(b) Replace $f(t)$ with 60 and solve for t.

$$f(t) = 20.57 \ln t + 10.58 \quad \text{Given function}$$

$$60 = 20.57 \ln t + 10.58 \quad \text{Let } f(t) = 60.$$

$$49.42 = 20.57 \ln t \quad \text{Subtract 10.58.}$$

$$\ln t = \frac{49.42}{20.57} \quad \text{Divide by 20.57 and rewrite.}$$

$$t = e^{49.42/20.57} \quad \text{Write in exponential form.}$$

$$t \approx 11.05 \quad \text{Use a calculator.}$$

Adding 11 to 2009 gives the year 2020. Based on this model, annual sales will reach 60 million in 2020.

Now Try Exercise 111.

4.5 Exercises

CONCEPT PREVIEW *Match each equation in Column I with the best first step for solving it in Column II.*

I	II
1. $10^x = 150$	**A.** Use the product rule for exponents.
2. $e^{2x-1} = 24$	**B.** Take the common logarithm on each side.
3. $\log_4(x^2 - 10) = 2$	**C.** Write the sum of logarithms as the logarithm of a product.
4. $e^{2x} \cdot e^x = 2e$	**D.** Let $u = e^x$ and write the equation in quadratic form.
5. $2e^{2x} - 5e^x - 3 = 0$	**E.** Change to exponential form.
6. $\log(2x - 1) + \log(x + 4) = 1$	**F.** Take the natural logarithm on each side.

CONCEPT PREVIEW *An exponential equation such as*

$$5^x = 9$$

can be solved for its exact solution using the meaning of logarithm and the change-of-base theorem. Because x is the exponent to which 5 must be raised in order to obtain 9, the exact solution is

$$\log_5 9, \quad \text{or} \quad \frac{\log 9}{\log 5}, \quad \text{or} \quad \frac{\ln 9}{\ln 5}.$$

For each equation, give the exact solution in three forms similar to the forms above.

7. $7^x = 19$ **8.** $3^x = 10$ **9.** $\left(\frac{1}{2}\right)^x = 12$ **10.** $\left(\frac{1}{3}\right)^x = 4$

Solve each equation. In Exercises 11–34, give irrational solutions as decimals correct to the nearest thousandth. In Exercises 35–40, give solutions in exact form. ***See Examples 1–4.***

11. $3^x = 7$ **12.** $5^x = 13$ **13.** $\left(\frac{1}{2}\right)^x = 5$

14. $\left(\frac{1}{3}\right)^x = 6$ **15.** $0.8^x = 4$ **16.** $0.6^x = 3$

17. $4^{x-1} = 3^{2x}$ **18.** $2^{x+3} = 5^{2x}$ **19.** $6^{x+1} = 4^{2x-1}$

20. $3^{x-4} = 7^{2x+5}$
21. $e^{x^2} = 100$
22. $e^{x^4} = 1000$
23. $e^{3x-7} \cdot e^{-2x} = 4e$
24. $e^{1-3x} \cdot e^{5x} = 2e$
25. $\left(\frac{1}{3}\right)^x = -3$
26. $\left(\frac{1}{9}\right)^x = -9$
27. $0.05(1.15)^x = 5$
28. $1.2(0.9)^x = 0.6$
29. $3(2)^{x-2} + 1 = 100$
30. $5(1.2)^{3x-2} + 1 = 7$
31. $2(1.05)^x + 3 = 10$
32. $3(1.4)^x - 4 = 60$
33. $5(1.015)^{x-1980} = 8$
34. $6(1.024)^{x-1900} = 9$
35. $e^{2x} - 6e^x + 8 = 0$
36. $e^{2x} - 8e^x + 15 = 0$
37. $2e^{2x} + e^x = 6$
38. $3e^{2x} + 2e^x = 1$
39. $5^{2x} + 3(5^x) = 28$
40. $3^{2x} - 12(3^x) = -35$

