

3.8 Suggested Problems

In problems 1-10 do the following

- Solve the given system by systematic elimination of unknowns using an augmented matrix.
- Check your answers.
- Give a geometric interpretation of your solution in terms of the graphs of the equations in the system.

$$\begin{array}{ll}
 1. \quad \begin{cases} 4x + 3y = 10 \\ 2x - 6y = -10 \end{cases} & 2. \quad \begin{cases} 2x - y = 7 \\ -8x + 4y = 11 \end{cases} \\
 3. \quad \begin{cases} 2x + 3y = -6 \\ -x + \frac{3}{2}y = 3 \end{cases} & 4. \quad \begin{cases} 3u - 2v = -1 \\ -5u + 3v = 2 \end{cases} \\
 5. \quad \begin{cases} x - z = 0 \\ 3x + y + z = 1 \\ -x + y + 2z = 2 \end{cases} & 6. \quad \begin{cases} 3r - 2s + 4t = 6 \\ 2r + 3s - 5t = -8 \\ 5r - 4s + 3t = 7 \end{cases} \\
 7. \quad \begin{cases} x + 2y - z = 1 \\ 2x + y + z = 1 \\ x - y + 2z = 1 \end{cases} & 8. \quad \begin{cases} x + 2y - z = 1 \\ 2x + y + z = 1 \\ 4x + 5y - z = 2 \end{cases} \\
 9. \quad \begin{cases} u + 2v - w = 2 \\ 2u + v + w = 1 \\ u - v + 2w = -1 \end{cases} & 10. \quad \begin{cases} r + 2s - t = 2 \\ 6r + 9s - 3t = 9 \\ r - s + 2t = -1 \end{cases}
 \end{array}$$

In Problems 11-16 evaluate the determinant.

$$\begin{array}{lll}
 11. \quad \begin{vmatrix} 1 & -1 \\ 3 & 2 \end{vmatrix} & 12. \quad \begin{vmatrix} 4 & 3 \\ 2 & 1 \end{vmatrix} & 13. \quad \begin{vmatrix} 6 & 3 \\ 4 & 2 \end{vmatrix} \\
 14. \quad \begin{vmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix} & 15. \quad \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} & 16. \quad \begin{vmatrix} 4 & -6 & 3 \\ -1 & 4 & 1 \\ 5 & -6 & 3 \end{vmatrix}
 \end{array}$$

In Problems 17-20 use determinants to decide whether the system has (a) a unique solution or (b) either no solution or infinitely many solutions. Do *not* solve the system.

$$\begin{array}{ll}
 17. \quad \begin{cases} 3x - 4y = 7 \\ x + 2y = -4 \end{cases} & 18. \quad \begin{cases} 2u - v = 5 \\ -8u + 4v = -11 \end{cases} \\
 19. \quad \begin{cases} 2x - 2y = 8 \\ -3x + y + 2z = -1 \\ x - 3y - z = 7 \end{cases} & 20. \quad \begin{cases} 4u + 2v + 4w = 2 \\ 2u + v + 2w = 1 \\ 3u - 4v + w = -5 \end{cases}
 \end{array}$$

In Problems 21-24 determine the value(s) of λ for which the homogeneous system will have nontrivial solutions.

$$21. \begin{cases} (1-\lambda)x + y = 0 \\ 4x + (1-\lambda)y = 0 \\ (3-\lambda)x + 2y + 4z = 0 \\ 2x - \lambda y + 2z = 0 \\ 4x + 2y + (3-\lambda)z = 0 \end{cases}$$

$$22. \begin{cases} (1-\lambda)x + 2y = 0 \\ 3x + (2-\lambda)y = 0 \\ -\lambda x - 2y - 3z = 0 \\ -x + (1-\lambda)y - z = 0 \\ 2x + 2y + (5-\lambda)z = 0 \end{cases}$$

25. (a) Show, without explicitly solving, that the homogeneous system

$$\begin{cases} -x + 2y - z = 0 \\ 2x + y + z = 0 \\ x - y + 2z = 0 \end{cases}$$

has a unique solution. (b) Based on (a) what is the unique solution to the system?