they can make selections and decisions. Problem solving requires that students know how to apply mathematics concepts and how to use computation skills in new or different settings.

How do students go about solving problems in mathematics? Research shows that first and second graders readily invent their own ways to solve simple word problems. However, by the middle grades, they stop their personal problem-solving attempts and begin to rely on rote procedures they have learned in school. Middle-grade students should be encouraged to continue to create and use their own ways to solve mathematics problems, as illustrated in Student Stories 14.2, "Encouraging a Problem-Solving Attitude."

Middle-grade students tend to automatically compute with whatever numbers are in the problems. To encourage a problem-solving attitude, teachers should help structure the students' responses to problems by talking with them about those responses. Encouraging such a discussion raises the reasoning level

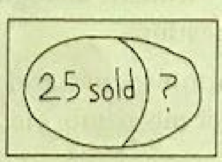
STUDENT STORIES 14.2

Encouraging a Problem-Solving Attitude

The following example of a word problem illustrates how teachers can encourage an inventive, problem-solving attitude (Lindquist, 1987).

Problem: Rebecca wants to sell 30 boxes of Girl Scout cookies. She has sold 25. How many more must she sell? The teacher asks if anyone can draw a picture to show this problem.

One student drew the figure at right to solve this problem.



REFLECTIVE QUESTION

1. How does drawing a picture help the student with problem solving?

of the students' answers. Teachers can help by listening to the students as they think aloud about the word problems. It is also important to encourage the use of different strategies to solve mathematics problems and to ask students, "How did you get your answer?"

Many of the mathematics textbooks that are used in general education classrooms today use a problem-solving approach. Because problem solving is often difficult for students with learning disabilities and related mild disabilities, they need extensive guidance and practice to learn to combine thinking and language with the calculation skills and concepts required in mathematics problem solving. To solve mathematics problems, students must analyze and interpret information so that they can make selections and decisions. The following 3 steps can help structure mathematics problem-solving lessons (Van de Walle, Karp, & Bay-Williams, 2010):

Step 1. Getting ready First, students attend to the problem and translate the problem into their own experiences and language. The teacher makes sure they understand what is expected.

Step 2. Students work It is at this stage that students have a chance to work without constant guidance. The teacher lets go, listens carefully, and provides hints.

Step 3. Class discussion In the final step, there is a discussion of the solutions. The teacher accepts student solutions without evaluation. Students justify and evaluate their results and methods. Teaching Tips 14.2 provides some problem-solving examples.

TEACHING TIPS 14.2

Some Problem-Solving Examples

1. Ask the students to view a pair of items, such as $5 - 2 = 3$ and $8 - 5 = 3$. Next, ask the students to explain how 2 different number combinations result in the same answer. For example, you arrived at the same answer because the answer represents the difference between the numbers in each combination (Cawley & Foley, 2001).
2. Ask students to compare the fractions $\frac{6}{8}$ and $\frac{4}{5}$. Then ask which fraction is larger. (Assume that the students have not been taught about common denominators.) One student answered, "I know that $\frac{4}{5}$ is the same as $\frac{8}{10}$ and that is $\frac{2}{10}$ *away from a whole.* Because tenths are smaller than eighths, $\frac{8}{10}$ must be closer to a whole, so $\frac{4}{5}$ is larger" (Van de Walle, 2004).
3. Ask students how the number 7 can be broken into different amounts. Then ask the students to draw pictures showing ways in which the number 7 can be broken into different amounts (Van de Walle, 2004).

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Did You Get It?

Which strategy is not recommended in a comprehensive, instructive mathematics curriculum?

a. theory trumping actuality
b. real-life application
c. much manipulation
d. action oriented

14.7 Assessing Mathematics Achievement

Information about a student's proficiency in mathematics can be gathered through (1) formal tests and (2) informal measures. Formal tests include standardized survey tests, group survey tests, individually administered achievement tests, and diagnostic math tests. Informal measures include informal inventories, analyzing mathematics errors, and curriculum-based assessment.

14.7a Formal Tests

Formal mathematics tests include standardized survey tests; some are designed for group administration, and some are individually administered achievement tests. There are also diagnostic mathematics tests. It is important to check the validity, reliability, and standardization procedures of tests before using them (Salvia, Ysseldyke, & Bolt, 2013).

Standardized Survey Tests Survey tests provide information on the general level of a student's mathematics performance.

Group Survey Tests Group survey tests are designed for group administration. Usually, data are available on a test's reliability, validity, and standardization procedures. Often there are accompanying manuals with tables for various kinds of score interpretations, including grade scores, age scores, standard scores, and percentiles. Most survey tests in mathematics are part of a general achievement test battery. Some of the most widely used tests are listed in Table 14.2.

TABLE 14.2

Formal Tests of Mathematics

Test	Grade or Age
Group Standardized Survey Tests	
• California Achievement Tests, CTB/McGraw-Hill *http://www.ctb.com*	Grades K–12
• Iowa Tests of Basic Skills, Riverside Publishing *http://www.riverpub.com*	Grades K–12
• Metropolitan Achievement Tests, Harcourt Assessment *http://www.harcourtassessment.com*	Grades K–12
Individually Administered Achievement Tests	
• Brigance Comprehensive Inventory of Basic Skills—Revised, Curriculum Associates *http://www.curriculumassociates.com*	Grades K–9
• Brigance Diagnostic Inventory of Essential Skills, Curriculum Associates *http://www.curriculumassociates.com*	Grades 6–Adult
• Kaufman Test of Educational Achievement—Normative Upgrade (K-TEA-NU), AGS *http://ags.pearsonassessments.com*	Grades K–12
• Peabody Individual Achievement Test—Revised (PIAT-R), AGS *http://ags.pearsonassessments.com*	Grades K–12
• Wide-Range Achievement Test—4(WRAT-4), PAR Inc. *http://www3.parinc.com*	Age 5–Adult
• Woodcock-Johnson Psychoeducational Battery—III, Riverside Publishing *http://www.riverpub.com*	Grades K–12
Diagnostic Math Tests	
• Key Math-Revised: A Diagnostic Inventory of Essential Mathematics, AGS *http://ags.pearsonassessments.com*	Grades K–6
• Stanford Diagnostic Mathematics Test 4, Harcourt Assessment *http://www.harcourtassessment.com*	Grades K–12
• Test of Mathematical Abilities—2, Pro-Ed *http://www.proedinc.com*	Grades 3–12

The standardized achievement tests are useful as screening instruments because they identify those students whose performance scores are below expected levels (Salvia, Ysseldyke, & Bolt, 2013). The major test batteries are well constructed, generally have excellent technical characteristics, and cover most items in the mathematics curriculum. However, because they are paper-and-pencil tests that rely on multiple-choice responses, the diagnostic information that can be obtained from them is limited. The group survey tests can also be given to individuals.

Individually Administered Achievement Tests These tests are designed for individual assessment. They can yield more diagnostic information than the group survey tests, providing information on specific areas of mathematics difficulty and more clues for planning instruction.

