Determine whether the given functions are inverses. **See Example 4.**

37.

x	$f(x)$
3	-4
2	-6
5	8
1	9
4	3

x	$g(x)$
-4	3
-6	2
8	5
9	1
3	4

38.

x	$f(x)$
-2	-8
-1	-1
0	0
1	1
2	8

x	$g(x)$
8	-2
1	-1
0	0
-1	1
-8	2

39. $f = \{(2, 5), (3, 5), (4, 5)\}$; $g = \{(5, 2)\}$

40. $f = \{(1, 1), (3, 3), (5, 5)\}$; $g = \{(1, 1), (3, 3), (5, 5)\}$

Use the definition of inverses to determine whether f and g are inverses. **See Example 3.**

41. $f(x) = 2x + 4, \quad g(x) = \frac{1}{2}x - 2$

42. $f(x) = 3x + 9, \quad g(x) = \frac{1}{3}x - 3$

43. $f(x) = -3x + 12, \quad g(x) = -\frac{1}{3}x - 12$

44. $f(x) = -4x + 2, \quad g(x) = -\frac{1}{4}x - 2$

45. $f(x) = \frac{x+1}{x-2}, \quad g(x) = \frac{2x+1}{x-1}$

46. $f(x) = \frac{x-3}{x+4}, \quad g(x) = \frac{4x+3}{1-x}$

47. $f(x) = \frac{2}{x+6}, \quad g(x) = \frac{6x+2}{x}$

48. $f(x) = \frac{-1}{x+1}, \quad g(x) = \frac{1-x}{x}$

49. $f(x) = x^2 + 3, \quad x \geq 0; \quad g(x) = \sqrt{x - 3}, \quad x \geq 3$

50. $f(x) = \sqrt{x + 8}, \quad x \geq -8; \quad g(x) = x^2 - 8, \quad x \geq 0$

Find the inverse of each function that is one-to-one. **See Example 4.**

51. $\{(-3, 6), (2, 1), (5, 8)\}$

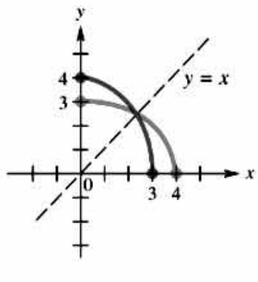
52. $\left\{(3, -1), (5, 0), (0, 5), \left(4, \frac{2}{3}\right)\right\}$

53. $\{(1, -3), (2, -7), (4, -3), (5, -5)\}$

54. $\{(6, -8), (3, -4), (0, -8), (5, -4)\}$

Determine whether each pair of functions graphed are inverses. **See Example 7.**

55.



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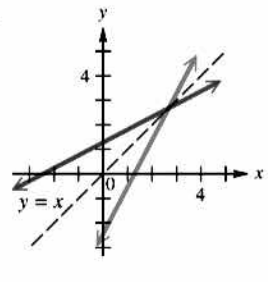

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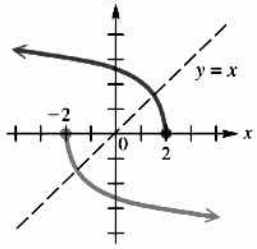

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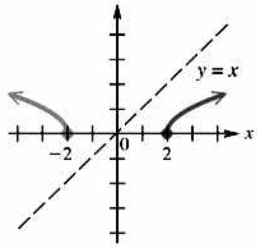


*For each function that is one-to-one, **(a)** write an equation for the inverse function, **(b)** graph f and f^{-1} on the same axes, and **(c)** give the domain and range of both f and f^{-1}. If the function is not one-to-one, say so. **See Examples 5–8.***

