# Experiment 8:

# Ballistic Pendulum and Projectile Motion

## Introduction

Among the most important of the laws of nature are the so-called ***conservation laws***, such as conservation of energy, momentum, electric charge, etc. To say that a quantity is conserved under certain circumstances means that if you measure the quantity, and then let nature go through whatever processes are involved, when you make a second measurement the result will be the same as before. This allows us to understand a great deal about our physical surroundings without knowing the details of the complicated processes that take place. Conservation laws also enable us to predict ahead of time the outcome of a given interaction.

In this experiment you will employ two of these laws - conservation of energy and conservation of momentum - to determine the velocity of a metal projectile fired from a gun into a device called the ballistic pendulum. You will also use the fact that the horizontal and vertical components of a projectile's motion are independent to measure this velocity, without reference to the conservation laws.

## Purpose

To determine the velocity of a projectile by means of (a) the conservation laws as applied to a ballistic pendulum, and (b) the horizontal and vertical components of its motion.

## Definitions

- **Kinetic Energy, KE** —is the energy that an object has due only to its motion where m is the mass of the object and v is velocity.

$$KE=\frac{1}{2}mv^2$$

- **Potential Energy, PE** —is the energy that an object has due to its position above some arbitrary reference point (here taken to be the lowest point in the pendulum's swing) where g is the acceleration due to gravity. In metric units $g = 9.8\ m/s^2 = 980\ cm/s^2$ and h is the distance between the reference level and the position of the object, measured *vertically*.

$$PE=mgh$$

- **Linear Momentum, p** —of a body, is the product of its mass and velocity, then

$$p=mv$$

- **Range, R** —is the horizontal distance through which a projectile travels.

## *Theory*

We consider as our system the metal ball of mass m, and the catcher that serves as the pendulum bob, having mass *M*. When the mechanism is fired, the ball leaves the gun traveling with an initial velocity v. Since the catcher is initially at rest, it contributes no momentum, so the *total* momentum of the system is that of the ball, *mv*. In the instant after the catcher catches the ball, the combination begins to move horizontally with a velocity V, giving (m + M)V as the total momentum of the system after the collision. Because there are no net forces in the horizontal direction, the conservation law for momentum applies to this situation, and we can write it as:

$$mv=(m+M)V \qquad \textbf{Equation 1}$$

i.e. the *total momentum* **before** the collision, mv, must be equal to the *total momentum* **after** the collision, (m + M)V. In this expression, *m* and *M* are measurable, so if we can find some method for determining V, then v, the velocity of the ball, can be calculated.

We do not have a direct way of measuring *V*, but we can infer it by applying a second conservation law, the conservation of energy. When the ball becomes lodged in the catcher and both share the common velocity *V*, the total energy of the system is its kinetic energy,

$$KE=\frac{1}{2}(m+M)V^2 \qquad \textbf{Equation 2}$$

Note that Equation 2 is the kinetic energy of the ball and pendulum *immediately after* the collision. Knowledge about the kinetic energy of the ball *before* the collision is not useful in our present situation because kinetic energy is *not* conserved during the collision (see Question 1). However, after the collision the losses in the system (resulting from air friction and friction at the pivot bearing) are negligible and we may assume that the total mechanical energy (PE + KE) *is* conserved.

After the collision, then, the kinetic energy is continuously changed into gravitational potential energy as the pendulum swings along the arc of its path, until it reaches its highest point.

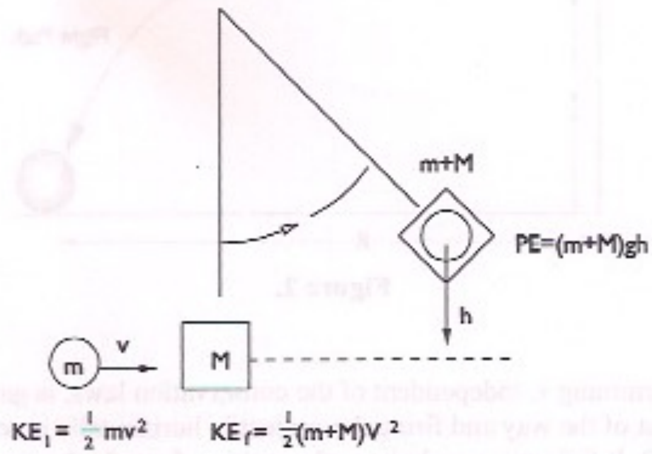


**Figure 1.**

The transformation is complete - all the energy of the system is now potential energy,

$$PE=(m+M)gh \qquad \textbf{Equation 3}$$

where $h$ is the vertical distance through which the catcher and ball have risen and $g$ is the gravitational acceleration. Now the conservation law for energy comes into play: the total mechanical energy of the system remains constant after the collision, so we can equate the kinetic energy just after the collision with the potential energy at the end:

$$\frac{1}{2}(m+M)V^2=(m+M)gh \qquad \textbf{Equation 4}$$

In this expression, the total mass (m + M) divides out and we get

$$V^2=2gh$$

or

$$V=\sqrt{2gh} \qquad \textbf{Equation 5}$$

The apparatus is designed to retain the catcher at the point where its height is a maximum, allowing us to measure $h$ easily. This gives us a value for $V$ that, when substituted into Equation 1, yields the desired result for $v$.

