5080 UNIT TWO

GUIDE

UNIT II STUDY GUIDE

Determining Probability

Probability

Who wants to calculate probability? Businesses (including farmers and ranchers raising crops and livestock), governments, and anyone wanting to quantify risks in life will calculate probability. This includes people involved in gaming as well. As you read, the textbook illustrates probability with coin tosses where the outcomes are just two possible ones—heads or tails (Render, Stair, Hanna, & Hale, 2015). You may know that gamblers have more complex probability problems to estimate a solution for—as in Texas Hold 'Em, where a cardholder may be calculating whether the remaining players are holding higher hands than his or her own. There are answers available to the cardholder's dilemma as well. Probability is "a numerical statement about the likelihood that an event will occur" (Render et al., 2015, p. 24). Mathematics can model this for us. Because some mathematical terms are equal to others, you can state the formulas for certain probabilities as you see in Chapter 2 of the textbook. In the physical world, the probability of anything is either 0 (cannot happen), 1 (100% chance of happening), or some fraction in between 0 and 1 (has a little/some/even/probable chance of happening). As something has to happen in every trial, the probabilities added up for identical trials equal 1 for the series. A tossed coin has a 50% or .5 chance of coming up heads, and the same 50% or .5 chance of coming up tails, but something will come up when the coin is tossed—or, .5 + .5 = 1.

So for probability of the event = P(event):

$0 \leq P(\text{event}) \leq 1$

The probability to be calculated is in that range somewhere! Now, how do you find it? Here are two types of approaches that fit what happens, the objective approach and subjective approach.

Types of Probability

□ *Objective approach*: The objective approach (when you can use numbers to calculate probability directly) uses two common methods (Render et al., 2015):

1. The *relative frequency method* is used when you know how often things happen (as in the coin example above, if you know how many times it was tossed), and
2. The *classical or logical method* is used when you know often things *should* happen (e.g., the number of ways a coin will land, heads or tails, without knowing the number of trials, or as a

better example, the chance of drawing an Ace in an American card deck – P = 4 / 52, or 1/13 = .077).
□ *Subjective approach*: The subjective approach is used to assess probabilities when logic cannot be applied and past trial outcomes are not known; then the probabilities are assessed subjectively. This also is done often in society for economic outlooks, weather, and prices. Opinion polls can be used for estimating candidate chances, and the *Delphi Method* (panel of experts) can be used for the best judgments likely to be received. (Render et al., 2015). The last unit (Unit VIII) explores this in more detail.

Types of Events
With the following methods, you will be able to determine some probability. A *mutually exclusive* event is when a trial is conducted and only one event can occur as the trial's outcome (Render et al., 2015). As you recall from coin tosses, the outcome will be either heads or tails. Heads and tails are mutually exclusive because both cannot occur in a single trial.
Intersections: Looking back to the card deck, you can try for an event that draws a seven and try for an event that draws a heart, and these are not mutually exclusive because you could draw a *seven of hearts*. Such outcomes that can be in both event "camps," as this example is the intersection of the probabilities of sevens and hearts:
P (Intersection of event 7s and event hearts) is written as
P = event 7s ∩ event hearts, or, to substitute,
P (A∩B) = P (AB). So when you multiply the probabilities of the events that intersected because an outcome could be both events, you get the probability of the intersection—the probability that an outcome will be both events.
Unions: How about all the event possibilities that are in either outcome and that probability? Events of all outcomes are called unions. Unions of all events that could be in either of two outcomes would be written as:
P(A or B) = P(AUB) = P (A) + P(B) – P(AB).
Why subtract out the small probability of the intersection P(AB)? The reason is that you don't want to double-count the events occurring in both outcomes.
Probability Rules
There is one more thing that you will often be asked to do: determine the probability that an event will occur (given that another event already occurred), or determine *conditional probability* (Render et al., 2015). Occurring events affect subsequent events, which is why this is an important truth of mathematics, and it supports figuring out this phenomenon. Write the probability that Event A will occur given that Event B already did as:
P(A| B) = P(AB)
P(B)
This is the conditional probability that Event A will occur as the probability of the intersection of A and B, divided by the probability of B. Note this, and the following things you can do because of mathematics:
Probability of the intersection of A and B = P (A∩B) = P (AB) = P(A| B)P(B)
So if you drew a heart from a deck of cards, what is the probability that it is a 7, or P(A), given that B, drawing a heart, already occurred?
P(A| B) = P(AB) = 1/52 = 1/13
P(B) 13/52

If one event will have no effect on another, then the events are *independent* of each other. Mathematics tells us that Event A and Event B are independent if:

$P(A|B) = P(A)$

Which they must be, as you can see you can come up with B all day long, and A still has the same probability. And so the intersection of independent events is:

$P(A \text{ and } B) = P(A)\,P(B)$

And this is why with a probability of 50%, or .5, of a coin showing heads or tails on a toss, the probability of two heads or two tails on two tosses A and then B (independent of each other because you can't come up both heads and tails) is:

$.5 \times .5 = .25$

With these probability skills, one can offer a variety of estimates to leaders. Mathematicians and scientists have pushed these fundamentals to reveal some additional capabilities:

Bayes' Theorem

Bayes' Theorem enables us to add new information to an existing probability calculation to determine the updated probability. If A is an event, and A' is the "other," complementary event, then

$P(A|B) = P(B|A)P(A)$.

$P(B|A)P(A) + P(B|A')P(A')$

If the business can afford it, the administrators can keep running trials to fine-tune the probability estimate. It may be best to be satisfied with just two or three trials, though. The differences between solutions may become too close together to matter, and one can show general awareness of such *probability distributions.*

Who finds it useful to calculate probability with the approaches explored so far? Early in this lesson, reasons of business and government were mentioned, and those remain areas that often are in need of gaining an indication of what might happen. Note that statistical analysis does not promise to show what WILL happen. All one can do without prescient powers is figure out what the chances are of something happening.

Those of you who partake in gaming for leisure may recall that casinos often forbid counting of cards and other calculating methods that give guest players a more-than-usual chance at winning. The usual chance is that the "House" (casino) has a slight edge in probability of winning, which is set by the specific rules of the game or by electronic or mechanical settings of gaming machines. But as you can see with Bayes' Theorem and other conditional probability theories, it is possible to negate the House advantage if you can (discretely) calculate probability after seeing a die or the first few cards. Note also that often what you see in casinos is not calculation at all but guesswork and a lot of hope! But casinos and gaming are supposed to be fun.

In this course, you must face the notion that luck and calculations are two different things. After accepting this, you are left with just the disturbing afterthought that business and government leaders may forego statistical analysis and strive for luck—consciously or unconsciously. This human tendency is part of why the science of statistical analysis was developed and leveraged to assist leaders.