59. $f(x) = 3x - 4$

60. $f(x) = 4x - 5$

61. $f(x) = -4x + 3$

62. $f(x) = -6x - 8$

63. $f(x) = x^3 + 1$

64. $f(x) = -x^3 - 2$

65. $f(x) = x^2 + 8$

66. $f(x) = -x^2 + 2$

67. $f(x) = \frac{1}{x}, \quad x \neq 0$

68. $f(x) = \frac{4}{x}, \quad x \neq 0$

69. $f(x) = \frac{1}{x - 3}, \quad x \neq 3$

70. $f(x) = \frac{1}{x + 2}, \quad x \neq -2$

71. $f(x) = \frac{x + 1}{x - 3}, \quad x \neq 3$

72. $f(x) = \frac{x + 2}{x - 1}, \quad x \neq 1$

73. $f(x) = \frac{2x + 6}{x - 3}, \quad x \neq 3$

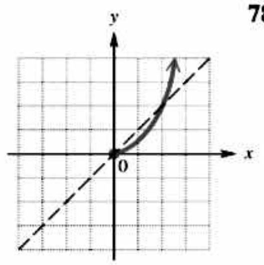
74. $f(x) = \frac{-3x + 12}{x - 6}, \quad x \neq 6$

75. $f(x) = \sqrt{x + 6}, \quad x \geq -6$

76. $f(x) = -\sqrt{x^2 - 16}, \quad x \geq 4$

*Graph the inverse of each one-to-one function. **See Example 7.***

77.



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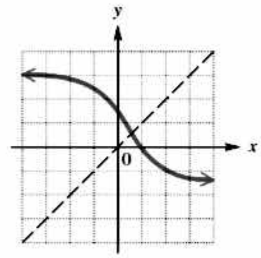

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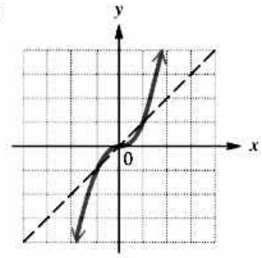

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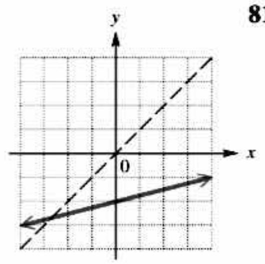
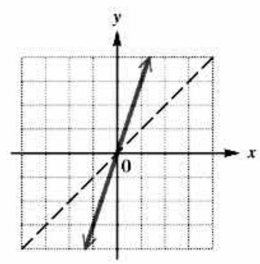
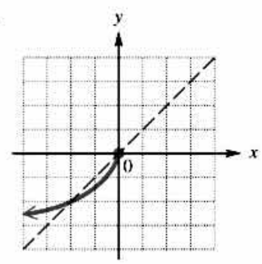


81.



82.



Concept Check The graph of a function f is shown in the figure. Use the graph to find each value.

83. $f^{-1}(4)$

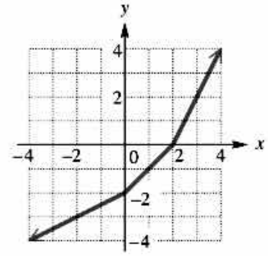
84. $f^{-1}(2)$

85. $f^{-1}(0)$

86. $f^{-1}(-2)$

87. $f^{-1}(-3)$

88. $f^{-1}(-4)$



Concept Check Answer each of the following.

89. Suppose $f(x)$ is the number of cars that can be built for x dollars. What does $f^{-1}(1000)$ represent?

90. Suppose $f(r)$ is the volume (in cubic inches) of a sphere of radius r inches. What does $f^{-1}(5)$ represent?

91. If a line has slope a, what is the slope of its reflection across the line $y = x$?

92. For a one-to-one function f, find $(f^{-1} \circ f)(2)$, where $f(2) = 3$.

4.1 Inverse Functions

- One-to-One Functions
- Inverse Functions
- Equations of Inverses
- An Application of Inverse Functions to Cryptography

One-to-One Functions

Suppose we define the following function F.

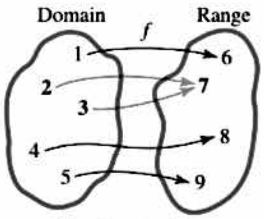
$$F = \{(-2, 2), (-1, 1), (0, 0), (1, 3), (2, 5)\}$$

(We have defined F so that each *second* component is used only once.) We can form another set of ordered pairs from F by interchanging the x- and y-values of each pair in F. We call this set G.

$$G = \{(2, -2), (1, -1), (0, 0), (3, 1), (5, 2)\}$$

G is the *inverse* of F. Function F was defined with each *second* component used only once, so set G will also be a function. (Each *first* component must be used only once.) In order for a function to have an inverse that is also a function, it must exhibit this one-to-one relationship.

