

GEO SC 397
PLATE TECTONICS
VECTOR OPERATIONS AND PLATE TECTONICS

Given: $eu\Omega_{na}$ and $na\Omega_{af}$ calculate: $eu\Omega_{af}$

	Latitude ($^{\circ}$ N)	Longitude ($^{\circ}$ E)	Angle (0-10 Ma)	Angular Vel. ω ($^{\circ}$ Ma $^{-1}$)
$eu\Omega_{na}$	68.0	137.0	2.5	0.25
$na\Omega_{af}$	69.7	-33.4	-3.6	-0.36

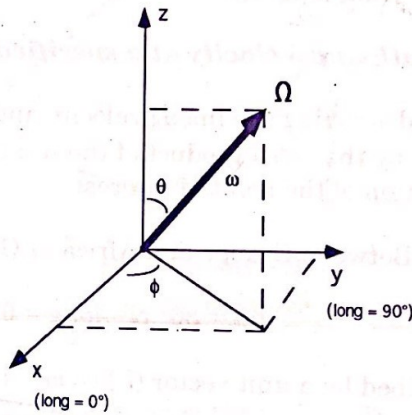
$$\Omega_x = \omega \sin\theta \cos\phi$$

$$= \omega \cos(\text{lat}) \cos(\text{long})$$

$$\Omega_y = \omega \sin\theta \sin\phi$$

$$= \omega \cos(\text{lat}) \sin(\text{long})$$

$$\Omega_z = \omega \cos\theta \leftarrow \omega \sin(\text{lat})$$



eu/na

$$\Omega_x = -0.0685$$

$$\Omega_y = 0.0639$$

$$\Omega_z = 0.2318$$

na/af

$$\Omega_x = -0.1043$$

$$\Omega_y = 0.0688$$

$$\Omega_z = -0.3377$$

$$eu\Omega_{af} = eu\Omega_{na} + na\Omega_{af}$$

[add components]

$$\Rightarrow eu\Omega_{af}$$

$$\Omega_x = (-0.0685) + (-0.1043) = -0.1728$$

$$\Omega_y = (0.0639) + (0.0688) = 0.1327$$

$$\Omega_z = (0.2318) + (-0.3377) = (-0.1059)$$

For usefulness this should be converted back to latitude, longitude, and angular velocity:

$$lat = \tan^{-1} [\Omega_z / (\Omega_x^2 + \Omega_y^2)^{1/2}]$$

$$long = \tan^{-1} [\Omega_y / \Omega_x]$$

means going west

$$\text{if } \Omega_x < 0 \text{ then } long = \text{sgn}(long) \times 180^\circ + long$$

$$\omega = (\Omega_x^2 + \Omega_y^2 + \Omega_z^2)^{1/2}$$

$$\Rightarrow eu\Omega_{af} = (-26, -217, .242)$$

(lat, long, ω)

$$\approx 217W = 143E$$

Calculate Azimuth and Velocity at a specified Point

The vector describing the linear velocity and azimuth at any point on the plate boundary is given by the cross-product of the rotation pole Ω with a unit vector \hat{r} which describes the location of the point of interest.

Example: Motion Between Europe and Africa at Gibraltar

$$lat = 36^\circ N \quad long = 6^\circ W \text{ (i.e. } -6^\circ)$$

Convert to xy, z

This will be described by a unit vector (\hat{r}) so $\omega = 1$

$$\hat{r} = r_x \hat{i} + r_y \hat{j} + r_z \hat{k}$$

basic vector

$$\hat{r} = (0.8046, -0.0846, 0.5878)$$

Vector describing the surface motion at Gibraltar then becomes:

$$V = eu\Omega_{af} \times \hat{r}$$

$$V = eu\Omega_{af} \times \hat{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -0.1728 & 0.1327 & -0.1059 \\ 0.8046 & -0.0846 & 0.5878 \end{vmatrix}$$

Determinant

$$(0.1327 \times 0.5878) - (-0.1059 \times -0.0846)$$

$$\times \hat{i} = 0.0691 \hat{i}$$

(ad - bc)

$$V = 0.0691 \hat{i} + 0.0164 \hat{j} + (-0.0922 \hat{k})$$

Note: V is in a coordinate system anchored to the center of the earth. We need to transform the coordinate system to a cartesian system anchored at our point of interest (i.e. on the Earth's surface) at the Latitude and Longitude of Gibraltar.