In addition, a number of commonly used criterion-referenced measures are available, such as the Brigance Comprehensive Inventory of Basic Skills—Revised, which provides extensive information about math achievement patterns. Table 14.2 lists some of the widely used individual tests.

Diagnostic Math Tests Diagnostic math tests are available for both group and individual administration. Group tests serve 2 purposes: (1) to provide diagnostic information for student program planning and (2) to assist in program evaluation for administrative purposes. Individual tests generally are used to evaluate patterns of strength and weakness or skills in mathematics that have been mastered and skills that have not been mastered.

14.7b Informal Measures

Informal measures offer another option for obtaining information about a student's performance and abilities in mathematics. Observations of a student's daily behavior in mathematics class and performance on homework assignments and on teacher-made tests or tests that accompany the textbook can provide information about the student's mathematics skills (Spinelli, 2006). Informal measures to assess mathematics include: (1) informal inventories, (2) analysis of mathematics errors, and (3) curriculum-based assessment.

Informal Inventories Informal tests can be devised by teachers to assess the student's mathematics skills (Bryant & Rivera, 1997). Once the general area of difficulty is determined, a more extensive diagnostic test of that area can be given. A sample informal arithmetic test appears in Figure 14.3. Teachers can easily construct informal tests to assess the student's achievement in a specific mathematics skill or in a sequence of mathematics skills. The informal test can be tailored for an individual student.

Analyzing Mathematics Errors Teachers should be able to detect the types of errors a student with mathematics difficulty is making so that instruction can be directed toward correcting those errors (Ashlock, 2006). This information is obtained by examining the students' work or by asking the student to explain how he or she went about solving a problem. When teachers observe the methods used by a student, they can deduce the thought processes the student is using. The four most common types of calculation errors are

FIGURE **14.3**
Informal Inventory of Arithmetic Skills

Addition

$\begin{array}{r} 3 \\ +5 \\ \hline \end{array}$ $\quad$ $\begin{array}{r} 8 \\ +0 \\ \hline \end{array}$ $\quad$ $\begin{array}{r} 25 \\ +71 \\ \hline \end{array}$ $\quad$ $\begin{array}{r} 20 \\ +49 \\ \hline \end{array}$ $\quad$ $\begin{array}{r} 15 \\ +\ 7 \\ \hline \end{array}$ $\quad$ $\begin{array}{r} 77 \\ +29 \\ \hline \end{array}$ $\quad$ $\begin{array}{r} 5 \\ 2 \\ +7 \\ \hline \end{array}$

$5 + 7 = \square$ $\quad$ $3 + \square = 12$ $\quad$ $\square + 7 = 15$

$\begin{array}{r} 233 \\ +\ 45 \\ \hline \end{array}$ $\quad$ $\begin{array}{r} 879 \\ +\ 48 \\ \hline \end{array}$ $\quad$ $\begin{array}{r} 648 \\ 745 \\ +286 \\ \hline \end{array}$

Subtraction

$\begin{array}{r} 7 \\ -5 \\ \hline \end{array}$ $\quad$ $\begin{array}{r} 25 \\ -\ 9 \\ \hline \end{array}$ $\quad$ $\begin{array}{r} 78 \\ -23 \\ \hline \end{array}$ $\quad$ $\begin{array}{r} 72 \\ -49 \\ \hline \end{array}$ $\quad$ $\begin{array}{r} 546 \\ -222 \\ \hline \end{array}$ $\quad$ $\begin{array}{r} 6762 \\ -4859 \\ \hline \end{array}$

$5 - 2 = \square$ $\quad$ $7 - \square = 4$ $\quad$ $\square - 3 = 5$

Multiplication

$\begin{array}{r} 5 \\ \times 3 \\ \hline \end{array}$ $\quad$ $\begin{array}{r} 6 \\ \times 7 \\ \hline \end{array}$ $\quad$ $\begin{array}{r} 24 \\ \times\ 2 \\ \hline \end{array}$ $\quad$ $\begin{array}{r} 86 \\ \times\ 7 \\ \hline \end{array}$ $\quad$ $\begin{array}{r} 59 \\ \times 34 \\ \hline \end{array}$ $\quad$ $\begin{array}{r} 25 \\ \times 79 \\ \hline \end{array}$

$6 \times 3 = \square$ $\quad$ $7 \times \square = 56$ $\quad$ $\square \times 5 = 20$

Division

$2\overline{)10}$ $\quad$ $4\overline{)16}$ $\quad$ $8\overline{)125}$ $\quad$ $11\overline{)121}$ $\quad$ $12\overline{)108}$

$12 \div 4 = \square$ $\quad$ $24 \div \square = 6$ $\quad$ $\square \div 9 = 6$

(1) place value, (2) computation facts, (3) using the wrong process, and (4) working from left to right. The following list shows examples of these common errors.

- **Place value.** Place value is the aspect of the number system that assigns specific significance to the position a digit holds in a numeral. Students who make this error do not understand the concepts of place value, regrouping, carrying, or borrowing and might make errors such as those shown here.

$$\begin{array}{r} 75 \\ -27 \\ \hline 58 \end{array} \qquad \begin{array}{r} 63 \\ +18 \\ \hline 71 \end{array}$$

  These students need concrete practice in the place value of 1s, 10s, 100s, and 1,000s. Effective tools for such practice are an abacus and a place-value box or chart with compartments. Students can sort objects such as sticks, straws, or chips into compartments to show place value.

- **Computation facts.** Students who make errors in basic adding, subtracting, multiplying, and dividing need more practice and drill. For example:

$$\begin{array}{r} 6 \\ \times 8 \\ \hline 46 \end{array} \qquad \begin{array}{r} 9 \\ \times 7 \\ \hline 62 \end{array}$$

  A handy multiplication chart, like the one shown in Figure 14.4, might be useful in checking their work.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

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FIGURE **14.4**
Multiplication Chart

- **Using the wrong process.** Some students make errors because they use the wrong mathematical process. For example:

$$\begin{array}{r} 6 \\ \times 2 \\ \hline 8 \end{array} \qquad \begin{array}{r} 15 \\ -3 \\ \hline 18 \end{array}$$

These students need work in recognizing symbols and signs.

- **Working from left to right.** Some students reverse the direction of calculations and work from left to right. For example:

$$\begin{array}{r} 35 \\ +81 \\ \hline 17 \end{array} \qquad \begin{array}{r} 56 \\ +71 \\ \hline 28 \end{array}$$

These students need work in place value.

In addition, poor writing skills cause many math errors. When students cannot read their own writing or fail to align their numbers in columns, they may not understand what to do.

Curriculum-Based Assessment The procedure of curriculum-based assessment or progress monitoring (see Chapter 2, "Assessment and the IEP Process") provides a useful way to measure mathematics progress. Curriculum-based assessment closely links assessment to the material that is being taught in the mathematics curriculum. The procedure may involve teacher-constructed tests that measure student progress on curricular objectives. In relation to mathematics, curriculum-based assessment consists of four steps (Baroody & Ginsburg, 1991; Shinn & Hubbard, 1992):

1. **Identify target skills.** For example, the skill might be math computation, such as adding two-digit numbers.
2. **Determine the objectives to be met.** For example, in a period of 4 weeks, the student will be able to write the answers to 20 two-digit addition problems correctly in 5 minutes.