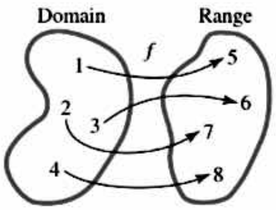
In a one-to-one function, each x-value corresponds to only one y-value, and each y-value corresponds to only one x-value.



Not One-to-One

Figure 1

The function f shown in **Figure 1** is not one-to-one because the y-value 7 corresponds to *two* x-values, 2 and 3. That is, the ordered pairs $(2, 7)$ and $(3, 7)$ both belong to the function. The function f in **Figure 2** is one-to-one.



One-to-One

Figure 2

One-to-One Function

A function f is a **one-to-one function** if, for elements a and b in the domain of f,

$$\boldsymbol{a \neq b} \quad \text{implies} \quad \boldsymbol{f(a) \neq f(b)}.$$

That is, different values of the domain correspond to different values of the range.

Using the concept of the *contrapositive* from the study of logic, the boldface statement in the preceding box is equivalent to

$$\boldsymbol{f(a) = f(b)} \quad \text{implies} \quad \boldsymbol{a = b}.$$

This means that if two range values are equal, then their corresponding domain values are equal. We use this statement to show that a function f is one-to-one in **Example 1(a).**

EXAMPLE 1 Deciding Whether Functions Are One-to-One

Determine whether each function is one-to-one.

(a) $f(x) = -4x + 12$ **(b)** $f(x) = \sqrt{25 - x^2}$

SOLUTION

(a) We can determine that the function $f(x) = -4x + 12$ is one-to-one by showing that $f(a) = f(b)$ leads to the result $a = b$.

$$
\begin{aligned}
f(a) &= f(b) \\
-4a + 12 &= -4b + 12 && f(x) = -4x + 12 \\
-4a &= -4b && \text{Subtract 12.} \\
a &= b && \text{Divide by } -4.
\end{aligned}
$$

By the definition, $f(x) = -4x + 12$ is one-to-one.

(b) We can determine that the function $f(x) = \sqrt{25 - x^2}$ is not one-to-one by showing that *different* values of the domain correspond to the *same* value of the range. If we choose $a = 3$ and $b = -3$, then $3 \neq -3$, but

$$f(3) = \sqrt{25 - 3^2} = \sqrt{25 - 9} = \sqrt{16} = 4$$

and $$f(-3) = \sqrt{25 - (-3)^2} = \sqrt{25 - 9} = 4.$$

Here, even though $3 \neq -3$, $f(3) = f(-3) = 4$. By the definition, f is *not* a one-to-one function.

✔ Now Try Exercises 17 and 19.

As illustrated in **Example 1(b),** a way to show that a function is *not* one-to-one is to produce a pair of different domain elements that lead to the same function value. There is a useful graphical test for this, the **horizontal line test.**

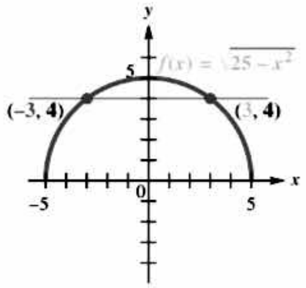


Figure 3

Horizontal Line Test

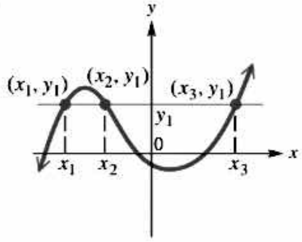
A function is one-to-one if every horizontal line intersects the graph of the function at most once.

NOTE In **Example 1(b),** the graph of the function is a semicircle, as shown in **Figure 3.** Because there is at least one horizontal line that intersects the graph in more than one point, this function is not one-to-one.

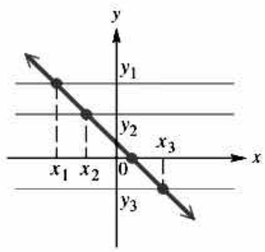
EXAMPLE 2 Using the Horizontal Line Test

Determine whether each graph is the graph of a one-to-one function.

(a)



(b)



SOLUTION

(a) Each point where the horizontal line intersects the graph has the same value of y but a different value of x. Because more than one different value of x (here three) lead to the same value of y, the function is not one-to-one.