Solve each equation. Give solutions in exact form. ***See Examples 5–9.***

41. $5 \ln x = 10$
42. $3 \ln x = 9$
43. $\ln 4x = 1.5$
44. $\ln 2x = 5$
45. $\log(2 - x) = 0.5$
46. $\log(3 - x) = 0.75$
47. $\log_6(2x + 4) = 2$
48. $\log_5(8 - 3x) = 3$
49. $\log_4(x^3 + 37) = 3$
50. $\log_7(x^3 + 65) = 0$
51. $\ln x + \ln x^2 = 3$
52. $\log x + \log x^2 = 3$
53. $\log_3[(x + 5)(x - 3)] = 2$
54. $\log_4[(3x + 8)(x - 6)] = 3$
55. $\log_2[(2x + 8)(x + 4)] = 5$
56. $\log_5[(3x + 5)(x + 1)] = 1$
57. $\log x + \log(x + 15) = 2$
58. $\log x + \log(2x + 1) = 1$
59. $\log(x + 25) = \log(x + 10) + \log 4$
60. $\log(3x + 5) - \log(2x + 4) = 0$
61. $\log(x - 10) - \log(x - 6) = \log 2$
62. $\log(x^2 - 9) - \log(x - 3) = \log 5$
63. $\ln(7 - x) + \ln(1 - x) = \ln(25 - x)$
64. $\ln(3 - x) + \ln(5 - x) = \ln(50 - 6x)$
65. $\log_8(x + 2) + \log_8(x + 4) = \log_8 8$
66. $\log_2(5x - 6) - \log_2(x + 1) = \log_2 3$
67. $\log_2(x^2 - 100) - \log_2(x + 10) = 1$
68. $\log_2(x - 2) + \log_2(x - 1) = 1$
69. $\log x + \log(x - 21) = \log 100$
70. $\log x + \log(3x - 13) = \log 10$
71. $\log(9x + 5) = 3 + \log(x + 2)$
72. $\log(11x + 9) = 3 + \log(x + 3)$
73. $\ln(4x - 2) - \ln 4 = -\ln(x - 2)$
74. $\ln(5 + 4x) - \ln(3 + x) = \ln 3$
75. $\log_5(x + 2) + \log_5(x - 2) = 1$
76. $\log_2(x - 7) + \log_2 x = 3$
77. $\log_2(2x - 3) + \log_2(x + 1) = 1$
78. $\log_5(3x + 2) + \log_5(x - 1) = 1$
79. $\ln e^x - 2 \ln e = \ln e^4$
80. $\ln e^x - \ln e^3 = \ln e^3$
81. $\log_2(\log_2 x) = 1$
82. $\log x = \sqrt{\log x}$
83. $\log x^2 = (\log x)^2$
84. $\log_2 \sqrt{2x^2} = \frac{3}{2}$

85. ***Concept Check*** Consider the following statement: "We must reject any negative proposed solution when we solve an equation involving logarithms." Is this correct? Why or why not?

86. ***Concept Check*** What values of x could not possibly be solutions of the following equation?

$$\log_a(4x - 7) + \log_a(x^2 + 4) = 0$$

Solve each equation for the indicated variable. Use logarithms with the appropriate bases. ***See Example 10.***

87. $p = a + \frac{k}{\ln x}$, for x

88. $r = p - k \ln t$, for t

89. $T = T_0 + (T_1 - T_0)10^{-kt}$, for t

90. $A = \frac{Pr}{1 - (1 + r)^{-n}}$, for n

91. $I = \frac{E}{R}(1 - e^{-Rt/2})$, for t

92. $y = \frac{K}{1 + ae^{-bx}}$, for b

93. $y = A + B(1 - e^{-Cx})$, for x

94. $m = 6 - 2.5 \log \frac{M}{M_0}$, for M

95. $\log A = \log B - C \log x$, for A

96. $d = 10 \log \frac{I}{I_0}$, for I

97. $A = P\left(1 + \frac{r}{n}\right)^{tn}$, for t

98. $D = 160 + 10 \log x$, for x

To solve each problem, refer to the formulas for compound interest.

$$A = P\left(1 + \frac{r}{n}\right)^{tn} \quad \text{and} \quad A = Pe^{rt}$$

99. *Compound Amount* If \$10,000 is invested in an account at 3% annual interest compounded quarterly, how much will be in the account in 5 yr if no money is withdrawn?