3. **Develop test items to sample each skill.** Assemble a collection of two-digit number problems.
4. **Develop criteria to measure achievement.** The student will write answers without errors to 20 randomly selected two-digit math problems in a 5-minute period.

Did You Get It?

A teacher who is paying particular attention to a student's homework, a quiz he just took, and his behaviors and performance as he works through several assigned problems in class is conducting what form of testing?

a. informal measures
b. an informal inventory test
c. an individually administered achievement test
d. a standardized survey test

14.8 Teaching Strategies to Improve Mathematics Difficulties

The "Teaching Strategies" section of this chapter highlights: (1) mathematics strategies for the general education classroom, (2) the mathematics curriculum, (3) principles of instruction for students with mathematics difficulties, (4) activities for teaching mathematics, and (5) assistive and instructional technology for mathematics instruction.

14.9 Mathematics Strategies for the General Education Classroom

Many students with learning disabilities and related mild disabilities who have mathematics difficulties receive their instruction in general education classrooms. General education teachers are therefore responsible for their instruction. General education teachers may not have enough training or background to address the mathematics difficulties of these students. Including Students in General Education 14.1, "Mathematics Strategies," provides several strategies for teaching mathematics to students with mathematics difficulties in the general education classroom.

Additional math strategies for teaching students with math difficulties in the general education classroom can be found on the following websites:

- Schrockguide, *http://www.school.discoveryeducation.com/schrockguide /math.html*
- PBS Teacher Source, *http://www.pbs.org/teachers*
- Illuminations, National Council of Teachers of Mathematics, *http://www .illuminations.nctm.org*
- Teacher Resources for the Classroom, *http://www.mathgoodies.com*
- Stuart Murphy *http://www.stuartmurphy.com*
- K-3 Teaching Resources, *http://www.k-3teachingresources.com*

Including Students in GENERAL EDUCATION 14.1

Mathematics Strategies

- Determine the students' basic computational skills in addition, subtraction, multiplication, and division.
- Have students use manipulatives to help them understand a concept.
- Teach the students mathematics vocabulary.
- Use visuals and graphics to illustrate concepts to the students.
- Have students make up their own word story problems.
- Teach students how to use a calculator.
- Teach money concepts by using either real money or play money.
- Teach time by using manipulative clocks.
- Provide many opportunities for practice and review.

(For additional information on teaching students with math difficulties in the general education classroom, see LD Online at *http://www.ldonline.org/indepth/math.*)

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- National Literacy of Virtual Manipulatives, *http://www.nationallibraryofvirtualmanipulatives.com*
- Cool Math 4 Kids, *http://www.coolmath4kids.com*

Did You Get It?

What is not a legitimate practice in a general education classroom?

a. the use of calculators
b. teaching about time constraints and limitations
c. the use of visuals to teach and reinforce concepts
d. performing work for a frustrated student

14.10 The Mathematics Curriculum

Both general education teachers and special education teachers should have a basic picture of the overall mathematics curriculum. It is important to know what the student has already learned in the mathematics curriculum and what mathematics learning lies ahead.

14.10a The Sequence of Mathematics: Grades K–8

Mathematics is a naturally cumulative subject typically taught in a sequence that introduces certain skills at each grade level. For example, learning multiplication depends on knowing addition. The major topics that are covered in the mathematics curriculum from kindergarten through grade 8 include numbers and numeration; whole numbers—addition and subtraction; whole numbers—multiplication and division; decimals; fractions; measurement; geometry; and computer education, a subject that is beginning to show up in many mathematics programs.

Although the sequence may vary somewhat in different programs, the general timetables of instruction are as follows:

Kindergarten Basic number meanings, counting, classification, seriation or order, recognition of numerals, and the writing of numbers

Grade 1 Addition through 20, subtraction through 20, place value of 1s and 10s, time to the half hour, money, and simple measurement

Grade 2 Addition through 100, subtraction through 100, counting from 0 to 100, skip-counting by 2s, place value of 100, and regrouping for adding and subtracting

Grade 3 Multiplication through 9s, odd or even skip-counting, place value of 1,000s, two- and three-place numbers for addition and subtraction, and telling time

Grade 4 Division facts, extended use of multiplication facts and related division facts through 9s, and two-place multipliers

Grade 5 Fractions, addition and subtraction of fractions, mixed numbers, long division, two-place division, and decimals

Grade 6 Percentages, three-place multipliers, two-place division, addition and subtraction of decimals and mixed decimals, multiplication and division of decimals, and mixed decimals by whole numbers

Grade 7 Geometry, rounding, ratios, and simple probability

Grade 8 Scientific notation, using graphs, complex fractions, complex applications, and word problems

14.10b The Secondary Mathematics Curriculum

The Common Core Standards at the secondary level (Grades 9 through 12) include these areas. (Common Core State Standards Initiative, 2010). These can be found at: *http://www.corestandards.org.*

- Number and Quantity
- Algebra
- Functions
- Modeling
- Geometry
- Statistics and Probability

Remember that the mathematical practices that were discussed earlier in the elementary standards are also in place in the secondary set of standards. (Common Core State Standards Initiative, 2010).

Did You Get It?

The math curriculum is heavily standardized, cumulative, and age specific concerning the grade when concepts are introduced, when assessment is done, and when students are expected to complete objectives. Multiplication through the "9s" is a task that your mathematics students should be tackling, and hopefully mastering, in which grade?

a. first
b. third
c. fourth
d. seventh

14.11 Principles of Instruction for Students With Mathematics Disabilities

Several principles of mathematics learning offer a guide for effective mathematics instruction. The principles discussed here include (1) early number learning, (2) progressing from the concrete to the abstract, (3) providing opportunity for practice and review, (4) generalizing the concepts and skills that have been learned, and (5) teaching mathematics vocabulary.

14.11a Early Number Learning

It is important to check into the previously acquired early number learning to ensure that the student is ready for what needs to be learned. Time and effort invested in building a firm foundation can prevent many later difficulties as the student tries to move on to more advanced and more abstract mathematics processes (Jordan et al., 2007). Table 14.3 gives descriptions of the essential basic early number-learning abilities. If they are lacking, they must be taught.

14.11b Progressing From the Concrete to the Abstract

Pupils can best understand a mathematics concept when teaching progresses from the concrete to the abstract. A teacher should plan three instructional stages: concrete, semiconcrete, and abstract (Miller & Hudson, 2007; Cass et al., 2003; Witzel et al., 2003).

1. In the concrete instruction stage, the student manipulates real objects in learning the skill. For example, the student could see, hold, and move 2 blocks and 3 blocks to learn that they equal 5 blocks.

   □□ + □□□ = 5

concrete instruction
Students manipulate actual materials for mathematics learning, such as blocks, cubes, and marbles.