(b) Every horizontal line will intersect the graph at exactly one point, so this function is one-to-one.

✔ Now Try Exercises 11 and 13.

The function graphed in **Example 2(b)** decreases on its entire domain.

In general, a function that is either increasing or decreasing on its entire domain, such as $f(x) = -x$, $g(x) = x^3$, and $h(x) = \frac{1}{x}$, must be one-to-one.

Tests to Determine Whether a Function Is One-to-One

1. Show that $f(a) = f(b)$ implies $a = b$. This means that f is one-to-one. **(See Example 1(a).)**
2. In a one-to-one function, every y-value corresponds to no more than one x-value. To show that a function is not one-to-one, find at least two x-values that produce the same y-value. **(See Example 1(b).)**
3. Sketch the graph and use the horizontal line test. **(See Example 2.)**
4. If the function either increases or decreases on its entire domain, then it is one-to-one. A sketch is helpful here, too. **(See Example 2(b).)**

Inverse Functions Certain pairs of one-to-one functions "undo" each other. For example, consider the functions

$$g(x) = 8x + 5 \quad \text{and} \quad f(x) = \frac{1}{8}x - \frac{5}{8}.$$

We choose an arbitrary element from the domain of g, say 10. Evaluate $g(10)$.

$$g(x) = 8x + 5 \qquad \text{Given function}$$
$$g(10) = 8 \cdot 10 + 5 \qquad \text{Let } x = 10.$$
$$g(10) = 85 \qquad \text{Multiply and then add.}$$

Now, we evaluate $f(85)$.

$$f(x) = \frac{1}{8}x - \frac{5}{8} \qquad \text{Given function}$$
$$f(85) = \frac{1}{8}(85) - \frac{5}{8} \qquad \text{Let } x = 85.$$
$$f(85) = \frac{85}{8} - \frac{5}{8} \qquad \text{Multiply.}$$
$$f(85) = 10 \qquad \text{Subtract and then divide.}$$

Starting with 10, we "applied" function g and then "applied" function f to the result, which returned the number 10. See **Figure 4.**

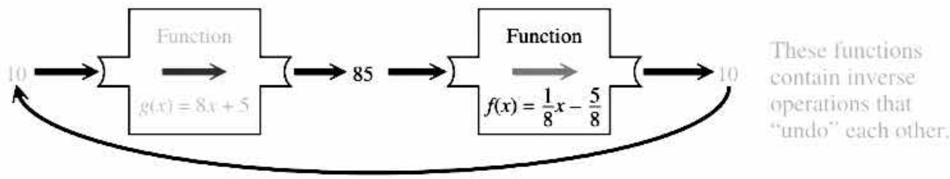


Figure 4

As further examples, confirm the following.

$$g(3) = 29 \quad \text{and} \quad f(29) = 3$$
$$g(-5) = -35 \quad \text{and} \quad f(-35) = -5$$
$$g(2) = 21 \quad \text{and} \quad f(21) = 2$$
$$f(2) = -\frac{3}{8} \quad \text{and} \quad g\left(-\frac{3}{8}\right) = 2$$

In particular, for the pair of functions $g(x) = 8x + 5$ and $f(x) = \frac{1}{8}x - \frac{5}{8}$,

$$f(g(2)) = 2 \quad \text{and} \quad g(f(2)) = 2.$$

In fact, for *any* value of x,

$$f(g(x)) = x \quad \text{and} \quad g(f(x)) = x.$$

Using the notation for composition of functions, these two equations can be written as follows.

$$(f \circ g)(x) = x \quad \text{and} \quad (g \circ f)(x) = x \qquad \text{The result is the identity function.}$$

Because the compositions of f and g yield the *identity* function, they are *inverses* of each other.

Inverse Function

Let f be a one-to-one function. Then g is the **inverse function** of f if

$$(f \circ g)(x) = x \quad \text{for every } x \text{ in the domain of } g,$$

and

$$(g \circ f)(x) = x \quad \text{for every } x \text{ in the domain of } f.$$

The condition that f is one-to-one in the definition of inverse function is essential. Otherwise, g will not define a function.

EXAMPLE 3 Determining Whether Two Functions Are Inverses

Let functions f and g be defined respectively by

$$f(x) = x^3 - 1 \quad \text{and} \quad g(x) = \sqrt[3]{x + 1}.$$

Is g the inverse function of f?

