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MAT 135 A

Homework #1

Due 01/15/2021 by 7 PM

via Canvas

## Problem 1

①

Roll a fair die 3 times.  
What is the probability that  
you get the same number all  
three times?

## Problem 2

Shuffle a deck of cards.

Compute

(a)  $P(\text{top card is a Queen})$

(b)  $P(\text{all Aces are together})$

### Problem 3

Pick an integer at random  
between 1 and 2000.

Compute the probability that  
it is divisible by 8, 12, or 13.

### Problem 4

Roll a die 16 times.

Compute the probability that  
a number occurs 6 times and  
the other numbers occur two times each.

### Problem 5

Choose each digit of a 4 digit number at random from digits 1, ..., 9. Compute the probability that no digit appears more than twice.

### Problem 6

Prove De Morgan's Laws:

$$\left(\bigcup_i A_i\right)^c = \bigcap_i A_i^c$$

$$\left(\bigcap_i A_i\right)^c = \bigcup_i A_i^c$$

## Problem 7

(4)

Prove that for any collection  $\mathcal{A}$  of subsets of  $\Omega$  there is the smallest field  $\mathcal{F}$  that contains all elements of  $\mathcal{A}$ .

Does the result still hold if you require  $\mathcal{F}$  to be a  $\sigma$ -field (i.e. for any collection  $\mathcal{A}$  there is the smallest  $\sigma$ -field  $\mathcal{F}$  that contains all elements of  $\mathcal{A}$ ) ?

## Problem 8

(5)

Toss a counterfeit coin  
(so the probability of Heads is 0.01)  
repeatedly. Show that, with  
probability one, a head turns up  
sooner or later.