100. *Compound Amount* If \$5000 is invested in an account at 4% annual interest compounded continuously, how much will be in the account in 8 yr if no money is withdrawn?

101. *Investment Time* Kurt wants to buy a \$30,000 truck. He has saved \$27,000. Find the number of years (to the nearest tenth) it will take for his \$27,000 to grow to \$30,000 at 4% interest compounded quarterly.

102. *Investment Time* Find t to the nearest hundredth of a year if \$1786 becomes \$2063 at 2.6%, with interest compounded monthly.

103. *Interest Rate* Find the interest rate to the nearest hundredth of a percent that will produce \$2500, if \$2000 is left at interest compounded semiannually for 8.5 yr.

104. *Interest Rate* At what interest rate, to the nearest hundredth of a percent, will \$16,000 grow to \$20,000 if invested for 7.25 yr and interest is compounded quarterly?

(Modeling) Solve each application. ***See Example 11.***

105. In the central Sierra Nevada (a mountain range in California), the percent of moisture that falls as snow rather than rain is approximated reasonably well by

$$f(x) = 86.3 \ln x - 680,$$

where x is the altitude in feet and $f(x)$ is the percent of moisture that falls as snow. Find the percent of moisture, to the nearest tenth, that falls as snow at each altitude.

(a) 3000 ft **(b)** 4000 ft **(c)** 7000 ft

106. Northwest Creations finds that its total sales in dollars, $T(x)$, from the distribution of x thousand catalogues is approximated by

$$T(x) = 5000 \log (x + 1).$$

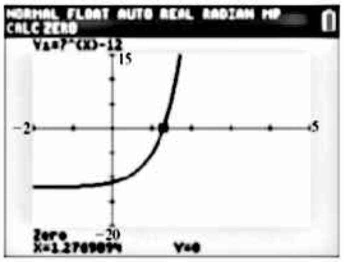
Find the total sales, to the nearest dollar, resulting from the distribution of each number of catalogues.

(a) 5000 **(b)** 24,000 **(c)** 49,000

EXAMPLE 1 Solving an Exponential Equation

Solve $7^x = 12$. Give the solution to the nearest thousandth.

SOLUTION The properties of exponents cannot be used to solve this equation, so we apply the preceding property of logarithms. While any appropriate base b can be used, the best practical base is base 10 or base e. We choose base e (natural) logarithms here.



As seen in the display at the bottom of the screen, when rounded to three decimal places, the solution of $7^x - 12 = 0$ agrees with that found in **Example 1.**

$$\begin{aligned}
7^x &= 12 \\
\ln 7^x &= \ln 12 && \text{Property of logarithms} \\
x \ln 7 &= \ln 12 && \text{Power property} \\
x &= \frac{\ln 12}{\ln 7} && \text{Divide by } \ln 7. \\
x &\approx 1.277 && \text{Use a calculator.}
\end{aligned}$$

This is *exact.*

This is *approximate.*

The solution set is $\{1.277\}$.

✔ Now Try Exercise 11.

CAUTION Do not confuse a quotient like $\frac{\ln 12}{\ln 7}$ in **Example 1** with $\ln \frac{12}{7}$, which can be written as $\ln 12 - \ln 7$. ***We cannot change the quotient of two logarithms to a difference of logarithms.***

$$\frac{\ln 12}{\ln 7} \neq \ln \frac{12}{7}$$

EXAMPLE 2 Solving an Exponential Equation

Solve $3^{2x-1} = 0.4^{x+2}$. Give the solution to the nearest thousandth.