TABLE 14.3

Early Number Learning

Ability	Description
Matching	Grouping similar objects together
Recognizing groups of objects	Recognizing a group of 3 objects without counting
Counting	Matching numerals to objects
Naming a number that comes after a given number	Stating what number comes after 7
Writing numerals from 0 to 10	Knowing the right sequence
Measuring and pairing	One-to-one correspondence, estimating, fitting objects
Sequential values	Arranging like objects in order by quantitative differences (e.g., by size)
Operations	Manipulation of the number facts to 10 without reference to concrete objects

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semiconcrete instruction
The level of mathematics instruction in which the students use representational objects to refer to math concepts, such as tallies, pictures, or marks, instead of the actual objects.

abstract instruction
At this level of mathematics instruction, students manipulate symbols without the help of concrete objects or representational pictures or tallies.

2. In the semiconcrete instruction stage, a graphic representation is substituted for actual objects. In the following example, circles represent objects in an illustration from a worksheet:

$$\bigcirc\bigcirc + \bigcirc\bigcirc\bigcirc = 5$$

3. At the abstract instruction stage, numerals finally replace the graphic symbols:

$$2 + 3 = 5$$

14.11c Provide Opportunity for Practice and Review

Students need many opportunities for review, drill, and practice to over-learn the math concepts because they must be able to use computation facts almost automatically. There are many ways to provide this practice, and teachers should vary the method as often as possible. Such techniques can include worksheets, flash cards, games, behavior management techniques (such as rewards for work completed), and computer practice (special software programs that give immediate feedback).

14.11d Teach Students to Generalize to New Situations

Students must learn to generalize a skill to many situations. For example, students can practice computation facts with many story problems that the teacher or students create and then exchange with each other. The goal is to gain skill in recognizing computational operations and applying them to various new situations.

14.11e Teach Mathematics Vocabulary

The vocabulary and concepts of mathematics are new to students and must be learned. The student may know the operation, but may not know the precise term applied to the operation. Table 14.4 shows the vocabulary for basic mathematics operations.

TABLE 14.4

Mathematics Vocabulary for Basic Operations

Operation	Terms		
Addition	3	→	addend
	+5	→	addend
	8	→	sum
Subtraction	9	→	minuend
	−3	→	subtrahend
	6	→	difference
Multiplication	7	→	multiplicand
	×5	→	multiplier
	35	→	product
Division	7	→	quotient
	6√42		
	↑	____	Divisor

Did You Get It?

If a teacher substitutes sticks for numbers to illustrate the process of addition, she is providing ______________ instruction.

a. semiconcrete
b. semiabstract
c. abstract
d. quasi-inferential

14.12 Activities for Teaching Mathematics

The instruction activities in this section are grouped into three areas: (1) teaching early number skills, (2) teaching computation skills, and (3) teaching word story problems.

14.12a Teaching Early Number Skills

Classification and Grouping

1. **Sorting games.** Give students objects that differ in only one attribute, such as color or texture, and ask them to sort the objects into two different boxes. For example, if the objects differ by color, have students put red items in one box and blue items in another box. At a more advanced level, increase the complexity of the classification of the attributes, asking students to sort, for example, movable objects from stationary objects. Another variation is to use objects that have several overlapping attributes, such as shape, color, and size. You might present the students with cutouts of triangles, circles, and squares in three colors (e.g., blue, yellow, and red) and two sizes (e.g., small and large). Ask the students to sort them according to shape and then according to color. Then ask the students to discover a third way of sorting.

2. **Matching and sorting.** A first step in the development of number concepts is the ability to focus on and to recognize a single object or shape. Have the student search through a collection of assorted objects to find a particular type of object. For example, the student might look in a box of colored beads or blocks for a red one, search through a collection of various kinds of coins for all the pennies, choose the forks from a box of silverware, look in a box of buttons for the oval ones, sort a bagful of cardboard shapes to pick out the circles, or look in a container of nuts and bolts for the square pieces.

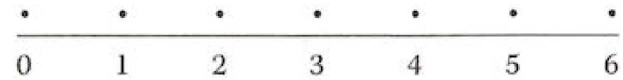
3. **Recognition of groups of objects.** Domino games, playing cards, concrete objects, felt boards, magnetic boards, and cards with colored disks all provide excellent materials for developing concepts of groups.

Ordering

1. **Serial order and relationships.** When teaching the concept of ordering, ask the student to tell the number that comes after 6 or before 5 or between 2 and 4. Also, ask the student to indicate the first, last, or third of a series of objects. Other measured quantities can be arranged by other dimensions, such as size, weight, intensity, color, or volume.

2. **Number lines.** A number line is a sequence of numbers forming a straight line that allows the student to manipulate computation directly. Number lines

and number blocks for the students to walk on are helpful in understanding the symbols and their relationships to one another.



3. **Arranging by size and length.** Have the student compare and contrast objects of different size, formulating concepts of smaller, bigger, taller, and shorter. Make cardboard objects, such as circles, trees, houses, and so forth; or collect objects, such as washers, paper clips, and screws. Have the student arrange the objects by size and then estimate the size of the objects by guessing whether certain objects would fit into certain spaces.

4. **One-to-one correspondence: Pairing.** One-to-one correspondence is a relationship in which one element of a set is paired with one, and only one, element of a second set. Pairing provides a foundation for counting. Activities designed to match or align one object with another are useful. Have the student arrange a row of pegs in a Peg-Board to match a prearranged row, or set a table and place one cookie on each dish, or plan the allocation of materials to the group so that each person receives one object.

Counting

1. **Motor activities for counting.** Some students learn to count verbally, but they do not attain the concept that each number corresponds to one object. Such students are helped by making strong motor and tactile responses along with the counting. Looking at visual stimuli or pointing to the objects may not be enough because such students will count erratically, skipping objects or saying 2 numbers for one object. Motor activities to help students establish the counting principle include placing a peg in a hole, clipping clothespins on a line, stringing beads onto a pipe cleaner, clapping 3 times, jumping 4 times, and tapping on the table 2 times. Use the auditory modality to reinforce visual counting by having students listen to the counts of a drumbeat with their eyes closed. The students may make a mark for each sound and then count the marks.

2. **Counting cups.** Take a set of containers, such as cups, and designate each with a numeral. Have the students fill each container with the correct number of items, using objects such as bottle caps, chips, buttons, screws, or washers.

Recognition of Numbers

1. **Visual recognition of numbers.** Students must learn to recognize both the printed numbers (7, 8, 3) and the words expressing these numbers (seven, eight, three). They must also learn to integrate the written forms with the spoken symbols. If students confuse one written number with another, color cues may help them to recognize the symbol. You might make, for example, the top of the number 3 green and the bottom of the number 3 red. Another activity is to have the students match the correct number with the correct set of objects; felt, cardboard, or sandpaper symbols or groups of objects can be used.

2. **Parking lot poster.** Draw a "parking lot" on a poster, numbering parking spaces with dots instead of numerals. Paint numerals on small cars and have the students park the cars in the correct spaces.

14.12b Teaching Computation Skills

The following list gives some strategies for teaching computation skills.