Coordinate Transformation

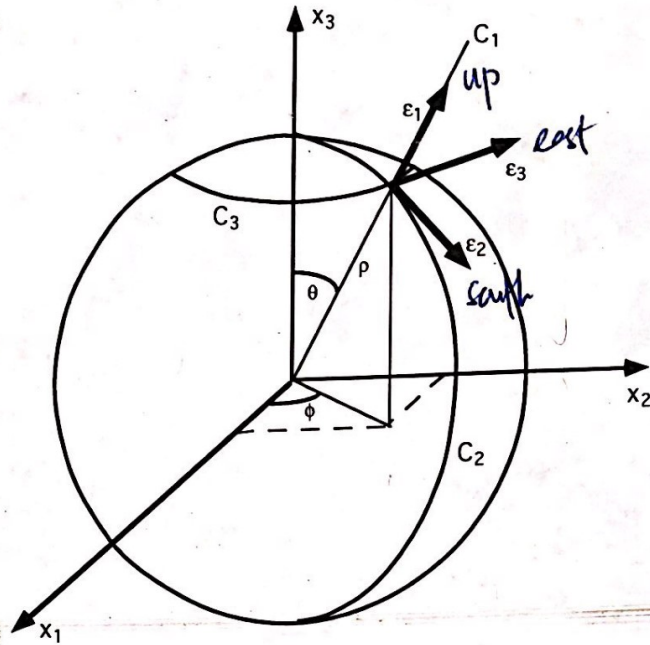
How is this coordinate system defined (i.e. what are the basis vectors)?

$$V = (V_{\epsilon_1}, V_{\epsilon_2}, V_{\epsilon_3})$$

$\epsilon_1 \perp$ to surface of earth
(pointing from center)

$\epsilon_2 \parallel$ to surface N-S
(+ south)

$\epsilon_3 \parallel$ to surface E-W
(+ east)



$$V_{\epsilon_1} = V_x \cos(\text{lat}) \cos(\text{long}) + V_y \cos(\text{lat}) \sin(\text{long}) + V_z \sin(\text{lat})$$

$$V_{\epsilon_2} = V_x \sin(\text{lat}) \cos(\text{long}) + V_y \sin(\text{lat}) \sin(\text{long}) - V_z \cos(\text{lat})$$

$$V_{\epsilon_3} = -V_x \sin(\text{long}) + V_y \cos(\text{long}) + V_z \cdot 0$$

NOTE: V_{ϵ_1} should = ZERO (i.e plates move along the surface of the Earth)
If it doesn't, you have made an arithmetic error!

Back to Gibraltar lat = 36° ; long = -6°

$$V_{\epsilon_1} = (0.0691) \cos(36^\circ) \cos(-6^\circ) + (0.0164) \cos(36^\circ) \sin(-6^\circ) + (-0.0922) \sin(36^\circ)$$

$$= 1.6 \times 10^{-5} \approx 0$$

$$V_{\epsilon_2} = (0.0691) \sin(36^\circ) \cos(-6^\circ) + (0.0164) \sin(36^\circ) \sin(-6^\circ) - (-0.0922) \cos(36^\circ)$$

$$= 0.114 \text{ + south}$$

$$V_{\epsilon_3} = -(0.0691) \sin(-6^\circ) + (0.0164) \cos(-6^\circ) + V_z \cdot 0$$

$$= 0.0235 \text{ + east}$$

Azimuth (from North) = $90^\circ + \tan^{-1}(V_{\epsilon_2}/V_{\epsilon_3})$
 = $90^\circ + \tan^{-1}(0.114/0.0235) = 168^\circ$

Velocity at Gibraltar = $|\omega_{\text{Eurasia}} \times r|$ = $(0.116^\circ \text{ Ma}^{-1}) (111.2 \text{ km}^\circ)$
 = $12.9 \text{ km/Ma [mm/yr]}$
 = 1.29 cm/yr