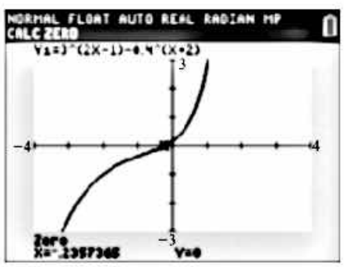
SOLUTION

$$\begin{aligned}
3^{2x-1} &= 0.4^{x+2} \\
\ln 3^{2x-1} &= \ln 0.4^{x+2} && \text{Take the natural logarithm on each side.} \\
(2x-1)\ln 3 &= (x+2)\ln 0.4 && \text{Power property} \\
2x\ln 3 - \ln 3 &= x\ln 0.4 + 2\ln 0.4 && \text{Distributive property} \\
2x\ln 3 - x\ln 0.4 &= 2\ln 0.4 + \ln 3 && \text{Write so that the terms with } x \text{ are on one side.} \\
x(2\ln 3 - \ln 0.4) &= 2\ln 0.4 + \ln 3 && \text{Factor out } x. \\
x &= \frac{2\ln 0.4 + \ln 3}{2\ln 3 - \ln 0.4} && \text{Divide by } 2\ln 3 - \ln 0.4. \\
x &= \frac{\ln 0.4^2 + \ln 3}{\ln 3^2 - \ln 0.4} && \text{Power property} \\
x &= \frac{\ln 0.16 + \ln 3}{\ln 9 - \ln 0.4} && \text{Apply the exponents.} \\
x &= \frac{\ln 0.48}{\ln 22.5} && \text{Product and quotient properties} \\
x &\approx -0.236 && \text{Use a calculator.}
\end{aligned}$$

This is *exact.*

This is *approximate.*

The solution set is $\{-0.236\}$.



This screen supports the solution found in **Example 2.**

✔ Now Try Exercise 19.

EXAMPLE 3 Solving Base e Exponential Equations

Solve each equation. Give solutions to the nearest thousandth.

(a) $e^{x^2} = 200$ **(b)** $e^{2x+1} \cdot e^{-4x} = 3e$

SOLUTION

(a)

$$\begin{aligned} e^{x^2} &= 200 \\ \ln e^{x^2} &= \ln 200 && \text{Take the natural logarithm on each side.} \\ x^2 &= \ln 200 && \ln e^{x^2} = x^2 \\ x &= \pm\sqrt{\ln 200} && \text{Square root property} \\ x &\approx \pm 2.302 && \text{Use a calculator.} \end{aligned}$$

Remember both roots.

The solution set is $\{\pm 2.302\}$.

(b)

$$\begin{aligned} e^{2x+1} \cdot e^{-4x} &= 3e \\ e^{-2x+1} &= 3e && a^m \cdot a^n = a^{m+n} \\ e^{-2x} &= 3 && \text{Divide by } e;\ \tfrac{e^{-2x+1}}{e^1} = e^{-2x+1-1} = e^{-2x}. \\ \ln e^{-2x} &= \ln 3 && \text{Take the natural logarithm on each side.} \\ -2x \ln e &= \ln 3 && \text{Power property} \\ -2x &= \ln 3 && \ln e = 1 \\ x &= -\frac{1}{2}\ln 3 && \text{Multiply by } -\tfrac{1}{2}. \\ x &\approx -0.549 && \text{Use a calculator.} \end{aligned}$$

The solution set is $\{-0.549\}$.

✔ **Now Try Exercises 21 and 23.**

EXAMPLE 4 Solving an Exponential Equation (Quadratic in Form)

Solve $e^{2x} - 4e^x + 3 = 0$. Give exact value(s) for x.

SOLUTION If we substitute $u = e^x$, we notice that the equation is quadratic in form.

$$\begin{aligned} e^{2x} - 4e^x + 3 &= 0 \\ (e^x)^2 - 4e^x + 3 &= 0 && a^{mn} = (a^n)^m \\ u^2 - 4u + 3 &= 0 && \text{Let } u = e^x. \\ (u-1)(u-3) &= 0 && \text{Factor.} \\ u - 1 = 0 \quad \text{or} \quad u - 3 &= 0 && \text{Zero-factor property} \\ u = 1 \quad \text{or} \quad u &= 3 && \text{Solve for } u. \\ e^x = 1 \quad \text{or} \quad e^x &= 3 && \text{Substitute } e^x \text{ for } u. \\ \ln e^x = \ln 1 \quad \text{or} \quad \ln e^x &= \ln 3 && \text{Take the natural logarithm on each side.} \\ x = 0 \quad \text{or} \quad x &= \ln 3 && \ln e^x = x;\ \ln 1 = 0 \end{aligned}$$

Both values check, so the solution set is $\{0, \ln 3\}$.