1. **Part–whole concepts.** The "big concept" idea is that addition and subtraction have a part–whole relationship; you add to find the whole or total of two or more parts, and you subtract from the whole to find the missing part. Use Figure 14.5 to help students see the part–whole relationship. Use counters or put the figure on an overhead projector to demonstrate the part–whole relationship to the entire group. Students can use counters to demonstrate the part–whole relationship.

part–whole relationship Addition and subtraction have a part-whole relationship. Add to the whole or total of 2 parts, and you subtract from the whole to find the missing part.

2. **Basic computation skills.** Many problems in mathematics are due to deficiencies in basic computation skills. To help students to overcome these deficiencies, teach the basic mathematics computation skills that the students lack: addition, subtraction, multiplication, division, fractions, decimals, and percentages. An inexpensive way to teach many mathematics computation skills is to obtain mathematics games and materials from a dollar store. Online dollar stores also can be a good source. At a dollar store, you can often get large stickers of multiplication tables. Students can put them on their file folder for math. (The online dollar store, Oriental Trading, is at *http://www.OrientalTrading.com.*) Mathematics games on CDs are also helpful for teaching computation skills. (Some games and CDs can be found at Planet CD Rom at *http://www.planetcdrom.com.*) A collection of these games and activities can be placed on a mathematics activities table.

mathematics computation The basic mathematical operations, consisting of addition, subtraction, multiplication, division, fractions, decimals, and percentages.

3. **Addition.** Knowledge of addition facts provides the foundation for all other computational skills. Addition is a short method of counting, and students should know that they can resort to counting when all else fails. Addition can be thought of as "part plus part equals whole." Important symbols to learn are + (plus, or "put together") and = (equals, or "the same as"). As with the other areas, begin by using concrete objects, then use cards with sets that represent numbers, and finally use the number sentence with the numbers alone: 3 + 2 = □. From this, the students can also learn that 2 + 3 = □; □ + 2 = 5; and 3 + □ = 5.

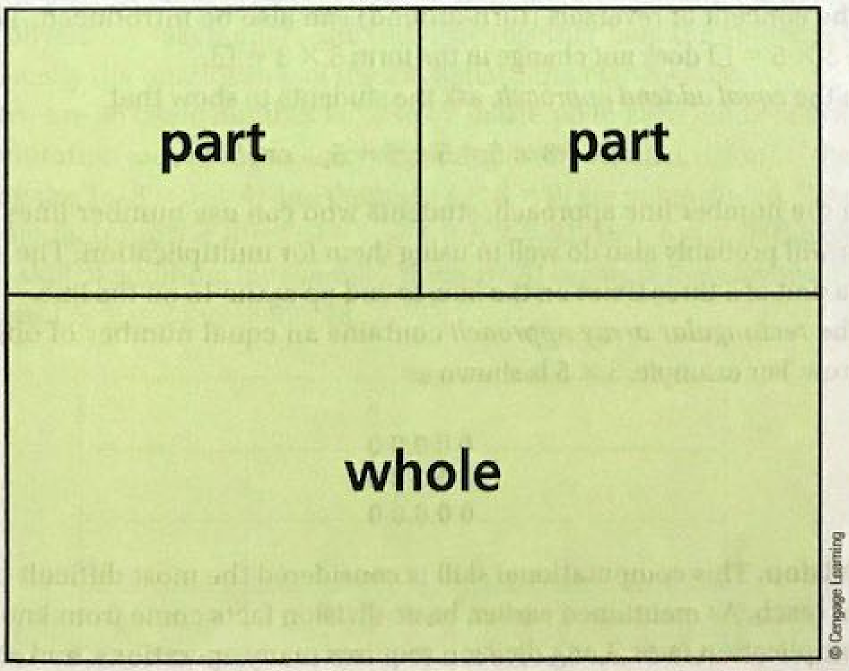
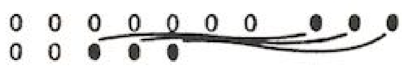


FIGURE **14.5**
Part-Whole Relationships

Teaching addition using sums between 10 and 20 is more difficult. There are several approaches. It is easier to start with doubles, such as $8 + 8 = 16$. Then ask what $9 + 8$ equals: one more than 16.

Another way is to "make a 10." For example, in $7 + 5$, the pupil takes 3 of the 5, and adds the 3 to the 7 to make 10. Now the students can see that 10 + the remaining $2 = 12$. Use movable objects so that the students can actually experience the process:



$7 + 3 = 10$
$10 + 2 = 12$

The number line provides another way to teach addition. With a number line, the students can visually perceive the addition process.

4. Subtraction. After the students have a firm basis in addition, introduce subtraction. An important new symbol is − (minus, or "take away"). A student places a set of objects on the desk and then takes away certain objects. How many are left? $6 - 2 = \square$. Then use cards with sets on them. Find 6 by using a card with a set of 2 and a card with a set of 4. Tell the students you have a set of 6 when the cards are joined. Take away the set of 2 and ask the students what is left.

The number line is also useful in subtraction.

Regrouping is an important concept that is introduced in subtraction, along with the ideas of "1s," "10s," and "100s."

5. Multiplication. Many students with a mathematics disability do not know multiplication facts (refer to Figure 14.4). Those students will be unable to learn division until they master multiplication facts.

Multiplication is a short method of adding. Instead of adding $2 + 2 + 2 + 2$, the students can learn $2 \times 4 = 8$. Subtraction is not a prerequisite of multiplication, and a student having difficulty with subtraction may do better with multiplication. The symbol to learn is x (times).

There are several ways of explaining multiplication. One way is the multiplication sentence. How much are 3 sets of 2? Using sets of objects, the students can find the total either by counting objects or by adding equal addends.

The concept of reversals (turn-around) can also be introduced. The sentence $3 \times 5 = \square$ does not change in the form $5 \times 3 = \square$.

In the *equal addend approach*, ask the students to show that

$$3 \times 5 = 5 + 5 + 5, \quad \text{or } 15$$

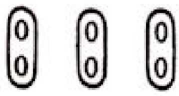
In the number line approach, students who can use number lines for addition will probably also do well in using them for multiplication. The student adds a unit of 5 three times on the line, to end up at the 15 on the line.

The *rectangular array approach* contains an equal number of objects in each row. For example, 3×5 is shown as

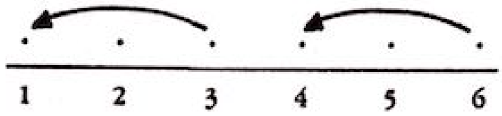
00000
00000
00000

6. Division. This computational skill is considered the most difficult to learn and to teach. As mentioned earlier, basic division facts come from knowledge of multiplication facts. Long division requires many operations, and students must be able to do all the steps before they can put them together. The new symbol is ÷ (divide).

There are a number of ways to approach division. Sets can be used: 6 ÷ 3 = ☐. Draw a set of 6 and enclose 3 equal sets. The missing factor is seen as 2:



How many subsets are there? How many objects are there in each set? The number line can also be used. By jumping back a unit of 3, how many jumps are needed?



The missing factors approach uses known multiplication facts and reverses the process: 3 × ☐ = 12. Then change to a division sentence: 12 ÷ 3 = ☐.