SOLUTION As shown in **Figure 5,** the horizontal line test applied to the graph indicates that f is one-to-one, so the function has an inverse. Because it is one-to-one, we now find $(f \circ g)(x)$ and $(g \circ f)(x)$.

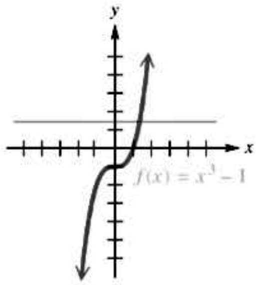


Figure 5

$(f \circ g)(x)$	$(g \circ f)(x)$
$= f(g(x))$	$= g(f(x))$
$= \left(\sqrt[3]{x+1}\right)^3 - 1$	$= \sqrt[3]{(x^3 - 1) + 1}$
$= x + 1 - 1$	$= \sqrt[3]{x^3}$
$= x$	$= x$

Since $(f \circ g)(x) = x$ and $(g \circ f)(x) = x$, function g is the inverse of function f.

✔ Now Try Exercise 41.

A special notation is used for inverse functions: If g is the inverse of a function f, then g is written as f^{-1} (read **"f-inverse"**).

$$f(x) = x^3 - 1 \quad \text{has inverse} \quad f^{-1}(x) = \sqrt[3]{x + 1}. \qquad \text{See Example 3.}$$

CAUTION ***Do not confuse the -1 in f^{-1} with a negative exponent.*** The symbol $f^{-1}(x)$ represents the inverse function of f, *not* $\frac{1}{f(x)}$.

By the definition of inverse function, the domain of f is the range of f^{-1}, and the range of f is the domain of f^{-1}. See **Figure 6.**

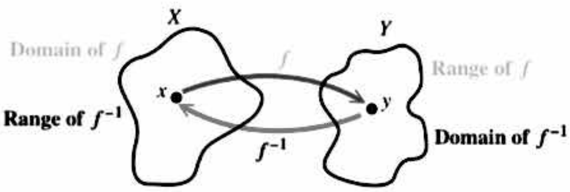


Figure 6

EXAMPLE 4 Finding Inverses of One-to-One Functions

Find the inverse of each function that is one-to-one.

(a) $F = \{(-2, 1), (-1, 0), (0, 1), (1, 2), (2, 2)\}$

(b) $G = \{(3, 1), (0, 2), (2, 3), (4, 0)\}$

(c) The table in the margin shows the number of hurricanes recorded in the North Atlantic during the years 2009–2013. Let f be the function defined in the table, with the years forming the domain and the numbers of hurricanes forming the range.

Year	Number of Hurricanes
2009	3
2010	12
2011	7
2012	10
2013	2

Source: www.wunderground.com

SOLUTION

(a) Each x-value in F corresponds to just one y-value. However, the y-value 2 corresponds to two x-values, 1 and 2. Also, the y-value 1 corresponds to both -2 and 0. Because at least one y-value corresponds to more than one x-value, F is not one-to-one and does not have an inverse.

(b) Every x-value in G corresponds to only one y-value, and every y-value corresponds to only one x-value, so G is a one-to-one function. The inverse function is found by interchanging the x- and y-values in each ordered pair.

$$G^{-1} = \{(1, 3), (2, 0), (3, 2), (0, 4)\}$$

Notice how the domain and range of G become the range and domain, respectively, of G^{-1}.

(c) Each x-value in f corresponds to only one y-value, and each y-value corresponds to only one x-value, so f is a one-to-one function. The inverse function is found by interchanging the x- and y-values in the table.

$$f^{-1}(x) = \{(3, 2009), (12, 2010), (7, 2011), (10, 2012), (2, 2013)\}$$

The domain and range of f become the range and domain of f^{-1}.

✔ **Now Try Exercises 37, 51, and 53.**

Equations of Inverses The inverse of a one-to-one function is found by interchanging the x- and y-values of each of its ordered pairs. The equation of the inverse of a function defined by $y = f(x)$ is found in the same way.

Finding the Equation of the Inverse of $y = f(x)$

For a one-to-one function f defined by an equation $y = f(x)$, find the defining equation of the inverse as follows. (If necessary, replace $f(x)$ with y first. Any restrictions on x and y should be considered.)