✔ **Now Try Exercise 35.**

Logarithmic Equations The following equations involve logarithms of variable expressions.

EXAMPLE 5 Solving Logarithmic Equations

Solve each equation. Give exact values.

(a) $7 \ln x = 28$ **(b)** $\log_2(x^3 - 19) = 3$

SOLUTION

(a)

$$7 \ln x = 28$$

$$\log_e x = 4 \quad \ln x = \log_e x; \text{ Divide by 7.}$$

$$x = e^4 \quad \text{Write in exponential form.}$$

The solution set is $\{e^4\}$.

(b)

$$\log_2(x^3 - 19) = 3$$

$$x^3 - 19 = 2^3 \quad \text{Write in exponential form.}$$

$$x^3 - 19 = 8 \quad \text{Apply the exponent.}$$

$$x^3 = 27 \quad \text{Add 19.}$$

$$x = \sqrt[3]{27} \quad \text{Take cube roots.}$$

$$x = 3 \quad \sqrt[3]{27} = 3$$

The solution set is $\{3\}$.

✔ **Now Try Exercises 41 and 49.**

EXAMPLE 6 Solving a Logarithmic Equation

Solve $\log(x + 6) - \log(x + 2) = \log x$. Give exact value(s).

SOLUTION Recall that logarithms are defined only for nonnegative numbers.

$$\log(x + 6) - \log(x + 2) = \log x$$

$$\log \frac{x + 6}{x + 2} = \log x \quad \text{Quotient property}$$

$$\frac{x + 6}{x + 2} = x \quad \text{Property of logarithms}$$

$$x + 6 = x(x + 2) \quad \text{Multiply by } x + 2.$$

$$x + 6 = x^2 + 2x \quad \text{Distributive property}$$

$$x^2 + x - 6 = 0 \quad \text{Standard form}$$

$$(x + 3)(x - 2) = 0 \quad \text{Factor.}$$

$$x + 3 = 0 \quad \text{or} \quad x - 2 = 0 \quad \text{Zero-factor property}$$

$$x = -3 \quad \text{or} \quad x = 2 \quad \text{Solve for } x.$$

The proposed negative solution (-3) is not in the domain of $\log x$ in the original equation, so the only valid solution is the positive number 2. The solution set is $\{2\}$.

✔ **Now Try Exercise 69.**

CAUTION Recall that the domain of $y = \log_a x$ is $(0, \infty)$. ***For this reason, it is always necessary to check that proposed solutions of a logarithmic equation result in logarithms of positive numbers in the original equation.***

EXAMPLE 7 Solving a Logarithmic Equation

Solve $\log_2[(3x-7)(x-4)] = 3$. Give exact value(s).

SOLUTION

$$\begin{aligned}
\log_2[(3x-7)(x-4)] &= 3 \\
(3x-7)(x-4) &= 2^3 && \text{Write in exponential form.} \\
3x^2 - 19x + 28 &= 8 && \text{Multiply. Apply the exponent.} \\
3x^2 - 19x + 20 &= 0 && \text{Standard form} \\
(3x-4)(x-5) &= 0 && \text{Factor.} \\
3x - 4 = 0 \quad \text{or} \quad x - 5 &= 0 && \text{Zero-factor property} \\
x = \frac{4}{3} \quad \text{or} \quad x &= 5 && \text{Solve for } x.
\end{aligned}$$

A check is necessary to be sure that the argument of the logarithm in the given equation is positive. In both cases, the product $(3x-7)(x-4)$ leads to 8, and $\log_2 8 = 3$ is true. The solution set is $\left\{\frac{4}{3}, 5\right\}$.

✔ Now Try Exercise 53.

EXAMPLE 8 Solving a Logarithmic Equation

Solve $\log(3x+2) + \log(x-1) = 1$. Give exact value(s).