7. Fractions. Geometric shapes are commonly used to introduce fractional numbers. The new symbol is shown next:

$$\frac{1}{2}\begin{array}{l}\rightarrow \text{number of special parts}\\ \rightarrow \text{total number of equal parts}\end{array}$$

Start with halves, then with quarters and then eighths. Cut shapes out of flannel or paper plates. Figure 14.6 illustrates common fractions.

8. Learning the computational facts. Once the concepts behind the facts are known, the students must memorize the facts themselves. Many different learning opportunities are needed. Students can write the facts, say them, play games with facts, take speed tests, and so forth. Also helpful are flash cards, rolling dice, playing cards, or learning a fact a day. A wide variety of methods should be used.

To learn computational skills, students with mathematics difficulties require much experience with concrete and manipulative materials before moving to the abstract and symbolic level of numbers. Objects and materials that can be physically taken apart and put back together help the students to observe visually the relationship of the fractional parts of the whole.

There are 56 basic number facts to be mastered in each mode of arithmetic computation (addition, subtraction, multiplication, and division), if the facts involving the 1s (3 + 1 = 4) and doubles (3 × 3 = 9) are not included. Examples of number facts are 3 + 4 = 7; 9 − 5 = 4; 3 × 7 = 21; 18 ÷ 6 = 3. In the computational skill of addition, for example, there are 81 separate facts involved in the span from 1 + 1 = 2 to 9 + 9 = 18.

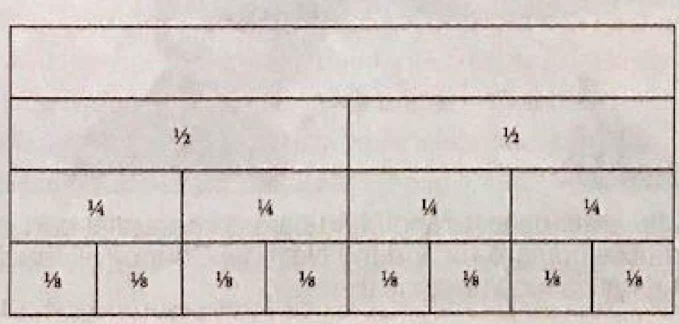


FIGURE **14.6**
Some Common Fractions

FIGURE **14.7**
Calendar for Learning Facts

Sun	Mon	Tue	Wed	Thur	Fri	Sat
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28

Few students have trouble with the 1s ($5 + 1 = 6$) or with the doubles ($2 + 2 = 4$). Therefore, if these facts are omitted, there are 56 basic addition facts to be mastered. Similarly, without the 1s and doubles, there are 56 facts to be mastered in each of the other computation areas—subtraction, multiplication, and division.

9. **The 2-weeks facts:** $7 + 7$. Students circle 2 full calendar weeks and count the number of days in each week, as shown in Figure 14.7, to learn that $7 + 7 = 14$.

10. **Subtraction of 9s from teen numbers.** One useful technique to help students learn subtraction of 9s from the teen numbers is to have students consider the following problem: $16 - 9 = \square$. Adding the 1 and 6 gives the correct answer of 7. This technique works with subtracting 9s from all teen numbers.

11. **Arrangements.** Present students with arrangement problems. For example, give students the numbers 1, 2, 3. Ask them in how many ways they can be arranged: 1-2-3; 1-3-2; 2-1-3; 2-3-1; 3-1-2; 3-2-1 (or $3 \times 2 \times 1 = 6$). Another example: If 4 children sit around a square table, in how many ways can they arrange themselves? ($4 \times 3 \times 2 \times 1 = 24$).

12. **Puzzle cards of combinations.** Make cardboard cards on which problems of addition, subtraction, multiplication, and division are worked. Cut each card in 2 so that the problem is on one part and the answer is on the other. Each card must be cut uniquely so that when the students try to assemble the puzzle, only the correct answer will fit.

13. **Playing cards.** An ordinary deck of cards becomes a versatile tool for teaching number concepts. Some activities are arranging suits in sequential order by number, matching sets of numbers, adding and subtracting with individual cards, and quickly recognizing the number in a set.

Annie Griffiths Belt/Corbis

Computer spreadsheets and charts are an essential part of the mathematics curriculum, and the National Council of Teachers of Mathematics recommends their use.

14.12c Teaching Word Story Problems

The goal of mathematics instruction is to apply the concepts and skills in problem solving. The National Council of Teachers of Mathematics (2000) calls for more emphasis on problem solving at all levels. Some suggestions for teaching word story problems are provided in the following list.

1. **Word story problems.** Use word story problems that are of interest to the students and within their experience.
2. **Posing problems orally.** This method is especially important for students with reading problems.
3. **Visual reinforcements.** Use concrete objects, drawings, graphs, or other visual reinforcements to clarify the problem, demonstrate solutions, and verify the answers. Have students act out the problem.
4. **Simplifying.** Have students substitute smaller and easier numbers for problems with larger or more complex numbers so that they can understand the problems and verify the solutions more readily.
5. **Restating.** Have students restate the problems in their own words. This verbalization helps the students to structure the problems for themselves and also shows whether they understand the problems.
6. **Supplementary problems.** Supplement textbook problems with your own, which could deal with classroom experiences. Including students' names makes the problem more realistic.
7. **Time for thinking.** Allow students enough time to think. Ask for alternative methods for solving the problems. Try to understand how the students thought about the problem and went about solving the problem.
8. **Steps in solving word problems.** Many students with learning disabilities have difficulty with word problems. Although problems in reading may be a factor, the difficulty is often in thinking through the math problems. Students tend to begin doing computations as soon as they see the numbers in the problems. The following steps are helpful in teaching word problem applications:

    a. **Seeing the situations.** Have the students first read the word problem and then relate the setting of the problem. The students do not need paper and pencil for this task. They should simply describe the setting or situation.

    b. **Determining the question.** Have the students decide what is to be discovered—What is the problem to be solved?

    c. **Gathering data.** The word problem often gives much data—some relevant, some not relevant to the solution. Ask the students to read the problem aloud, or silently, and then list the relevant and irrelevant data.

    d. **Analyzing relationships.** Help the students analyze the relationships among the data. For example, if the problem states that the down payment on an automobile costing $2,000 is 25%, the students must see the relationship between these two facts. Seeing relationships is a reasoning skill that students with mathematics disabilities often find difficult.

    e. **Deciding on a process.** Students must decide which computational process should be used to solve the problem. Students should be alert to key words, such as total or in all, which suggest addition, and is left or remains, which suggest subtraction. They should next put the problem into mathematical sentences.

f. **Estimating answers.** Have the students practice estimating what a reasonable answer might be. If the students understand the reasoning behind the problem, they should be able to estimate answers.

g. **Practice and generalization.** After students have thought through and worked out one type of problem, the teacher can give similar problems with different numbers.

time concepts
The sense of time, which is not easily comprehended by some students with learning disabilities, who may be poor at estimating the span of an hour, a minute, several hours, or a weekend and may have difficulty estimating how long a task will take. Trouble with time concepts characterizes students with mathematics disabilities.