Step 1 Interchange x and y.

Step 2 Solve for y.

Step 3 Replace y with $f^{-1}(x)$.

EXAMPLE 5 Finding Equations of Inverses

Determine whether each equation defines a one-to-one function. If so, find the equation of the inverse.

(a) $f(x) = 2x + 5$ **(b)** $y = x^2 + 2$ **(c)** $f(x) = (x - 2)^3$

SOLUTION

(a) The graph of $y = 2x + 5$ is a nonhorizontal line, so by the horizontal line test, f is a one-to-one function. Find the equation of the inverse as follows.

$$f(x) = 2x + 5 \quad \text{Given function}$$

$$y = 2x + 5 \quad \text{Let } y = f(x).$$

Step 1 $x = 2y + 5$ Interchange x and y.

Step 2 $x - 5 = 2y$ Subtract 5.

$$y = \frac{x-5}{2} \quad \text{Divide by 2. Rewrite.}$$

Solve for y.

Step 3 $f^{-1}(x) = \frac{1}{2}x - \frac{5}{2}$ Replace y with $f^{-1}(x)$. $\frac{a-b}{c} = \left(\frac{1}{c}\right)a - \frac{b}{c}$

Thus, the equation $f^{-1}(x) = \frac{x-5}{2} = \frac{1}{2}x - \frac{5}{2}$ represents a linear function. In the function $y = 2x + 5$, the value of y is found by starting with a value of x, multiplying by 2, and adding 5.

The equation $f^{-1}(x) = \frac{x-5}{2}$ for the inverse *subtracts* 5 and then *divides* by 2. An inverse is used to "undo" what a function does to the variable x.

(b) The equation $y = x^2 + 2$ has a parabola opening up as its graph, so some horizontal lines will intersect the graph at two points. For example, both $x = 3$ and $x = -3$ correspond to $y = 11$. Because of the presence of the x^2-term, there are many pairs of x-values that correspond to the same y-value. This means that the function defined by $y = x^2 + 2$ is not one-to-one and does not have an inverse.

Proceeding with the steps for finding the equation of an inverse leads to

$$y = x^2 + 2$$

$$x = y^2 + 2 \quad \text{Interchange } x \text{ and } y.$$

$$x - 2 = y^2 \quad \text{Solve for } y.$$

Remember *both* roots.

$$\pm\sqrt{x-2} = y. \quad \text{Square root property}$$

The last equation shows that there are two y-values for each choice of x greater than 2, indicating that this is not a function.

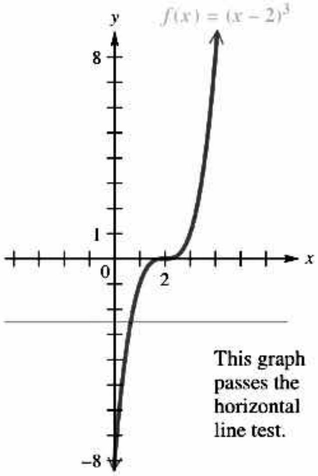


Figure 7

(c) **Figure 7** shows that the horizontal line test assures us that this horizontal translation of the graph of the cubing function is one-to-one.

$$f(x) = (x-2)^3 \qquad \text{Given function}$$

$$y = (x-2)^3 \qquad \text{Replace } f(x) \text{ with } y.$$

Step 1 $$x = (y-2)^3 \qquad \text{Interchange } x \text{ and } y.$$

Step 2
$$\left.\begin{aligned}\sqrt[3]{x} &= \sqrt[3]{(y-2)^3} && \text{Take the cube root on each side.}\\ \sqrt[3]{x} &= y-2 && \sqrt[3]{a^3} = a\\ \sqrt[3]{x}+2 &= y && \text{Add 2.}\end{aligned}\right\} \text{Solve for } y.$$

Step 3 $$f^{-1}(x) = \sqrt[3]{x}+2 \qquad \text{Replace } y \text{ with } f^{-1}(x). \text{ Rewrite.}$$

✔ Now Try Exercises 59(a), 63(a), and 65(a).