SOLUTION

$$\begin{aligned}
\log(3x+2) + \log(x-1) &= 1 \\
\log_{10}[(3x+2)(x-1)] &= 1 && \log x = \log_{10} x;\ \text{product property} \\
(3x+2)(x-1) &= 10^1 && \text{Write in exponential form.} \\
3x^2 - x - 2 &= 10 && \text{Multiply; } 10^1 = 10. \\
3x^2 - x - 12 &= 0 && \text{Subtract 10.}
\end{aligned}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{Quadratic formula}$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(3)(-12)}}{2(3)} \quad \text{Substitute } a = 3,\ b = -1,\ c = -12.$$

The two proposed solutions are

$$\frac{1 - \sqrt{145}}{6} \quad \text{and} \quad \frac{1 + \sqrt{145}}{6}.$$

The first proposed solution, $\frac{1-\sqrt{145}}{6}$, is negative. Substituting for x in $\log(x-1)$ results in a negative argument, which is not allowed. Therefore, this solution must be rejected.

The second proposed solution, $\frac{1+\sqrt{145}}{6}$, is positive. Substituting it for x in $\log(3x+2)$ results in a positive argument. Substituting it for x in $\log(x+1)$ also results in a positive argument. Both are necessary conditions. Therefore, the solution set is $\left\{\frac{1+\sqrt{145}}{6}\right\}$.

✔ Now Try Exercise 77.

NOTE We could have replaced 1 with $\log_{10} 10$ in **Example 8** by first writing

$$\log(3x+2) + \log(x-1) = 1 \quad \text{Equation from Example 8}$$
$$\log_{10}[(3x+2)(x-1)] = \log_{10} 10 \quad \text{Substitute.}$$
$$(3x+2)(x-1) = 10, \quad \text{Property of logarithms}$$

and then continuing as shown on the preceding page.

EXAMPLE 9 Solving a Base *e* Logarithmic Equation

Solve $\ln e^{\ln x} - \ln(x-3) = \ln 2$. Give exact value(s).

SOLUTION This logarithmic equation differs from those in **Examples 7 and 8** because the expression on the right side involves a logarithm.

$$\ln e^{\ln x} - \ln(x-3) = \ln 2$$
$$\ln x - \ln(x-3) = \ln 2 \quad e^{\ln x} = x$$
$$\ln \frac{x}{x-3} = \ln 2 \quad \text{Quotient property}$$
$$\frac{x}{x-3} = 2 \quad \text{Property of logarithms}$$
$$x = 2(x-3) \quad \text{Multiply by } x-3.$$
$$x = 2x - 6 \quad \text{Distributive property}$$
$$x = 6 \quad \text{Solve for } x.$$

Check that the solution set is $\{6\}$.

✔ **Now Try Exercise 79.**

Solving an Exponential or Logarithmic Equation

To solve an exponential or logarithmic equation, change the given equation into one of the following forms, where a and b are real numbers, $a > 0$ and $a \neq 1$, and follow the guidelines.

1. $\boldsymbol{a^{f(x)} = b}$

   Solve by taking logarithms on each side.

2. $\boldsymbol{\log_a f(x) = b}$

   Solve by changing to exponential form $a^b = f(x)$.

3. $\boldsymbol{\log_a f(x) = \log_a g(x)}$

   The given equation is equivalent to the equation $f(x) = g(x)$. Solve algebraically.

4. In a more complicated equation, such as

   $$e^{2x+1} \cdot e^{-4x} = 3e, \quad \text{See Example 3(b).}$$

   it may be necessary to first solve for $a^{f(x)}$ or $\log_a f(x)$ and then solve the resulting equation using one of the methods given above.

5. Check that each proposed solution is in the domain.

NOTE We could have replaced 1 with $\log_{10} 10$ in **Example 8** by first writing

$$\log(3x+2) + \log(x-1) = 1 \quad \text{Equation from Example 8}$$
$$\log_{10}[(3x+2)(x-1)] = \log_{10} 10 \quad \text{Substitute.}$$
$$(3x+2)(x-1) = 10, \quad \text{Property of logarithms}$$

and then continuing as shown on the preceding page.