9. **Time.** Time concepts involve a difficult dimension for many students with mathematics disabilities to grasp, so they may require specific instructions to learn how to tell time. Real clocks or teacher-made clocks are needed to teach this skill. A teacher-made clock can be created by using a paper fastener to attach cardboard hands to a paper plate. A sequence for teaching time might be the hour (1:00), the half hour (4:30), the quarter hour (7:15), 5-minute intervals (2:25), before and after the hour, minute intervals, and seconds. Use television schedules of programs or classroom activities and relate them to clock time.

10. **Money.** The use of real money and lifelike situations is an effective way to teach number facts to some students. Have them play store, make change, or order a meal from a restaurant menu and then add up the cost and pay for it. All of these situations provide concrete and meaningful practice for learning arithmetic.

14.12d Secondary Mathematics Strategies

Teaching Algebra Through Active Learning High school students used algebra to analyze variable pricing of local cell phone plans. They used advertisements from cell phone companies to sell monthly charges and they also looked at additional charges, such as texting charges, to fund the real cost of the phone plan. They created an algebraic equation to reflect the real cost. They then compared various cell phone costs. They created electronic presentations, using charts, graphs, and PowerPoint to show their results to parents and fellow students (Boss, 2009).

Word Problems STAR is a strategy for word problems for students with learning disabilities and related mild disabilities (Maccini & Hughes, 2000).

S- Search the word problem. Read the problem, ask yourself questions, and write down facts.

T- Translate the words into an equation. For example, identify the operation, representing the equation through manipulative objects or drawings.

A- Answer the problem.

R- Review the solution and check that the solution is reasonable.

Order of Operations Strategy Knowing the order of operations is an important prerequisite for learning algebra. The ORDER strategy helps students with learning disabilities and related mild disabilities remember the order of operations (Minskoff & Allsopp, 2003).

O- Observe the problem. Read the problem and look for multiple operation signs.

R- Read the signs. Look at each sign and identify the operation it represents.

D- Decide which operation to do first. Operations must be performed in a particular order.

E- Execute the rules of order. The phrase "many Dogs are Smelly" reminds students that multiplication and division come before addition and subtraction.

R- Relax. You are done.

Did You Get It?

What basic skillset serves as the foundation for all other mathematical computational skills?

a. ordering
b. sorting
c. addition
d. matching

14.13 Using Technology for Mathematics Instruction

14.13a Calculators

Students must be required to learn the computation facts, but there are times for using the calculator as well. Calculators are available as stand alone low cost products but are also a part of an iPhone, iPad, or computers. Students in school should be taught how to make efficient use of the calculator. In doing a mathematics-reasoning problem, students often become so bogged down in computation that they never get to the reasoning aspects of the lesson. By using calculators, students can put their energies into understanding the mathematical concept rather than on performing the underlying calculation process (Center for Implementing Technology in Education, 2007).

A low-cost pocket calculator is easily accessible and handy. It can be used to compute basic facts, as well as more complicated math processes, and it is also useful for self-checking. Because it is more socially acceptable than other counting systems, it is particularly helpful for adults who have not memorized basic computation facts. Students do need instruction in the proper way to use a calculator, so lessons must be designed to teach calculation skills.

Students with mathematics difficulties may find talking calculators useful. The talking calculator is a calculator with a speech synthesizer. When a number, symbol, or operation is pressed, it is vocalized by the speech synthesizer. The user gets auditory feedback and can double-check the answers.

mathematics difficulties
Difficulty using quantitative and number concepts.

Secondary students and adults are likely to need programmable calculators to perform more complex math functions.

14.13b Computers

The rapid pace of change in computer applications has made computer technology especially useful for teaching mathematics.

Beyond calculators and computer games, teachers today have the opportunity to utilize virtual manipulatives as discussed previously. Interactive whiteboards are also found in many of today's schools and can be used very effectively to build math skills.

TeachSource Video Case Activity

Watch the TeachSource Video Case entitled "Using Technology to Promote Discovery Learning: High School Geometry Class."

In this video, Gary Simons, the geometry teacher, uses "discovery learning" to help students learn how to make conjectures in geometry class. They use a technology tool called "sketchpad" to study angles in geometry. The geometry teacher explains that discovery learning is an essential part of the teacher's repertoire. He also points out that "traditional mathematics" teaching is also a part of the teacher's repertoire.

QUESTIONS

1. What is "discovery learning" in geometry?
2. How does "traditional mathematics" teaching differ from "discovery learning"?

There are a number of math apps that are available for the iPad. Aronin and Floyd (2013) provide some examples of apps for preschoolers.

Monkey Math School Sunshine
My First Tangrams

When selecting apps for math, it is important to match children's preferences, strengths, specific needs, and developmental levels with appropriate levels (Aronin and Floyd, 2013).

Now there is increased emphasis on STEM (science, technology, engineering, and mathematics). The National Aeronautics and Space Administration (*http://www.nasa.gov/*) has many resources available for free. These lesson plans and resources provide an array of integrated activities for science, technology, engineering, and math. Other websites that might be helpful include:

Math Fact Fluency—*www.reflexmath.com*
Online Math Learning—*www.onlinemathlearning.com*

Many mathematics software programs, although not specifically designed for students with mathematics disabilities, may be useful. Computers motivate students, and the mathematics software programs can individualize, provide feedback, and offer repetition (Belson, 2003; Lewis, 1998; Raskind & Higgins, 1998a). These programs should have as little clutter as possible and should offer concise, clear directions, moving from simple and concrete directions to longer and more complex ones. The programs should question the student frequently (asking, for example, "Are you sure? Do you want to change your answer?"). They should also provide immediate feedback to the student.

Mathematics programs range from drill-and-practice programs to problem-solving programs. A good source for mathematics software programs for students with learning disabilities is Closing the Gap's Resource Directory. The website for Closing the Gap is at *http://www.closingthegap.com.*

14.13c **Spreadsheets**

Computer spreadsheets are an essential part of the mathematics curriculum, and their use is recommended by the National Council of Teachers of Mathematics (2000). Moreover, students with learning disabilities often do very well with spreadsheet applications, possibly because spreadsheets are a visual task, rather than a linguistic task. A spreadsheet displays numeric information through a grid of columns and rows. The intersection of a column and a row is called a cell. When numbers are placed in the cells, they can be used in mathematics computations or in mathematics formulas, such as averages. Charts and