EXAMPLE 6 Finding the Equation of the Inverse of a Rational Function

The following rational function is one-to-one. Find its inverse.

$$f(x) = \frac{2x+3}{x-4}, \quad x \neq 4$$

SOLUTION $$f(x) = \frac{2x+3}{x-4}, \quad x \neq 4 \qquad \text{Given function}$$

$$y = \frac{2x+3}{x-4} \qquad \text{Replace } f(x) \text{ with } y.$$

Step 1 $$x = \frac{2y+3}{y-4}, \quad y \neq 4 \qquad \text{Interchange } x \text{ and } y.$$

Step 2

Pay close attention here.

$$\left.\begin{aligned}x(y-4) &= 2y+3 && \text{Multiply by } y-4.\\ xy-4x &= 2y+3 && \text{Distributive property}\\ xy-2y &= 4x+3 && \text{Add } 4x \text{ and } -2y.\\ y(x-2) &= 4x+3 && \text{Factor out } y.\\ y &= \frac{4x+3}{x-2}, \quad x \neq 2 && \text{Divide by } x-2.\end{aligned}\right\} \text{Solve for } y.$$

In the final line, we give the condition $x \neq 2$. (Note that 2 is not in the *range* of f, so it is not in the domain of f^{-1}.)

Step 3 $$f^{-1}(x) = \frac{4x+3}{x-2}, \quad x \neq 2 \qquad \text{Replace } y \text{ with } f^{-1}(x).$$

✔ Now Try Exercise 71(a).

One way to graph the inverse of a function f whose equation is known follows.

Step 1 Find some ordered pairs that are on the graph of f.

Step 2 Interchange x and y to find ordered pairs that are on the graph of f^{-1}.

Step 3 Plot those points, and sketch the graph of f^{-1} through them.

Another way is to select points on the graph of f and use symmetry to find corresponding points on the graph of f^{-1}.

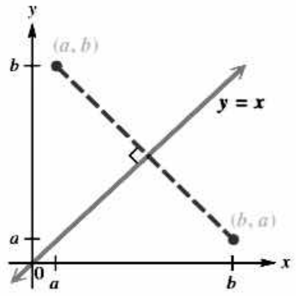


Figure 8

For example, suppose the point (a, b) shown in **Figure 8** is on the graph of a one-to-one function f. Then the point (b, a) is on the graph of f^{-1}. The line segment connecting (a, b) and (b, a) is perpendicular to, and cut in half by, the line $y = x$. The points (a, b) and (b, a) are "mirror images" of each other with respect to $y = x$.

Thus, we can find the graph of f^{-1} from the graph of f by locating the mirror image of each point in f with respect to the line $y = x$.

EXAMPLE 7 Graphing f^{-1} Given the Graph of f

In each set of axes in **Figure 9,** the graph of a one-to-one function f is shown in blue. Graph f^{-1} in red.

SOLUTION In **Figure 9,** the graphs of two functions f shown in blue are given with their inverses shown in red. In each case, the graph of f^{-1} is a reflection of the graph of f with respect to the line $y = x$.

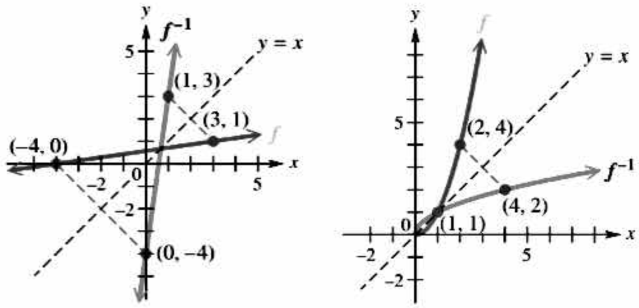


Figure 9

✔ **Now Try Exercises 77 and 81.**

EXAMPLE 8 Finding the Inverse of a Function (Restricted Domain)

Let $f(x) = \sqrt{x+5}$, $x \geq -5$. Find $f^{-1}(x)$.