EXAMPLE 9 Solving a Base *e* Logarithmic Equation

Solve $\ln e^{\ln x} - \ln(x-3) = \ln 2$. Give exact value(s).

SOLUTION This logarithmic equation differs from those in **Examples 7 and 8** because the expression on the right side involves a logarithm.

$$\ln e^{\ln x} - \ln(x-3) = \ln 2$$
$$\ln x - \ln(x-3) = \ln 2 \quad e^{\ln x} = x$$
$$\ln \frac{x}{x-3} = \ln 2 \quad \text{Quotient property}$$
$$\frac{x}{x-3} = 2 \quad \text{Property of logarithms}$$
$$x = 2(x-3) \quad \text{Multiply by } x-3.$$
$$x = 2x - 6 \quad \text{Distributive property}$$
$$x = 6 \quad \text{Solve for } x.$$

Check that the solution set is $\{6\}$.

Now Try Exercise 79.

Solving an Exponential or Logarithmic Equation

To solve an exponential or logarithmic equation, change the given equation into one of the following forms, where a and b are real numbers, $a > 0$ and $a \neq 1$, and follow the guidelines.

1. $a^{f(x)} = b$

   Solve by taking logarithms on each side.

2. $\log_a f(x) = b$

   Solve by changing to exponential form $a^b = f(x)$.

3. $\log_a f(x) = \log_a g(x)$

   The given equation is equivalent to the equation $f(x) = g(x)$. Solve algebraically.

4. In a more complicated equation, such as

   $$e^{2x+1} \cdot e^{-4x} = 3e, \quad \text{See Example 3(b).}$$

   it may be necessary to first solve for $a^{f(x)}$ or $\log_a f(x)$ and then solve the resulting equation using one of the methods given above.

5. Check that each proposed solution is in the domain.

Applications and Models

EXAMPLE 10 Applying an Exponential Equation to the Strength of a Habit

The strength of a habit is a function of the number of times the habit is repeated. If N is the number of repetitions and H is the strength of the habit, then, according to psychologist C.L. Hull,

$$H = 1000(1 - e^{-kN}),$$

where k is a constant. Solve this equation for k.

SOLUTION

$$H = 1000(1 - e^{-kN})$$ First solve for e^{-kN}.

$$\frac{H}{1000} = 1 - e^{-kN}$$ Divide by 1000.

$$\frac{H}{1000} - 1 = -e^{-kN}$$ Subtract 1.

$$e^{-kN} = 1 - \frac{H}{1000}$$ Multiply by -1 and rewrite.

Now solve for k.

$$\ln e^{-kN} = \ln\left(1 - \frac{H}{1000}\right)$$ Take the natural logarithm on each side.

$$-kN = \ln\left(1 - \frac{H}{1000}\right)$$ $\ln e^x = x$

$$k = -\frac{1}{N}\ln\left(1 - \frac{H}{1000}\right)$$ Multiply by $-\frac{1}{N}$.

With the final equation, if one pair of values for H and N is known, k can be found, and the equation can then be used to find either H or N for given values of the other variable.

✔ **Now Try Exercise 91.**

EXAMPLE 11 Modeling PC Tablet Sales in the U.S.

Year	Sales (in millions)
2010	10.3
2011	24.1
2012	35.1
2013	39.8
2014	42.1

Source: Forrester Research.

The table gives U.S. tablet sales (in millions) for several years. The data can be modeled by the function

$$f(t) = 20.57 \ln t + 10.58, \quad t \geq 1,$$

where t is the number of years after 2009.

(a) Use the function to estimate the number of tablets sold in the United States in 2015.

(b) If this trend continues, approximately when will annual sales reach 60 million?

SOLUTION

(a) The year 2015 is represented by $t = 2015 - 2009 = 6$.

$$f(t) = 20.57 \ln t + 10.58$$ Given function

$$f(6) = 20.57 \ln 6 + 10.58$$ Let $t = 6$.

$$f(6) \approx 47.4$$ Use a calculator.

Based on this model, 47.4 million tablets were sold in 2015.