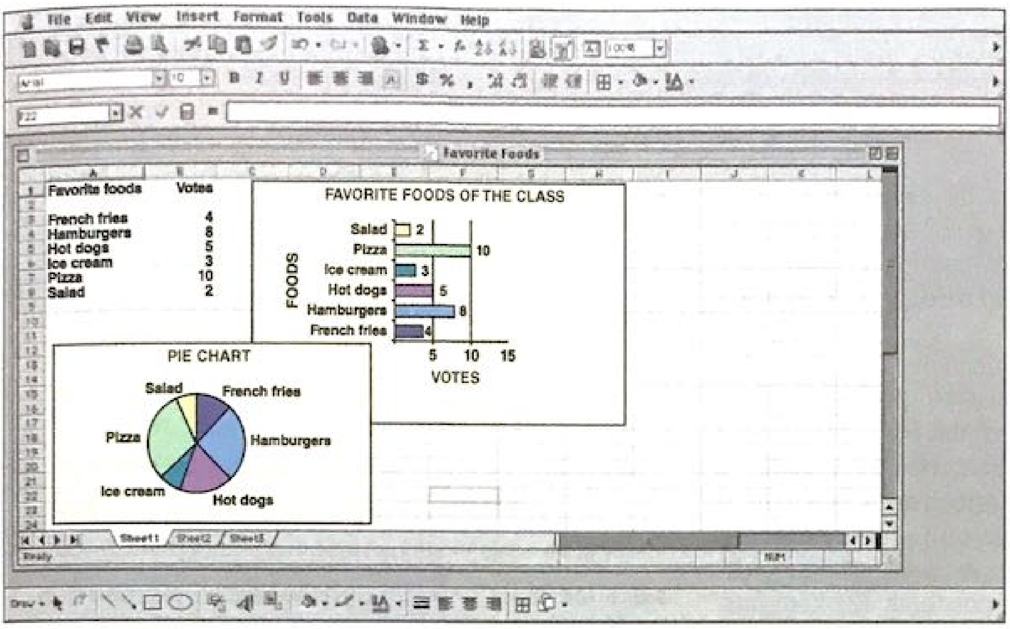


FIGURE **14.8**
Spreadsheet, Pie Chart, and Bar Graph

graphs, such as pie charts, bar graphs, or line graphs, are electronically made, based on the numbers in the spreadsheets (see Figure 14.8).

A pie chart is a circular chart cut into segments illustrating magnitudes or frequencies. A bar graph is a type of chart in which different values are represented by rectangular bars. A wide variety of student activities can be accomplished with spreadsheets, such as planning a budget, keeping records of grades, compiling an inventory of items used in a hobby, or tracking election results.

pie chart
A circular chart showing segments that illustrate magnitude or frequencies.

bar graph
A type of chart in which different values are represented by rectangular bars.

In one activity, students noted their favorite foods. These foods were then listed on a chalkboard and each student voted for his or her favorite food. The class votes were tallied (e.g., pizza, 10 votes; hot dogs, 5 votes). The favorite foods and votes were then put into Columns A and B on a spreadsheet, and the students made pie charts and bar graphs from their spreadsheets. Figure 14.8 shows the spreadsheet, pie chart, and bar graph that resulted from this activity.

spreadsheets
A display of numeric information through rows and columns, usually used in a computer program.

Did You Get It?

Spreadsheets, a valuable and effective tool for teaching mathematical computation, are helpful because they are visual. The use of spreadsheets in mathematics is

a. mandated by CORE.
b. recommended by the National Council of Teachers of Mathematics.
c. frowned upon by the National Education Association.
d. mandated by No Child Left Behind Act.

I Have a Kid Who...

SAM: *A Student Who Has Difficulty in Algebra*

Sam is age 14 years 8 months, he is in the ninth grade, and he is identified through his IEP as a student with learning disabilities. Sam enjoys reading and does well in English, but he has encountered difficulty in mathematics since his elementary years.

Sam is in the general education algebra class. At the beginning of the school year, Mr. Zero, Sam's algebra teacher, reviewed some of the basic concepts of basic pre-algebra that the students had been taught the previous year in eighth grade. After a quick review, most students were ready to move on to more advanced algebra concepts. Sam, however, was still having difficulty with the basic algebra concepts. Mr. Zero realizes these early algebra concepts must be mastered before moving on to more advanced concepts. He collaborates with the special education teacher and they both work with Sam to teach him basic pre-algebra skills. They will teach Sam to solve the following types of algebra problems:

- Addition, subtraction, multiplication, and division problems involving integers. For example: $4 + 6$; $4 + -6$; 4×5; $-18 \div -3$.
- Simplify addition, subtraction, and division equations. For example: $(2x + 6) + (4x + 7) = 6x + 13$.
- Solve expressions with variables. For example: $3x = -24$.
- Solve two-step equations. For example: $3x - 4 = 2$.
- Solve multi-step equations. For example: $5x - 4 = 2x + 5$.

Source: Adapted from The Iris Center for Faculty Enhancement, Case Study Unit, Algebra (Part 1), *http://www.iris.peabody.vanderbilt.edu.*

QUESTIONS

1. How is the strategy of collaboration being used for Sam?
2. Do you think that other students in the algebra class would benefit from this collaborative teaching?

Chapter Summary

- Some students with learning disabilities and related mild disabilities have severe difficulty in learning mathematics. For others, mathematics seems to be an area of strength.
- Dyscalculia is a severe disability in learning and using mathematics that is associated with a neurological dysfunction.
- Early number learning in young children includes abilities in spatial relations, visual-motor and visual-perception processing, and concepts of time and direction.
- Characteristics of mathematics disabilities are related to information-processing difficulties, language and reading abilities, and math anxiety.
- Views about teaching mathematics have changed over the years in response to national concerns. Today's approach is to require high standards and annual testing.
- There are several learning theories of mathematics instruction for students with mathematics disabilities, which include the progression from concrete learning to abstract learning, direct instruction, learning strategies instruction, and problem-solving approaches.
- Students' mathematics skills can be assessed through formal tests and informal measures. Each provides a different kind of information about mathematics performance.
- The content of the mathematics curriculum is sequential and cumulative. Different elements of mathematics are taught at different grade levels.
- Principles of instruction in mathematics stress that the students should have early number learning. Instruction should progress from the concrete to the abstract, with ample opportunity for practice and review. The students must learn to generalize concepts that have been learned, and they should also know the vocabulary for basic mathematics operations.

- Students should learn basic computational facts, but they should be allowed to use calculators for some purposes in the classroom. Calculator use should be part of the mathematics curriculum.
- Computers have many useful applications in teaching mathematics to students with learning disabilities and related mild disabilities.

Questions for Discussion and Reflection

1. The Individuals With Disabilities Education Improvement Act (IDEA-2004) recognizes 2 areas in which students can have mathematics disabilities. Identify these 2 areas and discuss the implications for services.
2. Characteristics of learning disabilities and related mild disabilities can affect the learning of mathematics. Select two characteristics of students with mathematics disabilities, and describe how these characteristics can affect mathematics learning.
3. Do you think calculators should be used in mathematics instruction? Why or why not? Discuss how they could be used.
4. How can computers be used in the teaching of mathematics?
5. Describe how students can be instructed to go from concrete learning to abstract learning.

Key Terms

abstract instruction (p. 444)
bar graph (p. 455)
concrete instruction (p. 443)
direct instruction (p. 429)
dyscalculia (p. 424)
early number learning (p. 425)
math anxiety (p. 427)
mathematics computation (p. 447)
mathematics difficulties (p. 453)
mathematics learning disabilities (p. 423)
number lines (p. 425)
one-to-one correspondence (p. 424)
part–whole relationship (p. 447)
pie chart (p. 455)
place value (p. 427)
problem solving (p. 423)
semiconcrete instruction (p. 444)
spatial relationships (p. 425)
spreadsheet (p. 455)
time concepts (p. 452)