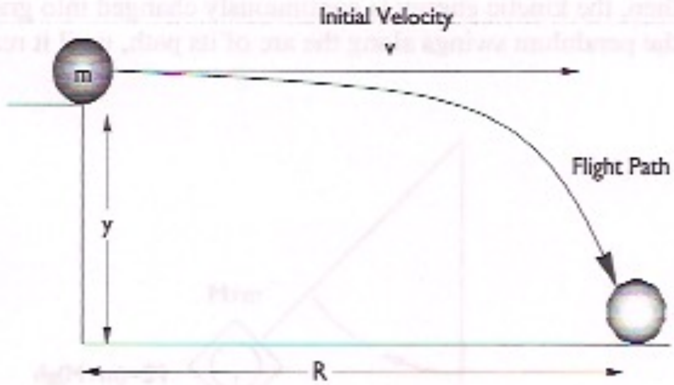


**Figure 2.**

Another way of determining *v*, independent of the conservation laws, is gained by swinging the pendulum bob out of the way and firing the projectile horizontally onto the floor, as illustrated in Figure 2. It follows a parabolic path resulting from the horizontal and vertical parts of the motion, which are independent. As before, the projectile leaves the gun with a horizontal velocity v, which, because there is no acceleration in the horizontal direction, remains constant, and is given by

$$V=\frac{R}{t}$$ **Equation 6**

where R is the horizontal distance traveled by the projectile and t is the time required to make the trip. The vertical distance the ball falls is the same as if we has simply dropped it, and is

$$y=\frac{1}{2}gt^2$$ **Equation 7**

Solving for *t* we obtain

$$t=\sqrt{\frac{2y}{g}}$$ **Equation 8**

Since the time *t* is the same in each case, we can combine equations (6) and (8) to get

$$v=R\sqrt{\frac{g}{2y}}$$ **Equation 9**

By using the measured values for $R$ and $y$ one can then calculate a value for the velocity $v$. This value for $v$ is to be compared with that obtained by doing the collisions experiment.

## *Apparatus*

- Ballistic Pendulum
- Plumb bob
- Balance and set of weights
- Carbon paper
- Meter stick

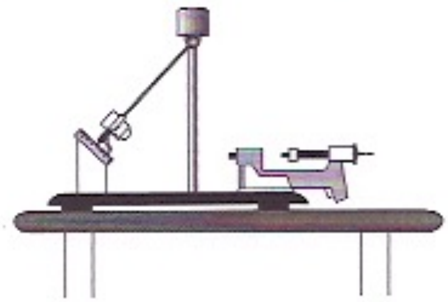

**Figure 3. Ballistic Pendulum**

Last/First Name (print): ______________________

PHYS 1730-Section ______ Username ID (xxx9999): ______

### *Procedure*

Caution: Do NOT remove the pendulum from the apparatus.

1. Determine the mass of the steel ball. Obtain the mass of the pendulum bob from your instructor. Record these on the data sheet.

2. Prepare the gun for firing. With the pendulum at rest and free to swing, pull the trigger, thereby firing the ball into the pendulum bob. Record the notch on the curved scale reached by the pawl corresponding to the stopping point of the pendulum. Carefully depress the spring retainer and remove the ball from the pendulum.

3. Repeat procedure (2) four more times, each time recording the position of the pendulum on the rack.

4. Measure the flat portion of the swinging index arm at rest, and after each shot, record the differences in height.

5. With the pendulum bob in its lowest position, measure the distance, $h_2$, from the index point to the base of the apparatus. Record to the nearest 0.1 centimeter.

6. Set the apparatus near the edge of the laboratory table. Swing the pendulum up onto the rack so that it does not interfere with the flight of the ball.

7. Fire the ball into the box provided to determine approximately where it will land. Place a sheet of white paper on the bottom of the box to correspond to your estimate of the landing point. Cover the paper with carbon paper so that the ball will leave a mark on the white paper when it strikes. Fire the ball five times.

8. For each firing, measure the range, r, along the floor from where the ball left the gun to the point at which it struck the paper. You can use the plumb bob to locate the point on the floor directly below the ball as it leaves the gun.

9. Measure the vertical distance of the point where the ball leaves the gun.

## *Data*

Mass of steel ball 7.8 g

Mass of pendulum 78.5 g

Readings of curved scale for maximum pendulum height:

1. 8.7
2. 8.6
3. 8.8
4. 8.6
5. 8.5

Initial height 8 cm

Average reading 8.64 cm

Distance through which pendulum was raised .64 cm

Velocity of ball and pendulum (V) (Equation 5) 35.42 cm/s

Velocity of ball (v) (Equation 1) 391.9 cm/s

Vertical distance of ball above floor (y) 98 cm

Range Values

1. 211 cm
2. 210 cm
3. 212 cm
4. 218 cm
5. 212 cm

Average Range 213 cm

Velocity of ball from range measurements (v) (Equation 9) 476.28 cm/s

Percent difference in the two values of v 19.4 %