SOLUTION The domain of f is restricted to the interval $[-5, \infty)$. Function f is one-to-one because it is an increasing function and thus has an inverse function. Now we find the equation of the inverse.

$$f(x) = \sqrt{x+5}, \quad x \geq -5 \qquad \text{Given function}$$

$$y = \sqrt{x+5}, \quad x \geq -5 \qquad \text{Replace } f(x) \text{ with } y.$$

Step 1 $$x = \sqrt{y+5}, \quad y \geq -5 \qquad \text{Interchange } x \text{ and } y.$$

Step 2 $$x^2 = \left(\sqrt{y+5}\right)^2 \qquad \text{Square each side.}$$

$$x^2 = y + 5 \qquad \left(\sqrt{a}\right)^2 = a \text{ for } a \geq 0$$

$$y = x^2 - 5 \qquad \text{Subtract 5. Rewrite.}$$

Solve for y.

However, we cannot define $f^{-1}(x)$ as $x^2 - 5$. The domain of f is $[-5, \infty)$, and its range is $[0, \infty)$. The range of f is the domain of f^{-1}, so f^{-1} must be defined as follows.

Step 3 $$f^{-1}(x) = x^2 - 5, \quad x \geq 0$$

As a check, the range of f^{-1}, $[-5, \infty)$, is the domain of f.

Graphs of f and f^{-1} are shown in **Figures 10 and 11.** The line $y = x$ is included on the graphs to show that the graphs of f and f^{-1} are mirror images with respect to this line.

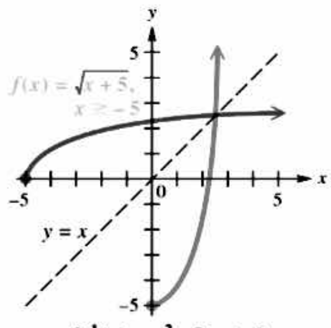


Figure 10

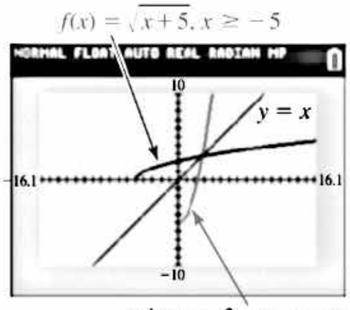
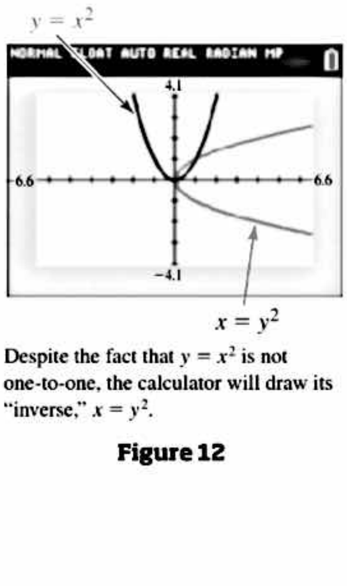


Figure 11

Now Try Exercise 75.

Important Facts about Inverses

1. If f is one-to-one, then f^{-1} exists.
2. The domain of f is the range of f^{-1}, and the range of f is the domain of f^{-1}.
3. If the point (a, b) lies on the graph of f, then (b, a) lies on the graph of f^{-1}. The graphs of f and f^{-1} are reflections of each other across the line $y = x$.
4. To find the equation for f^{-1}, replace $f(x)$ with y, interchange x and y, and solve for y. This gives $f^{-1}(x)$.



Despite the fact that $y = x^2$ is not one-to-one, the calculator will draw its "inverse," $x = y^2$.

Figure 12

Some graphing calculators have the capability of "drawing" the reflection of a graph across the line $y = x$. This feature does not require that the function be one-to-one, however, so the resulting figure may not be the graph of a function. See **Figure 12.** ***It is necessary to understand the mathematics to interpret results correctly.***

An Application of Inverse Functions to Cryptography A one-to-one function and its inverse can be used to make information secure. The function is used to encode a message, and its inverse is used to decode the coded message. In practice, complicated functions are used.

EXAMPLE 9 Using Functions to Encode and Decode a Message

Use the one-to-one function $f(x) = 3x + 1$ and the following numerical values assigned to each letter of the alphabet to encode and decode the message BE MY FACEBOOK FRIEND.

A	1	**H**	8	**O**	15	**V**	22
B	2	**I**	9	**P**	16	**W**	23
C	3	**J**	10	**Q**	17	**X**	24
D	4	**K**	11	**R**	18	**Y**	25
E	5	**L**	12	**S**	19	**Z**	26
F	6	**M**	13	**T**	20		
G	7	**N**	14	**U**	21		